

# Strong decays of $P_{\psi}^N(4312)^+$ to $J/\psi(\eta_c)p$ and $\bar{D}^{(*)}\Lambda_c$ within the Bethe-Salpeter framework

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**ABSTRACT:** Based on the effective Lagrangian in the heavy quark limit, we calculate the one-boson-exchange interaction kernel of  $P_{\psi}^N(4312)^+$  as the  $\bar{D}\Sigma_c$  molecular state in isospin- $\frac{1}{2}$ . We present the Bethe-Salpeter equation and wave function for the constituent particles to be a (pseudo)scalar meson and a  $\frac{1}{2}$  baryon. By solving the Bethe-Salpeter equation, we obtain  $P_{\psi}^N(4312)^+$  as the  $\bar{D}\Sigma_c$  molecular state with  $J^P = (\frac{1}{2})^-$ . Combining the effective Lagrangian and the obtained BS wave function, the partial decay widths of  $P_{\psi}^N(4312)^+$  to  $J/\psi p$ ,  $\eta_c p$ ,  $\bar{D}^{*0}\Lambda_c^+$  and  $\bar{D}^0\Lambda_c^+$  are calculated to be 0.17, 0.085, 8.8, and 0.026 MeV, respectively, which are roughly consistent with the LHCb experimental measurements and some other theoretical researches. The obtain results indicate the fraction of  $\bar{D}^{*0}\Lambda_c^+$  channel amounts to  $\sim 90\%$  of  $P_{\psi}^N(4312)^+$ , and is a highly promising channel to be discovered in the near future experiments. Our results favor the interpretation of  $P_{\psi}^N(4312)^+$  as the  $\bar{D}\Sigma_c$  molecular state with  $J^P = (\frac{1}{2})^-$  and isospin  $I = \frac{1}{2}$ .

**KEYWORDS:** Hadronic Spectroscopy, Structure and Interactions, Properties of Hadrons, Effective Field Theories of QCD

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## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b><math>P_\psi^N(4312)^+</math> as the <math>\bar{D}\Sigma_c</math> molecular state</b>	<b>3</b>
2.1	Bethe-Salpeter equation of a $J^P = 0^-$ meson and a $\frac{1}{2}^+$ baryon	3
2.2	Interaction kernel from the one-boson exchange	5
2.3	Salpeter wave function for the $J^P = \frac{1}{2}^-$ pentaquark states	7
<b>3</b>	<b>Strong decays of <math>P_\psi^N(4312)^+</math> to <math>J/\psi(\eta_c)p</math> and <math>\bar{D}^{(*)0}\Lambda_c^+</math> within the BS wave function</b>	<b>7</b>
3.1	Amplitude for $P_\psi^N(4312)^+ \rightarrow J/\psi p$	9
3.1.1	Amplitude with $D$ exchange	9
3.1.2	Amplitude with $D^*$ exchange	11
3.2	Amplitude for $P_\psi^N(4312)^+ \rightarrow \eta_c p$	12
3.3	$P_\psi^N(4312)^+ \rightarrow \bar{D}^{*0}\Lambda_c^+$	13
3.4	$P_\psi^N(4312)^+ \rightarrow \bar{D}^0\Lambda_c^+$ by exchanging a $\rho$	14
3.5	Partial decay widths	14
<b>4</b>	<b>Numerical results and discussions</b>	<b>16</b>
4.1	Numerical parameters	16
4.2	Numerical results and theoretical uncertainties	16
4.3	Summary	19
<b>A</b>	<b>Expressions of the decay form factors</b>	<b>20</b>

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## 1 Introduction

In 2019, a narrow pentaquark state  $P_c(4312)^+$  is first observed in the  $J/\psi p$  invariant mass spectrum [1] by the LHCb collaboration, which indicates this state at least to contain five valence quarks, namely,  $[c\bar{c}uud]$  quark contents. This pentaquark state will be labeled as  $P_\psi^N(4312)^+$  in this work following the new naming scheme proposed by the LHCb collaboration [2], where the superscript  $N$  denotes the isotopic spin  $I = \frac{1}{2}$  and the subscript  $\psi$  denotes the hidden charm flavor. The measured mass and total width are  $M_{P_\psi^N(4312)^+} = 4312 \text{ MeV}$  and  $\Gamma_{P_\psi^N(4312)^+} = 9.8 \pm 2.7_{-4.5}^{+3.7} \text{ MeV}$  [1] respectively. The proximity to the  $\bar{D}\Sigma_c$  threshold of the observed narrow peak suggests that they play an important role in the dynamics of  $P_\psi^N(4312)^+$  state, and makes the  $\bar{D}\Sigma_c$  molecular state picture a natural interpretation to this exotic particle.

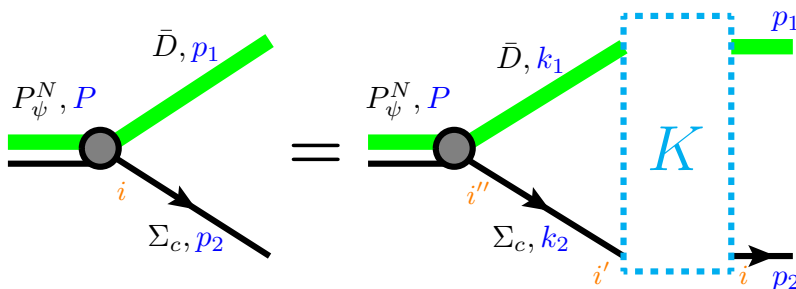
The hidden charm molecular pentaquark states have been proposed before the experimental confirmation [3–9]. After the LHCb discoveries, lots of literature explored these

newly observed pentaquark states from different aspects within different approaches, such as refs. [10–27]. Although the properties of the  $P_\psi^N(4312)^+$  are most likely to be the  $S$ -wave combination of  $\bar{D}\Sigma_c$  with  $I(J^P) = 1/2(1/2^-)$  [10–16, 26, 27], the contrary view [28], or the possibilities of the compact pentaquark state [29, 30] or kinematical effects [31, 32] still exist. Though suggested by the LHCb to be labeled as  $P_\psi^N(4312)^+$ , the essence of this pentaquark state is still an open question.

Besides the spectrum or electromagnetic properties [33–35], the strong decay properties play important roles in determining the nature of the pentaquark states. The decay to  $J/\psi p$  is the discovery channel and also the only detected decay mode of  $P_\psi^N(4312)^+$  so far, and hence this decay channel should be paid more attention to explore the property of  $P_\psi^N(4312)^+$ . Several approaches are used to study the decay properties of these pentaquark states [10, 15, 30, 36–41], including the effective Lagrangian methods [10, 15], the flavor-spin and heavy quark spin symmetry [30, 36], the chiral constituent quark model [37], QCD sum rules [40, 41], etc. Most of the previous studies are based on the nonrelativistic Schrodinger or Lippmann-Schwinger equation and the results are dependent on several introduced free parameters, especially the cutoff value in the form factors. These undetermined parameters weaken the prediction power of the theories and bring ambiguity in interpreting the nature of  $P_\psi^N(4312)^+$ . Some researches also suggest the  $\eta_c p$  channel can be an important decay mode of  $P_\psi^N(4312)^+$  [37, 38], especially, the methods by using the heavy quark symmetry predict that the decay ratio of  $P_\psi^N(4312)^+$  to  $\eta_c p$  over  $J/\psi p$  can reach about three [20, 36]. However, no experimental evidence is reported in a recent search for pentaquark state  $P_\psi^N(4312)^+$  in  $\Lambda_b^0 \rightarrow \eta_c p K^-$  decay channel [42]. Also some studies suggest the  $\bar{D}^{*0}\Lambda_c^+$  may be the dominant decay channels of  $P_\psi^N(4312)^+$  [15, 37]. Though there are already some calculations on these decays, the relevant researches are still relatively scarce and the current predictions are not well consistent with each other. More studies on the decay behaviors of  $P_\psi^N(4312)^+$  can be important and helpful to explore its inner structure and dynamics.

In this work, we will calculate the partial decay widths of  $P_\psi^N(4312)^+$  to  $J/\psi(\eta_c)p$  and  $\bar{D}^{(*)0}\Lambda_c^+$  by combing the Bethe-Salpeter (BS) framework with the effective Lagrangian. The Bethe-Salpeter equation (BSE) is a relativistic two-body bound state equation. Another advantage is that the constructed BS wave functions only depend on the good quantum number spin-parity and Lorentz covariance. The BS methods have already been successfully used to cope with mass spectra of the doubly heavy baryons [43, 44], producing the recently observed molecular pentaquarks [26] and the fully heavy tetraquark  $T_{QQ\bar{Q}\bar{Q}}$  states [45], and also the hadronic transitions and decays [46–52]. The theoretical calculations from BS methods achieve satisfactory consistences with the experimental measurements.

This paper is organized as follows. After the introduction, we start with the Bethe-Salpeter equation for  $P_\psi^N(4312)^+$  as the molecular state of a (pseudo)scalar meson and a baryon, including the interaction kernel and the relevant Salpeter wave function (section 2), then we calculate the strong decay widths of  $P_\psi^N(4312)^+ \rightarrow J/\psi(\eta_c)p$  and  $\bar{D}^{(*)0}\Lambda_c^+$  (section 3). We finally present the numerical results, discussion and summaries in section 4.



**Figure 1.** Bethe-Salpeter equation of the molecular state consisting of the constituent (pseudo)scalar meson and  $J = \frac{1}{2}$  baryon. The orange letters denote the Dirac indices. The blue symbols  $P$ ,  $p_1(k_1)$ ,  $p_2(k_2)$  denote the momenta of the pentaquark, constituent meson, and the constituent baryon respectively.

## 2 $P_\psi^N(4312)^+$ as the $\bar{D}\Sigma_c$ molecular state

In this part, we will first briefly review Bethe-Salpeter equation of a scalar meson and a baryon under the instantaneous approximation. Then we introduce the pentaquark interaction kernel based on the one-boson exchange. The relativistic BS wave functions of the  $J^P = (\frac{1}{2})^- P_\psi^N$  state will be introduced and solved numerically to prepare for the next decay calculations.

### 2.1 Bethe-Salpeter equation of a $J^P = 0^-$ meson and a $\frac{1}{2}^+$ baryon

Figure 1 schematically depicts the Bethe-Salpeter equation for the bound state consisting of a constituent meson and a constituent baryon, which can be expressed as

$$\Gamma(P, q, r) = \int \frac{d^4k}{(2\pi)^4} (-i)K(k, q)[S(k_2)\Gamma(P, k, r)D(k_1)], \quad (2.1)$$

where  $\Gamma(P, q, r)$  denotes the vertex of the pentaquark, constituent meson and baryon; we used  $P$ ,  $q$ , and  $r$  to represent the pentaquark total momentum, inner relative momentum, and spin state respectively; the inner relative momentum  $q$  and  $k$  are defined as  $q \equiv \alpha_2 p_1 - \alpha_1 p_2$ ,  $k \equiv \alpha_2 k_1 - \alpha_1 k_2$ , with  $\alpha_{1(2)} \equiv \frac{m_{1(2)}}{m_1 + m_2}$ ,  $k_{1(2)}$  denoting the momentum of the constituent meson (baryon), and  $m_{1(2)}$  is the corresponding mass;  $S(k_2) = \frac{i}{\not{k}_2 - m_2}$  is the free Dirac propagator of the baryon;  $D(k_1) = \frac{i}{k_1^2 - m_1^2}$  denotes the usual scalar propagator. The  $i\epsilon$  should be implied in all the propagators. Since both the two constituent particles, namely  $\bar{D}$  and  $\Sigma_c$ , contain a heavy charm quark, the relative velocity would be small. Then the interaction kernel  $K(k, q)$  is assumed to be instantaneous and is not dependent on the time component of the exchanged momentum ( $k - q$ ), namely,  $K(k, q) \sim K(k_\perp - q_\perp)$ , where  $k_\perp = k - k_P \hat{P}$  with  $k_P \equiv k \cdot \hat{P}$  and  $\hat{P} = \frac{P}{M}$ , and  $M$  is the pentaquark mass. The spacelike momentum  $q_\perp$  is defined similarly. Throughout this work, this instantaneous approximation is assumed for the pentaquark kernel.

The four-dimensional Bethe-Salpeter wave function is defined as

$$\psi(q) = S(p_2)\Gamma(q)D(p_1), \quad (2.2)$$

where the dependence on  $P$  and  $r$  omitted for simplicity. Since the interaction kernel  $K(k_\perp - q_\perp)$  is instantaneous, the integral over the time component of  $q$  can be absorbed into the wave function and it is useful to define the three-dimensional Salpeter wave function as

$$\varphi(q_\perp) \equiv -i \int \frac{dq_P}{2\pi} \psi(q). \quad (2.3)$$

where the factor  $(-i)$  is just a convention for later convenience.

Performing the contour integral over  $q_P$  on both sides of eq. (2.2), we can obtain the Salpeter equation (SE) for meson-baryon bound state [26],

$$\varphi(q_\perp) = \frac{1}{2w_1} \left[ \frac{\Lambda^+(p_{2\perp})}{M - w_1 - w_2} + \frac{\Lambda^-(p_{2\perp})}{M + w_1 + w_2} \right] \Gamma(q_\perp), \quad (2.4)$$

where  $w_i = (m_i^2 - p_{i\perp}^2)^{1/2}$  ( $i = 1, 2$ ) denotes the kinetic energy of the constituent meson and baryon respectively. The projector operators  $\Lambda^\pm(p_{2\perp})$  are defined as

$$\Lambda^\pm(p_{2\perp}) = \frac{1}{2} [1 \pm H_2(p_{2\perp})] \gamma_0, \quad (2.5)$$

$$H_2(p_{2\perp}) = \frac{1}{w_2} (\not{p}_{2\perp} + m_2) \gamma^0. \quad (2.6)$$

Notice that  $H_2(p_{2\perp})$  is just the corresponding Dirac Hamiltonian divided by the kinetic energy  $w_2$ . Using the projector operator, we can further define the positive and negative energy wave functions as  $\varphi^\pm \equiv \Lambda^\pm \gamma^0 \varphi$ , and we also have  $\varphi = \varphi^+ + \varphi^-$ . The SE above can be further rewritten as the following type

$$M\varphi = (w_1 + w_2)H_2(p_{2\perp})\varphi + \frac{1}{2w_1}\gamma_0\Gamma(q_\perp). \quad (2.7)$$

where the vertex  $\Gamma(q_\perp)$  is now expressed as the integral of the Salpeter wave function,

$$\Gamma(q_\perp) = \int \frac{d^3k_\perp}{(2\pi)^3} K(k_\perp - q_\perp) \varphi(k_\perp). \quad (2.8)$$

The Salpeter eq. (2.7) is in fact an eigenvalue equation about the Salpeter wave function  $\varphi_\alpha(q_\perp)$ , where the pentaquark mass  $M$  behaves as the eigenvalue. The three-dimensional BSE, namely, eq. (2.7), indicates that the mass of the pentaquark state consists of two parts, the kinetic energy and the potential energy.

The normalization condition of the BS wave function is generally expressed as,

$$-i \int \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \bar{\psi}(P, q, \bar{r}) \frac{\partial}{\partial P^0} I(P, k, q) \psi(P, k, r) = 2M \delta_{r\bar{r}},$$

where  $\bar{\psi} = \psi^\dagger \gamma^0$  and  $\delta_{r\bar{r}}$  is the Kronecker symbol; the integral kernel  $I$  in the normalization condition reads,

$$I(P, k, q) = (2\pi)^2 \delta^4(k - q) S^{-1}(p_2) D^{-1}(p_1) + iK(P, k, q).$$

Notice in this work, under the instantaneous approximation, the interaction kernel has no dependence on  $P^0$ , which indicates the normalization would only involve the inverses of

the two propagators. Performing the contour integral, the normalization condition can be further expressed by the Salpeter wave function as

$$\int \frac{d^3 q_\perp}{(2\pi)^3} (2w_1) \bar{\varphi}(q_\perp, \bar{r}) \gamma^0 \varphi(q_\perp, r) = 2M \delta_{r\bar{r}}, \quad (2.9)$$

where  $\bar{\varphi} = \varphi^\dagger \gamma^0$  and the symbol  $\bar{r}$  just denotes the spin state; also notice both the BS wave function  $\psi(q)$  and the Salpeter wave function  $\varphi(q_\perp)$  are four-component spinor.

## 2.2 Interaction kernel from the one-boson exchange

The  $P_\psi^N(4312)^+$  are consistent with the  $\bar{D}\Sigma_c$  molecular state with isospin  $I = \frac{1}{2}$  and  $I_3 = +\frac{1}{2}$ , which can be expressed in the uncoupled representation as

$$|\frac{1}{2}, \frac{1}{2}\rangle = \frac{\sqrt{2}}{\sqrt{3}} |1, +1\rangle |\frac{1}{2}, -\frac{1}{2}\rangle - \frac{1}{\sqrt{3}} |1, 0\rangle |\frac{1}{2}, \frac{1}{2}\rangle = \frac{\sqrt{2}}{\sqrt{3}} |\Sigma_c^{++}\rangle |D^-\rangle - \frac{1}{\sqrt{3}} |\Sigma_c^+\rangle |\bar{D}^0\rangle. \quad (2.10)$$

In the molecular state scenario of  $P_\psi^N(4312)^+$ , the interaction kernel between the two constituents  $\Sigma_c$  and  $\bar{D}$  can be realized by the one-boson exchange. Notice the usual one-pion exchange is not possible in the  $\bar{D}\Sigma_c$  bound state for the parity. We only need to consider the light scalar and vector meson exchange.

Considering the heavy quark spin-flavor symmetry, hidden local symmetry and the light quark chiral symmetry, the involved Lagrangian describing the charmed anti-heavy-light meson and a light scalar and vector meson reads [6, 53]

$$\mathcal{L}_{HHV} = -\rho_{V1} \langle \bar{H}_c v_\alpha V^\alpha H_c \rangle - \rho_{T1} \langle \bar{H}_c \sigma^{\alpha\beta} (\partial_\alpha V_\beta - \partial_\beta V_\alpha) H_c \rangle + \sigma_1 \langle \bar{H}_c \sigma H_c \rangle. \quad (2.11)$$

Here  $\langle \rangle$  denotes taking the Dirac trace, and  $\sigma$  denotes field of the light scalar meson.  $\rho_{V1}$ ,  $\rho_{T1}$ , and  $\sigma_1$  denote the corresponding coupling constants.  $H_{\bar{c}}$  represents the field of the  $(\bar{D}, \bar{D}^*)$  doublet in the heavy quark limit,

$$H_{\bar{c}} = \left( \bar{D}^{*\mu} \gamma_\mu + i \bar{D} \gamma_5 \right) \frac{1 - \not{v}}{2}, \quad (2.12)$$

where  $\bar{D} = (\bar{D}^0, D^-, D_s^-)$  denotes anti-charmed heavy-light meson fields in flavor triplet, and  $\bar{D}^{*\mu}$  is the corresponding vector state;  $\bar{H}_{\bar{c}} = \gamma^0 H_{\bar{c}}^\dagger \gamma_0$  is the usual conjunction in Dirac space; and  $v$  denotes four-velocity of the heavy-light meson. The symbol  $V$  denotes the  $3 \times 3$  matrix consisting of the 9 light vector meson fields [6, 53]

$$V = \begin{bmatrix} \frac{(\rho^0 + \omega)}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{(\rho^0 - \omega)}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{bmatrix}. \quad (2.13)$$

Considering the heavy quark symmetry, hidden local symmetry and chiral symmetry, the effective Lagrangian of the heavy-light baryon and light mesons reads [6, 54–56]

$$\mathcal{L}_{B_6 B_6 V} = \rho_{V2} \langle \bar{S}_\mu v_\alpha V^\alpha S^\mu \rangle + i \rho_{T2} \langle \bar{S}_\mu (\partial_\mu V_\nu - \partial_\nu V_\mu) S_\nu \rangle + \sigma_2 \langle \bar{S}_\mu \sigma S^\mu \rangle. \quad (2.14)$$

Here  $\langle \rangle$  denotes taking trace in the  $3 \times 3$  flavor space. The baryon spin doublet are incorporated in field

$$S_\mu = -\frac{1}{\sqrt{3}}(\gamma_\mu + v_\mu)\gamma^5 B_6 + B_{6\mu}^*, \quad (2.15)$$

where the systematic baryon sextet  $B_6$  in  $3 \times 3$  matrix reads

$$B_6 = \begin{bmatrix} \Sigma_c^{++} & \frac{1}{\sqrt{2}}\Sigma_c^+ & \frac{1}{\sqrt{2}}\Xi_c'^+ \\ \frac{1}{\sqrt{2}}\Sigma_c^+ & \Sigma_c^0 & \frac{1}{\sqrt{2}}\Xi_c'^0 \\ \frac{1}{\sqrt{2}}\Xi_c'^+ & \frac{1}{\sqrt{2}}\Xi_c'^0 & \Omega_c^0 \end{bmatrix}. \quad (2.16)$$

The conjugation defines as usual for the spinor field  $\bar{S}_\mu^{mn} = (S_\mu^{mn})^\dagger \gamma_0$ . An asterisk on the symbol denotes the corresponding spin- $\frac{3}{2}$  baryon, which is not involved in this work.

Using above relevant Lagrangian and based on the one-boson exchange, we calculate the interaction kernel of  $\bar{D}\Sigma_c$  in isospin- $\frac{1}{2}$  as

$$K(s_\perp) = F^2(s_\perp^2) \left( V_1 + V_2 \frac{\not{s}_\perp}{|s_\perp|} \right), \quad (2.17)$$

where  $F(s_\perp^2)$  denotes the regulator in the heavy hadron ( $\bar{D}$  or  $\Sigma_c$  here) vertex; and the potential  $V_1$  and  $V_2$  is specifically expressed as,

$$\begin{aligned} V_1 &= -2\sigma_1\sigma_2 M_D \frac{1}{E_\sigma^2} + \rho_{V1}\rho_{V2} M_D \left( \frac{1}{E_\rho^2} - \frac{1}{2E_\omega^2} \right), \\ V_2 &= -\frac{1}{3}\rho_{V1}\rho_{T2} M_D |s| \left( \frac{2}{E_\rho^2} - \frac{1}{E_\omega^2} \right), \end{aligned} \quad (2.18)$$

where  $E_\rho = (s^2 + m_\rho^2)^{1/2}$  denotes the energy of the inter-mediator  $\rho$  meson, and similar for  $E_\sigma$  and  $E_\omega$ . The influence of the potential strength on the decay widths will be discussed later.

There is no general method to choose the regulator functions. In this work, we use the following propagator-type form factor, namely,

$$F(s^2) = \frac{m_\Lambda^2}{s^2 + m_\Lambda^2}, \quad (2.19)$$

where  $m_\Lambda$  is the introduced cutoff parameter to characterize the regulator function. Notice  $m_\Lambda$  is the only free parameter in this analysis and can be determined by fitting bound state mass to the experimental data, which is found to be  $m_\Lambda = 1.25$  GeV for  $P_\psi^N(4312)^+$  and close to the mass scale of the exchanged particle. In the limit  $s^2 \rightarrow 0$ , the heavy hadron is seen by the inter-mediator mesons as a point-like particle, and hence the form factor is normalized to 1. The cutoff value  $m_\Lambda$  is usually believed to be much larger than the typical energy scale  $\sqrt{2\mu\epsilon} \sim 0.1$  GeV for  $P_\psi^N(4312)^+$  [15, 57], where  $\mu = \frac{m_1 m_2}{m_1 + m_2}$  is the reduced mass of the two-hadron system and  $\epsilon = (m_1 + m_2 - M)$  denotes the bound energy. Our determined cutoff value is consistent with this universal estimation. The obtained  $V_1$  and  $V_2$  for isospin- $\frac{1}{2}$  are displayed graphically in 3a.

### 2.3 Salpeter wave function for the $J^P = \frac{1}{2}^-$ pentaquark states

According to the spin-parity properties, and also considering the proper Lorentz structures, the Salpeter wave function of  $J^P = \frac{1}{2}^-$  pentaquarks consisting of a  $0^-$  meson and  $\frac{1}{2}^+$  baryon can be generally constructed as

$$\varphi(P, q_\perp, r) = \left( f_1 + f_2 \frac{\not{q}_\perp}{q} \right) \gamma^5 u(P, r), \quad (2.20)$$

where the radial wave function  $f_{1(2)}(|\vec{q}|)$  only explicitly depend on  $|\vec{q}|$ ;  $u(P, r)$  denotes Dirac spinor with spin state  $r$ . In terms of the spherical harmonics  $Y_l^m$ , the wave function can be rewritten as

$$\varphi(P, q_\perp, r) = 2\sqrt{\pi} \left[ f_1 Y_0^0 + \frac{1}{\sqrt{3}} f_2 \left( Y_1^1 \gamma^- + Y_1^{-1} \gamma^+ - Y_1^0 \gamma^3 \right) \right] \gamma^5 u(P, r), \quad (2.21)$$

where  $\gamma^\pm = \mp \frac{1}{\sqrt{2}} (\gamma^1 \pm i\gamma^2)$ . Then it is obvious to see that  $f_1$  and  $f_2$  represent the  $S$ - and  $P$ -wave components, respectively. Inserting the wave function into eq. (2.9), we obtain the normalization satisfied by the radial wave functions as

$$\int \frac{d^3 q_\perp}{(2\pi)^3} 2w_1 (f_1^2 + f_2^2) = 1. \quad (2.22)$$

Inserting the Salpeter wave function eq. (2.20) into the Salpeter equation (2.7), eliminating the spinor, calculating the trace, we can obtain two coupled eigenvalue equations with the pentaquark mass  $M$  as the eigenvalue and  $f_{1(2)}$  as the eigen wave functions (see refs. [26, 43, 44] for details). Solving the eigenvalue equations numerically, we can obtain the corresponding mass spectra and numerical wave functions, which are also graphically displayed in 3b.

### 3 Strong decays of $P_\psi^N(4312)^+$ to $J/\psi(\eta_c)p$ and $\bar{D}^{(*)0}\Lambda_c^+$ within the BS wave function

In this section, we first present the relevant effective Lagrangian; then we give the decay amplitude by using the BS wave function combining with the effective Lagrangian; finally, the expressions of the partial decay widths are presented in terms of the relevant form factors.

For  $P_\psi^N(4312)^+ \rightarrow J/\psi(\eta_c)p$ , the involved interactions are  $J/\psi DD^{(*)}$ ,  $\eta_c DD^*$ , and  $\Sigma_c ND^{(*)}$ , which involve the Lagrangian of the doubly heavy meson and the heavy-light meson. The heavy-light charmed mesons in  $S$ -wave can be represented by [53, 58, 59]

$$H_c = \frac{1 + \not{\psi}}{2} (D^{*\mu} \gamma_\mu + iD\gamma_5), \quad (3.1)$$

where  $D^{*\mu}$  and  $D$  denote the corresponding vector and pseudoscalar charmed  $D$  mesons respectively. The anti-heavy-light meson doublet  $H_{\bar{c}}$  has been presented in eq. (2.12).

For doubly heavy mesons, the heavy quark flavor symmetry does not hold any longer, while the heavy quark spin symmetry still holds. In the ground states, the charmonium



forms a doublet consisting of a pseudoscalar  $\eta_c$  and a vector state  $J/\psi$ , which can be represented by [60]

$$R = \frac{1 + \not{\psi}}{2} (\psi^\mu \gamma_\mu + i\eta_c \gamma_5) \frac{1 - \not{\psi}}{2}, \quad (3.2)$$

where  $\psi^\mu$  and  $\eta_c$  denotes the fields of the corresponding mesons. Here all the hadron fields in above equations contain a factor of  $\sqrt{M_H}$  with  $M_H$  the corresponding meson mass.

By assuming the invariance under independent rotations of the heavy quark spins, it is possible to write down the effective coupling between the  $S$ -wave charmonia and the heavy-light mesons as [61]

$$\mathcal{L}_2 = g_2 \text{Tr} (R \overleftrightarrow{H}_c \not{\partial} \overleftrightarrow{H}_c) + \text{H.c.}, \quad (3.3)$$

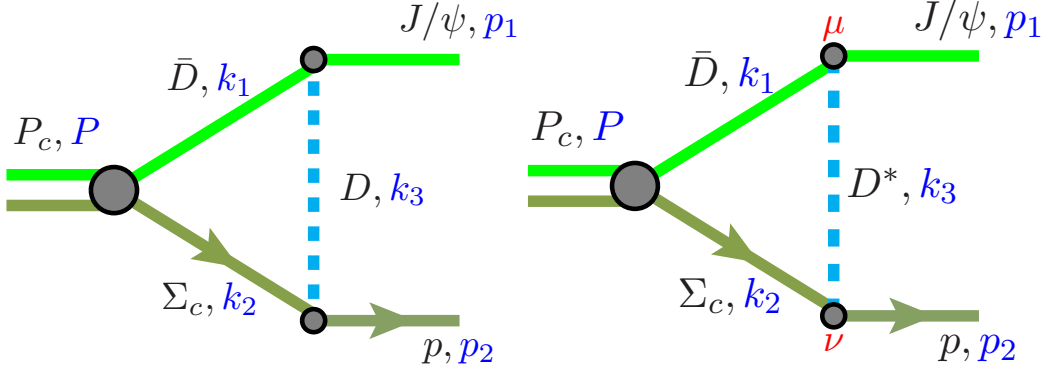
which is invariant under independent heavy quark spin symmetry; and the notation  $A \overleftrightarrow{\partial} B \equiv A \partial B - \partial A B$  is used. Consequently, we obtain the following effective Lagrangian describing  $J/\psi$  and  $\eta_c$  coupling to the  $DD^*$ ,

$$\begin{aligned} \mathcal{L}_2 = & + g_{\psi DD} \psi^\dagger{}^\mu \bar{D} \partial_\mu D \\ & - i g_{\psi DD^*} \frac{1}{M_\psi} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \psi^\dagger{}_\nu (\bar{D} \partial_\alpha D^*_\beta + \bar{D}^*_\alpha \partial_\beta D) \\ & + g_{\psi D^* D^*} \psi^\dagger{}^\mu (\bar{D}^{*\nu} \partial_\nu D^*_\mu - \partial_\nu \bar{D}^{*\mu} D^{*\nu} + 2 \partial_\mu \bar{D}^{*\nu} D^*_\nu) \\ & + g_{DD^* \eta_c} \eta_c^\dagger (\partial_\mu \bar{D} D^{*\mu} - \bar{D}^{*\mu} \partial_\mu D) \\ & + i g_{D^* D^* \eta_c} \frac{1}{M_{\eta_c}} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \eta_c^\dagger \bar{D}^*_\nu \partial_\alpha D^*_\beta + \text{H.c.}, \end{aligned} \quad (3.4)$$

where we have divide a meson mass in the second and the last Lagrangians to keep all the coupling constants dimensionless. The symbol  $\epsilon^{\mu\nu\alpha\beta}$  denotes the totally antisymmetric Levi-Civita tensor with  $\epsilon^{\mu\nu\alpha\beta} = -\epsilon_{\mu\nu\alpha\beta}$  and convention  $\epsilon^{0123} = 1$ . All these coupling constants are related to a single coupling  $g_2$ , which is determined to be  $g_2 = \sqrt{M_\psi} / (2M_D f_\psi)$  with  $f_\psi$  denoting the  $J/\psi$  decay constant [61]. Then all other coupling constants can also be expressed in terms of the  $g_{\psi DD}$  as

$$\begin{aligned} g_{\psi DD} &= \frac{2M_\psi}{f_\psi}, \\ g_{\psi DD^*} &= \left( \frac{M_{D^*}}{M_D} \right)^{1/2} g_{\psi DD}, \\ g_{\psi D^* D^*} &= \frac{1}{2} \left( \frac{M_{D^*}}{M_D} \right) g_{\psi DD}, \\ g_{DD^* \eta_c} &= \frac{1}{2} \left( \frac{M_{\eta_c} M_{D^*}}{M_\psi M_D} \right)^{1/2} g_{\psi DD}, \\ g_{D^* D^* \eta_c} &= \left( \frac{M_{\eta_c}}{M_\psi} \right)^{1/2} \frac{M_{D^*}}{M_D} g_{\psi DD}. \end{aligned} \quad (3.5)$$

In next section, we will also discuss the effects of these coupling constants on the final decay widths.



**Figure 2.** Strong decay of  $P_\psi^N(4312)^+$  to the  $J/\psi p$  by exchanging a virtual mediator  $D$  (left panel) and  $D^*$  (right panel).  $P$ ,  $k_1$ ,  $k_2$ ,  $P_1$ ,  $P_2$  denote the momenta of  $P_\psi^N$ , constituent meson, constituent baryon, the final  $J/\psi$ , and the final  $p$  respectively.

### 3.1 Amplitude for $P_\psi^N(4312)^+ \rightarrow J/\psi p$

$P_\psi^N(4312)^+$  as the  $\bar{D}\Sigma_c$  molecular state can decay to  $J/\psi p$  by exchanging either a  $D$  or a  $D^*$  virtual meson, and the total amplitude is the sum of the two.

#### 3.1.1 Amplitude with $D$ exchange

The left panel of Figure 2 shows the Feynman diagram of  $P_\psi^N(4312)^+ \rightarrow J/\psi p$  by exchanging the  $D$  meson. Besides the pentaquark vertex, we also need two other effective Lagrangian to obtain the decay width. From above results, the effective Lagrangian describing the  $DDJ/\psi$  interaction read

$$\mathcal{L}_{\psi DD} = g_{\psi DD}(\psi^\mu)^\dagger \bar{D} \partial_\mu D, \quad (3.6)$$

where  $D = (D^0, D^+, D_s^+)^T$  represents the charmed meson fields in flavor triplet, and  $\bar{D}$  represents fields of the corresponding anti-charmed mesons. Whereas the effective Lagrangian for  $ND\Sigma_c$  interaction behaves as [10, 62]

$$\mathcal{L}_{ND\Sigma_c} = (-i)g_{ND\Sigma_c} \bar{N} \gamma_5 \Sigma_c \bar{D}^\dagger + \text{H.c.}, \quad (3.7)$$

where  $N$  stands for nucleon field doublet;  $\Sigma_c = \boldsymbol{\sigma} \cdot \boldsymbol{\Sigma}_c$  with  $\boldsymbol{\sigma}$  denoting the Pauli matrix and  $\boldsymbol{\Sigma}_c$  denoting the  $\Sigma_c$  baryon isospin triplet.

The invariant amplitude for  $P_\psi^N(4312)^+ \rightarrow J/\psi p$  by exchanging a  $D$  can then be expressed by the Bethe-Salpeter vertex as

$$\mathcal{A}_1 = \int \frac{d^4 k}{(2\pi)^4} \bar{u}_2(-i g_{ND\Sigma_c}) \gamma^5 [S(k_2) \Gamma(P, k, r) D(k_1)] (g_{\psi DD}) D(k_3) (i) e_1^{*\alpha} k_{1\alpha}, \quad (3.8)$$

where  $u_2$  is short for  $u_{(r_2)}(P_2)$  with  $r_2$  representing the proton spin state;  $e_1$  is short for  $e_{(r_1)}(P_1)$  representing the polarization vector of the final  $J/\psi$  with  $P_1$  denoting the  $J/\psi$  momentum and  $r_1 = 0, \pm 1$  representing the 3 possible polarization states. The polarization vector  $e_1$  fulfills the Lorentz condition

$$e_1^\alpha P_{1\alpha} = 0. \quad (3.9)$$

The momentum of the exchanged virtual charmed meson is denoted as  $k_3 = (k_1 - P_1)$ . We will use  $M_1$  and  $M_2$  to denote the masses of the final  $J/\psi$  and proton respectively.

Also notice  $k_1 = (\alpha_1 P + k)$  and  $k_3$  are involved in the four-dimensional integration over  $k$ . To simplify this amplitude, first we strip off the triangle amplitude involved the integral over  $k$  as

$$T_{1\alpha} u(P, r) = \gamma_5 \int \frac{d^4 k}{(2\pi)^4} [S(k_2) \Gamma(P, k, r) D(k_1)] D(k_3) k_{1\alpha}, \quad (3.10)$$

where the Lorentz condition of the vector meson is utilized, and we also strip off the spinor  $u(P, r)$  for later convenience. The decay amplitude  $\mathcal{A}_1$  can then be simplified as

$$\mathcal{A}_1 = (g_{ND\Sigma_c} g_{\psi DD}) e_1^{*\alpha} [\bar{u}_2 T_{1\alpha} u(P, r)]. \quad (3.11)$$

Then we perform the contour integral over  $k_P$  on eq. (3.10), and obtain

$$T_1^\alpha u(P, r) = \gamma_5 \int \frac{dk_\perp^3}{(2\pi)^3} \frac{1}{2w_3} (a_1^\alpha \varphi^+ + a_2^\alpha \varphi^-), \quad (3.12)$$

where we used the expression of the positive(negative) energy wave functions  $\varphi^\pm = \Lambda^\pm \gamma^0 \varphi$ ; the two coefficients  $a_1$  and  $a_2$  behaves as

$$a_1^\alpha = c_1 x_1^\alpha + c_2 x_2^\alpha + c_3 x_3^\alpha, \quad (3.13)$$

$$a_2^\alpha = c_4 x_4^\alpha + c_5 x_5^\alpha + c_6 x_6^\alpha, \quad (3.14)$$

where  $x_i = k_1(k_P = k_{P_i})$  with  $(i = 1, \dots, 6)$ , and  $k_{P_i}$ s are defined as

$$k_{P1} = \zeta_1^+, k_{P2} = \zeta_2^+, k_{P3} = \zeta_3^+, k_{P4} = \zeta_1^-, k_{P5} = \zeta_2^-, k_{P6} = \zeta_3^-, \quad (3.15)$$

where the abbreviations  $\zeta_1^\pm \equiv -(\alpha_1 M \mp w_1)$ ,  $\zeta_2^\pm \equiv (\alpha_2 M \mp w_2)$ , and  $\zeta_3^\pm \equiv (E_1 - \alpha_1 M \pm w_3)$  are used. The coefficients  $c_i$ s ( $i = 1, \dots, 6$ ) are defined as

$$\begin{aligned} c_{1(4)} &= \frac{1}{w_1 + w_3 \mp E_1}, \\ c_{2(5)} &= \frac{(-1)}{w_2 + w_3 \mp E_2}, \\ c_{3(6)} &= \frac{(w_1 + w_2 \mp M)}{(w_1 + w_3 \pm E_1)(w_2 + w_3 \mp E_2)}. \end{aligned} \quad (3.16)$$

Now the amplitude  $\mathcal{A}_{1T;\alpha}$  has been expressed by the three-dimensional Salpeter wave function  $\varphi(k_\perp)$ , and can be further simplified as the two form factors

$$T_{1\alpha} = (s_{1P} \gamma_\alpha + s_{2P} \hat{P}_\alpha), \quad (3.17)$$

Inserting the obtained  $P_\psi^N(4312)^+$  wave function, namely, eq. (2.20), into eq. (3.12) and then calculating the three-dimensional integral numerically, we can obtain the amplitude  $T_{1\alpha}$  in terms of  $s_{1P}$  and  $s_{2P}$ . In the appendix A we collect the specific expressions of the two form factors in terms of the Salpeter wave functions  $f_1$  and  $f_2$ . The amplitude  $\mathcal{A}_1$  then behaves as

$$\mathcal{A}_1 = (g_{ND\Sigma_c} g_{\psi DD}) e_1^{*\alpha} \bar{u}_2 (s_{1P} \gamma_\alpha + s_{2P} \hat{P}_\alpha) u(P, r). \quad (3.18)$$

### 3.1.2 Amplitude with $D^*$ exchange

For  $P_\psi^N(4312)^+$  decaying by exchanging  $D^*$  in the lowest level, the relevant Feynman diagram is displayed in the right panel of [Figure 2](#), and the two involved interaction vertexes are  $DD^*J/\psi$  and  $\Sigma_c D^* p$ . The  $DD^*J/\psi$  interaction is represented by the following effective Lagrangian

$$\mathcal{L}_{\psi DD^*} = (-i)g_{\psi DD^*}\epsilon^{\mu\nu\alpha\beta}\frac{1}{M_\psi}\partial_\mu\psi_\nu^\dagger\partial_\alpha D_\beta^*\bar{D}, \quad (3.19)$$

Notice here the coupling constant  $g_{\psi DD^*}$  is defined to have the same dimension with the pseudoscalar coupling constant  $g_{\psi DD}$ . The effective Lagrangian of  $ND^*\Sigma_c$  reads [\[62–64\]](#)

$$\mathcal{L}_{ND^*\Sigma_c} = ig_{ND^*\Sigma_c}\bar{N}\gamma^\alpha\Sigma_c D_\alpha^{*\dagger} + \text{H.c.} \quad (3.20)$$

All the coupling constants in these effective Lagrangian will be specified in next section.

The invariant amplitude for  $P_\psi^N(4312)^+ \rightarrow J/\psi p$  by exchanging a  $D^*$  can be expressed by the Bethe-Salpeter vertex as

$$\mathcal{A}_2 = (g_{ND^*\Sigma_c}g_{\psi DD^*})\bar{u}_2\gamma^\nu\int\frac{d^4k}{(2\pi)^4}[S(k_2)\Gamma(P,k,r)D(k_1)]D_{\mu\nu}(k_3)\frac{-i^3}{M_1}\epsilon^{P_1\alpha\beta\mu}e_{1\alpha}^*k_{1\beta}, \quad (3.21)$$

where the propagator of the exchanged  $D^*$  meson behaves as

$$D_{\mu\nu}(k_3) = i\frac{-g_{\mu\nu} + k_{3\mu}k_{3\nu}/m_3^2}{k_3^2 - m_3^2 + i\epsilon}, \quad (3.22)$$

Here the propagator mass  $m_3$  is  $M_{D^*}$ . Notice the contraction with Levi-Civita tensor forces the momentum part in numerator of  $D_{\mu\nu}(k_3)$  to be zero. The amplitude can be further simplified as

$$\mathcal{A}_2 = (g_{\Sigma_c ND^*}g_{\psi DD^*})e_1^{*\alpha}\bar{u}_2\gamma^\mu(-i)\frac{1}{M_1}\epsilon_{P_1\alpha\beta\mu}T_2^\beta u(P,r), \quad (3.23)$$

where we have stripped off the amplitude involved the integral over  $k$  as before

$$T_2^\beta u(P,r) = \int\frac{d^4k}{(2\pi)^4}[S(k_2)\Gamma(P,k,r)D(k_1)]D(k_3)k_1^\beta. \quad (3.24)$$

In order to express the amplitude by the three-dimensional Salpeter wave function, we perform the contour integration over  $k_P$  on eq. [\(3.24\)](#) as usual and obtain

$$T_2^\beta u(P,r) = \int\frac{d^3k_\perp}{(2\pi)^3}\frac{1}{2w_3}\left(a_1^\beta\varphi^+ + a_2^\beta\varphi^-\right). \quad (3.25)$$

Combining  $T_2^\beta u(P,r)$  with  $\gamma^\mu\frac{(-i)}{M_1}\epsilon_{P_1\alpha\beta\mu}$ , and using the following identity of the Levi-Civita symbol,

$$i\gamma_\mu\epsilon^{\mu\alpha\beta\nu} = \gamma^5(\gamma^\alpha\gamma^\beta\gamma^\nu - \gamma^\alpha g^{\beta\nu} + \gamma^\beta g^{\alpha\nu} - \gamma^\nu g^{\alpha\beta}), \quad (3.26)$$

we can express the decay amplitude  $\mathcal{A}_2$  for  $D^*$  exchange by the following form factors

$$\mathcal{A}_2 = (g_{\psi DD^*}g_{\Sigma_c ND^*})e_1^{*\alpha}\bar{u}_2(s_{1V}\gamma_\alpha + s_{2V}\hat{P}_\alpha)u(P,r). \quad (3.27)$$

Namely, the amplitude  $\mathcal{A}_2$  can be expressed by the same form as  $\mathcal{A}_1$ , which is just what it should be. In above equations, the specific expressions of  $s_{1V}$  and  $s_{2V}$  can be obtained by inserting the Salpeter wave functions into eq. [\(3.25\)](#) and performing the integral numerically. The specific expressions are presented in the appendix [A](#).

### 3.2 Amplitude for $P_\psi^N(4312)^+ \rightarrow \eta_c p$

Since  $\eta_c$  with  $J^P = 0^-$ , the decay to  $\eta_c p$  can only happen by exchanging a  $D^*$  while the mode of exchanging a  $D$  is forbidden. From eq. (3.4), the effective Lagrangian responsible for  $DD^*\eta_c$  interaction reads

$$\mathcal{L}_{DD^*\eta_c} = g_{DD^*\eta_c} \eta_c^\dagger \partial_\mu \bar{D} D^{*\mu}. \quad (3.28)$$

The effective Lagrangian describing  $ND^*\Sigma_c$  interaction has been presented in eq. (3.20). The corresponding Feynman diagram is similar with that for the decay to  $J/\psi p$  with  $D^*$  exchange. The decay amplitude for  $P_\psi^N(4312)^+ \rightarrow \eta_c p$  behaves as

$$\mathcal{A}_3 = \bar{u}_2 (i g_{ND^*\Sigma_c}) \gamma^\alpha \int \frac{d^4 k}{(2\pi)^4} [S(k_2) \Gamma(k, r) D(k_1)] D_{\alpha\beta}(k_3) g_{D^*D\eta_c} (-i P_1^\beta). \quad (3.29)$$

As usual, it is convenient to strip off the part involved the integral over  $k$  as

$$T_3 u(P, r) = \int \frac{d^4 k}{(2\pi)^4} (\gamma^\alpha P_1^\beta) [S(k_2) \Gamma(k, r) D(k_1)] D_{\alpha\beta}(k_3),$$

Performing the contour integral over  $k_P$ , we can express  $T_3$  by the three-dimensional Salpeter wave function

$$T_3 u(P, r) = \int \frac{d^3 k_\perp}{(2\pi)^3} \frac{1}{2w_3} (\gamma^\alpha P_1^\beta) \sum_{i=1}^3 [c_i d_{\alpha\beta}(y_i) \varphi^+ + c_{i+3} d_{\alpha\beta}(y_{i+3}) \varphi^-], \quad (3.30)$$

where the positive (negative) energy wave function is related to the Salpeter wave function by  $\varphi^\pm = \Lambda^\pm \gamma_0 \varphi$ ; and we define  $d_{\alpha\beta}$  and  $y_i$  as

$$\begin{aligned} d_{\alpha\beta}(y_i) &= -g_{\alpha\beta} + \frac{y_{i\alpha} y_{i\beta}}{m_3^2}, \\ y_i &= k_3(k_P = k_{P_i}) = x_{iP} \hat{P} + k_\perp - P_1. \end{aligned} \quad (3.31)$$

Notice that contribution of the momentum part in  $d_{\alpha\beta}$  will be suppressed when the exchanged particle is heavy. Inserting the Salpeter wave function eq. (2.20) of  $P_\psi^N(4312)^+$ , we obtain  $T_3$  expressed by one form factor,

$$T_3 = s_{3V} \gamma_5. \quad (3.32)$$

Finally, we obtain the amplitude for decay to  $\eta_c p$  by form factor  $s_{3V}$  with a simple form

$$\mathcal{A}_3 = (g_{ND^*\Sigma_c} g_{\eta_c DD^*}) \bar{u}_2 (s_{3V} \gamma_5) u(P, r). \quad (3.33)$$

The expression of  $s_{3V}$  is also listed in appendix A as the integral over Salpeter wave functions.

### 3.3 $P_\psi^N(4312)^+ \rightarrow \bar{D}^{*0}\Lambda_c^+$

The strong decay of  $P_\psi^N(4312) \rightarrow \bar{D}^{*0}\Lambda_c^+$  is similar with the decay to  $J/\psi p$ , just the vector meson  $J/\psi$  replaced by  $\bar{D}^{*0}$ , the proton replaced by the  $\Lambda_c^+$  baryon, and the propagator  $D^{(*)}$  replaced by the  $\pi(\rho)$  respectively. The effective Lagrangian describing the interaction of  $\bar{D}^*\bar{D}\phi$  and  $\Lambda_c\Sigma_c\phi$  are [65–68]

$$\mathcal{L}_{\bar{D}\bar{D}^*\phi} = g_{\bar{D}\bar{D}^*\phi}(\bar{D}^{*\mu})^\dagger \partial_\mu \phi \bar{D}, \quad (3.34)$$

$$\mathcal{L}_{\Sigma_c\Lambda_c\phi} = (-i)g_{\Lambda_c\Sigma_c\phi}\bar{\Lambda}_c\gamma_5\Sigma_c\phi, \quad (3.35)$$

where  $\phi$  is the  $3 \times 3$  traceless hermitian matrix consisting of eight pseudo-scalar meson fields,

$$\phi = \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{bmatrix}. \quad (3.36)$$

The coupling constants can be obtained under the heavy quark spin-flavor symmetry. Here we use the coupling constant  $g_{\bar{D}\bar{D}^*\phi} = \frac{2g}{f_\pi}\sqrt{M_D M_{D^*}}$  with the  $\pi$  decay constant  $f_\pi = 0.132 \text{ GeV}$  and the coupling constant  $g = 0.59$  [66], and the coupling constant  $g_{\Lambda_c\Sigma_c\phi} = 19.3$  [68].

Combining above effective Lagrangian and the BS vertex, we can express the decay amplitude of  $P_\psi^N(4312)^+ \rightarrow \bar{D}^{*0}\Lambda_c^+$  by exchanging a  $\pi$  as

$$\mathcal{A}_4 = \int \frac{d^4k}{(2\pi)^4} (-ig_{\Lambda_c\Sigma_c\phi})\bar{u}_2\gamma_5[S(k_2)\Gamma(k, r)D(k_1)]D(k_3)(g_{D^*D\phi})(ik_{1\alpha})(e_1^\alpha)^*. \quad (3.37)$$

By taking a similar calculation procedure with that in decay to  $J/\psi p$ , we can further express this amplitude by two form factors,

$$\mathcal{A}_4 = (g_{\Lambda_c\Sigma_c\phi}g_{D^*D\phi})(e_1^\alpha)^*\bar{u}_2 \left( s_{4P}\gamma_\alpha + s_{5P}\hat{P}_\alpha \right) u(P, r), \quad (3.38)$$

where the form factor  $s_{4P}$  and  $s_{5P}$  has exactly the same expressions with  $s_{1P}$  and  $s_{2P}$ , respectively, just the masses of  $m_3$ ,  $M_1$ , and  $M_2$  changed from  $M_D$ ,  $M_{J/\psi}$ , and  $M_p$  to  $M_\pi$ ,  $M_{\bar{D}^{*0}}$ , and  $M_{\Lambda_c}$ , respectively.

The decay to  $\bar{D}^{*0}\Lambda_c^+$  by exchanging a  $\rho$  is similar with the decay to  $J/\psi p$  by exchanging a  $D^*$ , and the relevant effective Lagrangian are

$$\mathcal{L}_{\bar{D}\bar{D}^*V} = -g_{\bar{D}\bar{D}^*V}\frac{i}{M_{\bar{D}^*}}\epsilon^{\mu\nu\alpha\beta}\partial_\mu(\bar{D}_\nu^*)^\dagger\partial_\alpha V_\beta\bar{D}, \quad (3.39)$$

$$\mathcal{L}_{\Sigma_c\Lambda_c V} = ig_{\Sigma_c\Lambda_c V}\bar{\Lambda}_c\gamma_\alpha\Sigma_c V^\alpha, \quad (3.40)$$

where the coupling constants  $g_{\bar{D}\bar{D}^*V} = 4\rho_{T1} = 9.3 \text{ GeV}^{-1}$ , and  $g_{\Sigma_c\Lambda_c V} = 0.56$  [68]. Then we can express the corresponding amplitude as

$$\mathcal{A}_5 = (g_{\Lambda_c\Sigma_c V}g_{\bar{D}\bar{D}^*V})\bar{u}_2\gamma^\alpha \int \frac{d^4k}{(2\pi)^4} [S(k_2)\Gamma(P, k, r)D(k_1)]D_{\alpha\beta}(k_3)\frac{(-i^3)}{M_1}\epsilon^{P_1\mu\nu\beta}e_{1\mu}^*(ik_{3\nu}). \quad (3.41)$$

Here the propagator mass  $m_3$  is  $M_\rho$ . Again by taking a similar calculation procedure as in subsection 3.1.2, the invariant amplitude is finally expressed by the form factors

$$\mathcal{A}_5 = (g_{D^*D\rho}g_{\Lambda_c\Sigma_c\rho})e_1^{*\alpha}\bar{u}_2(s_{4V}\gamma_\alpha + s_{5V}\hat{P}_\alpha)u(P, r), \quad (3.42)$$

where the form factor  $s_{4(5)V} = s_{1(2)V}(m_3 = M_\rho, M_1 = M_{\bar{D}^*0}, M_2 = M_{\Lambda_c})$ .

Finally, we can express the total amplitude for decay to  $\bar{D}^{*0}\Lambda_c^+$  as

$$\mathcal{A}[P_\psi^N(4312)^+\bar{D}^*\Lambda_c] = \mathcal{A}_4 + \mathcal{A}_5 \quad (3.43)$$

### 3.4 $P_\psi^N(4312)^+ \rightarrow \bar{D}^0\Lambda_c^+$ by exchanging a $\rho$

The decay of  $P_\psi^N(4312)^+ \rightarrow \bar{D}^0\Lambda_c^+$  is quite similar with the decay to  $\eta_cp$ . The involved interaction Lagrangian is

$$\mathcal{L}_{\bar{D}\bar{D}V} = g_{\bar{D}\bar{D}V}\partial_\mu(\bar{D}^\dagger)V^\mu\bar{D}, \quad (3.44)$$

where the coupling constant  $g_{\bar{D}\bar{D}V} = 2\rho_{V1} = 7.51$ . The corresponding decay amplitude behaves as

$$\mathcal{A}_6 = g_{\Lambda_c\Sigma_cV}\bar{u}_2\gamma^\alpha \int \frac{d^4k}{(2\pi)^4}[S(k_2)\Gamma(k, r)D(k_1)]D_{\alpha\beta}(k_3)g_{\bar{D}\bar{D}V}(iP_1^\beta). \quad (3.45)$$

As usual, it is convenient to strip off the part involved the integral over  $k$  as

$$T_6u(P, r) = \int \frac{d^4k}{(2\pi)^4}(\gamma^\alpha P_1^\beta)[S(k_2)\Gamma(k, r)D(k_1)]D_{\alpha\beta}(k_3),$$

which can be further simplified by finishing the integral involved the Salpeter vertex as

$$T_6 = s_{6V}\gamma_5, \quad (3.46)$$

where the only form factor  $s_{6V}$  has the same expression with  $s_{3V}$ , just the taking the parameter values  $m_3 = M_\rho, M_1 = M_{\bar{D}^0}, M_2 = M_{\Lambda_c}$  in the form factor expressed by the Salpeter wave functions. Finally, the decay amplitude of  $P_\psi^N(4312)^+ \rightarrow \bar{D}^0\Lambda_c^+$  can be expressed as

$$\mathcal{A}_6 = i(g_{\Lambda_c\Sigma_cV}g_{\bar{D}\bar{D}V})\bar{u}_2(s_{6V}\gamma_5)u(P, r). \quad (3.47)$$

### 3.5 Partial decay widths

Combing the two amplitudes from  $D$  and  $D^*$  mediators together, we obtain the full invariant amplitude for  $P_\psi^N(4312)^+ \rightarrow J/\psi p$  decay by two form factors,

$$\mathcal{A}[P_\psi^N(4312)^+ \rightarrow J/\psi p] = \mathcal{A}_1 + \mathcal{A}_2 = e_1^{*\alpha}\bar{u}_2(s_1\gamma_\alpha + s_2\hat{P}_\alpha)u(P, r). \quad (3.48)$$

where  $s_1$  and  $s_2$  are related to the coupling constants and are expressed as

$$\begin{aligned} s_1 &= g_{\psi DD}g_{ND\Sigma_c}s_{1P} + g_{\psi DD^*}g_{ND^*\Sigma_c}s_{1V}, \\ s_2 &= g_{\psi DD}g_{ND\Sigma_c}s_{2P} + g_{\psi DD^*}g_{ND^*\Sigma_c}s_{2V}. \end{aligned} \quad (3.49)$$

Similarly, the total invariant amplitude for  $P_\psi^N(4312)^+ \rightarrow \bar{D}^{*0}\Lambda_c^+$  can also be obtained as

$$\mathcal{A}[P_\psi^N(4312)^+ \rightarrow \bar{D}^{*0}\Lambda_c^+] = \mathcal{A}_4 + \mathcal{A}_5 = e_1^{*\alpha}\bar{u}_2 \left( s_4\gamma_\alpha + s_5\hat{P}_\alpha \right) u(P, r). \quad (3.50)$$

where  $s_4$  and  $s_5$  are related to the coupling constants and are expressed as

$$\begin{aligned} s_4 &= g_{\bar{D}\phi\bar{D}^*}g_{\Sigma_c\phi\Lambda_c} s_{4P} + g_{\bar{D}V\bar{D}^*}g_{\Sigma_c V\Lambda_c} s_{4V}, \\ s_5 &= g_{\bar{D}\phi\bar{D}^*}g_{\Sigma_c\phi\Lambda_c} s_{5P} + g_{\bar{D}V\bar{D}^*}g_{\Sigma_c V\Lambda_c} s_{5V}. \end{aligned} \quad (3.51)$$

For the decays with final vector meson  $J/\psi$  or  $\bar{D}^{*0}$ , squaring the amplitude and summing all the polarization states, we obtain

$$\sum_{r_1, r_2, r} |\mathcal{A}|^2 = \left( -g^{\alpha\beta} + \frac{P_1^\alpha P_1^\beta}{M_1^2} \right) \text{Tr} \left( \not{P}_2 + M_2 \right) T_\alpha[J/\psi(\bar{D}^{*0})] (\not{P} + M) \bar{T}_\beta, \quad (3.52)$$

where

$$T_\alpha[J/\psi(\bar{D}^{*0})] = \left( s_{1(4)}\gamma_\alpha + s_{2(5)}\hat{P}_\alpha \right), \quad (3.53)$$

and  $\bar{T}_\beta = \gamma^0 T_\beta^\dagger \gamma_0$  is defined as the usual conjugation variable; we also used the relationship of the summation over the vector polarization states  $r_1$ ,

$$\sum_{r_1} e_{(r_1)}^\alpha e_{(r_1)}^{*\beta} = -g^{\alpha\beta} + \frac{P_1^\alpha P_1^\beta}{M_1^2}; \quad (3.54)$$

and the summation over the polarization states of the spinors

$$\sum_{r_2} u_{(r_2)}(P_2)\bar{u}_{(r_2)}(P_2) = (\not{P}_2 + M_2), \quad (3.55)$$

$$\sum_r u(P, r)\bar{u}(P, r) = (\not{P} + M). \quad (3.56)$$

For  $P_\psi^N(4312)^+ \rightarrow \eta_c p(\bar{D}^0\Lambda_c^+)$  decay, the squared amplitude behaves as

$$\sum_{r_2, r} |\mathcal{A}_{3(6)}|^2 = \text{Tr} \left( \not{P}_2 + M_2 \right) \mathcal{A}_{3(6)} (\not{P} + M) \bar{\mathcal{A}}_{3(6)} = 4M(E_2 - M_2)s_{3(6)}^2, \quad (3.57)$$

where  $s_3 = g_{ND^*\Sigma_c}g_{\eta_c D^*D}s_{3V}$  and  $s_6 = g_{\Lambda_c\Sigma_c V}g_{\bar{D}\bar{D}V}s_{6V}$ . The squared amplitude is proportional to the kinetic energy of the final baryon.

Finally, the partial decay width of  $P_\psi^N(4312)^+$  to  $J/\psi(\eta_c)p$  or  $\bar{D}^{(*)0}\Lambda_c^+$  is expressed as

$$\Gamma[P_\psi^N(4312)^+ \rightarrow M_{P(V)}B] = \frac{|\mathbf{P}_1|}{8\pi M^2} C_1 \frac{1}{2} \sum_{r, r_1, r_2} |\mathcal{A}|^2, \quad (3.58)$$

where  $C_1$  denotes the isospin factor, and the three momentum of the final meson,  $J/\psi(\eta_c)$  or  $\bar{D}^{(*)0}$ , is given by

$$|\mathbf{P}_1| = \frac{1}{2M} \left[ \left( M^2 - (M_1 + M_2)^2 \right) \left( M^2 - (M_1 - M_2)^2 \right) \right]^{\frac{1}{2}}. \quad (3.59)$$



## 4 Numerical results and discussions

### 4.1 Numerical parameters

Before giving the decay widths, we first summarize the effective interaction coefficients we used in above effective Lagrangian. The interaction coefficients between the heavy hadron and the light bosons are obtained under the heavy quark symmetry, which read [6, 9, 12, 14, 26]:  $\rho_{V1} = \frac{\beta g_V}{\sqrt{2}} = 3.75$ ,  $\rho_{T1} = \frac{\lambda g_V}{\sqrt{2}} = 2.34 \text{ GeV}^{-1}$ , and  $\sigma_1 = 0.76$ ;  $\rho_{V2} = \frac{\beta s g_V}{\sqrt{2}} = 7.26$ ,  $\rho_{T2} = \frac{\lambda s g_V}{\sqrt{2}} = 13.81 \text{ GeV}^{-1}$ , and  $\sigma_2 = 6.2$ . In the heavy quark limit, the coupling constants between the heavy hadrons read [61]  $g_{DD\psi} = \frac{2M_\psi}{f_\psi}$  and  $g_{DD^*\psi} = (\frac{M_{D^*}}{M_D})^{1/2} g_{DD\psi}$  with the  $J/\psi$  decay constant  $f_\psi = 0.416 \text{ GeV}$  estimated from the dilepton decay width [69]; the  $DD^*\eta_c$  coupling constant reads  $g_{DD^*\eta_c} = \frac{1}{2}(\frac{M_{\eta_c} M_{D^*}}{M_\psi M_D})^{1/2} g_{DD\psi}$ . Combined with the total amplitude eq. (3.48), it can be found that the partial decay width is proportional to  $\frac{1}{f_\psi^2}$ . The coupling constants related to the baryons used are  $g_{ND\Sigma_c} = 2.69$  and  $g_{ND^*\Sigma_c} = 3.0$  [10, 62]. These values are the standard parameters used in this work, and we will also vary the standard parameters to explore their influence on the wave functions and the final decay widths.

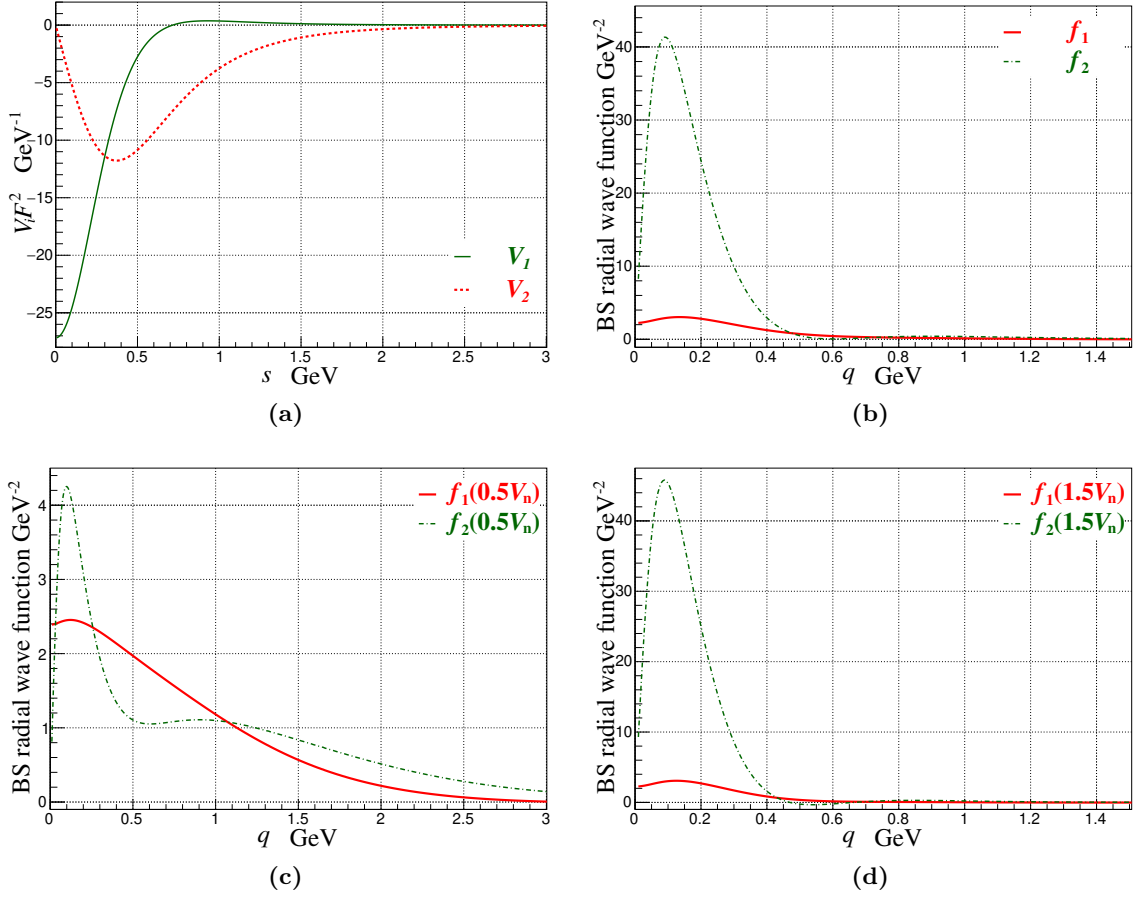
The hadron masses used are  $M_{P_\psi^N(4312)^+} = 4.312 \text{ GeV}$ ,  $M_\psi = 3.097 \text{ GeV}$ ,  $M_{\eta_c} = 2.983 \text{ GeV}$ ,  $M_p = 0.938 \text{ GeV}$ ,  $M_{\bar{D}^*0} = 2.007 \text{ GeV}$ ,  $M_{\Lambda_c^+} = 2.286 \text{ GeV}$  [70].

### 4.2 Numerical results and theoretical uncertainties

The only free parameter in this work is the regulator  $m_\Lambda$  in the form factor  $F(\mathbf{s}^2)$  in eq. (2.19). All the other parameters have been determined by the previous experimental data. By solving the relevant BS eigenvalue equation, we find proper cutoff values of  $m_\Lambda$  can produce bound state of  $\bar{D}\Sigma_c$  based on the one-boson exchange kernel in isospin- $\frac{1}{2}$ . Then by fitting the bound state mass  $M$  to the experimental measurement  $M_{P_\psi^N(4312)^+} = 4.312 \text{ GeV}$ , we fix  $m_\Lambda$  in eq. (2.19) to be  $0.87 \text{ GeV}$ . Then the obtained  $V_1$  and  $V_2$  in the interaction kernel are graphically shown in 3a.

In 3b we show the obtained BS wave functions  $f_1$  and  $f_2$  for  $P_\psi^N(4312)^+$ . On the other hand, the obtained radial wave functions depend on the obtained potential  $V_{1(2)}$ , which is directly related to the coupling constants  $\sigma_{1(2)}$ ,  $\rho_{V1(2)}$  and  $\rho_{T2}$ . To reflect the influence of these parameters on the wave functions and decay widths, we vary the numerical values of  $V_{1(2)}$  under standard parameters by  $\mp 50\%$ . Under these variations, the obtained regulator values are then  $m_\Lambda = 1.288 \text{ GeV}$  and  $0.73 \text{ GeV}$  respectively, and the corresponding wave functions obtained are displayed in 3c and 3d. As  $V_{1(2)}$  decreases, the fitted regulator parameter  $m_\Lambda$  increases, and also the role of wave function  $f_1$  becomes more important.

Our results of the mass spectra for  $I(J^P) = 1/2(1/2)^- \bar{D}\Sigma_c$  molecule indicate that there only exists one bound state, namely,  $P_\psi^N(4312)^+$  as the ground state of  $\bar{D}\Sigma_c$  molecule. Our results do not support any radially excited states. This conclusion is robust even under the  $\pm 50\%$  change of the interaction kernel  $V_{1(2)}$ .



**Figure 3.** The figure (a) shows the isospin- $\frac{1}{2}$  potentials  $V_n F^2$  ( $n = 1, 2$ ). Figure (b) is the Bethe-Salpeter radial wave function  $f_1$  and  $f_2$  for  $P_\psi^N(4312)^+$  as the  $\bar{D}\Sigma_c$  molecular state based on the one-boson exchange; while (c) and (d) are the radial wave functions when the interaction potential  $V_n$  in eq. (2.18) is reduced and increased by 50% based, respectively, where the corresponding regulator values are  $m_\Lambda = 1.288 \text{ GeV}$  and  $0.73 \text{ GeV}$  respectively.

The obtained numerical values of the form factors for decays to  $J/\psi p$  and  $\bar{D}^{*0}\Lambda_c^+$  channels in eq. (3.48) are

$$\begin{aligned}
 s_{1P} &= 1.1 \times 10^{-3}, & s_{2P} &= 1.9 \times 10^{-3}, & s_{1V} &= -4.1 \times 10^{-3}, & s_{2V} &= 7.6 \times 10^{-3}, \\
 s_{4P} &= 7.5 \times 10^{-3}, & s_{5P} &= 6.0 \times 10^{-3}, & s_{4V} &= -8.7 \times 10^{-3}, & s_{5V} &= 1.2 \times 10^{-2}.
 \end{aligned}$$

For decays to  $\eta_c p$  and  $\bar{D}^0\Lambda_c^+$ , the obtained form factors  $s_{3V}$  in eq. (3.33) and  $s_{6V}$  in eq. (3.47) are

$$s_{3V} = 5.1 \times 10^{-3}, \quad s_{6V} = 3.6 \times 10^{-2}.$$

Inserting above form factors into the decay width expressions, we obtain the partial decay widths as  $\Gamma[P_\psi^N(4312)^+ \rightarrow J/\psi p] = 0.11 \text{ MeV}$  and  $\Gamma[P_\psi^N(4312)^+ \rightarrow \eta_c p] = 0.085 \text{ MeV}$ . The obtained partial width for decay to  $J/\psi p$  is  $\sim 1\%$  of the total width  $\Gamma_{\text{Tot}} = 9.8 \pm 2.7_{-4.5}^{+3.7} \text{ MeV}$  [1] reported by the LHCb collaboration. The  $J/\psi p$  channel is also the only

Channel	This	[38]	[40]	[15]	[37]	[10]	[24]
$J/\psi p$	$0.17_{-0.04}^{+0.04}$	$0.32 \pm 0.08$	$1.67_{-0.56}^{+0.92}$	$10^{-3} \sim 0.1$	0.033	(3 ~ 8)	$9.3_{-9.3}^{+19.5}$
$\eta_c p$	$0.085_{-0.018}^{+0.016}$	$0.98 \pm 0.25$	$5.54_{-0.50}^{+0.75}$	$10^{-2} \sim 0.4$	0.066	—	$0.26_{-0.55}^{+0.24}$
$\bar{D}^{*0} \Lambda_c^+$	$8.8_{-1.9}^{+1.6}$	—	—	10.7	6.16	—	—
$\bar{D}^0 \Lambda_c^+$	$0.026_{-0.06}^{+0.05}$	—	—	0.3	0	—	—

**Table 1.** Comparison of partial decay widths of  $P_\psi^N(4312)^+$  to  $J/\psi(\eta_c)p$  and  $\bar{D}^{(*)0}\Lambda_c^+$  with other works in units of MeV, where our theoretical uncertainties are induced by varying the relevant coupling constants by  $\pm 5\%$  in the effective Lagrangian.

observed decay mode of  $P_\psi^N(4312)^+$  currently. While the decay fraction of  $P_\psi^N(4312)^+$  to  $\eta_c p$  is about 50% smaller than the  $J/\psi p$  channel. There is still no evident signal in recent experimental search of  $P_\psi^N(4312)^+$  in  $\eta_c p$  channel [42]. Notice the obtained results are totally predictive and there are no any free adjustable parameters since the regulator  $m_\Lambda$  has been fixed by the mass of  $P_\psi^N(4312)^+$ .

Our result of  $P_\psi^N(4312)^+ \rightarrow \bar{D}^{*0}\Lambda_c^+$  is 8.8 MeV, which can amount to  $\sim 90\%$  of the total width. This prediction is consistent with the calculations in refs. [15, 37]. The partial decay width  $\Gamma[P_\psi^N(4312)^+ \rightarrow \bar{D}^0\Lambda_c^+]$  is about 0.026 MeV, which is comparable with decay to  $\eta_c p$  but negligible with  $\bar{D}^{*0}\Lambda_c^+$ . The huge difference between  $\bar{D}^{*0}$  and  $\bar{D}^0$  channel mainly stems from the difference of the involved coupling constants. Our results support  $\bar{D}^{*0}\Lambda_c^+$  to be the dominant channels for  $P_\psi^N(4312)^+$ , which would be a most promising decay channel to be detected in experiments. The sum of these four decay widths are  $\sim 9.1$  MeV, which amounts to  $\sim 93\%$  of the total width  $\Gamma = 9.8$  MeV reported by the LHCb.

A comparison of our results with other works is listed in Table 1. Our obtained partial decay widths are roughly consistent with those in refs. [15, 37, 38]. Notice the theoretical results for decay widths of  $P_\psi^N(4312) \rightarrow J/\psi(\eta_c)p$  are quite different from each other for the complication of this problem. Also besides the result in ref. [24], the partial decay width to  $\eta_c p$  is great than  $J/\psi p$  in refs. [15, 37, 38, 40], which is roughly consistent with a simple analysis in the heavy quark limit in ref. [20]. However, our result of the partial decay width to  $\eta_c p$  is about the half of that to  $J/\psi p$ . The reason for this difference are analyzed as follows. First, the phase space of  $J/\psi p$  channel is almost the same with that of  $\eta_c p$  channel. While after summing over all polarization states, we obtain the squared amplitudes as

$$\sum_{r,r_1,r_2} |\mathcal{A}(J/\psi p)|^2 = 17.5s_1^2 + 8.2s_1s_2 + 1.6s_2^2, \quad (4.1)$$

$$\sum_{r,r_1,r_2} |\mathcal{A}(\eta_c p)|^2 = 5.0s_3^2, \quad (4.2)$$

where the form factor  $s_1$  part dominates in  $J/\psi p$  channel, which corresponds to the  $J/\psi p$  channel amplitude obtained in ref. [20] in the nonrelativistic format. It is easy to see the coefficient in front of  $s_1^2$  in  $J/\psi p$  channel is  $\sim 3$  times of that in  $\eta_c p$  channel, which steps from the vector polarization states of  $J/\psi$ . On the other hand, the coupling constants  $g_{\psi DD^{(*)}}$  is about 2 times of  $g_{\eta_c DD^*}$ ; and the obtained form factor  $s_{3V}$  for  $\eta_c p$  decay channel

is roughly near with the dominant item  $s_{1V}$  for  $J/\psi p$ , which finally makes our partial decay width for  $\eta_c p$  channel is smaller than that for  $J/\psi p$  channel. Notice in the  $\bar{D}^{(*)}\Lambda_c$  decay channels, the obtained form factor  $s_{6V}$  in pseudoscalar mode is about 3 times of that in the vector mode, which is consistent with the nonrelativistic estimation in the heavy quark limit in ref. [20]. The main difference between  $J/\psi(\eta_c)p$  and  $\bar{D}^{(*)}\Lambda_c$  channels stems from the much heavier exchanged particle  $D^*$  in the former one, which would suppress the contribution of the momentum part in the numerator of the propagator  $D_{\alpha\beta}(k_3)$  and then make  $s_{3V} \sim s_{1V}$ . However, more researches are still needed.

Since the obtained partial decay widths are also directly dependent on the coupling constants  $g_{\psi DD}$ ,  $g_{\psi DD^*}$ ,  $g_{ND\Sigma_c}$ ,  $g_{ND^*\Sigma_c}$ , and  $g_{DD^*\eta_c}$  in the relevant effective Lagrangian. To see the sensitivity of the our partial decay widths on these parameters, we calculate the theoretical uncertainties by varying the every coupling constant by  $\pm 5\%$ , and then searching the parameter space to find the maximum deviation. The obtained theoretical errors are also listed in above Table 1, where the relative uncertainties induced from the coupling constants amount to about  $\sim 20\%$  for the four channels we calculated here. Another theoretical uncertainties come from the interaction strength in the BS kernel, which can be collected into the kernel potential  $V_{1(2)}$ . When the interaction kernel  $V_{1(2)}$  varies by  $\pm 50\%$  based on the standard parameters, the obtained decay widths are 4.3 MeV and 0.04 MeV for  $J/\psi p$  channel, respectively; while for  $\bar{D}^{*0}\Lambda_c^+$  channel, the results are 1.1 MeV and 9.1 MeV, respectively; while for  $\bar{D}^0\Lambda_c^+$  channel, the results are 2.5 MeV and 0.005 MeV, respectively.

Also it is worthy to notice that the obtained partial decay widths for  $J/\psi(\eta_c)p$  channels are both proportional to  $\frac{1}{f_\psi^2}$ , while the values of  $f_\psi$  used in ref. [10] and ref. [40] are 0.426 and 0.481 GeV respectively, under which our result for  $J/\psi(\eta_c)p$  channel would be 5% and 25% smaller respectively.

### 4.3 Summary

We give a brief summary. In this work, firstly, based on the effective Lagrangian in the heavy quark limit, we calculate the one-boson-exchange interaction kernel of  $\bar{D}\Sigma_c$  in the isospin- $\frac{1}{2}$  state. Then by using the Bethe-Salpeter equation, we obtain the mass spectrum and wave functions of the experimental  $P_\psi^N(4312)^+$  as the  $\bar{D}\Sigma_c$  molecular state with  $J^P = (\frac{1}{2})^-$ . Then combining the effective Lagrangian and the obtained BS wave function, we calculate the partial decay width to be 0.17 MeV, 0.085 MeV, 8.8 MeV, and 0.026 MeV for  $P_\psi^N(4312)^+ \rightarrow J/\psi, \eta_c p, \bar{D}^{*0}\Lambda_c^+$  and  $\bar{D}^0\Lambda_c^+$ , respectively. The obtained numerical results indicate that the fraction of  $\bar{D}^{*0}\Lambda_c^+$  channel can amount to  $\sim 90\%$  of  $P_\psi^N(4312)^+$ , which makes  $\bar{D}^{*0}\Lambda_c^+$  to be a much more promising decay channel to be discovered in experiments. This result can also serve as an important test for the molecular interpretation of  $P_\psi^N(4312)^+$ . Our results are roughly consistent with some other calculations and also the LHCb experimental measurements. However, more theoretical analysis and experimental measurements are necessary to determine the properties of the pentaquark state  $P_\psi^N(4312)^+$ . The interpretation of  $P_\psi^N(4312)^+$  as the  $\bar{D}\Sigma_c$  molecular state with  $J^P = (\frac{1}{2})^-$  and isospin  $I = \frac{1}{2}$  is favored by this work.

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## A Expressions of the decay form factors

For completeness, we list the specific expressions of the relevant form factors here, which are all represented by the integral over the radial Salpeter wave functions  $f_1$  and  $f_2$ . Parts of following expressions are calculated with the help of the `FeynCalc` package [71–73]. The four form factors for decay  $P_\psi^N(4312)^+ \rightarrow J/\psi p$  in eq. (3.49) are

$$\begin{aligned}
 s_{1P} &= - \int \frac{d^3 k_\perp}{(2\pi)^3} \frac{1}{2w_2} c_{22} k [k v_0 f_1 + (w_2 u_0 + m_2 v_0) f_2], \\
 s_{2P} &= \int \frac{d^3 k_\perp}{(2\pi)^3} \frac{1}{2w_2 P_1^2} (Y_1 f_1 + Y_2 f_2), \\
 s_{1V} &= - \int \frac{d^3 k_\perp}{(2\pi)^3} \frac{1}{2w_2 P_1^2} (Z_1 f_1 + Z_2 f_2), \\
 s_{2V} &= - \int \frac{d^3 k_\perp}{(2\pi)^3} \frac{1}{2w_2 P_1^2} (Z_3 f_1 + Z_4 f_2),
 \end{aligned} \tag{A.1}$$

where  $Y_1$ ,  $Y_2$ , and  $Z_1 \sim Z_4$  read

$$\begin{aligned}
 Y_1 &= P_1^2 w_2 u_1 + c_1 E_1 k P_1 v_1 + c_1 k M_2 P_1 v_1 - c_1 k M P_1 v_1 - m_2 P_1^2 v_1 - c_1 E_1 k P_1 w_2 u_0 \\
 &\quad - c_{21} E_1^2 k^2 v_0 - c_{21} E_1 k^2 M_2 v_0 + c_{21} E_1 k^2 M v_0 + c_1 E_1 k m_2 P_1 v_0 + c_{22} k^2 P_1^2 v_0,
 \end{aligned} \tag{A.2}$$

$$\begin{aligned}
 Y_2 &= c_1 E_1 P_1 w_2 u_1 - c_1 M P_1 w_2 u_1 + c_1 M_2 P_1 w_2 u_1 - c_1 E_1 m_2 P_1 v_1 - 2c_1 E_2 m_2 P_1 v_1 \\
 &\quad + c_1 m_2 M_2 P_1 v_1 + c_1 m_2 M P_1 v_1 + k P_1^2 v_1 - c_{21} E_1^2 k w_2 u_0 + c_{21} E_1 k M w_2 u_0 \\
 &\quad - c_{21} E_1 k M_2 w_2 u_0 + c_{22} k P_1^2 w_2 u_0 + c_{21} E_1^2 k m_2 v_0 + 2c_{21} E_1 E_2 k m_2 v_0 \\
 &\quad - c_1 E_1 k^2 P_1 v_0 - c_{21} E_1 k m_2 M_2 v_0 - c_{21} E_1 k m_2 M v_0 + c_{22} k m_2 P_1^2 v_0,
 \end{aligned} \tag{A.3}$$

$$\begin{aligned}
 Z_1 &= -E_1 P_1^2 w_2 u_1 + M P_1^2 w_2 u_1 + M_2 P_1^2 w_2 u_1 - c_1 E_1^2 k P_1 v_1 + c_1 k M_1^2 P_1 v_1 + E_1 m_2 P_1^2 v_1 \\
 &\quad - m_2 M P_1^2 v_1 - m_2 M_2 P_1^2 v_1 + c_1 E_1^2 k P_1 w_2 u_0 - c_1 E_1 k M P_1 w_2 u_0 - c_1 E_1 k M_2 P_1 w_2 u_0 \\
 &\quad + c_{21} E_1^3 k^2 v_0 - c_1 E_1^2 k m_2 P_1 v_0 - c_{22} E_1 k^2 P_1^2 v_0 - c_{21} E_1 k^2 M_1^2 v_0 \\
 &\quad + c_1 E_1 k m_2 M P_1 v_0 + c_1 E_1 k m_2 M_2 P_1 v_0 - c_{22} k^2 M P_1^2 v_0 - c_{22} k^2 M_2 P_1^2 v_0,
 \end{aligned} \tag{A.4}$$

$$\begin{aligned}
 Z_2 &= c_1 M_1^2 P_1 w_2 u_1 + c_1 E_1^2 m_2 P_1 v_1 + 2c_1 E_1 E_2 m_2 P_1 v_1 - 2c_1 E_1 m_2 M P_1 v_1 \\
 &\quad + c_1 m_2 M_1^2 P_1 v_1 + k M P_1^2 v_1 + k M_2 P_1^2 v_1 + c_{21} E_1^3 k w_2 u_0 + c_1 E_1^2 k^2 P_1 v_0 \\
 &\quad - c_{21} E_1 k M_1^2 w_2 u_0 - c_{22} E_1 k P_1^2 w_2 u_0 - c_{22} k M P_1^2 w_2 u_0 - c_{22} k M_2 P_1^2 w_2 u_0 \\
 &\quad + 2c_{21} E_1^2 k m_2 M v_0 - c_1 E_1 k^2 M P_1 v_0 - c_1 E_1 k^2 M_2 P_1 v_0 + c_{22} E_1 k m_2 P_1^2 v_0
 \end{aligned}$$

$$\begin{aligned}
 & -c_{21}E_1km_2M_1^2v_0 + 2c_{22}E_2km_2P_1^2v_0 - 3c_{22}km_2MP_1^2v_0 - c_{22}km_2M_2P_1^2v_0 \\
 & - 2c_{21}E_1^2E_2km_2v_0 - c_{21}E_1^3km_2v_0 - E_1kP_1^2v_1 - c_1E_1^2P_1w_2u_1,
 \end{aligned} \tag{A.5}$$

$$\begin{aligned}
 Z_3 = & -MP_1^2w_2u_1 - M_2P_1^2w_2u_1 + c_1E_1kMP_1v_1 - c_1E_1kM_2P_1v_1 - c_1kM_1^2P_1v_1 \\
 & + m_2M_2P_1^2v_1 + c_1E_1kMP_1w_2u_0 + c_1E_1kM_2P_1w_2u_0 - c_{21}E_1^2k^2Mv_0 \\
 & + c_{21}E_1^2k^2M_2v_0 + c_{21}E_1k^2M_1^2v_0 - c_1E_1km_2MP_1v_0 - c_1E_1km_2M_2P_1v_0 \\
 & + m_2MP_1^2v_1 - c_{22}k^2M_2P_1^2v_0 + 3c_{22}k^2MP_1^2v_0,
 \end{aligned} \tag{A.6}$$

$$\begin{aligned}
 Z_4 = & c_1E_1MP_1w_2u_1 - c_1E_1M_2P_1w_2u_1 - c_1M_1^2P_1w_2u_1 + c_1E_1m_2MP_1v_1 \\
 & - c_1m_2M_1^2P_1v_1 - kMP_1^2v_1 - kM_2P_1^2v_1 - c_{21}E_1^2kMw_2u_0 + c_{21}E_1^2kM_2w_2u_0 \\
 & + c_{21}E_1kM_1^2w_2u_0 + 3c_{22}kMP_1^2w_2u_0 - c_{22}kM_2P_1^2w_2u_0 - c_{21}E_1^2km_2Mv_0 \\
 & + c_{21}E_1^2km_2M_2v_0 + c_1E_1k^2MP_1v_0 + c_1E_1k^2M_2P_1v_0 + c_{21}E_1km_2M_1^2v_0 \\
 & + 3c_{22}km_2MP_1^2v_0 - c_{22}km_2M_2P_1^2v_0 - c_1E_1m_2M_2P_1v_1.
 \end{aligned} \tag{A.7}$$

In above expressions,  $P_1 = |\mathbf{P}_1|$ , and

$$c = \cos \theta, \quad c_{21} = \frac{1}{2}(3 \cos^2 \theta - 1), \quad c_{22} = \frac{1}{2}(\cos^2 \theta - 1), \tag{A.8}$$

where  $\theta$  denotes the angle between  $\mathbf{k}$  and  $\mathbf{P}_1$ . We also define  $u_n$  and  $v_n$  ( $n = 0, 1, 2$ ) for later convenience

$$\begin{aligned}
 u_n &= (c_1x_{1P}^n + c_2x_{2P}^n + c_3x_{3P}^n) + (c_4x_{4P}^n + c_5x_{5P}^n + c_6x_{6P}^n), \\
 v_n &= (c_1x_{1P}^n + c_2x_{2P}^n + c_3x_{3P}^n) - (c_4x_{4P}^n + c_5x_{5P}^n + c_6x_{6P}^n).
 \end{aligned} \tag{A.9}$$

The expressions of  $c_i$  are listed in eq. (3.16).

The form factor  $s_{3V}$  in eq. (3.33) for  $P_\psi^N(4312)^+ \rightarrow \eta_c p$  decay behaves

$$s_{3V} = \int \frac{d^3k_\perp}{(2\pi)^3} \frac{1}{4m_3^2w_2w_3P_1^2} \left[ (P_1^2X_1 + kc_1X_3)f_1 + (kP_1^2X_2 + c_1X_4)f_2 \right], \tag{A.10}$$

where  $X_1 \sim X_4$  read

$$\begin{aligned}
 X_1 = & -ckMP_1w_2u_0 - ckM_2P_1w_2u_0 + m_3^2Mw_2u_0 + m_3^2M_2w_2u_0 - MM_1^2w_2u_0 \\
 & - M_1^2M_2w_2u_0 + ck m_3P_1w_2u_1 + E_1m_3Mw_2u_1 + E_1m_3M_2w_2u_1 + m_3M_1^2w_2u_1 \\
 & - E_1m_3^2w_2u_2 + ck^3P_1v_0 + k^2M_1^2v_0 + ck m_2MP_1v_0 + ck m_2M_2P_1v_0 \\
 & - ck m_2m_3P_1v_1 - m_2m_3^2Mv_0 + m_2MM_1^2v_0 - m_2m_3^2M_2v_0 + m_2M_1^2M_2v_0 \\
 & - E_1k^2m_3v_1 - E_1m_2m_3Mv_1 - E_1m_2m_3M_2v_1 + E_1m_2m_3^2v_2 - m_2m_3M_1^2v_1,
 \end{aligned} \tag{A.11}$$

$$\begin{aligned}
 X_2 = & ckP_1w_2u_0 + M_1^2w_2u_0 - E_1m_3w_2u_1 + ck m_2P_1v_0 + ck m_3P_1v_1 + m_3M_1^2v_1 \\
 & - ckMP_1v_0 - ckM_2P_1v_0 + m_3^2Mv_0 + m_3^2M_2v_0 + m_2M_1^2v_0 - MM_1^2v_0 \\
 & - M_1^2M_2v_0 + E_1m_3Mv_1 + E_1m_3M_2v_1 - E_1m_2m_3v_1 - E_1m_3^2v_2,
 \end{aligned} \tag{A.12}$$

$$\begin{aligned}
 X_3 = & -cE_1kP_1w_2u_0 - E_1M_1^2w_2u_0 + ckMP_1w_2u_0 + ckM_2P_1w_2u_0 + MM_1^2w_2u_0 \\
 & + M_2M_1^2w_2u_0 + E_1^2m_3w_2u_1 - E_1m_3Mw_2u_1 - E_1m_3M_2w_2u_1 + cE_1km_2P_1v_0 \\
 & - cE_1km_3P_1v_1 - cE_1kMP_1v_0 - cE_1kM_2P_1v_0 + E_1m_2M_1^2v_0 + E_1m_3^2Mv_0
 \end{aligned}$$

$$\begin{aligned}
& + E_1 m_3^2 M_2 v_0 - E_1 M M_1^2 v_0 - E_1 M_2 M_1^2 v_0 + c k m_3 M P_1 v_1 + c k m_3 M_2 P_1 v_1 \\
& - c k m_2 M P_1 v_0 - c k m_2 M_2 P_1 v_0 + c k M_1^2 P_1 v_0 - m_3^2 M_1^2 v_0 - m_2 M M_1^2 v_0 \\
& - m_2 M_2 M_1^2 v_0 + M_1^4 v_0 + E_1^2 m_3 M v_1 + E_1^2 m_3 M_2 v_1 - E_1^2 m_2 m_3 v_1 + E_1^2 m_3^2 v_2 \\
& - 2 E_1 m_3 M_1^2 v_1 + E_1 m_2 m_3 M v_1 + E_1 m_2 m_3 M_2 v_1 - E_1 m_3^2 M v_2 - E_1 m_3^2 M_2 v_2 \\
& + m_3 M M_1^2 v_1 + m_3 M_2 M_1^2 v_1, \tag{A.13}
\end{aligned}$$

$$\begin{aligned}
X_4 = & v_0 m_2 M_1^4 + u_0 w_2 M_1^4 - v_0 m_2 m_3^2 M_1^2 + E_1 k^2 v_0 M_1^2 + 2 E_2 k^2 v_0 M_1^2 + m_2 m_3 M_2 v_1 M_1^2 \\
& - k^2 M v_0 M_1^2 - E_1 M v_0 m_2 M_1^2 + k^2 v_0 M_2 M_1^2 - E_1 v_0 m_2 M_2 M_1^2 + c k v_0 m_2 P_1 M_1^2 \\
& - u_0 m_3^2 w_2 M_1^2 - E_1 M u_0 w_2 M_1^2 - E_1 u_0 M_2 w_2 M_1^2 + c k u_0 P_1 w_2 M_1^2 + 2 E_2 m_3 w_2 u_1 M_1^2 \\
& - M m_3 w_2 u_1 M_1^2 + m_3 M_2 w_2 u_1 M_1^2 - 2 E_1 m_2 m_3 v_1 M_1^2 + M m_2 m_3 v_1 M_1^2 \\
& + E_1 M v_0 m_2 m_3^2 + E_1 v_0 m_2 m_3^2 M_2 + c E_1 k^3 v_0 P_1 + 2 c E_2 k^3 v_0 P_1 - c k^3 M v_0 P_1 \\
& - c E_1 k M v_0 m_2 P_1 + c k^3 v_0 M_2 P_1 - c E_1 k v_0 m_2 M_2 P_1 + E_1 M u_0 m_3^2 w_2 + E_1 u_0 m_3^2 M_2 w_2 \\
& - c E_1 k M u_0 P_1 w_2 - c E_1 k u_0 M_2 P_1 w_2 + E_1^2 M m_3 w_2 u_1 + E_1^2 m_3 M_2 w_2 u_1 + c E_1 k m_3 P_1 w_2 u_1 \\
& + 2 c E_2 k m_3 P_1 w_2 u_1 - c k M m_3 P_1 w_2 u_1 + c k m_3 M_2 P_1 w_2 u_1 - E_1^2 m_3^2 w_2 u_2 - 2 E_1 E_2 m_3^2 w_2 u_2 \\
& + E_1 M m_3^2 w_2 u_2 - E_1 m_3^2 M_2 w_2 u_2 - E_1^2 k^2 m_3 v_1 - 2 E_1 E_2 k^2 m_3 v_1 + E_1 k^2 M m_3 v_1 \\
& + E_1^2 M m_2 m_3 v_1 - E_1 k^2 m_3 M_2 v_1 + E_1^2 m_2 m_3 M_2 v_1 - c E_1 k m_2 m_3 P_1 v_1 + c k M m_2 m_3 P_1 v_1 \\
& + c k m_2 m_3 M_2 P_1 v_1 + E_1^2 m_2 m_3^2 v_2 - E_1 M m_2 m_3^2 v_2 - E_1 m_2 m_3^2 M_2 v_2. \tag{A.14}
\end{aligned}$$

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