Flavor-violating charged lepton decays in the little Randall-Sundrum model

A. Akshay[*] and Mathew Thomas Arun[†]
School of Physics, Indian Institute of Science Education and Research, Thiruvananthapuram 695551, India

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The little Randall-Sundrum (little RS) model receives significantly stronger constraints from the flavor observables in comparison to the Randall-Sundrum (RS) model. In this paper, we analyze the effect of the electroweak sector in little RS on flavor-changing decays of charged leptons. We compare the predictions of the model with the current limits on the flavor-violating branching ratios of $\mu \to eee$, $\tau \to eee$, $\tau \to \mu\mu\mu$, $\tau \to \mu ee$, $\tau \to e\mu\mu$, $\mu \to e\gamma$, and $\mu Ti \to eTi$, and we show that the dominant constraint arises from the $\mu Ti \to eTi$ process which strongly limits the KK-1 gauge boson mass (M_{KK}) to be $\gtrsim$30.7 TeV. We then derive and show that generalizing the electroweak gauge sector to include the brane localized kinetic term relaxes this constraint to $\gtrsim$12 TeV. Toward the conclusion, we comment on the possibility that the large flavor-violating currents can be mitigated by relaxing the assumption regarding the unnatural thinness and rigidity of the UV brane and discuss the possibility of suppression of these currents in the presence of fat fluctuating branes.

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I. INTRODUCTION

The charged lepton flavor violation (CLFV) has been in focus ever since intermediate vector bosons were proposed [1,2]. Even with the inclusion of the neutrino oscillation phenomena in the Standard Model (SM) of particle physics, the CLFV processes are predicted to be very small. Hence, any evidence of CLFV inevitably points toward new physics. As of now, these processes are strongly constrained at 90% C.L. by the branching ratios $B_{\mu\to e\gamma} < 4.2\times10^{-13}$ [3], $B_{\mu\to eee} < 1.0\times10^{-12}$ [4], $B_{\mu Ti\to eTi} < 4.3\times10^{-12}$ [5], $B_{\tau\to eee} < 2.7\times10^{-8}$ [6], $B_{\tau\to e\gamma} < 3.3\times10^{-8}$ [7], $B_{\pi^0\to\mu e} < 3.6\times10^{-10}$ [8], and $B_{Z\to\mu e} < 7.5\times10^{-7}$ [9]. On the other hand, various beyond-SM scenarios do predict such flavor-violating processes.

One of the successful extensions of the SM in warped extra dimensions has been the little Randall-Sundrum (little RS) model [10,11] with a fundamental scale of $\sim10^3$ TeV. Recently, the author studied [12] the correction to ϵ_K in little RS arising from the contribution of tree-level KK-1 gluon exchange in K-$\bar{K}$ oscillation. There, it was shown that the strong bound on the imaginary part of the C_4^{sd} operator in the effective Hamiltonian [13] ruled out the mass scale lower than $\sim$32 TeV. This constraint was shown to soften significantly in the presence of brane localized kinetic terms (BLKTs) and minimal flavor protection [14] flavor symmetry.

In this paper, we discuss flavor violation in the charged lepton sector of little RS. We choose to work in a basis that does not mix the left-handed charged leptons. Rather, the entire mixing is taken to be in the right-handed sector while keeping the 5D Yukawa coupling anarchic. There can, in principle, exist mixing in the left-handed charged sector through the tribimaximal Cabibbo mechanism [15], but such a choice will affect the neutrino mixing matrix as well. For brevity, we do not consider such models here. Doing this, we can study the flavor violations in the charged lepton sector independent of the neutrino parameters.

We subject the lepton flavor-violating decay predictions of the anarchic little RS model to observations from rare μ decays such as $\mu \to e\gamma$, $\mu \to 3e$, and $\mu \to e$ conversion in the presence of Ti nuclei and the rare trilepton decays of τ. Among these, we will show that $\mu \to e$ conversion proves to be the most constraining, and this constrains the lower limit of the KK-1 gauge boson mass to be $\gtrsim$30.7 TeV. For comparison, the lower mass limit of the gauge boson in the Randall-Sundrum model [16,17] that satisfies the $\mu \to e$ conversion is $M_{KK} \gtrsim 5.9$ TeV [18]. In line with the discussion on kaon oscillation in Ref. [12], here, we study the effect of the BLKT in the electroweak sector of the model and, in particular, on the Z boson wave function. We show that this modification relaxes the strong bounds on the KK-1 mass of the gauge boson to $\gtrsim$12 TeV, making the Kaluza-Klein (KK) gauge boson available at the upcoming high-energy colliders.

[*]akshay17@iisertvm.ac.in
[†]mathewthomas@iisertvm.ac.in

For anarchic 5D Yukawa couplings, since the light fermions are localized close to the ultraviolet (UV) brane, it is possible that the unphysical assumption of rigid thin branes in little RS might have a role to play in these large flavor violations. If we replace this thin rigid UV brane with a fat brane, along with its fluctuations, namely, branons [19–21], then we show that the flavor-violating couplings get suppressed, lowering the bound on the KK-1 gauge boson mass to $\gtrsim 10$ TeV.

The paper is organized as follows. In the next section, we briefly review the little RS model, gauge and lepton field KK decomposition, and their interactions. In Sec. III, we will derive the trilepton decay branching ratios and $\mu \to e$ conversion rate in the model. Then, we will discuss the effect of BLKT on the interactions and recompute the constraints in Sec. IV. In Sec. V, we summarize the work and discuss the effects of fat branes on lepton flavor violation.

II. THE LITTLE RS MODEL

In this section, we briefly recap the little RS model. Our four-dimensional space-time is assumed to emerge from a 5D anti–de Sitter (AdS) space-time, with a fundamental scale $M = 10^3$ TeV [10,22], upon orbifolding on $M_4 \times S^1/Z_2$. The line element of this 5D space-time is given by

$$ds^2 = g_{MN}dx^M dx^N = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \qquad (1)$$

where M and N are five-dimensional space-time indices, $\eta_{\mu\nu} = \mathrm{diag}(-1,+1,+1,+1)$, and $0 \leq y \leq L$. The warp factor is taken to be $kL \sim 7$ so that the warped down scale at the infrared (IR) brane becomes $M_5 e^{-kL} \sim \mathcal{O}(1\ \mathrm{TeV})$.

The Higgs field, in order to stabilize its vacuum expectation value from quantum fluctuations, is assumed to be localized on this IR brane. To avoid large localized flavor violating and proton decay operators on the brane, we assume that the gauge fields and fermions propagate in the bulk. Moreover, we consider these fields to transform under the adjoint and fundamental representations of the Standard Model gauge group $SU(3)_C \times SU(2)_W \times U(1)_Y$, respectively.

Below, we describe the five-dimensional gauge and fermion fields, relevant to the process, and their KK decompositions. We begin our discussion with a review of the electroweak sector coupled with the Higgs boson in five-dimensional space-time.

A. Bulk gauge fields

1. Action of the 5D theory

Let us consider the bulk gauge fields W_M^a and B_M, where $M = \{0,1,2,3,5\}$, of $SU(2)_L \times U(1)_Y$, coupled to the scalar localized on the IR brane. The 5D action for this system is given by

$$S_{\mathrm{gauge}} = \int d^4x \int_0^L dy(\mathcal{L}_{W,B} + \mathcal{L}_{\mathrm{Higgs}}). \qquad (2)$$

The kinetic part of the 5D gauge theory ($\mathcal{L}_{W,B}$) and the Higgs-sector Lagrangian ($\mathcal{L}_{\mathrm{Higgs}}$) are given by, respectively,

$$\mathcal{L}_{W,B} = \sqrt{g}g^{KM}g^{LN}\left(-\frac{1}{4}W_{KL}^a W_{MN}^a - \frac{1}{4}B_{KL}B_{MN}\right) \qquad (3)$$

and

$$\begin{aligned}\mathcal{L}_{\mathrm{Higgs}} &= \delta(|y|-L)[(D_M\Phi)^\dagger(D^M\Phi) - V(\Phi)]\\ V(\Phi) &= -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2. \end{aligned} \qquad (4)$$

The field strength tensor of the gauge field is denoted by W_{MN}, and the covariant derivative by

$$D_M = \partial_M - ig_5\tau_a W_M^a - ig_5' I B_M, \qquad (5)$$

where g_5 and g_5' are the 5D gauge couplings of W and B bosons, respectively, and the Higgs doublet can be decomposed as

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} -i\sqrt{2}\varphi^+(x) \\ v + h(x) + i\varphi^3(x)\end{pmatrix}, \qquad (6)$$

with the Higgs vacuum expectation value denoted by v, and $\varphi^\pm = (\varphi^1 \mp i\varphi^2)/\sqrt{2}$.

The δ function in Eq. (4) ensures that the Higgs vacuum expectation value is stabilized to $\sim\mathcal{O}(1\ \mathrm{TeV})$. For simplicity, we also perform the usual redefinitions of the gauge fields:

$$\begin{aligned} W_M^\pm &= \frac{1}{\sqrt{2}}(W_M^1 \mp iW_M^2),\\ Z_M &= \frac{1}{\sqrt{g_5^2 + g_5'^2}}(g_5 W_M^3 - g_5' B_M),\\ A_M &= \frac{1}{\sqrt{g_5^2 + g_5'^2}}(g_5' W_M^3 + g_5 B_M). \end{aligned} \qquad (7)$$

2. KK decomposition

After compactification, the Kaluza-Klein decomposition of 5D gauge field becomes

$$V_\mu(x,y) = \sum_n V_\mu^{(n)}(x) f_V^{(n)}(y), \qquad (8)$$

where $V_\mu^{(n)}(x) = \{A_\mu^{(n)}(x), Z_\mu^{(n)}(x), W_\mu^{\pm(n)}(x)\}$ are the four-dimensional KK modes of the photon, Z boson, and the $W^\pm$ boson and $f_V^{(n)}(y)$ their wave profiles in the bulk. The Euler-Lagrange equation of motion of these bulk modes are given by

$$-\partial_5(e^{-2ky}\partial_5 f_V^{(n)}) = m_n^2 f_V^{(n)}. \qquad (9)$$

In this equation, we have used $\partial_\mu\partial^\mu V_\mu^{(n)}(x) = m_n^2 V_\mu^{(n)}(x)$. These fields are set to satisfy the boundary condition $(\delta V^\mu \partial_y V_\mu)|_{0,L} = 0$ and the orthonormality condition

$$\int_0^L dy f_V^{(n)} f_V^{(m)} = \delta_{nm}. \qquad (10)$$

Solving Eq. (9), for $m_n = 0$, we find that the zero-mode profile of the gauge boson is flat and is given by

$$f_V^{(0)}(y) = \frac{1}{\sqrt{L}}, \qquad (11)$$

For the higher KK modes ($m_n \neq 0$), the solution to Eq. (9) is given in terms of the Bessel J and Bessel Y functions and is of the form

$$f_V^{(n)}(y) = N_V^{(n)} e^{ky}\left[J_1\left(\frac{m_n e^{ky}}{k}\right) + b_V^{(n)} Y_1\left(\frac{m_n e^{ky}}{k}\right)\right], \qquad (12)$$

where $N_V^{(n)}$ and $b_V^{(n)}$ are the two constants of integration. While $N_V^{(n)}$ is fixed by the orthonormality condition, $b_V^{(n)}$ is determined using the boundary condition. Demanding Neumann boundary condition at $y = 0$ and $y = L$, we get

$$b_V^{(n)} = -\frac{J_0(\frac{m_n}{k})}{Y_0(\frac{m_n}{k})} \quad \text{at } y = 0,$$
$$b_V^{(n)} = -\frac{J_0(\frac{m_n}{k}e^{kL})}{Y_0(\frac{m_n}{k}e^{kL})} \quad \text{at } y = L. \qquad (13)$$

The mass spectrum of the KK modes ($m_n = x_n k e^{-kL}$) can be computed from the solutions to the equation $J_0(x_n)Y_0(x_n e^{-kL}) - Y_0(x_n)J_0(x_n e^{-kL}) = 0$, obtained by equating the relations given above. For future convenience we denote the KK-1 gauge boson mass by $M_{KK} = m_1$.

B. Bulk fermion fields

The Clifford algebra in five-dimensional space-time is defined by gamma matrices, $\Gamma^A = \{\gamma^0, \gamma^1, \gamma^2, \gamma^3, \gamma^5\}$, that satisfy $\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}$, where η^{AB} is the flat metric defined on the 5D tangent space. Since the algebra is irreducible, one cannot construct a chirality projection operator in five dimensions. Thus, the fermion fields constructed in this geometry have four complex degrees of freedom, which on compactification leads to vectorlike four-dimensional fermions. Moreover, due to the lack of chiral symmetry, the geometry does not prohibit a mass term in the bulk for the fermions. These mass terms will become crucial for the geometric Froggatt-Nielsen mechanism to generate four-dimensional fermion mass hierarchy. Let us start our discussion by constructing the five-dimensional fermionic action.

1. 5D fermionic action

The 5D action for doublet ($\hat{\ell}$) and singlet ($\hat{e}$) leptons can be written as

$$S_{\text{fermion}} = S_{\text{kin}} + S_{\text{Yuk}}, \qquad (14)$$

$$S_{\text{kin}} = \int d^5x\sqrt{-g}\,\bar{\hat{\ell}}(\Gamma^A E_A^M D_M + m_\ell)\hat{\ell}$$
$$+ \int d^5x\sqrt{-g}\sum_{e=e,\mu,\tau}\bar{\hat{e}}(\Gamma^A E_A^M D_M + m_e)\hat{e},$$
$$S_{\text{Yuk}} = \int d^5x\sqrt{-g}\,\bar{\hat{\ell}}_i(\tilde{Y}_{5D})^{ij}\hat{e}_j H(x^\mu)\delta(y-L) + \text{H.c.}, \qquad (15)$$

where m_ℓ and m_e are the bulk masses for the doublet and singlet lepton fields, respectively, and E_A^M the inverse fünfbeins. We denote the tangent space indices with A, B and the five-dimensional space-time index with M, N. Since the fünfbeins satisfy the condition $e_M^A \eta_{AB} e_N^B = g_{MN}$, for the geometry given in Eq. (1), they become $e_M^A = (e^{-ky}\delta_\mu^\alpha, 1)$. The five-dimensional anarchic Yukawa matrix is denoted as $(\tilde{Y}_{5D})^{ij}$, with i, j representing the generational indices. In the above equation, D_M represent the covariant derivative in five dimensions given by $D_M = \partial_M + \omega_M$, where $\omega_M = \frac{1}{8}\omega_{MAB}[\Gamma^A, \Gamma^B]$ and the spin connection given by

$$\omega_{MAB} = g_{RN} E_A^N(\partial_M E_B^R + \tilde{\Gamma}_{MT}^R E_B^T), \qquad (16)$$

where E_M^A are the inverse fünfbeins and $\tilde{\Gamma}_{MS}^R$ are the Christoffel connections.

Upon compactification, the 5D Dirac fermions decompose to two 4D Weyl spinors. To ensure that only the correct chiral projections survive at the zero mode, we use boundary conditions

$$\hat{\ell}_L(++), \quad \hat{\ell}_R(--), \quad \hat{e}_L(--), \quad \hat{e}_R(++), \qquad (17)$$

at the orbifold fixed points ($y = 0, y = L$). Here, L and R stand for the left and right chiral fields, respectively, under the four-dimensional chiral projection operator, and $+(-)$ stands for the Neumann (Dirichlet) boundary conditions. For example, $\hat{\ell}_L(++)$ means that we apply the Neumann boundary conditions at both $y = 0$ and $y = L$.

2. The Kaluza-Klein decompositions

After compactification, the KK expansion of a generic fermion field (Ψ) becomes

$$\Psi(x,y)_{L,R} = \sum_{n=0}^{\infty} \frac{1}{\sqrt{L}} \psi_{L,R}^{(n)}(x) f_{L,R}^{(n)}(y,c), \tag{18}$$

where $\psi_{L,R}^{(n)}(x)$ denotes the corresponding four-dimensional KK modes and $f_{L,R}(y)$ their extra-dimensional profiles in the bulk. These wave profiles are set to satisfy the orthonormality condition

$$\int_0^L dy e^{-3ky} f_{L,R}^{(n)} f_{L,R}^{(m)} = \delta_{n,m}. \tag{19}$$

The normalized zero-mode profile for doublets and singlets, with their respective bulk mass parameters $c_{\ell_i} = m_{\ell_i}/k$ and $c_{e_i} = -m_{e_i}/k$, can be derived as [12,13]

$$f_L^{(0)}(y, c_{\ell_i}) = \sqrt{k} f^0(c_{\ell_i}) e^{ky(2-c_{\ell_i})} e^{(c_{\ell_i}-0.5)kL}, \tag{20}$$

$$f_R^{(0)}(y, c_{e_i}) = \sqrt{k} f^0(c_{e_i}) e^{ky(2-c_{e_i})} e^{(c_{e_i}-0.5)kL}, \tag{21}$$

where

$$f^0(c) = \sqrt{\frac{(1-2c)}{1-e^{-(1-2c)kL}}}. \tag{22}$$

Using the boundary conditions given in Eq. (17), the lightest and next-to-lightest modes of chiral leptons in four dimensions become

$$\Psi_L = \left(\hat{\ell}_L^{i(0)}, \hat{\ell}_L^{i(1)}, \hat{e}_L^{i(1)}\right),$$
$$\Psi_R = \left(\hat{e}_R^{i(0)}, \hat{e}_R^{i(1)}, \hat{\ell}_R^{I(1)}\right). \tag{23}$$

3. Yukawa interaction

Using the action in Eq. (15) and the wave function for the zero-mode leptons given in Eq. (21), the four-dimensional Yukawa matrix can be derived in terms of the 5D anarchic Yukawa as

$$\begin{aligned} Y_{4D}^{ij} &= f_L^{(0)}(L, c_{\ell_i}) Y_{5D}^{ij} f_R^{(0)}(L, c_{e_j}) \\ &= \sqrt{\frac{(1-2c_{\ell_i})(1-2c_{e_j})}{(e^{(1-2c_{\ell_i})kL}-1)(e^{(1-2c_{e_j})kL}-1)}} e^{(1-(c_{\ell_i}+c_{e_j}))kL} Y_{5D}^{ij}. \end{aligned} \tag{24}$$

For the fermion KK modes given in Eq. (23), the above mass matrix becomes

$$\mathcal{M} = \begin{pmatrix} M_0 & M_0 F_R & 0 \\ F_L M_0 & F_L M_0 F_R & M_{KK} \\ 0 & M_{KK} & 0 \end{pmatrix}, \tag{25}$$

where $F_{L,R}^i = \frac{f_{L,R}^{(1)}(L, c_{\ell_i, e_i})}{f_{L,R}^{(0)}(L, c_{\ell_i, e_i})}$ and $M_0^{ij} = \frac{v}{\sqrt{2}} Y_{4D}^{ij}$. Since the fermions are in the flavor basis, we need to rotate this mass matrix to obtain the physical states. In order to do that, it will be easier if we first diagonalize the SM part M_0 with a biunitary transformation (U_L, U_R). Acting on the mass matrix $\mathcal{M}$ with diag$(U_L, 1, 1)$ on the left and on the right with diag$(U_R^\dagger, 1, 1)$, we get

$$\mathcal{M} = \begin{pmatrix} M_D & \frac{v}{\sqrt{2}}\Delta_R & 0 \\ \frac{v}{\sqrt{2}}\Delta_L & \Delta_1 & M_{KK} \\ 0 & M_{KK} & 0 \end{pmatrix}, \tag{26}$$

where $M_D = U_L M_0 U_R^\dagger$, $\frac{v}{\sqrt{2}}\Delta_R = U_L M_0 F_R = M_D U_R F_R$, $\frac{v}{\sqrt{2}}\Delta_L = F_L M_0 U_R^\dagger = F_L U_L^\dagger M_D$, and $\Delta_1 = F_L M_0 F_R = F_L U_L^\dagger M_D U_R F_R$.

Moreover, since the mixing of higher KK modes with the zero mode is suppressed by $\frac{v}{M_{KK}}$, it is convenient and informative to diagonalize the lower 2×2 part of the mass matrix. With that, to the leading-order expansion in $x = \frac{\Delta_1}{M_{KK}}$, the diagonal mass matrix becomes [18]

$$\mathcal{M}_D = \begin{pmatrix} M_D & x_R \frac{1}{2}(1+\frac{x}{4}) & x_R \frac{1}{2}(1-\frac{x}{4}) \\ x_L \frac{1}{2}(1+\frac{x}{4}) & M_{KK} + \frac{\Delta_1}{2} & 0 \\ x_L \frac{1}{2}(1-\frac{x}{4}) & 0 & -M_{KK} + \frac{\Delta_1}{2} \end{pmatrix}, \tag{27}$$

where $x_{L,R} = \frac{v}{\sqrt{2}}\Delta_{L,R}$. Note that the off-diagonal elements in this matrix are very small compared to M_{KK}. Now, the degeneracy in the KK-1 mode is lifted to

$$M_{KK}^{(1)} = M_{KK} + \frac{\Delta_1}{2},$$
$$M_{KK}^{(2)} = -M_{KK} + \frac{\Delta_1}{2}. \tag{28}$$

4. Couplings with Z boson

Before discussing the flavor-violating effects, we need to identify the relevant couplings of the gauge boson. For simplicity, we will consider the couplings of fermion bilinear with an Abelian gauge field in the bulk of AdS. Generalization to flavor-violating interactions of the non-Abelian gauge field is then straightforward. A five-dimensional action for the $U(1)$ gauge field can be written as

$$\mathcal{S} = -\frac{1}{4g_5^2} \int d^5x \sqrt{-g} (g^{CM} g^{DN} F_{CD} F_{MN}), \tag{29}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ is the field strength tensor and g_5 the 5D gauge coupling.

In the unitary gauge ($A_5 = 0$), the vector field can be Fourier expanded as

$$A_\mu(x,y) = \sum_n A_\mu^{(n)}(x) f_A^{(n)}(y), \tag{30}$$

and the coupling of the zero-mode lepton bilinear with the gauge KK modes can be computed from the overlap integral:

$$g_{L,R}^{(n)}(c_{\ell,e}) = \frac{g_5}{L}\int_0^L dy e^{-3ky} f_A^{(n)}(y) f_{L,R}^{(0)}(y, c_{\ell,e}) f_{L,R}^{(0)}(y, c_{\ell,e}). \tag{31}$$

In the above equation, $f_A^{(n)}$ and $f_{L,R}^{(0)}$ are as given in Eqs. (12) and (22), respectively. Since the geometric Froggatt-Nielsen mechanism requires distinct bulk mass values "$c_{\ell,e}$" for the leptons, the couplings $g_{L,R}^{(n)}$ are different depending on the localization of the fermion zero mode. Replacing the Abelian gauge field with the Z boson, we can explicitly write these interactions as

$$\mathcal{L} = g_L^{(n)}(c_{\ell_i})\bar{\hat{\ell}}_L^{i(0)}\gamma_\mu Z^{\mu(n)}\hat{\ell}_L^{i(0)} + g_R^{(n)}(c_{e_i})\bar{\hat{e}}_R^{i(0)}\gamma_\mu Z^{\mu(n)}\hat{e}_R^{i(0)}. \tag{32}$$

These couplings, being dependent on the bulk mass parameter $c_{\ell,e}$, generate flavor violations in the interactions of the gauge boson KK modes on rotating the fermions to their mass basis. With this understanding, we can now address the consequences of such terms in little RS.

III. FLAVOR-VIOLATING DECAYS IN LITTLE RS

In this section, we focus on the charged lepton flavor-violating decay processes such as $\mu \to e\gamma$, $\mu^- \to e^+e^-e^-$, $\mu Ti \to eTi$, $\tau^- \to e^-e^-e^+$, $\tau^- \to \mu^-e^-e^+$, and $\tau^- \to e^-\mu^-\mu^+$, in the little RS framework. Among these, the only loop process is $\mu \to e\gamma$. Though this decay is divergent in the Randall-Sundrum model with brane localized Higgs, since little RS is an effective theory with much lower cutoff $\sim 10^3$ TeV, we expect a need to reanalyze this decay. The dominant contribution to the process $\mu \to e\gamma$ proceeds through a one-loop Feynman diagram with brane localized Higgs and KK-fermion fields as shown in Fig. 1. The amplitude of the process, assuming $M_{KK}^{(i)}$ to be much greater than the energy scales involved, thus becomes

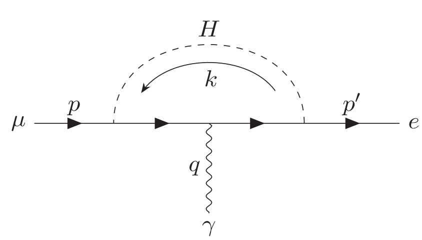


FIG. 1. The diagram that generates the process $\mu \to e\gamma$.

$$\begin{aligned} A_{\mu\to e\gamma} &= \bar{u}(p')\Bigg\{eA_\mu \sum_i \frac{1}{M_{KK}^{(i)4}} Y_{ei} \\ &\quad\times \int \frac{d^4k}{(2\pi)^4}\frac{(\not{p}' + \not{k} + M_{KK}^{(i)})\gamma^\mu(\not{p}' + \not{k} + M_{KK}^{(i)})}{k^2 - m_H^2} \\ &\quad\times Y_{i\mu}\Bigg\}u(p) \\ &= \frac{1}{2m_\mu}\bar{u_L}(p')\sigma^{\mu\nu}F_{\mu\nu}u_R(p)C_L(q^2) \\ &\quad + \frac{1}{2m_\mu}\bar{u_R}(p')\sigma^{\mu\nu}F_{\mu\nu}u_L(p)C_R(q^2). \end{aligned} \tag{33}$$

The divergence in this amplitude come through the large number of fermion KK modes that contribute to this loop $\frac{C_{L,R}}{m_\mu^2} \sim \frac{1}{16\pi^2}\log(N_{KK})$. Their contributions worsen at two loops. Thus, the cutoff-dependent part of the Wilson coefficient has the form [18]

$$\frac{C_{L,R}}{m_\mu^2} \sim \frac{1}{16\pi^2}\left(\frac{Y_{5D}}{M_{KK}}\right)^2\left\{\log\left(\frac{\Lambda_{5D}}{k}\right) + \frac{1}{16\pi^2}Y_{5D}\frac{\Lambda_{5D}^2}{k^2} + \cdots\right\}, \tag{34}$$

where the first and second terms are the one-loop and two-loop contributions, respectively.

The Randall-Sundrum model, being UV complete, requires $\Lambda_{5D} = M_{pl} = 10^{16}$ TeV to avoid hierarchy. Moreover, to avoid quantum gravity effects, the curvature should satisfy the condition $k/M_{pl} \ll 0.1$. Hence, assuming $k \lesssim 10^{15}$ TeV, the number of KK modes that contribute to the process becomes $N_{KK} \gtrsim 10$. At two loops, dimension analysis suggests that the amplitude becomes $\sim N_{KK}^2 = (\frac{\Lambda_{5D}}{k})^2 \gtrsim 100$ [18], which means that the one-loop and two-loop contributions in Eq. (34) are of the same order $[\mathrm{Log}(\frac{\Lambda_{5D}}{k}) \sim \frac{Y_{5D}}{16\pi^2}\frac{\Lambda_{5D}^2}{k^2} \sim 1]$. And the higher loops contribute strongly and the result is not convergent. This feature threatens any reliable calculations of the $\mu \to e\gamma$ process in the Randall-Sundrum model.

In comparison, the little Randall-Sundrum model is an effective theory with a scale 10^3 TeV, such that quantum gravity effects are insignificant. It is straightforward to find a parameter space, for example, $\Lambda_{5D} \sim 10^3$ TeV and $k \sim 800$ TeV, in which the number of KK fermions contributing to the loop can be much smaller

$[N_{KK} \sim \frac{1000}{800} \sim \mathcal{O}(1)]$, and the cutoff-dependent part of the Wilson coefficient becomes

$$\frac{C_{L,R}}{m_\mu^2} \sim \frac{1}{16\pi^2}\left(\frac{Y_{5D}}{M_{KK}}\right)^2 \{0.2 + 0.008 + \cdots\}, \quad (35)$$

where the two-loop term is much smaller than the one-loop with the higher-loop terms further suppressed. Though the amplitude is calculable now, still, the result is sensitive to the cutoff scale of the model, but this can be cured if we can dynamically stabilize this scale. Being phenomenological in nature, little RS requires new physics to do this. One way this could be achieved is by embedding the model in a six-dimensional $S^1/Z_2 \times S^1/Z_2$ scenario, for which all the radii are stabilized [23,24].

The cutoff-independent part of the Wilson coefficient, $C_{L,R}(q^2 = 0)$, can be derived from Eq. (33) as

$$C_{L,R}(q^2 = 0) = \frac{em_\mu}{32\pi^2}\sum_i Y_{ei}\frac{m_H^2}{M_{KK}^{(i)3}}Y_{i\mu}, \quad (36)$$

where $Y = \mathcal{M}_{\mathcal{D}}/(v/\sqrt{2})$ is the rotated Yukawa coupling and $M_{KK}^{(i)}$ are the masses of the first KK mode given in Eqs. (27) and (28), respectively. The branching ratio for this process now becomes [25,26]

$$B(\mu \to e\gamma) = \frac{12\pi^2}{(G_F m_\mu^2)^2}[|C_L(0)|^2 + |C_R(0)|^2], \quad (37)$$

where $C_L(0)$ and $C_R(0)$ can be derived by using Eqs. (27) and (28) in Eq. (36):

$$\begin{aligned} C_L(0) &= e\frac{m_\mu m_H^2}{32\pi^2}\left[Y_{e1}Y_{1\mu}\frac{1}{M_{KK}^{(1)3}} + Y_{e2}Y_{2\mu}\frac{1}{M_{KK}^{(2)3}}\right] \\ &= e\frac{m_\mu m_H^2}{32\pi^2 M_{KK}^4}[\Delta_R\Delta_1\Delta_L]_{e\mu}, \\ C_R(0) &= e\frac{m_\mu m_H^2}{32\pi^2}\left[Y_{e1}Y_{1\mu}\frac{1}{M_{KK}^{(1)3}} + Y_{e2}Y_{2\mu}\frac{1}{M_{KK}^{(2)3}}\right]^\dagger \\ &= e\frac{m_\mu m_H^2}{32\pi^2 M_{KK}^4}[\Delta_R\Delta_1\Delta_L]^\dagger_{e\mu}. \end{aligned} \quad (38)$$

Δ_L and Δ_R are the off-diagonal terms in the fermion mass matrix, and Δ_1 is the Yukawa mass of the KK-1 leptons given in Eq. (26).

This finite part can be computed, and, on comparing it with the experimental bound on the branching ratio $B_{\rm expt}(\mu \to e\gamma) \lesssim 0.042 \times 10^{-11}$ [27], we obtain the lower limit on the mass scale $M_{KK} \gtrsim 1.4$ TeV.

A. Trilepton decays and $\mu - e$ conversions

Before we discuss the trilepton decays in the little RS model, to study the flavor-violating effects, it is important and insightful if we identify the model-independent four-fermion interactions that contribute to the processes. Adopting the parametrization in Refs. [25,28], the most general low-energy effective, dimension-6, Lagrangian responsible for these processes can be written as

$$\begin{aligned} -\mathcal{L}_{\rm eff} = \frac{4G_F}{\sqrt{2}}[&g_3^{ij}(\bar{e}_{iR}\gamma^\mu e_{jR})(\bar{e}_{kR}\gamma_\mu e_{kR}) \\ &+ g_4^{ij}(\bar{\ell}_{iL}\gamma^\mu \ell_{jL})(\bar{\ell}_{kL}\gamma_\mu \ell_{kL}) \\ &+ g_5^{ij}(\bar{e}_{iR}\gamma^\mu e_{jR})(\bar{\ell}_{kL}\gamma_\mu \ell_{kL}) \\ &+ g_6^{ij}(\bar{\ell}_{iL}\gamma^\mu \ell_{jL})(\bar{e}_{kR}\gamma_\mu e_{kR})] + \text{H.c.}, \end{aligned} \quad (39)$$

where $g_{3,4,5,6}$ are dimensionless Wilson coefficients. Note that we have considered only vector operators and not scalar or pseudoscalar ones. This is because the Higgs contribution to the flavor-violating process is suppressed by small masses of the fermions involved. Moreover, the next-to-leading-order effects from KK fermions mixing are further suppressed.

Like fermions, the electroweak symmetry breaking with brane localized Higgs boson also mixes the KK levels of the Z boson. Details of the symmetry breaking and the mass matrices of electroweak gauge bosons are given in Appendix A. To diagonalize this mass matrix, we need to rotate the gauge basis $(Z^{(0)}, Z^{(1)})$ to the physical basis $(Z_{(0)}, Z_{(1)})$, wherein the admixture enters as

$$Z_{(0)} = Z^{(0)} + f\frac{m_Z^2}{M_{KK}^2}Z^{(1)}, \qquad Z_{(1)} = Z^{(1)} - f\frac{m_Z^2}{M_{KK}^2}Z^{(0)}, \quad (40)$$

where $f(\sim\sqrt{2kL})$ parametrizes the mixing between the zero and first KK level. From Eq. (31) it is clear that, since the extra-dimensional wave profile of $Z^{(0)}$ is flat, it couples democratically to all the lepton generations, whereas couplings of the fermion zero mode bilinear with the $Z^{(1)}$ are determined by the appropriate overlap integral and are dependent on the bulk mass parameter c_{ℓ_i,e_i}:

$$\alpha_i = 2\sqrt{2\pi}\int_0^L dy e^{-3ky} f_Z^{(1)}(y)[f_{L,R}^{(0)}(y, c_{\ell_i,e_i})]^2. \quad (41)$$

In the above equation, α_e, α_μ, and α_τ denote the ratios of couplings given in Eq. (32) to the SM ones. On rotating to the mass basis of leptons, the matrix which describes the $Z_\mu^{(1)}$ couplings takes the form

$$\mathcal{L}_{\rm int} = g_{L,R}\overline{\Psi}U_{L,R}^\dagger \alpha U_{L,R}\gamma^\mu \Psi Z_\mu^{(1)}, \quad (42)$$

where $g_{L,R}$ are the SM gauge couplings, $U_{L,R}$ are the unitary mixing matrices for charged SM leptons, $\alpha = \mathrm{diag}(\alpha_e, \alpha_\mu, \alpha_\tau)$, and

$$\Psi = \begin{pmatrix} e_{L,R} \\ \mu_{L,R} \\ \tau_{L,R} \end{pmatrix}. \tag{43}$$

It can easily be seen that the generational dependence of the $Z^{(1)}$ boson couplings, in Eq. (41), conspire to create off-diagonal elements that generate flavor violations. Moreover, this property of the KK-1 mode of the Z boson is inherited by the physical $Z_{(0)}$ boson due to the admixture. Thus, the dominant flavor violation in the electroweak sector is mediated by the zero mode of the physical Z boson, and integrating them out from low-energy processes results in the Wilson coefficients shown in Eq. (39). A detailed discussion is given in Appendix B.

Now, using the operators in Eq. (39) in Eq. (B4), the branching ratio for the process $\mu \to 3e$ can be written as

$$\mathrm{BR}(\mu \to 3e) = 2(|g_3^{\mu e}|^2 + |g_4^{\mu e}|^2) + |g_5^{\mu e}|^2 + |g_6^{\mu e}|^2, \tag{44}$$

where we have assumed $\mathrm{BR}(\mu \to e\nu\nu) = 1$.

The $\mu - e$ conversion rate [28] becomes

$$B_{conv} = \frac{2p_e E_e G_F^2 m_\mu^3 \alpha_{\mathrm{QED}}^3 Z_{\mathrm{eff}}^4 Q_N^2}{\pi^2 Z \Gamma_{capt}} [|g_R^{\mu e}|^2 + |g_L^{\mu e}|^2], \tag{45}$$

where the couplings $g_{3,4,5,6}$ and $g_{L,R}$ are given in Appendix B, α_{QED} is the QED coupling strength, and the remaining atomic physics constants are given in Ref. [28].

In order to constraint the little RS model, we impose the following current PDG limits: $\mathrm{BR}(\mu \to 3e) < 10^{-12}$ [4,29] and $B_{\mu Ti \to eTi} < 4.3 \times 10^{-12}$ [5,29]. And for the rare tau decays, we employ the constraints $\mathrm{BR}(\tau \to e^+e^-e^+) < 2.7 \times 10^{-8}$, $\mathrm{BR}(\tau \to \mu^+\mu^-\mu^+) < 2.1 \times 10^{-8}$, and $\mathrm{BR}(\tau \to \mu^-e^-e^+) < 1.8 \times 10^{-8}$ [29]. For computing the $\mu \to e$ conversion, we use the numerical values for titanium from Ref. [30].

Comparing these limits with Eqs. (44) and (45), we can derive the lower bound on the mass scale in the model. We present the constraints on M_{KK} for little RS and compare it with the bounds obtained in the Randall-Sundrum model in Table I. The details regarding our numerical analysis is given in Appendix C.

IV. BRANE LOCALIZED GAUGE KINETIC TERMS

Going beyond the simplest possible extension of SM in little RS, the action of the gauge field in five dimensions, given in Eq. (29), can be generalized [12,31–33] as

TABLE I. Constraints on the first KK-mode mass, M_{KK}, coming from various measurements for a brane Higgs field in both the RS and the little RS, except for the $\mathrm{BR}(\mu \to e\gamma)$ in the RS model where we have used bulk Higgs, since the brane Higgs is not computable in the RS.

Model	$\mathrm{BR}(\mu \to 3e)$	B_{conv}	$\mathrm{BR}(\mu \to e\gamma)$
RS	2.5 TeV	5.9 TeV	*8 TeV
Little RS	20.8 TeV	30.7 TeV	1.4 TeV

Model	$\mathrm{BR}(\tau \to 3e)$	$\mathrm{BR}(\tau \to 3\mu)$	$\mathrm{BR}(\tau \to \mu ee)$
RS	0.1 TeV	0.4 TeV	0.36 TeV
Little RS	2.48 TeV	2.43 TeV	2.50 TeV

$$\mathcal{S} = -\frac{1}{4g_5^2}\int d^5x\sqrt{-g}\{(g^{AM}g^{BN}F_{AB}F_{MN}) + (l_{\mathrm{UV}}\delta(y) + l_{\mathrm{IR}}\delta(y-L))g^{\alpha\mu}g^{\beta\nu}F_{\alpha\beta}F_{\mu\nu}\}, \tag{46}$$

where l_{UV} and l_{IR} are the localized kinetic term strengths at the UV and the IR branes, respectively. The origin of these terms is for the time being unknown, but it is understood that for correct renormalization of the model such terms are necessary [34,35]. These terms can be hypothesized to have their origin in the unperturbative effects of brane localized matter coupled to the gauge field.

Varying the action in Eq. (46) with respect to the field, the equation of motion becomes

$$-\partial_5(e^{-2ky}\partial_5 f_A^{(n)}) = (1 + l_{\mathrm{IR}}\delta(y-L) + l_{\mathrm{UV}}\delta(y))m_n^2 f_A^{(n)}, \tag{47}$$

where $f_A^{(n)}(y)$ are the wave functions that satisfy the orthonormality condition:

$$\int_0^L dy[1 + l_{\mathrm{IR}}\delta(y) + l_{\mathrm{UV}}\delta(y-L)]f_A^{(n)}f_A^{(m)} = \delta_{nm}. \tag{48}$$

The solution to Eq. (47) is given by

$$f_A^{(n)}(y) = N_A^{(n)}e^{ky}\left[J_1\left(\frac{m_n}{ke^{-ky}}\right) + b_A^{(n)}Y_1\left(\frac{m_n}{ke^{-ky}}\right)\right], \tag{49}$$

where $N_A^{(n)}$ is the normalization constant and $b_A^{(n)}$ the integration constant fixed by the modified boundary conditions

$$\partial_y f_A^{(n)}\Big|_{y=0} = -l_{\mathrm{UV}}m_n^2 f_A^{(n)}(0),$$
$$\partial_y f_A^{(n)}\Big|_{y=L} = +e^{2kL}l_{\mathrm{IR}}m_n^2 f_A^{(n)}(L). \tag{50}$$

Using the above relations, $b_A^{(n)}$ becomes

$$b_A^{(n)} = -\frac{J_0(\frac{m_n}{k}) + m_n l_{\rm UV} J_1(\frac{m_n}{k})}{Y_0(\frac{m_n}{k}) + m_n l_{\rm UV} Y_1(\frac{m_n}{k})} \quad \text{at } y=0,$$

$$b_A^{(n)} = -\frac{J_0(\frac{m_n}{k}e^{kL}) - m_n l_{\rm IR} e^{kL} J_1(\frac{m_n}{k}e^{kL})}{Y_0(\frac{m_n}{k}e^{kL}) - m_n l_{\rm IR} e^{kL} Y_1(\frac{m_n}{k}e^{kL})} \quad \text{at } y=0, \qquad (51)$$

where $m_n = x_n k e^{-kL}$ and x_n are the roots of the master equation obtained by equating the two relations in Eq. (51).

From Eqs. (47) and (48), the zero-mode wave function can be derived as

$$f_A^{(0)}(y) = \frac{1}{\sqrt{L + l_{\rm IR} + l_{\rm UV}}}. \qquad (52)$$

Since the BLKT modifies the bulk gauge field wave profile, their overlap with the lepton bilinear becomes

$$g_{L,R}^{(n)}(c_{\ell,e}) = \frac{g_5}{L}\int_0^L dy e^{-3ky} f_A^{(n)}(y) f_{L,R}^{(0)}(y, c_{\ell,e}) f_{L,R}^{(0)}(y, c_{\ell,e}), \qquad (53)$$

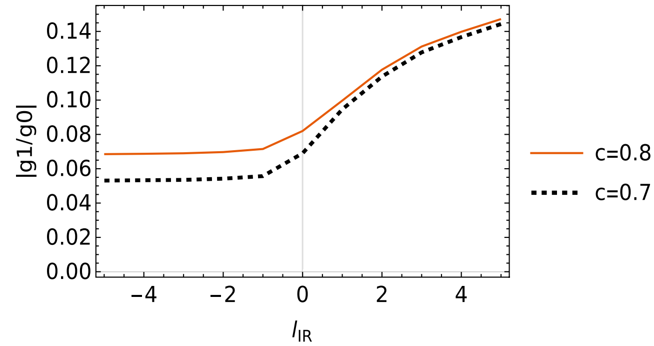
where $g_5 = g_0\sqrt{L + l_{\rm IR} + l_{\rm UV}}$, with g_0 denoting the coupling of the $Z^{(0)}$ boson with the fermion. This generalization of the gauge field does not affect the gauge zero-mode couplings. On the other hand, the higher KK-mode couplings are modified significantly. Since the CLFV is mediated by the $Z_\mu^{(1)}$, to capture the effect of BLKT, it is instructive to define the quantity

$$\left|\frac{\Delta g_1}{g_0}\right| = \left|\frac{g_{L,R}^{(1)}(c_2) - g_{L,R}^{(1)}(c_1)}{g_0}\right|, \qquad (54)$$

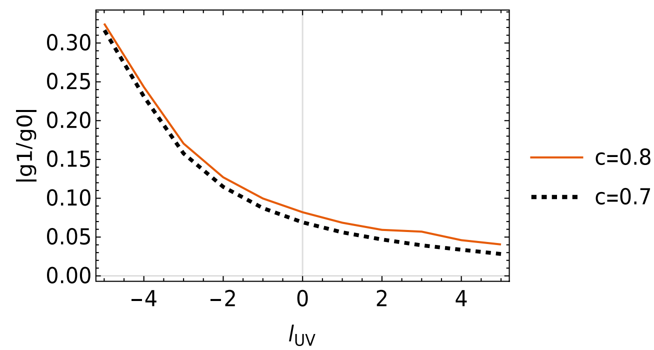
where $g_{L,R}^{(1)}(c)$ is given in Eq. (53).

For illustration, in Fig. 2 we display the coupling strengths of the KK-1 partner of the Z boson, $|\frac{g_1}{g_0}|$ [we denote $g_{L,R}^{(1)}(c) = g_1$], and lepton bilinears, with the bulk mass parameters $c_1 = 0.8$ and $c_2 = 0.7$, as a function of the BLKT strengths. The values of $|\frac{\Delta g_1}{g_0}|$ for different BLKTs are given in Table II.

The numerical analysis is similar to the scenario without BLKT but using the modified couplings in Eq. (53). The probability distribution function of simulated data, for the processes $\mu \to eee$ and $\mu Ti \to eTi$, is presented in Fig. 3. For comparison, we have also shown the scenario without BLKT. These plots clearly show that the branching ratio and conversion rate constraints relax on imposing BLKT, bringing down the lower limit on the KK-1 gauge boson mass scale to ∼12 TeV, thus making the model relevant at upcoming hadronic collider searches. The bounds on these processes in the presence of BLKT are summarized in Table III.



(a)



(b)

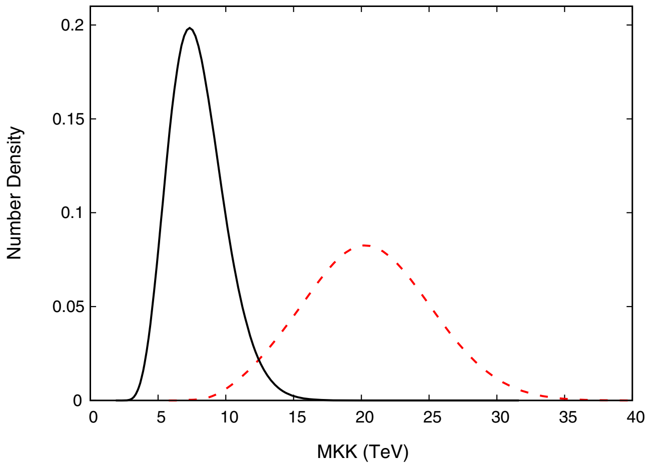
FIG. 2. (a) The coupling of KK-1Z boson, $|g_1/g_0|$, with leptons having bulk mass parameters $c = 0.8$ (red solid line) and $c = 0.7$ (black dotted line) as a function of $kl_{\rm IR}$ (assuming $kl_{\rm UV} = 0$). (b) The coupling of KK-1Z boson, $|g_1/g_0|$,with fermions having bulk mass parameters $c = 0.8$ (red solid line) and $c = 0.7$ (black dotted line) as a function of $kl_{\rm UV}$ (assuming $kl_{\rm IR} = 0$).

V. SUMMARY

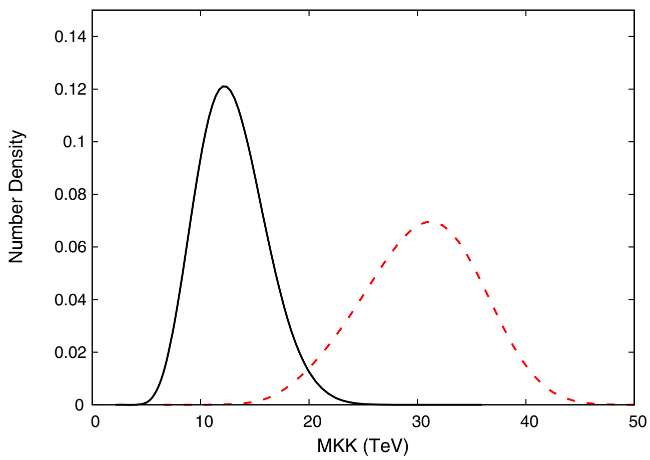
As an effective theory below 10^3 TeV, the little Randall-Sundrum model has been quite successful in relaxing the strong constraints from the electroweak precision observables without introducing custodial

TABLE II. $|\frac{\Delta g_1}{g_0}|$ values for the three cases of BLKTs computed with $c_1 = 0.8$ and $c_2 = 0.7$.

BLKT	$kl_{\rm IR} = kl_{\rm UV} = 0$	$kl_{\rm IR} = -5$, $kl_{\rm UV} = 0$	$kl_{\rm IR} = 0$, $kl_{\rm UV} = -5$
$\lvert\frac{\Delta g_1}{g_0}\rvert$	0.013	0.015	0.008
BLKT	$kl_{\rm IR} = 5$, $kl_{\rm UV} = 0$	$kl_{\rm IR} = 0$, $kl_{\rm UV} = 5$	$kl_{\rm IR} = 5$, $kl_{\rm UV} = -5$
$\lvert\frac{\Delta g_1}{g_0}\rvert$	0.003	0.0123	0.002



(a)



(b)

FIG. 3. Probability distribution function satisfying (a) the experimental branching ratio for the process $\mu \to eee$ with $kl_{\rm IR} = 5, kl_{\rm UV} = -5$ (black solid line) and $kl_{\rm UV} = 0, kl_{\rm IR} = 0$ (red dashed line) and (b) the experimental $\mu Ti \to eTi$ conversion rate $kl_{\rm IR} = 5, kl_{\rm UV} = -5$ (black solid line) and $kl_{\rm UV} = 0$, $kl_{\rm IR} = 0$ (red dashed line).

symmetry. As an added advantage, it also predicts an enhanced signal at LHC [12] compared to its UV complete counterpart. On the other hand, the flavor predictions turn for the worse. In a previous article by the author [12], the effects of little RS on kaon oscillation were discussed, where they observed that the contribution to the ϵ_K parameter was enhanced by a tree-level KK-1 gluon exchange diagram. The lower limit on the compactification scale was computed to be ~32 TeV. On the

TABLE III. Bounds on the M_{KK} for the different BLKTs considered.

	$kl_{\rm IR} = kl_{\rm UV} = 0$	$kl_{\rm IR} = 5,\ kl_{\rm UV} = -5$
$B_{(\mu\to 3e)}$	20.8 TeV	7.49 TeV
$B_{\mu Ti\to eTi}$	30.7 TeV	12.03 TeV

other hand, these limits were relaxed on including the BLKT for gluons and the scale was lowered to ~5 TeV. The relaxation of the constraints was also achieved by imposing minimal flavor protection, $U(3)$ flavor symmetry. This brings us to believe that the simplistic structure of gauge kinetic terms, considered so far in the literature, may not be the correct nature of the Universe. Instead, we need to introduce BLKT. These generalized gauge kinetic terms are also necessary to correctly renormalize the gauge sector [34–38].

In this paper, we extend the study of flavor violation to the charged lepton sector. The anarchic little RS model was subjected to a set of experimental constraints from the rare decays of $\mu \to eee$, $\tau \to eee$, $\tau \to \mu\mu\mu$, $\tau \to \mu ee$, $\tau \to e\mu\mu$, $\mu \to e\gamma$, and $\mu Ti \to eTi$. Unlike the case for hadrons, here, the flavor violations are mediated by the SM Z boson through mixing with the KK-1 partners in gauge basis. We note that the little RS suffers stronger bounds from the lepton flavor-violating sector, which constrains the lower limit on the KK-1 gauge boson mass to $M_{KK} \sim 30.7$ TeV. To mitigate such large constraints and make the model viable at colliders, in this article, we proposed brane localized kinetic terms for electroweak gauge bosons. Here, we considered nonminimal kinetic terms in both UV and IR branes and found that positive values of BLKT on IR brane and negative BLKT on UV brane relax the bounds, effectively reducing the lower limit to $M_{KK} \geq 12$ TeV. The electroweak sector in little RS exhibits a very rich flavor phenomenology. These interactions, discussed in our paper, can also contribute significantly to processes involving lepton universality violation.

Before concluding, we note that the flavor violations discussed here are generated due to the difference in the localizing wave profiles of fermion zero modes at the UV brane. If the wave profiles were degenerate, the KK modes of the Z boson would have coupled democratically to all the leptons. It is interesting to investigate whether these large corrections can be mitigated by relaxing our assumption on the unnatural thinness and rigidity of the brane. Though beyond the scope of this paper, if we consider fat branes, we can show that the lower limit on M_{KK} softens significantly. For demonstration, we choose two scenarios with the ratio of brane width to compactification radius 0.1 and 0.2, and we have plotted the probability distribution function satisfying the constraints on the branching ratio of $\mu \to eee$ and $\mu \to e$ conversion in Fig. 4. For ratio 0.2, it can be seen that the constraints from $\mu \to eee$ limits $M_{KK} \gtrsim 8$ TeV, while $\mu Ti \to eTi$ limits $M_{KK} \gtrsim 12$ TeV. Moreover, for flavor violations arising on the brane, the pseudo-Nambu-Goldstone bosons of the spontaneously broken translational symmetry (branons) are understood [19,39] to suppress the coupling of gauge boson KK modes with the fermion bilinear. Hence, a major part of the flavor violation

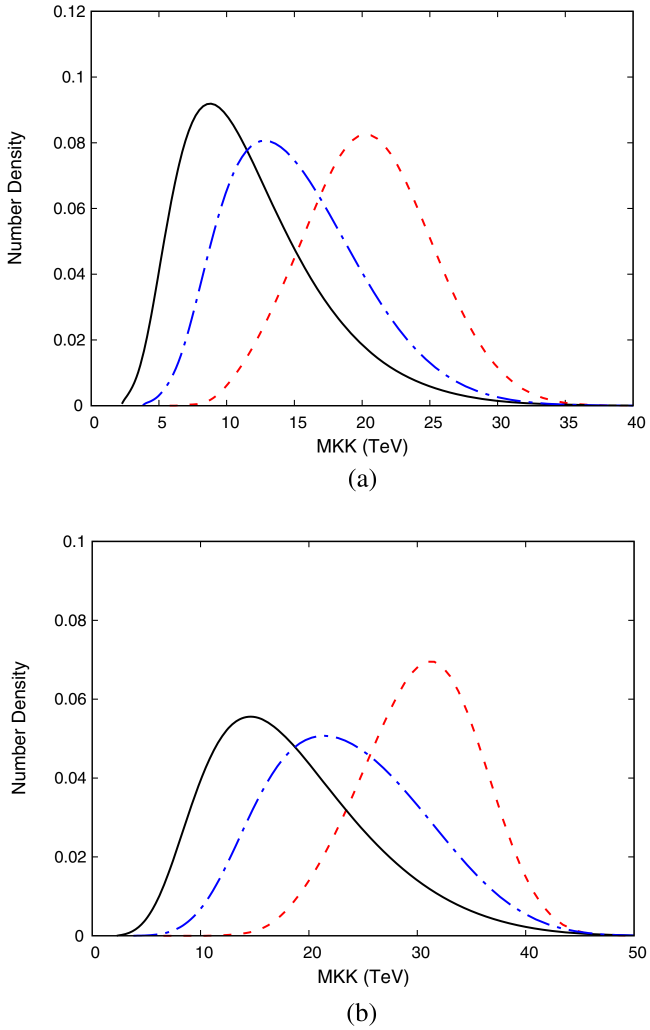


FIG. 4. Probability distribution function satisfying (a) constraints on the branching ratio of $\mu \to eee$ with a fat brane of size ratio 0.2 (black solid line) and 0.1 (blue dot-dashed line) and a thin brane (red dashed line) and (b) constraints on the $\mu - e$ conversion with a fat brane of size ratio 0.2 (black solid line) and 0.1 (blue dot-dashed line) and a thin brane (red dashed line).

appears outside the brane. This interesting feature disappears as the brane gets thinner and more rigid.

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APPENDIX A: MASS MATRIX OF GAUGE FIELDS

After spontaneous symmetry breaking of the Higgs, the mass terms from the Lagrangian given in Eqs. (3) and (4) become

$$\begin{aligned}\mathcal{L}_m &= \sum_n m_w^{(n)2} W_\mu^{+(n)} W^{-\mu(n)} + m_A^{(n)2} A_\mu^{(n)} A^{\mu(n)} \\ &\quad + m_z^{(n)2} Z_\mu^{(n)} Z^{\mu(n)} + \frac{v^2}{2}\int dy\,\delta(y-L) g_5^2 e^{-2ky} \\ &\quad \times \left(\sum_{m,n} W_\mu^{+(m)} W^{-\mu(n)} f_w^{(m)}(y) f_w^{(n)}(y)\right) + \frac{v^2}{2}(g_5^2 + {g_5}'^2) \\ &\quad \times \int e^{-2ky} \sum_{m,n} Z_\mu^{(m)} Z^{\mu(n)} f_z^{(m)}(y) f_z^{(n)}(y)\delta(y-L)dy,\end{aligned} \tag{A1}$$

where $m_w^{(n)}$, $m_A^{(n)}$, and $m_z^{(n)}$ are the nth KK masses of the W boson, photon, and Z boson, respectively.

Mass matrix of these gauge fields computed from the Lagrangian Eq. (A1): The mass term of A_μ is

$$\left[A_\mu^{(0)} A_\mu^{(1)}\right]\begin{bmatrix} 0 & 0 \\ 0 & m_A^{(1)2}\end{bmatrix}\left[A_\mu^{(0)} A_\mu^{(1)}\right]. \tag{A2}$$

The zero mode of the photon does not couple with the Higgs, and, hence, it is massless.

The mass term of $W_\mu^{\pm}$ is

$$\begin{aligned}M_w &= m_w^{(0)2} W_\mu^{+(0)} W^{-\mu(0)} + m_w^{(1)2} W_\mu^{+\mu(1)} W^{-\mu(1)} \\ &\quad + \frac{v^2}{2}\int g_5^2 e^{-2ky} \sum_{m,n} W_\mu^{+(m)} W^{-\mu(n)} f_w^{(m)}(y) \\ &\quad \times f_w^{(n)}(y)\delta(y-L)dy.\end{aligned} \tag{A3}$$

This can be represented by the matrix form

$$M_W = \left[W_\mu^{+(0)} W_\mu^{+(1)}\right]\begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} + m_w^{(1)2}\end{bmatrix}\begin{bmatrix} W_\mu^{-(0)} \\ W_\mu^{-(1)}\end{bmatrix}, \tag{A4}$$

where $a_{m,n} = g_5^2 \frac{v^2}{2} e^{-2kL} f_w^{(m)}(L) f_w^{(n)}(L)$.

Similarly, the mass term of the Z boson is

$$\begin{aligned}M_Z &= m_z^{(0)2} Z_\mu^{(0)} Z^{\mu(0)} + m_z^{(1)2} Z_\mu^{(1)} Z^{\mu(1)} + \frac{v^2}{2}(g_5^2 + {g_5}'^2) \\ &\quad \times \int e^{-2ky} \sum_{m,n} Z_\mu^{(m)} Z^{\mu(n)} f_z(m)(y) f_z^{(n)}(y)\delta(y-L)dy,\end{aligned} \tag{A5}$$

$$M_Z = \left[Z_\mu^{(0)} Z_\mu^{(1)}\right]\begin{bmatrix} b_{00} & b_{01} \\ b_{10} & b_{11} + m_z^{(1)2}\end{bmatrix}\begin{bmatrix} Z_\mu^{(0)} \\ Z_\mu^{(1)}\end{bmatrix}, \tag{A6}$$

where $b_{m,n} = g_5^2 \frac{v^2}{2} e^{-2kL} f_z^{(m)}(L) f_z^{(n)}(L)$ and $Z^{\mu(0)}$ and $Z^{\mu(1)}$ are in the gauge basis of the Z boson. This is a feature of IR-brane localized electroweak symmetry breaking.

APPENDIX B: COUPLINGS AND $l_i \to 3l_j$ BRANCHING RATIO

On going to mass basis, the flavor-basis coupling matrices $C^F_{L,R} = g_{L,R}\mathrm{diag}(\alpha_e, \alpha_\mu, \alpha_\tau)$ get rotated to $C_{L,R} = U_{L,R} C^F_{L,R} U^\dagger_{L,R}$. Since $\alpha_e \neq \alpha_\mu \neq \alpha_\tau$, in the rotated basis $C_{L,R}$ generate off-diagonal entries that lead to flavor violation. Using the unitarity of $U_{L,R}$, we get

$$g^{(1)\mu e}_{L,R} = g_{L,R}(U^{L,R}_{1,2} U^{L,R*}_{2,2}(\alpha_\mu - \alpha_e) + U^{L,R}_{1,3} U^{L,R*}_{2,3}(\alpha_\tau - \alpha_e)),$$
$$g^{(1)\tau\mu}_{L,R} = g_{L,R}(U^{L,R}_{2,1} U^{L,R*}_{3,1}(\alpha_e - \alpha_\mu) + U^{L,R}_{2,3} U^{L,R*}_{3,3}(\alpha_\tau - \alpha_e)),$$
$$g^{(1)\tau e}_{L,R} = g_{L,R}(U^{L,R}_{1,2} U^{L,R*}_{3,2}(\alpha_\mu - \alpha_e) + U^{L,R}_{1,3} U^{L,R*}_{3,3}(\alpha_\tau - \alpha_e)), \tag{B1}$$

where $g_{L,R}$ are the usual SM couplings. Using Eq. (40), the couplings to Z_0 are obtained via multiplication by $f\frac{m_Z^2}{M_{KK}^2}$; that is,

$$g^{l_i l_j}_{L,R} = \frac{-f m_z^2}{M_{KK}^2} g^{(1)l_i l_j}_{L,R}. \tag{B2}$$

With the above equations, it is now relatively simple to derive the effective Lagrangian given in Eq. (39). The Wilson coefficients $g_{3,4,5,6}$ given in the effective Lagrangian arise from the processes with the exchange of $Z_{(0)}$ and $Z_{(1)}$ bosons. These can be derived as

$$g_3^{l_i l_j} = 2g_R[f - \alpha_j]\frac{m_Z^2}{M_{KK}^2} g_R^{(1)l_i l_j},$$
$$g_4^{l_i l_j} = 2g_L[f - \alpha_j]\frac{m_Z^2}{M_{KK}^2} g_L^{(1)l_i l_j},$$
$$g_5^{l_i l_j} = 2g_L[f - \alpha_j]\frac{m_Z^2}{M_{KK}^2} g_R^{(1)l_i l_j},$$
$$g_6^{l_i l_j} = 2g_R[f - \alpha_j]\frac{m_Z^2}{M_{KK}^2} g_L^{(1)l_i l_j}, \tag{B3}$$

where i and j are (e, μ, τ). The first term in the above equation is computed from the Z_0 exchange, while the second is from Z_1 exchange.

The relevant branching fractions for the process $l_i \to 3l_j$ now become [18,25]

$$\mathrm{BR}(l_i \to 3l_j) = 2(|g_3^{l_i l_j}|^2 + |g_4^{l_i l_j}|^2) + |g_5^{l_i l_j}|^2 + |g_6^{l_i l_j}|^2. \tag{B4}$$

APPENDIX C: NUMERICAL ANALYSIS

Here, we present the numerical analysis of the little RS parameter space, to determine how accurately the little RS geometric origin of flavor can be tested in current and future experiments. We will discuss the full parameter scan of the bulk mass parameters, namely, "c" values, that fit the lepton mass within the experimental error. We have assumed anarchic Yukawa in the lepton sector. We choose the basis in which the left-handed mixing matrix $U_L = I$ and U_R, the right-handed mixing matrix, contains the mixing elements. This means that the flavor violation is generated in the right-handed sector, and, hence, the model is independent from fitting the U_{PMNS} matrix, which should happen once neutrino phenomenology is modeled in little RS.

The 3×3 complex matrices of five-dimensional Yukawa couplings Y_e contain nine real and nine complex elements. Since we have assumed the left-handed sector to be aligned with SM, the five-dimensional Yukawa should also be assumed to have some symmetry, which reduces the total free parameters to six real and three complex phases. For simplicity, we also choose the basis in which the bulk mass parameters are diagonal and real.

We restrict the five-dimensional Yukawa couplings to the range $0.1 \leq |Y_{i,j}| \leq 3$ so that the values do not introduce unnatural hierarchies and remain below the perturbative limit in the model. Ignoring the mixing of fermionic KK modes, for the time being, we can write

$$\zeta = U_L^\dagger Y_{4D} U_R, \tag{C1}$$

where Y_{4D} are the 4D Yukawa coupling defined in Eq. (25) and ζ is defined as

$$\zeta = \frac{\sqrt{2}}{v}\mathrm{diag}(m_e, m_\mu, m_\tau). \tag{C2}$$

Using this, we compute the five-dimensional Yukawa coupling as

$$(Y_{5D})_{i,j} = f^{(0)-1}(c_{\ell i})(U_L \zeta U_R^\dagger) f^{(0)-1}(c_{ej}), \tag{C3}$$

where U_L and U_R are the lepton mixing matrices. We have run the scan over 10^6 iterations and collected 5000 points in the little RS parametric space satisfying the anarchic Yukawa conditions. The range of c values satisfying the above condition is given in Table IV. Using these points, we calculated the bounds on M_{KK} for all decays mentioned in Sec. III.

TABLE IV. The bulk mass parameter c used in our scan.

$c_{\ell 1}$	$c_{\ell 2}$	$c_{\ell 3}$	c_{e1}	c_{e2}	c_{e3}
2.15–2.25	1.1–1.2	1–1.05	0.8–0.95	0.65–0.8	0.55–0.70

[1] G. Feinberg, Decays of the μ meson in the intermediate-meson theory, Phys. Rev. **110**, 1482 (1958).

[2] B. W. Lee and R. E. Shrock, Natural suppression of symmetry violation in gauge theories: Muon—lepton and electron lepton number nonconservation, Phys. Rev. D **16**, 1444 (1977).

[3] A. M. Baldini *et al.*, Search for the lepton flavour violating decay $\mu^+ \to e^+\gamma$ with the full dataset of the MEG experiment, Eur. Phys. J. C **76**, 434 (2016).

[4] U. Bellgardt *et al.*, Search for the decay $\mu^+ \to e^+e^+e^-$, Nucl. Phys. **B299**, 1 (1988).

[5] C. Dohmen *et al.*, Test of lepton flavor conservation in $\mu \to e$ conversion on titanium, Phys. Lett. B **317**, 631 (1993).

[6] K. Hayasaka *et al.*, Search for lepton flavor violating tau decays into three leptons with 719 million produced $\tau^+\tau^-$-pairs, Phys. Lett. B **687**, 139 (2010).

[7] B. Aubert *et al.*, Searches for Lepton Flavor Violation in the Decays $\tau^\pm \to e^\pm\gamma$ and $\tau^\pm \to \mu^\pm\gamma$, Phys. Rev. Lett. **104**, 021802 (2010).

[8] R. Appel *et al.*, Search for lepton flavor violation in k^+ decays into a charged pion and two leptons, Phys. Rev. Lett. **85**, 2877 (2000).

[9] G. Aad *et al.*, Search for high-mass dilepton resonances in pp collisions at $\sqrt{s} = 8$ TeV with the atlas detector, Phys. Rev. D **90**, 052005 (2014).

[10] H. Davoudiasl, G. Perez, and A. Soni, The little Randall-Sundrum model at the Large Hadron Collider, Phys. Lett. B **665**, 67 (2008).

[11] H. Davoudiasl, S. Gopalakrishna, and A. Soni, Big signals of little Randall-Sundrum models, Phys. Lett. B **686**, 239 (2010).

[12] G. D'Ambrosio, M. T. Arun, A. Kushwaha, and S. K. Vempati, Taming εK in little Randall-Sundrum models, Phys. Rev. D **104**, 055012 (2021).

[13] C. Csaki, A. Falkowski, and A. Weiler, The flavor of the composite Pseudo-Goldstone Higgs, J. High Energy Phys. 09 (2008) 008.

[14] J. Santiago, Minimal flavor protection: A new flavor paradigm in warped models, J. High Energy Phys. 12 (2008) 046.

[15] S. F. King, Tri-bimaximal-Cabibbo mixing, Phys. Lett. B **718**, 136 (2012).

[16] L. Randall and R. Sundrum, A Large Mass Hierarchy from a Small Extra Dimension, Phys. Rev. Lett. **83**, 3370 (1999).

[17] L. Randall and R. Sundrum, An Alternative to Compactification, Phys. Rev. Lett. **83**, 4690 (1999).

[18] K. Agashe, A. E. Blechman, and F. Petriello, Probing the Randall-Sundrum geometric origin of flavor with lepton flavor violation, Phys. Rev. D **74**, 053011 (2006).

[19] M. Bando, T. Kugo, T. Noguchi, and K. Yoshioka, Brane Fluctuation and Suppression of Kaluza-Klein Mode Couplings, Phys. Rev. Lett. **83**, 3601 (1999).

[20] J. Hisano and N. Okada, On effective theory of brane world with small tension, Phys. Rev. D **61**, 106003 (2000).

[21] J. A. R. Cembranos, A. Dobado, and A. L. Maroto, Branon phenomenology, in *Proceedings of the 2005 ILC and Detector Workshop and 2nd ILC Accelerator Workshop* (2005), 12, arXiv:hep-ph/0512302.

[22] B. M. Dillon and V. Sanz, Kaluza-Klein gravitons at LHC2, Phys. Rev. D **96**, 035008 (2017).

[23] M. T. Arun and D. Choudhury, Bulk gauge and matter fields in nested warping: I. The formalism, J. High Energy Phys. 09 (2015) 202.

[24] M. T. Arun and D. Choudhury, Stabilization of moduli in spacetime with nested warping and the UED, Nucl. Phys. **B923**, 258 (2017).

[25] W.-F. Chang and J. N. Ng, Lepton flavor violation in extra dimension models, Phys. Rev. D **71**, 053003 (2005).

[26] G. Moreau and J. I. Silva-Marcos, Flavor physics of the RS model with KK masses reachable at LHC, J. High Energy Phys. 03 (2006) 090.

[27] R. L. Workman *et al.*, Review of particle physics, Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).

[28] Y. Kuno and Y. Okada, Muon decay and physics beyond the standard model, Rev. Mod. Phys. **73**, 151 (2001).

[29] P. Zyla *et al.*, Review of particle physics, Prog. Theor. Exp. Phys. **2020**, 083C01 (2020).

[30] P. Wintz, in *Proceedings of the 29th International Conference on High-Energy Physics (ICHEP 98)* (Vancouver, Canada, 1998), pp. 23–29.

[31] H. Georgi, A. K. Grant, and G. Hailu, Brane couplings from bulk loops, Phys. Lett. B **506**, 207 (2001).

[32] M. Carena, T. M. P. Tait, and C. E. M. Wagner, Branes and orbifolds are opaque, Acta Phys. Pol. B **33**, 2355 (2002), https://www.actaphys.uj.edu.pl/R/33/9/2355.

[33] S. Fichet, Braneworld effective field theories—holography, consistency and conformal effects, J. High Energy Phys. 04 (2020) 016.

[34] H. Georgi, A. K. Grant, and G. Hailu, Chiral fermions, orbifolds, scalars and fat branes, Phys. Rev. D **63**, 064027 (2001).

[35] N. Arkani-Hamed, A. G. Cohen, and H. Georgi, Anomalies on orbifolds, Phys. Lett. B **516**, 395 (2001).

[36] M. Carena, E. Ponton, T. M. P. Tait, and C. E. M. Wagner, Opaque branes in warped backgrounds, Phys. Rev. D **67**, 096006 (2003).

[37] S. Groot Nibbelink and M. Hillenbach, Renormalization of supersymmetric gauge theories on orbifolds: Brane gauge couplings and higher derivative operators, Phys. Lett. B **616**, 125 (2005).

[38] A. S. Cornell, A. Deandrea, L.-X. Liu, and A. Tarhini, Renormalisation running of masses and mixings in UED models, Mod. Phys. Lett. A **28**, 1330007 (2013).

[39] M. T. Arun, Relaxing the W' constraint on compactified extra-dimension, Adv. High Energy Phys. **2022**, 8784084 (2022).