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# Mass spectra of hidden heavy-flavor tetraquarks with two and four heavy quarks

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**Abstract** Inspired by the observation of the $X(6900)$ by LHCb and the $X(6600)$ (with mass $6552 \pm 10 \pm 12$ MeV) recently by CMS and ATLAS experiments of the LHC in the di-$J/\Psi$ invariant mass spectrum, we systemically study masses of all ground-state configurations of the hidden heavy-flavor tetraquarks $q_1 Q_2 \bar{q}_3 \bar{Q}_4$ and $Q_1 Q_2 \bar{Q}_3 \bar{Q}_4$ ($Q = c, b$; $q = u, d, s$) containing two and four heavy quarks in the MIT bag model with chromomagnetic interaction and enhanced binding energy. Considering color-spin mixing due to chromomagnetic interaction, our mass computation indicates that the observed $X(6600)$ is likely to be the $0^{++}$ ground states of hidden-charm tetraquark $cc\bar{c}\bar{c}$ with computed masses 6572 MeV, which has a $0^{++}$ color partner around 6469 MeV. The fully bottom system of tetraquark $bb\bar{b}\bar{b}$ has masses of 19685 MeV and 19717 MeV for the $0^{++}$ ground states. Further calculation of the tetraquark systems $sc\bar{s}\bar{c}$, $sb\bar{s}\bar{b}$, $cb\bar{c}\bar{b}$, $nc\bar{n}\bar{c}$ and $nb\bar{n}\bar{b}$ shows that $Z_c(4200)$ is a $1^{+-}$ state of tetraquark $nc\bar{n}\bar{c}$ and $Z(4020)$ is a $1^{+-}$ state of tetraquark $nc\bar{n}\bar{c}$ with a mass of 4079 MeV. All of these tetraquarks are above their lowest thresholds of two mesons and unstable against the strong decays.

## 1 Introduction

All known strongly interacting particles (mesons and baryons) could be classified as bound states made of a quark–antiquark pair or three quarks for a long time based on the conventional scheme of the quark model by Gell-Mann [1] and Zweig [2]. Meanwhile, they also suggested possible existence of the hadron states of multiquarks like tetraquarks (with quark configuration $q^2\bar{q}^2$) and pentaquarks ($q^4\bar{q}$). In the 1970s, multiquark states (the exotic light mesons like the $a_0$ and $f_0$) are calculated by Jaffe based on the dynamical framework of the MIT bag model [3,4]. Despite that multiquarks are considered to be exotic in the sense that they go beyond the conventional scheme of quark model, they are, in principle, allowed by the quantum chromodynamics (QCD), the theory of the strong force that binds quarks into hadrons.

Since observation of the first exotic hadron $X(3872)$ [5] in 2003 by the Belle, many (more than 20) tetraquark candidates have been observed among charmonium-like or bottomonium-like $XYZ$ states, which include the charmonium-like states the $Z_c(3900)$ [6], the $Z_c(4200)$ [7], the $Z_c(4430)$ [8–11]. Some of the observed $XYZ$ states, like the charged state $Z_c(3900)$ [6], are undoubtedly exotic. In 2020, a candidate of fully charm tetraquark, the $X(6900)$, has been observed by LHCb in the di-$J/\Psi$ invariant mass spectrum around the mass of 6905 MeV, which is later confirmed by CMS and ATLAS of the LHC at CERN [12–14]. Meanwhile in the same di-$J/\Psi$ invariant mass spectrum, a new structure, the $X(6600)$, are also found by CMS with mass of $6552 \pm 10 \pm 12$ MeV, which is very likely to be the fully charm tetraquark.

The purpose of this work is to use the MIT bag model with enhanced binding energy to systemically study the ground-state masses of the hidden heavy-flavor tetraquarks containing two or four heavy quarks. Based on color-spin wavefunctions constructed for the hidden heavy-flavor tetraquarks, we solve the bag model and diagonalize the chromomagnetic interaction (CMI) to take into account the possible color-spin mixing of the states with same quantum numbers. We find that the computed masses of the fully charmed tetraquark $cc\bar{c}\bar{c}$ is in a good agreement with the mass measurement by the CMS experiment [13]. Further mass computation is performed for hidden heavy-flavor systems of the tetraquarks $bb\bar{b}\bar{b}$, $cb\bar{c}\bar{b}$, $sc\bar{s}\bar{c}$, $sb\bar{s}\bar{b}$, $nc\bar{n}\bar{c}$, $nb\bar{n}\bar{b}$, with a suggestion that

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the particle $Z_c(4200)$ reported by [7] is likely to be the hidden-charm tetraquark $nc\bar{n}\bar{c}$ with $J^{PC} = 1^{+-}$.

In Sect. 2, we present the allowed wavefunctions of hidden heavy-flavor tetraquarks with two or four heavy quarks. In Sect. 3, We describe the framework of MIT bag model to be used in this work. The mass matrix evaluation for the CMI and its diagonalization are detailed in Sect. 4. The masses of the hidden heavy-flavor tetraquarks are computed numerically for the systems ($cc\bar{c}\bar{c}$, $bb\bar{b}\bar{b}$), $cb\bar{c}\bar{b}$ , ($sc\bar{s}\bar{c}$, $sb\bar{s}\bar{b}$) and ($nc\bar{n}\bar{c}$, $nb\bar{n}\bar{b}$) in Sect. 5. We end with conclusions and remarks in Sect. 6.

## 2 Wavefunctions of hidden-flavor tetraquarks

We consider hidden heavy-flavor tetraquarks containing two or four heavy quarks ($q_1Q_2\bar{q}_3\bar{Q}_4$ and $Q_1Q_2\bar{Q}_3\bar{Q}_4$, $Q = c, b$, $q = u, d, s$), which include seven flavor combinations of four quark systems: $cc\bar{c}\bar{c}$, $bb\bar{b}\bar{b}$, $sc\bar{s}\bar{c}$, $sb\bar{s}\bar{b}$, $cb\bar{c}\bar{b}$, $nc\bar{n}\bar{c}$, $nb\bar{n}\bar{b}$, with $n = u, d$. In this section, we describe the wavefunctions of the hidden heavy-flavor tetraquarks in the flavor and the color-spin space.

In the flavor space, we utilized $\delta^S_{12}$ if $q_1q_2$ is symmetric and $\delta^A_{12} \equiv 1 - \delta^S_{12}$ if $q_1q_2$ is antisymmetric to restrict the wavefunction [15]. If the wavefunction has no flavor symmetry (beyond the isospin symmetry $SU(2)_I$) under the exchange of $q_1$ and $q_2$, then $\delta^S_{12} = \delta^A_{12} = 1$. The notation "1" stands for the integer of unit numerically.

In color space, the hidden heavy-flavor tetraquark $q_1q_2\bar{q}_3\bar{q}_4$ can be in two color states: $6_c \otimes \bar{6}_c$ and $\bar{3}_c \otimes 3_c$, with the respective wave functions (superscript stands for color representation),

$$\phi^T_1 = \left|(q_1q_2)^6(\bar{q}_3\bar{q}_4)^{\bar{6}}\right\rangle, \quad \phi^T_2 = \left|(q_1q_2)^{\bar{3}}(\bar{q}_3\bar{q}_4)^3\right\rangle. \tag{1}$$

With the help of the color $SU(3)_c$ symmetry, one can write the two configurations $\phi^T_{1,2}$ here in terms of the fundamental representations, i.e., of the color bases $c_n = |r\rangle$, $|b\rangle$ and $|g\rangle$ of the $SU(3)_c$ group (see Appendix A).

In the spin space, there are six states of a tetraquark state allowed (Appendix A), with the wavefunctions (subscript stands for spin),

$$\begin{aligned}
\chi^T_1 &= \left|(q_1q_2)_1(\bar{q}_3\bar{q}_4)_1\right\rangle_2, \quad \chi^T_2 = \left|(q_1q_2)_1(\bar{q}_3\bar{q}_4)_1\right\rangle_1, \\
\chi^T_3 &= \left|(q_1q_2)_1(\bar{q}_3\bar{q}_4)_1\right\rangle_0, \quad \chi^T_4 = \left|(q_1q_2)_1(\bar{q}_3\bar{q}_4)_0\right\rangle_1, \\
\chi^T_5 &= \left|(q_1q_2)_0(\bar{q}_3\bar{q}_4)_1\right\rangle_1, \quad \chi^T_6 = \left|(q_1q_2)_0(\bar{q}_3\bar{q}_4)_0\right\rangle_0.
\end{aligned} \tag{2}$$

Based on the Pauli's principle, one can construct twelve color-spin wavefunctions for the lowest S-wave (in coordinate space) tetraquarks:

$$\begin{aligned}
\phi^T_1\chi^T_1 &= \left|(q_1q_2)^6_1(\bar{q}_3\bar{q}_4)^{\bar{6}}_1\right\rangle_2 \delta^A_{12}\delta^A_{34}, \\
\phi^T_2\chi^T_1 &= \left|(q_1q_2)^{\bar{3}}_1(\bar{q}_3\bar{q}_4)^3_1\right\rangle_2 \delta^S_{12}\delta^S_{34}, \\
\phi^T_1\chi^T_2 &= \left|(q_1q_2)^6_1(\bar{q}_3\bar{q}_4)^{\bar{6}}_1\right\rangle_1 \delta^A_{12}\delta^A_{34}, \\
\phi^T_2\chi^T_2 &= \left|(q_1q_2)^{\bar{3}}_1(\bar{q}_3\bar{q}_4)^3_1\right\rangle_1 \delta^S_{12}\delta^S_{34}, \\
\phi^T_1\chi^T_3 &= \left|(q_1q_2)^6_1(\bar{q}_3\bar{q}_4)^{\bar{6}}_1\right\rangle_0 \delta^A_{12}\delta^A_{34}, \\
\phi^T_2\chi^T_3 &= \left|(q_1q_2)^{\bar{3}}_1(\bar{q}_3\bar{q}_4)^3_1\right\rangle_0 \delta^S_{12}\delta^S_{34}, \\
\phi^T_1\chi^T_4 &= \left|(q_1q_2)^6_1(\bar{q}_3\bar{q}_4)^{\bar{6}}_0\right\rangle_1 \delta^A_{12}\delta^S_{34}, \\
\phi^T_2\chi^T_4 &= \left|(q_1q_2)^{\bar{3}}_1(\bar{q}_3\bar{q}_4)^3_0\right\rangle_1 \delta^S_{12}\delta^A_{34}, \\
\phi^T_1\chi^T_5 &= \left|(q_1q_2)^6_0(\bar{q}_3\bar{q}_4)^{\bar{6}}_1\right\rangle_1 \delta^S_{12}\delta^A_{34}, \\
\phi^T_2\chi^T_5 &= \left|(q_1q_2)^{\bar{3}}_0(\bar{q}_3\bar{q}_4)^3_1\right\rangle_1 \delta^A_{12}\delta^S_{34}, \\
\phi^T_1\chi^T_6 &= \left|(q_1q_2)^6_0(\bar{q}_3\bar{q}_4)^{\bar{6}}_0\right\rangle_0 \delta^S_{12}\delta^S_{34}, \\
\phi^T_2\chi^T_6 &= \left|(q_1q_2)^{\bar{3}}_0(\bar{q}_3\bar{q}_4)^3_0\right\rangle_0 \delta^A_{12}\delta^A_{34}.
\end{aligned} \tag{3}$$

We choose these wavefunctions to be the bases (zero-order approximation) of the tetraquark eigenstates for which the chromomagnetic interaction (CMI) are ignored. We are going to employ these bases to take into account the chromomagnetic mixing when the CMI added. For example, for the $J^{PC} = 0^{++}$ state of the $cc\bar{c}\bar{c}$ tetraquark, one can employ two bases of the wavefunctions $\phi^T_2\chi^T_3$, $\phi^T_1\chi^T_6$ in Eq. (3), which satisfy the required symmetry in the color-spin space, to explore the mixing of the color-spin states due to the CMI. Note that the flavor symmetry of $q_1q_2 = cc$ and $\bar{q}_3\bar{q}_4 = \bar{c}\bar{c}$ implies that the wavefunction $\phi_2\chi_6$ is symmetric as a whole due to the antisymmetry of the colortriplet 3 or $\bar{3}$ as well as the spin singlet (0) under the fermion exchange $q_1 \leftrightarrow q_2$ or $\bar{q}_3 \leftrightarrow \bar{q}_4$. So the $\phi_2\chi_6$ is forbidden by Pauli's principle.

With respect to given flavor compositions of the tetraquarks, one can write the allowed color-spin states that may mix due to the CMI for each choice of the quantum number $J^{PC}$ in Table 1, where $Q'$ denoting heavy quark differing with $Q$. Note that for the flavor composition $QQ\bar{Q}\bar{Q}$ with quantum numbers $J^{PC} = 1^{+-}$ and $2^{++}$, there is only one color-spin state for each of them, that is, the $\phi^T_2\chi^T_2$ associated with $1^{+-}$ and $\phi^T_2\chi^T_1$ associated with $2^{++}$, for which not mixing occurs in reality.

## 3 The MIT bag model

We use the MIT bag model which includes enhanced binding energy and the CMI in the interaction correction $\Delta M$. The mass formula for the MIT bag model is [16]

$$M(R) = \sum_i \omega_i + \frac{4}{3}\pi R^3 B - \frac{Z_0}{R} + \Delta M, \tag{4}$$

**Table 1** Allowed state mixing of the hidden heavy-flavor tetraquarks due to chromomagnetic interaction

| State | $J^{PC}$ | Allowed states for mixing |
|---|---|---|
| $QQ\bar{Q}\bar{Q}$ | $0^{++}$ | $(\phi_2^T\chi_3^T, \phi_1^T\chi_6^T)$ |
| | $1^{+-}$ | $(\phi_2^T\chi_2^T)$ |
| | $2^{++}$ | $(\phi_2^T\chi_1^T)$ |
| $QQ'\bar{Q}\bar{Q}', qQ\bar{q}\bar{Q}$ | $0^{++}$ | $(\phi_2^T\chi_3^T, \phi_2^T\chi_6^T, \phi_1^T\chi_3^T, \phi_1^T\chi_6^T)$ |
| | $1^{++}$ | $(\frac{1}{\sqrt{2}}(\phi_2^T\chi_4^T+\phi_2^T\chi_5^T), \frac{1}{\sqrt{2}}(\phi_1^T\chi_4^T+\phi_1^T\chi_5^T))$ |
| | $1^{+-}$ | $(\phi_2^T\chi_2^T, \frac{1}{\sqrt{2}}(\phi_2^T\chi_4^T-\phi_2^T\chi_5^T), \phi_1^T\chi_2^T, \frac{1}{\sqrt{2}}(\phi_1^T\chi_4^T-\phi_1^T\chi_5^T))$ |
| | $2^{++}$ | $(\phi_2^T\chi_1^T, \phi_1^T\chi_1^T)$ |

$$\omega_i = \left(m_i^2 + \frac{x_i^2}{R^2}\right)^{1/2}, \tag{5}$$

with the first term describes (relativistic) kinetic motion of each quark $i$ in tetraquark, the second is the volume energy of bag with bag constant $B$, the third is the zero-point-energy with coefficient $Z_0$ and $R$ the bag radius to be determined variationally. In Eq. (5), the dimensionless parameters $x_i = x_i(mR)$ are related to $R$ through an transcendental equation

$$\tan x_i = \frac{x_i}{1 - m_i R - \left(m_i^2 R^2 + x_i^2\right)^{1/2}}. \tag{6}$$

In Eq. (4), we denote the sum of the first three terms to be $M(T)$. The interaction correction $\Delta M$ includes the enhanced binding energy $M_B$ among the quarks in tetraquark and the mass splitting $M_{CMI}$ corresponding to the CMI:

$$\Delta M = M_B + M_{CMI} = \sum_{i<j} B_{ij} + \langle H_{CMI}\rangle, \tag{7}$$

where $B_{ij}$ stands for the binding energy [17,18] between quarks $i$ and $j$, described below at the end of this section, and the chromomagnetic interaction $H_{CMI}$ is given by

$$H_{CMI} = -\sum_{i<j} \left(\boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j\right)\left(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right) C_{ij}. \tag{8}$$

where $\boldsymbol{\lambda}_i$ and $\boldsymbol{\sigma}_i$ are the Gell-Mann and Pauli matrices of the quark $i$, respectively, and $C_{ij}$ the CMI coupling parameters, given by [19]

$$C_{ij} = 3\frac{\alpha_s(R)}{R^3}\bar{\mu}_i\bar{\mu}_j I_{ij}, \tag{9}$$

with $\alpha_s(R)$ is the running coupling given in Ref. [16], $\bar{\mu}_i$ the reduced magnetic moment of quark $i$,

$$\alpha_s(R) = \frac{0.296}{ln\left[1 + (0.281R)^{-1}\right]}. \tag{10}$$

$$\bar{\mu}_i = \frac{R}{6}\frac{4\omega_i R + 2\lambda_i - 3}{2\omega_i R(\omega_i R - 1) + \lambda_i}, \tag{11}$$

and

$$I_{ij} = 1 + 2\int_0^R \frac{dr}{r^4}\bar{\mu}_i\bar{\mu}_j = 1 + F\left(x_i, x_j\right), \tag{12}$$

where $\lambda_i \equiv m_i R$. The function $F\left(x_i, x_j\right)$ is given by

$$\begin{aligned} F\left(x_i, x_j\right) = &\left(x_i \sin^2 x_i - \frac{3}{2}y_i\right)^{-1}\left(x_j \sin^2 x_j - \frac{3}{2}y_j\right)^{-1} \\ &\times\Big\{-\frac{3}{2}y_i y_j - 2x_i x_j sin^2 x_i sin^2 x_j + \frac{1}{2}x_i x_j\left[2x_i Si\left(2x_i\right)\right. \\ &+2x_j Si\left(2x_j\right) - \left(x_i + x_j\right) Si\left(2\left(x_i + x_j\right)\right) \\ &\left.-\left(x_i - x_j\right) Si\left(2\left(x_i - x_j\right)\right)\right]\Big\} \end{aligned} \tag{13}$$

where $y_i = x_i - \cos(x_i)\sin(x_i)$, $x_i$ is the solution of Eq. (6), and

$$Si(x) = \int_0^x \frac{\sin(t)}{t}dt. \tag{14}$$

Note that the functional of the running coupling $\alpha_s(R)$ in Eq. (9) and other parameters (the quark mass $m_i$, zero-point energy coefficient $Z_0$, bag constant $B$) are evaluated in Ref. [16] via mapping the model mass prediction to the ground-state masses of the observed mesons and baryons. The obtained values for these model parameters are [16]

$$\left\{\begin{array}{cc} m_n = 0\,\text{GeV}, & m_s = 0.279\,\text{GeV}, \\ m_c = 1.641\,\text{GeV}, & m_b = 5.093\,\text{GeV}, \\ Z_0 = 1.84, & B^{1/4} = 0.145\,\text{GeV}. \end{array}\right\} \tag{15}$$

We will use these parameters to analyze the heavy tetraquarks in this work, with the bag radius $R$ determined variationally via the MIT bag model, instead of fixing the parameters from the measured tetraquark data. One is due to that the available data of the tetraquarks are few till now and some of the states, like the $X_{0,1}(2900)$, the $Z(3900)$, the $Z_c(4020)$, the $Z(4430)$ and $T_{cc}(3875)$, seem to be complex and lack a simple explanation in terms of a pure compact state or pure molecule. The other is to keep the unified feature of the MIT bag model in dealing with both of the light hadrons and heavy hadrons [16].

The binding energy $M_B$ in Eq. (7) measures the short-range chromoelectric interaction between quarks and/or antiquarks. For the massive quarks of $i$ and $j$, this energy, which scales like $-\alpha_s(r_{ij})/r_{ij}$, becomes sizable when both quarks ($i$ and $j$) are massive, moving nonrelativistically. We treat this energy as the sum of the pair binding energies, $B_{QQ'}(B_{Qs})$, between heavy quarks ($Q$ and $Q'$) and between heavy quarks $Q$ and the strange quarks $s$ [17,18]. This leads to five binding energies $B_{cs}$, $B_{cc}$, $B_{bs}$, $B_{bb}$, and $B_{bc}$ for any quark pair in the color configuration $\bar{3}_c$, which are extractable from heavy mesons and can be scaled to other color configurations.

Assuming two quarks $QQ'$ to be in the color anti-triplet $\bar{3}_c$ inside baryon, the binding energy $B_{QQ'} \equiv B_{QQ'}[\bar{3}_c]$ are extracted in the MIT bag model [16] (Appendix A) for the combination of $QQ' = cc, bb, bc, bs$ and $cs$, so that a unified parameter setup was established for the ground states of meson, baryons and heavy hadrons (including doubly baryon and tetraquarks). The results are [16]

$$\left\{\begin{array}{l} B_{cs} = -0.025\,\text{GeV},\ B_{cc} = -0.077\,\text{GeV}, \\ B_{bs} = -0.032\,\text{GeV},\ B_{bb} = -0.128\,\text{GeV}, \\ B_{bc} = -0.101\,\text{GeV}. \end{array}\right\} \tag{16}$$

## 4 Color and spin factors for tetraquarks

To determine the mass splitting $M_{CMI} = \langle H_{CMI}\rangle$ via the CMI Hamiltonian $H_{CMI}$ in Eq. (8), one has to evaluate the chromomagnetic matrices $H_{CMI}$ of the tetraquarks $T$ for a given quantum number $J^{PC}$. For this, one can firstly work out the color factors $\langle \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j\rangle$ and spin factors $\langle \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\rangle$ as matrices over the color and spin bases, respectively, the allowed states of tetraquarks with given $J^{PC}$ in Table 1. In this section, we present the color and spin factors as a matrix elements in the color and spin space, and give an unified expressions for binding energy $M_B = \sum_{i<j} F_c B_{ij}(\bar{3}_c)$ for the both colors, where $F_c = F_c[R]$ is the color factor relative to the color factor $\langle \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j\rangle_B = -8/3$ for baryon in which all of quark pairs $(i, j)$ are in $\bar{3}_c$.

Color factor in the color states $|n\rangle$ and $|m\rangle$:

$$\langle \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j\rangle_{nm} = \sum_{\alpha=1}^{8} Tr\left(c_{in}^{\dagger}\lambda^{\alpha}c_{im}\right) Tr\left(c_{jn}^{\dagger}\lambda^{\alpha}c_{jm}\right), \tag{17}$$

and spin factor in the spin states $|x\rangle$ and $|y\rangle$:

$$\langle \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\rangle_{xy} = \sum_{\alpha=1}^{3} Tr\left(\chi_{ix}^{\dagger}\sigma^{\alpha}\chi_{iy}\right) Tr\left(\chi_{jx}^{\dagger}\sigma^{\alpha}\chi_{jy}\right), \tag{18}$$

where $c_{in}$ stands for the color basis of a given quark $i$, which consist of three colors $r$, $g$ and $b$: $c_{i1} = r$, $c_{i2} = g$, $c_{i3} = b$, and $\chi_{ix}$ represents its spin basis which consist of two spin states of $\uparrow$ and $\downarrow$.

In color-spin wavefunction of the tetraquark $T$, one can compute explicitly the expectation values of $H_{CMI}$,

$$\langle T|H_{CMI}|T\rangle = -\sum_{i<j}\langle \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j\rangle_{TT}\langle \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\rangle_{TT} C_{ij}, \tag{19}$$

to obtain the color and spin factor, writing the mass formula for $M_{CMI}$ in terms of the CMI couplings $C_{ij}$, which are given further by Eq. (9) in the MIT bag model. Here the state of $T$ are the mixed states listed in Table 1, with the mixed weight $w = (w_1, w_2, \ldots, w_f)$ solved (as eigenvector during the CMI diagonalization) numerically in Tables 2, 5, 6 and 7 in the Sect. 5.

Given the two formula (17) and (18), one can compute the color factors $\langle \phi_1^T, \phi_2^T|\boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j|\phi_1^T, \phi_2^T\rangle$ as 2 by 2 matrix in the color subspace of $(\phi_1^T, \phi_2^T)$, via applying Eqs. (30) and (31) in Appendix A. The result are obtained to be

$$\begin{aligned} \langle \boldsymbol{\lambda}_1 \cdot \boldsymbol{\lambda}_2\rangle &= \langle \boldsymbol{\lambda}_3 \cdot \boldsymbol{\lambda}_4\rangle = \begin{bmatrix} \frac{4}{3} & 0 \\ 0 & -\frac{8}{3} \end{bmatrix}, \\ \langle \boldsymbol{\lambda}_1 \cdot \boldsymbol{\lambda}_3\rangle &= \langle \boldsymbol{\lambda}_2 \cdot \boldsymbol{\lambda}_4\rangle = \begin{bmatrix} -\frac{10}{3} & 2\sqrt{2} \\ 2\sqrt{2} & -\frac{4}{3} \end{bmatrix}, \\ \langle \boldsymbol{\lambda}_1 \cdot \boldsymbol{\lambda}_4\rangle &= \langle \boldsymbol{\lambda}_2 \cdot \boldsymbol{\lambda}_3\rangle = \begin{bmatrix} -\frac{10}{3} & -2\sqrt{2} \\ -2\sqrt{2} & -\frac{4}{3} \end{bmatrix}. \end{aligned} \tag{20}$$

From the above matrices, we see that the color configurations $\phi_1^T$ and $\phi_2^T$ may mix for a tetraquark state $T$ due to the chromomagnetic interaction.

We further consider the binding energy $M_B$ based on Eq. (16), which corresponds to the binding energy $B_{ij} \equiv B_{ij}[\bar{3}_c]$ in baryons with the quark pair $(i, j)$ in $\bar{3}_c$. Let us then consider the binding energy $M_B$ for a given color configurations of the tetraquark $T = (q_1q_2)^R(\bar{q}_3\bar{q}_4)^{\bar{R}}$(with representation $R = 6_c$ and $\bar{3}_c$). First of all, one can scale the pair binding energy $B_{ij} \equiv B_{ij}[\bar{3}_c]$ of the pair in baryon to $F_c[R]B_{ij}[\bar{3}_c]$ of the pair in tetraquark $T$. At last, applying to all quark pair $(i, j)$ of the tetraquark $T$ with configurations $\phi_1^T$ and $\phi_2^T$, one can obtain the pair binding energies $F_c[R]B_{ij}$, whose sums are,

$$\begin{aligned} M_B(\phi_1^T) &= -\frac{1}{2}B_{12} - \frac{1}{2}B_{34} + \frac{5}{4}B_{13} + \frac{5}{4}B_{14} \\ &\quad + \frac{5}{4}B_{23} + \frac{5}{4}B_{24}, \end{aligned} \tag{21}$$

$$\begin{aligned} M_B(\phi_2^T) &= B_{12} + B_{34} + \frac{1}{2}B_{13} + \frac{1}{2}B_{14} \\ &\quad + \frac{1}{2}B_{23} + \frac{1}{2}B_{24}, \end{aligned} \tag{22}$$

for the tetraquark $T$, respectively, where $B_{ij}$ is the binding energy with $(i, j)$ in $\bar{3}_c$.

For color sextets of the pair $(1, 2)$ and $(3, 4)$, for instance, the binding energy is $-B_{12}/2$ and $-B_{34}/2$, respectively, with $F_c = (4/3)/(-8/3) = -1/2$. For any of representation of the quark $i$ and antiquark $j$, the binding energies in $T$ are either $-5B_{ij}/4$ or $B_{ij}/2$. We note that $B_{ij}$ vanishes if both of quark $i$ and $j$ are light quarks or one of them is non-strange light quark ($B_{nQ}=0$, $B_{nn}=0$, $B_{n\bar{n}}=0$, $B_{s\bar{s}}=0$) since the short range interactions between $(i,j)$ quarks are small and thereby ignorable averagely for quark pair $(i,j)=(n,n)$ or $(i,j)=(s,s)$, due to their relativistic motion.

We come to consider the spin factors, which is given by $\langle\chi^T|\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j|\chi^T\rangle$. In the subspace spanned by $\{\chi^T_{1\sim 6}\}$ in Eq. (32), the direct computation yields the following matrices,

$$\langle\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\rangle_{\chi_1^T}=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&-3&0\\0&0&0&0&0&-3\end{bmatrix}, \tag{23}$$

$$\langle\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3\rangle_{\chi_2^T}=\begin{bmatrix}1&0&0&0&0&0\\0&-1&0&\sqrt{2}&-\sqrt{2}&0\\0&0&-2&0&0&-\sqrt{3}\\0&\sqrt{2}&0&0&1&0\\0&-\sqrt{2}&0&1&0&0\\0&0&-\sqrt{3}&0&0&0\end{bmatrix}, \tag{24}$$

$$\langle\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_4\rangle_{\chi_3^T}=\begin{bmatrix}1&0&0&0&0&0\\0&-1&0&-\sqrt{2}&-\sqrt{2}&0\\0&0&-2&0&0&\sqrt{3}\\0&-\sqrt{2}&0&0&-1&0\\0&-\sqrt{2}&0&-1&0&0\\0&0&\sqrt{3}&0&0&0\end{bmatrix}, \tag{25}$$

$$\langle\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3\rangle_{\chi_4^T}=\begin{bmatrix}1&0&0&0&0&0\\0&-1&0&\sqrt{2}&\sqrt{2}&0\\0&0&-2&0&0&\sqrt{3}\\0&\sqrt{2}&0&0&-1&0\\0&\sqrt{2}&0&-1&0&0\\0&0&\sqrt{3}&0&0&0\end{bmatrix}, \tag{26}$$

$$\langle\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_4\rangle_{\chi_5^T}=\begin{bmatrix}1&0&0&0&0&0\\0&-1&0&-\sqrt{2}&\sqrt{2}&0\\0&0&-2&0&0&-\sqrt{3}\\0&-\sqrt{2}&0&0&1&0\\0&\sqrt{2}&0&1&0&0\\0&0&-\sqrt{3}&0&0&0\end{bmatrix}, \tag{27}$$

$$\langle\boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_4\rangle_{\chi_6^T}=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&-3&0&0\\0&0&0&0&1&0\\0&0&0&0&0&-3\end{bmatrix}. \tag{28}$$

Combing the spin factors in Eqs. (23)–(28) with Eq. (20), we are the position to use Eqs. (19), (17) and (18) to compute the mass splitting $M_{CMI}$ duo to chromomagnetic interaction. Using Eqs. (21) and (22), one can compute the mass sum $\Delta M=M_B+M_{CMI}$ in Eq. (7) and further obtain, via adding mass of the bag $M_{bag}=\sum_i\omega_i+(4/3)\pi R^3B-Z_0/R$, a complete mass formula for the hidden heavy-flavor tetraquark systems $T$ addressed in this work,

$$M(T)=M_{bag}+M_B+M_{CMI}\ (C_{ij}), \tag{29}$$

in which $M_{CMI}\ (C_{ij})$ are linear functions of the CMI couplings $C_{ij}$, with the linear coefficients given by the color and spin factors shown in this section.

## 5 Masses of hidden heavy-flavor tetraquarks

Given the input parameters in Eq. (15), one can numerically solve Eq. (4) variationally, with the mass splitting $M_{CMI}$ and the CMI couplings $C_{ij}$ given by Eqs. (9), (10), (11) and (12), to obtain bag radius $R$ and numerically give the masses $M(T)$ of the hidden heavy-flavor tetraquarks $T$. Meanwhile, we show the corresponding numerical results for the bag radius $R_0$, the mixing weights (eigenvectors of the CMI matrix $H_{CMI}$), the tetraquark masses $M(T)$ and thresholds of two mesons as a final states in the Tables 2, 5, 6 and 7. In the following, we present the results and discussions with respect to the tetraquark systems addressed below in order.

### 5.1 Fully heavy tetraquark systems

In the case of fully charmed systems of the tetraquarks $cc\bar{c}\bar{c}$, we show the numerical results for $R_0$, the state-mixing weights (eigenvectors of $H_{CMI}$), the tetraquark masses $M(T)$ and thresholds of two mesons final states in the Table 2, with the later two plotted in Fig. 1. For $J^{PC}=0^{++}$ states in Table 2, there are two $cc\bar{c}\bar{c}$ states with masses of 6572 MeV and 6469 MeV, splitted by 103 MeV. We suggest that the former, the $0^{++}$ $cc\bar{c}\bar{c}$ state with mass 6572, is the candidate of the newly observed fully-charmed resonance $X(6600)$ with mass $6552\pm10\pm12$ MeV seen in the CMS measurement in the di-$J/\Psi$ spectrum [12,13]. The tetraquark $(cc\bar{c}\bar{c})$ states with $J^{PC}=1^{+-}$ and $J^{PC}=2^{++}$ have the masses within a similar mass region, as shown in Fig. 1. We find that all these $cc\bar{c}\bar{c}$ states relatively far above their two mesons thresholds shown explicitly. For instance, the $0^{++}$ state are all above the thresholds of the $J/\psi J/\psi$ and $\eta_c\eta_c$, about 275−605 MeV, indicating that they are not stable against strong decays through quark rearrangement to the final state of $J/\psi J/\psi$ as well as $\eta_c\eta_c$. For the $1^{+-}$ state, there is one state, and its mass is above the thresholds of the two mesons $\eta_c J/\psi$ and $J/\psi J/\psi$ about 325−440 MeV, unstable against the strong decay to the later. For the $2^{++}$ state, there is one state with the mass above the threshold $(J/\psi J/\psi)$ about 350 MeV, also strongly unstable.

We also compare our calculations with other works cited and list the results in Table 3, Our mass predictions for the $cc\bar{c}\bar{c}$ states, around 6.4−6.5 GeV, are in consistent with other calculations cited in Table 3, with the masses about 6.1−6.5 GeV, lower than that of the $X(6900)$. This implies

**Table 2** The numerical results for the bag radius $R_0$, the state-mixing weights (eigenvectors of $H_{CMI}$), the tetraquark masses $M(T)$ and thresholds of two mesons final states for the hidden heavy-flavor tetraquarks ($cc\bar{c}\bar{c}$, $bb\bar{b}\bar{b}$)

| State | $J^{PC}$ | Eigenvector | $R_0$ (GeV$^{-1}$) | $M(T)$ (MeV) | Threshold (MeV) |
|---|---|---|---|---|---|
| $cc\bar{c}\bar{c}$ | $0^{++}$ | (0.58, 0.81) | 4.74 | 6572 | $J/\psi J/\psi = 6194$; $\eta_c\eta_c = 5967$ |
| | | (−0.81, 0.58) | 4.44 | 6469 | |
| | $1^{+-}$ | 1.00 | 4.59 | 6519 | $\eta_c J/\psi = 6080$; $J/\psi J/\psi = 6194$ |
| | $2^{++}$ | 1.00 | 4.66 | 6545 | $J/\psi J/\psi = 6194$ |
| $bb\bar{b}\bar{b}$ | $0^{++}$ | (0.58, 0.81) | 3.15 | 19717 | $\Upsilon\Upsilon = 18921$; $\eta_b\eta_b = 18798$ |
| | | (−0.81, 0.58) | 2.99 | 19685 | |
| | $1^{+-}$ | 1.00 | 3.07 | 19700 | $\eta_b\Upsilon = 18859$; $\Upsilon\Upsilon = 18921$ |
| | $2^{++}$ | 1.00 | 3.11 | 19708 | $\Upsilon\Upsilon = 18921$ |

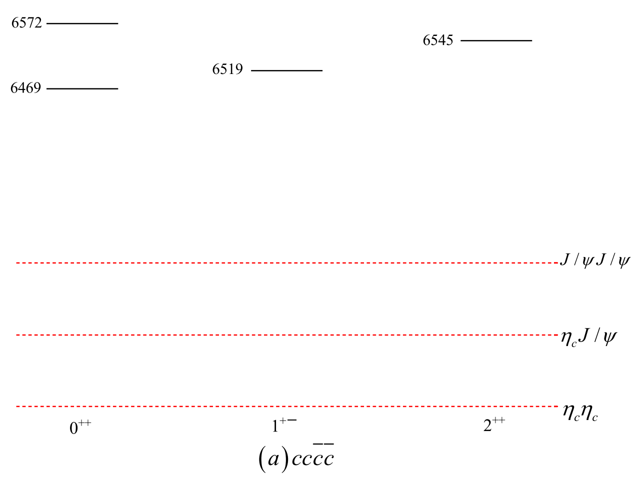


**Fig. 1** The computed masses (MeV the solid lines) of the $cc\bar{c}\bar{c}$ tetraquark system in their ground-states, as well as two meson thresholds (MeV the dotted lines)

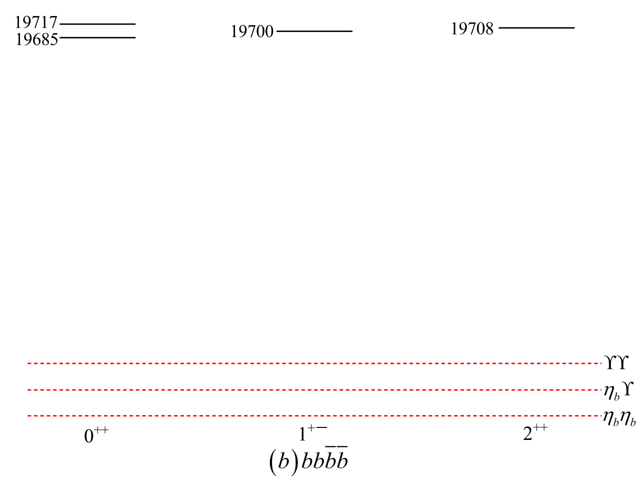


**Fig. 2** The computed masses (MeV the solid lines) of the $bb\bar{b}\bar{b}$ tetraquark system in their ground-states, as well as two meson thresholds (MeV the dotted lines)

that the $X(6900)$ is likely to be an excitation of the $cc\bar{c}\bar{c}$ state [20].

For fully bottom systems of the tetraquarks $bb\bar{b}\bar{b}$, the solved results of the model are shown in Table 2. We find that all these $bb\bar{b}\bar{b}$ states (with $J^{PC} = 0^{++}$, $1^{+-}$ and $2^{++}$) are close to each other and strongly unstable as they are far above their two mesons final states shown. For instance, two of the $0^{++}$ states have the masses of 19717 MeV and 19685 MeV (with mass splitting 32 MeV). As seen in Fig. 2, the two of the $0^{++}$ states are above thresholds ($\Upsilon\Upsilon$, $\eta_b\eta_b$) about 764−919 MeV. For the $1^{+-}$ state of $bb\bar{b}\bar{b}$, its mass is higher than the threshold ($\eta_b\Upsilon$ and $\Upsilon\Upsilon$) about 780−840 MeV. For the $2^{++}$ state, it is above the threshold ($\Upsilon\Upsilon$), about 787 MeV. By the way, our results for the $bb\bar{b}\bar{b}$ systems are also compared to other works cited, as shown in Table 4.

### 5.2 The bottom-charmed system($cb\bar{c}\bar{b}$)

For bottom-charmed systems of the tetraquarks $cb\bar{c}\bar{b}$, we show in Table 5 the computed results for $R_0$, the mixing weights (the CMI eigenvectors), the tetraquark masses $M(T)$ and thresholds (two mesons), with the later two plot-

**Table 3** Comparison of our results for the $cc\bar{c}\bar{c}$ systems with other calculations cited. All masses are in unit of MeV

| State | $J^{PC}$ | This work | [21] | [22] | [23] | [24] | [25] | [26] | [27] | [28] |
|---|---|---|---|---|---|---|---|---|---|---|
| ($cc\bar{c}\bar{c}$) | $0^{++}$ | 6469 | 6487 | 6477 | 6797 | 6440−6820 | 6437 | 6200 | 6192 | 6038−6115 |
| | $0^{++}$ | 6572 | 6518 | 6695 | 7016 | 6440−6820 | 6383 | ... | ... | ... |
| | $1^{+-}$ | 6519 | 6500 | 6528 | 6899 | 6370−6510 | 6437 | ... | ... | 6101−6176 |
| | $2^{++}$ | 6545 | 6524 | 6573 | 6956 | 6370−6510 | 6437 | ... | ... | 6172−6216 |

**Table 4** Comparison of our results for the $bb\bar{b}\bar{b}$ systems with other calculations cited. All masses are in unit of MeV

| State | $J^{PC}$ | This work | [21] | [23] | [29,30] | [27] | [31] |
|---|---|---|---|---|---|---|---|
| $(bb\bar{b}\bar{b})$ | $0^{++}$ | 19685 | 19322 | 20155 | 18840 | 18826 | 18754 |
| | $0^{++}$ | 19717 | 19338 | 20275 | ... | ... | ... |
| | $1^{+-}$ | 19700 | 19329 | 20212 | 18840 | ... | 18808 |
| | $2^{++}$ | 19708 | 19341 | 20243 | 18850 | ... | 18916 |

**Table 5** Computed results for the bottom-charmed tetraquark states $cb\bar{c}\bar{b}$. The thresholds of two mesons are also listed

| State | $J^{PC}$ | Eigenvector | $R_0$ (GeV$^{-1}$) | $M(T)$ (MeV) | Threshold (MeV) |
|---|---|---|---|---|---|
| $cb\bar{c}\bar{b}$ | $0^{++}$ | $(-0.21, -0.52, 0.82, 0.16)$ | 3.76 | 13076 | $B_c^*B_c^* = 12652$; $\Upsilon J/\psi = 12557$ |
| | | $(-0.77, 0.24, -0.16, 0.58)$ | 3.90 | 13117 | $B_cB_c = 12550$; $\eta_b\eta_c = 12382$ |
| | | $(-0.14, -0.82, -0.55, 0.01)$ | 4.02 | 13147 | |
| | | $(0.59, -0.06, -0.05, 0.80)$ | 4.12 | 13176 | |
| | $1^{+-}$ | $(-0.43, 0.39, 0.14, 0.80)$ | 3.95 | 13941 | $B_c^*B_c^* = 12652$; $B_c^*B_c = 12597$ |
| | | $(0.63, 0.74, -0.25, 0.02)$ | 4.0 | 13959 | $\eta_bJ/\psi = 12496$; $\Upsilon\eta_c = 12444$ |
| | | $(0.57, -0.26, 0.71, 0.32)$ | 4.04 | 13966 | |
| | | $(0.30, -0.48, -0.65, 0.51)$ | 4.18 | 14011 | |
| | $1^{++}$ | $(-0.58, 0.82)$ | 3.94 | 13944 | $B_c^*B_c = 12597$; $\Upsilon J/\psi = 12557$ |
| | | $(0.82, 0.58)$ | 4.07 | 13974 | |
| | $2^{++}$ | $(0.74, 0.67)$ | 4.06 | 13158 | $B_c^*B_c^* = 12652$; $\Upsilon J/\psi = 12557$ |
| | | $(-0.68, 0.74)$ | 4.08 | 13165 | |

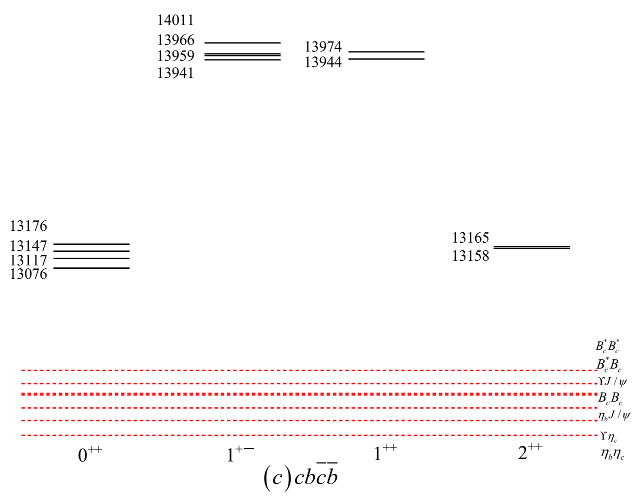


**Fig. 3** Computed masses (MeV the solid lines) of the $cb\bar{c}\bar{b}$ systems of tetraquarks in their ground states, and the thresholds (MeV the dotted lines) of the two meson final states

ted in Fig. 3. We find that there are four $J^{PC} = 0^{++}$ states for the $cb\bar{c}\bar{b}$ systems, all above the thresholds ($B_c^*B_c^*$, $\Upsilon J/\psi$, $B_cB_c$ and $\eta_b\eta_c$) about 424−794 MeV. There are four states of the $cb\bar{c}\bar{b}$ systems with $J^{PC} = 1^{+-}$, all highly above the thresholds ($B_c^*B_c^*$, $B_c^*B_c$, $\eta_bJ/\psi$ and $\Upsilon\eta_c$) about 1289−1567 MeV, and two states of the $cb\bar{c}\bar{b}$ systems with $J^{PC} = 1^{++}$, all highly above the thresholds ($B_c^*B_c$ and $\Upsilon J/\psi$) about 1347−1417 MeV. There are also two states with $J^{PC} = 2^{++}$, both above the thresholds ($B_c^*B_c^*$ and $\Upsilon J/\psi$) about 506−608 MeV. This indicates that the $cb\bar{c}\bar{b}$ systems are unstable against strong decay to the final states of the mesons.

### 5.3 The strange-heavy systems ($sc\bar{s}\bar{c}$ and $sb\bar{s}\bar{b}$)

For strange-charmed systems of the tetraquarks $sc\bar{s}\bar{c}$, we show in Table 6 the computed results for $R_0$, the mixing weights, the masses $M(T)$ and thresholds (two mesons), with the later two plotted in Fig. 4. We find that there are four $J^{PC} = 0^{++}$ states of the $sc\bar{s}\bar{c}$ systems, all below the threshold of $D_{s1}^*D_{s1}^*$, in which three states with masses (4492, 4378, 4254) MeV are above the thresholds of $D_sD_s$ and $\phi$ (1020) $J/\psi$ about 137−556 MeV and unstable against strong decay to them. The lowest state with mass of 4091 MeV is above the threshold of $D_sD_s$ about 155 MeV while it is near to the threshold of $\phi(1020)J/\psi$, far below the threshold of $D_{s1}^*D_{s1}^*$. It is uncertain whether the lowest state is above or below the threshold of $\phi$ (1020) $J/\psi$ as the model uncertainty is as large as ±40 MeV [16]. In the case of the $J^{PC} = 1^{+-}$ states, there are four states, with three of them having the mass of (4529, 4596, 4638) MeV and all below the thresholds of $D_sD_{s1}^*$ and $D_{s1}^*D_{s1}^*$ about 110–1031 MeV and one state, with mass of 4843 MeV, above

**Table 6** Computed results for the strange-heavy tetraquark states $sc\bar{s}\bar{c}$ and $sb\bar{s}\bar{b}$. The thresholds of two mesons are also listed

| State | $J^{PC}$ | Eigenvector | $R_0$ (GeV$^{-1}$) | $M(T)$ (MeV) | Threshold (MeV) |
|---|---|---|---|---|---|
| $sc\bar{s}\bar{c}$ | $0^{++}$ | (−0.18, −0.51, 0.83, 0.14) | 4.70 | 4091 | $D_s D_s = 3936$; $D_{s1}^* D_{s1}^* = 5560$ |
| | | (−0.76, 0.21, −0.15, 0.60) | 4.93 | 4254 | $\phi(1020)\, J/\psi = 4117$ |
| | | (−0.12, −0.84, −0.54, 0.01) | 5.11 | 4378 | |
| | | (0.62, −0.06, −0.04, 0.78) | 5.33 | 4492 | |
| | $1^{+-}$ | (−0.33, 0.57, 0.03, 0.75) | 5.21 | 4529 | $D_s D_{s1}^* = 4748$; $D_{s1}^* D_{s1}^* = 5560$ |
| | | (0.87, 0.49, 0.08, 0.02) | 5.30 | 4596 | |
| | | (0.18, −0.46, 0.77, 0.41) | 5.38 | 4638 | |
| | | (0.32, −0.47, −0.64, 0.52) | 5.46 | 4843 | |
| | $1^{++}$ | (−0.58, 0.82) | 5.22 | 4572 | $\phi(1020)\, J/\psi = 4117$; $D_s D_{s1}^* = 4748$ |
| | | (0.82, 0.58) | 5.41 | 4655 | |
| | $2^{++}$ | (0.55, 0.83) | 5.39 | 4421 | $D_{s1}^* D_{s1}^* = 5560$; $\phi(1020)\, J/\psi = 4117$ |
| | | (−0.83, 0.55) | 5.39 | 4450 | |
| $sb\bar{s}\bar{b}$ | $0^{++}$ | (−0.36, −0.35, 0.80, 0.33) | 4.43 | 10843 | $B_s^0 B_s^0 = 10734$; $B_s^* B_s^* = 10830$ |
| | | (−0.58, 0.39, −0.36, 0.62) | 4.60 | 11023 | $\phi(1020)\, \Upsilon = 10480$ |
| | | (0.35, 0.81, 0.47, 0.10) | 4.74 | 11111 | |
| | | (0.64, −0.30, −0.13, 0.70) | 4.86 | 11158 | |
| | $1^{+-}$ | (−0.41, 0.67, −0.20, 0.58) | 4.88 | 11344 | $B_s^0 B_s^* = 10782$; $B_s^* B_s^* = 10830$ |
| | | (0.76, 0.39, 0.45, 0.26) | 4.94 | 11388 | |
| | | (−0.31, −0.47, 0.63, 0.53) | 4.88 | 11457 | |
| | | (0.38, −0.43, −0.59, 0.56) | 5.11 | 11727 | |
| | $1^{++}$ | (0.82, 0.58) | 4.98 | 11408 | $B_s^0 B_s^* = 10782$; $\phi(1020)\, \Upsilon = 10480$ |
| | | (−0.58, 0.82) | 4.85 | 11448 | |
| | $2^{++}$ | (0.51, 0.86) | 4.97 | 11093 | $B_s^* B_s^* = 10830$; $\phi(1020)\, \Upsilon = 10480$ |
| | | (−0.86, 0.51) | 4.99 | 11154 | |

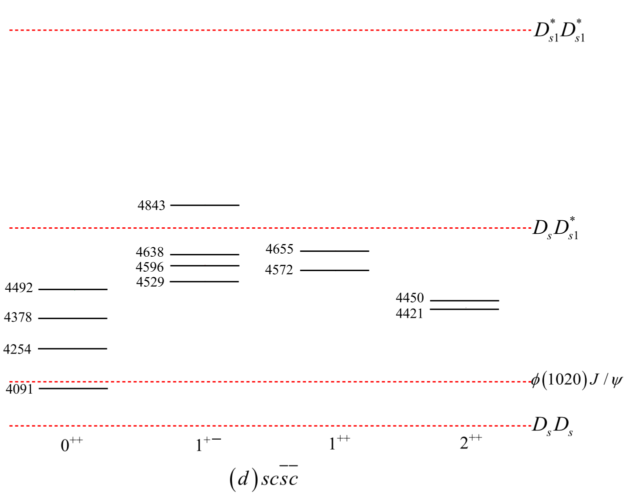


**Fig. 4** Computed masses (MeV the solid lines) of the $sc\bar{s}\bar{c}$ tetraquarks and corresponding two meson thresholds (MeV the dotted lines)

the threshold of $D_s D_{s1}^*$ about 95 MeV but below the threshold of $D_{s1}^* D_{s1}^*$ about 717 MeV. There are two $sc\bar{s}\bar{c}$ systems with $J^{PC} = 1^{++}$, both of which are above the threshold of $\phi(1020)\, J/\psi$ about 455−538 MeV and below the threshold of $D_s D_{s1}^*$ about 93−176 MeV. There are two $sc\bar{s}\bar{c}$ systems with $J^{PC} = 2^{++}$, both above the threshold of $\phi(1020)\, J/\psi$ about 304−333 MeV and below the threshold of $D_{s1}^* D_{s1}^*$ about 1110−1139 MeV, unstable to strong decay to $\phi(1020)\, J/\psi$.

For strange-bottom systems $sb\bar{s}\bar{b}$, we show in Table 6 the computed results for $R_0$, the mixing weights, the masses $M(T)$ and thresholds, with the later two plotted in Fig. 5. Similarly, there are four states for each of $J^{PC} = 0^{++}$ and $J^{PC} = 1^{+-}$, and two states for each of $J^{PC} = 1^{++}$ and $J^{PC} = 2^{++}$. All of the $sb\bar{s}\bar{b}$ systems are above the thresholds except for the lowest one with mass of 10843 MeV which is near to thresholds (13 MeV) of the $B_s^* B_s^*$. The $0^{++}$ states of the $sb\bar{s}\bar{b}$ systems are above the thresholds of $B_s^0 B_s^0$, $B_s^* B_s^*$ and $\phi(1020)\, \Upsilon$. Among them, the minimum mass of 10843 MeV can be strongly decayed into $B_s^0 B_s^0$ and $\phi(1020)\, \Upsilon$. Because of the error in the model, it is uncertain whether it is above or below the threshold of $B_s^* B_s^*$. The four $J^{PC} = 1^{+-}$ states are all highly above the thresholds of $B_s^0 B_s^*$, $B_s^* B_s^*$ (about 562−945 MeV and 514−897 MeV, respectively). There are two states $J^{PC} = 1^{++}$, which are higher than $B_s^0 B_s^*$ and $\phi(1020)\, \Upsilon$ (about 626−666 MeV and

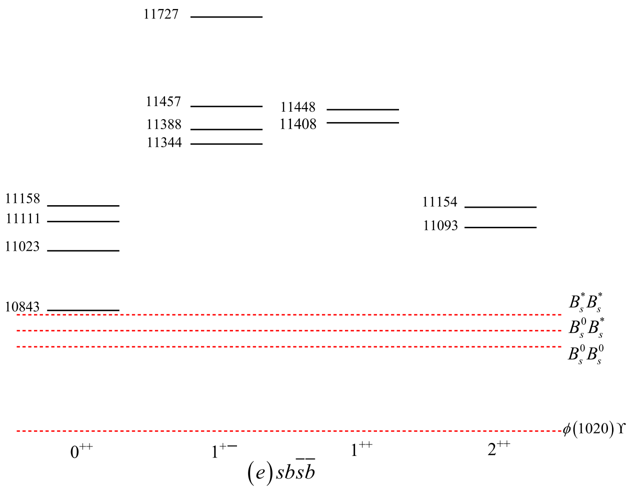


**Fig. 5** Computed masses (MeV the solid lines) of the $sb\bar{s}\bar{b}$ tetraquarks and corresponding two meson thresholds (MeV the dotted lines)

928−968 MeV, respectively). $J^{PC} = 2^{++}$ has two states, which are higher than $B_s^*B_s^*$ and $\phi(1020)\Upsilon$ thresholds (about 263−324 MeV and 613−674 MeV), indicating they are unstable.

### 5.4 The heavy-light(non-strange) systems ($nc\bar{n}\bar{c}$ and $nb\bar{n}\bar{b}$)

For hidden charmed systems of the tetraquarks $nc\bar{n}\bar{c}$, we show the computed results for $R_0$, the mixing weights, the masses $M(T)$ and thresholds in Table 7, with the later two plotted in Fig. 6. There are four states for each of $J^{PC} = 0^{++}$ and $J^{PC} = 1^{+-}$, and two states for each of $J^{PC} = 1^{++}$ and $J^{PC} = 2^{++}$. For the $0^{++}$ states, two higher states (4259 MeV, 4127 MeV) are all above the thresholds of $D^0D^0$, $D^*D^*$, $\omega(782)\,J/\psi$ and $\pi^0\eta_c$ (about 397−529 MeV, 110−242 MeV, 248−380 MeV and 1008−1140 MeV). The lower state with mass 3954 MeV, which is above the thresholds of $D^0D^0$, $\omega(782)\,J/\psi$, $\pi^0\eta_c$ and below the threshold of $D^*D^*$, can strongly decay to the three former final states. The lowest state, which is below the thresholds of $D^*D^*$, $\omega(782)\,J/\psi$, $D^0D^0$ and above the thresholds of $\pi^0\eta_c$, can decay to two final states of $\pi^0\eta_c$. Further, all states with $J^{PC} = 1^{+-}$, $1^{++}$ and $2^{++}$ are above the thresholds of $D^0D^*$, $D^*D^*$, $\pi^0 J/\psi$, $\omega(782)\,J/\psi$, $\omega(782)\,\eta_c$, can decay to the laters with same quantum numbers. For instance, the $1^{++}$ states can decay to $\omega(782)\,J/\psi$ and $D^0D^*$, the $2^{++}$ states can decay to $D^*D^*$, $\omega(782)\,J/\psi$. Our mass spectrum

**Table 7** Computed results for the hidden heavy-flavor tetraquark $nc\bar{n}\bar{c}$ and $nb\bar{n}\bar{b}$, with respective thresholds of two mesons shown also

| State | $J^{PC}$ | Eigenvector | $R_0$ (GeV$^{-1}$) | $M(T)$ (MeV) | Threshold (MeV) |
|---|---|---|---|---|---|
| $nc\bar{n}\bar{c}$ | $0^{++}$ | (−0.24, −0.45, 0.83, 0.20) | 4.90 | 3715 | $D^0D^0 = 3730$; $D^*D^* = 4017$ |
| | | (−0.71, 0.26, −0.22, 0.62) | 5.01 | 3954 | $\omega(782)\,J/\psi = 3879$ |
| | | (0.17, 0.85, 0.50, 0.00) | 5.17 | 4127 | $\pi^0\eta_c = 3119$ |
| | | (0.65, −0.11, −0.06, 0.75) | 5.38 | 4259 | |
| | $1^{+-}$ | (−0.31, 0.65, −0.06, 0.69) | 5.15 | 4079 | $D^0D^* = 3874$; $D^*D^* = 4017$ |
| | | (0.87, 0.33, 0.34, 0.11) | 5.26 | 4172 | $\pi^0 J/\psi = 3264$; $\omega(782)\,\eta_c = 3767$ |
| | | (−0.15, −0.50, 0.70, 0.48) | 5.33 | 4263 | |
| | | (0.35, −0.46, −0.61, 0.54) | 5.32 | 4626 | |
| | $1^{++}$ | (−0.58, 0.82) | 5.18 | 4210 | $D^0D^* = 3874$; $\omega(782)\,J/\psi = 3879$ |
| | | (0.82, 0.58) | 5.36 | 4233 | |
| | $2^{++}$ | (0.46, 0.89) | 5.34 | 4152 | $D^*D^* = 4017$; $\omega(782)\,J/\psi = 3879$ |
| | | (−0.89, 0.46) | 5.31 | 4219 | |
| $nb\bar{n}\bar{b}$ | $0^{++}$ | (−0.37, −0.31, 0.80, 0.36) | 4.65 | 10484 | $B^0B^0 = 10560$; $B^*B^* = 10650$ |
| | | (−0.51, 0.37, −0.40, 0.67) | 4.70 | 10760 | $\omega(782)\,\Upsilon = 10242$ |
| | | (0.42, 0.78, 0.43, 0.14) | 4.82 | 10887 | $\pi^0\eta_b = 9533$ |
| | | (0.65, −0.40, −0.14, 0.63) | 4.90 | 10943 | |
| | $1^{+-}$ | (−0.43, 0.67, −0.25, 0.54) | 4.79 | 10865 | $B^0B^* = 10605$; $B^*B^* = 10650$ |
| | | (0.73, 0.41, 0.47, 0.29) | 4.84 | 10925 | $\pi^0\Upsilon = 9594$; $\omega(782)\,\eta_b = 10182$ |
| | | (−0.36, −0.45, 0.61, 0.55) | 4.81 | 11097 | |
| | | (0.39, −0.42, −0.59, 0.57) | 4.95 | 11509 | |
| | $1^{++}$ | (0.82, 0.58) | 4.89 | 10949 | $B^0B^* = 10605$; $\omega(782)\,\Upsilon = 10242$ |
| | | (−0.58, 0.82) | 4.78 | 11090 | |
| | $2^{++}$ | (0.45, 0.89) | 4.88 | 10835 | $B^*B^* = 10650$; $\omega(782)\,\Upsilon = 10242$ |
| | | (−0.89, 0.45) | 4.88 | 10944 | |

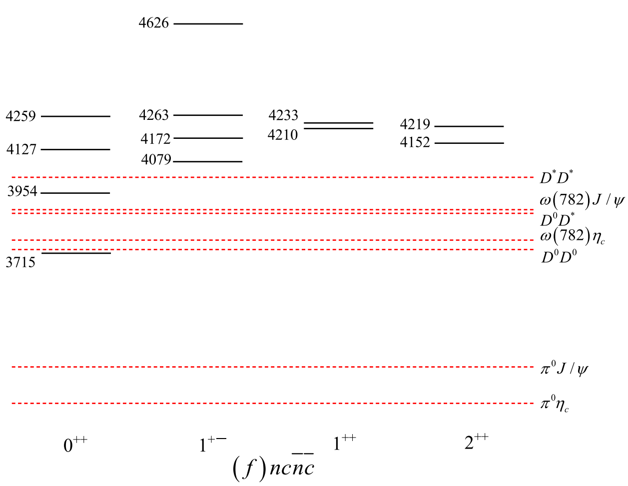


**Fig. 6** Computed masses(MeV the solid lines) of the hidden-bottom tetraquarks $nc\bar{n}\bar{c}$ and the two meson thresholds (MeV the dotted lines)

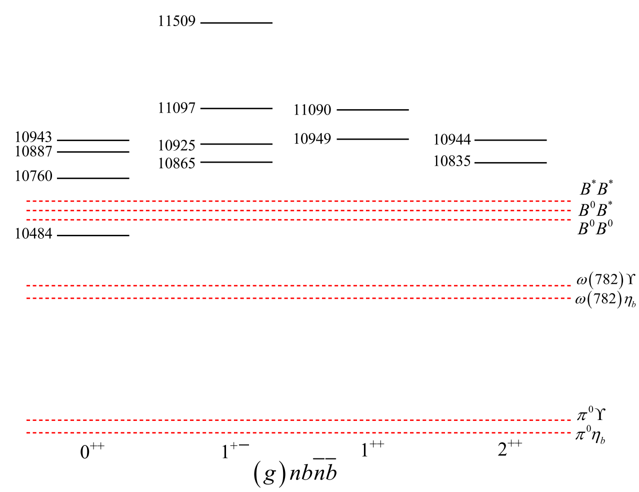


**Fig. 7** Computed masses(MeV the solid lines) of the hidden-bottom tetraquark $nb\bar{n}\bar{b}$ and the two meson thresholds (MeV the dotted lines)

for the $1^{++}$ sector do not contain the observed mass of the $X(3872)$. This can be due to that the $X(3872)$, being not a pure tetraquark state, mixes some other configurations like $c\bar{c}$ or/and $D\bar{D}^*$ molecules [32] (see Ref. [33] for a review). When considered these mixings our prediction 4210 MeV for the compact $nc\bar{n}\bar{c}$ state with $J^{PC} = 1^{++}$ may shift its mass down to near 3872 MeV. Other possibility may be that the parameters extracted from mesons and baryons do not apply to the tetraquark sector due to substantially different color configuration between tetraquark and ordinary mesons. In other word, the effective parameters might change for tetraquarks in a way beyond the simple adjustment of bag radius $R$.

For hidden-bottom systems of tetraquarks $nb\bar{n}\bar{b}$, we show the computed results for $R_0$, the mixing weights, the masses $M(T)$ and thresholds in Table 7, with the later two plotted in Fig. 7. We find from Fig. 7 that all states of $nb\bar{n}\bar{b}$ systems are above the thresholds of their final states of two mesons, except for the lowest state (10,484 MeV), which is below the thresholds of $B^0B^0$ and $B^*B^*$ only and it can decay into $\omega\,(782)\,\Upsilon$, $\pi^0\eta_b$. The possible decays are, for instance,the $nb\bar{n}\bar{b}(0^{++})$ to $B^0B^0$, $B^*B^*$, $\omega\,(782)\,\Upsilon$ and $\pi^0\eta_b$, the $nb\bar{n}\bar{b}(1^{+-})$ to $B^0B^*$, $B^*B^*$, $\pi^0\Upsilon$ and $\omega\,(782)\,\eta_b$, the $nb\bar{n}\bar{b}(1^{++})$ to $B^0B^*$, $\omega\,(782)\,\Upsilon$, the $nb\bar{n}\bar{b}(2^{++})$ to $B^*B^*$, $\omega\,(782)\,\Upsilon$.

As shown in Tables 2, 5, 6 and 7, the typical sizes of the resonances, in unit of the radius $R_0$ (GeV$^{-1}$), range from 2.99 to 5.46 GeV$^{-1}$, namely, from 0.6 to 1.1 fm. For instance,

$$cc\bar{c}\bar{c} : 0.88-0.95\,\text{fm},$$
$$bb\bar{b}\bar{b} : 0.60-0.62\,\text{fm},$$
$$nc\bar{n}\bar{c} : 0.98-1.08\,\text{fm}.$$

In addition, the tetraquarks in the bottom sector is normally smaller in size than that in the charm sector.

## 6 Summary

Stimulated by observations of the $X(6900)$ by LHCb and the recent observations of the $X(6600)$ by CMS and ATLAS experiments of the LHC, we have systematically investigated the ground-state masses of hidden heavy-flavor tetraquarks with two and four hidden heavy-flavor within a unified framework of MIT bag model which incorporates chromomagnetic interactions and enhanced binding energy. Based on color-spin wavefunctions constructed for the hidden heavy-flavor tetraquarks, we solve the MIT bag model and diagonalize the chromomagnetic interaction (CMI) to predict masses of the color-spin multiplets of hidden heavy-flavor tetraquarks in their ground states with spin-parity quantum numbers $J^{PC} = 0^{++}$, $1^{++}$, $2^{++}$, and $1^{+-}$. We find that the fully charmed tetraquark $cc\bar{c}\bar{c}$ with $J^{PC} = 0^{++}$ has mass about 6572 MeV and is very likely to be the $X(6600)$ reported by CMS and ATLAS experiments of the LHC, with the measured mass $6552 \pm 10 \pm 12$ MeV. We have further computed masses of the tetraquark systems $bb\bar{b}\bar{b}$, $cb\bar{c}\bar{b}$, $sc\bar{s}\bar{c}$, $sb\bar{s}\bar{b}$, $nc\bar{n}\bar{c}$ and $nb\bar{n}\bar{b}$ in their color-spin multiplets and suggested that the particle $Z_c(4200)$ reported by [7] is likely to be the hidden-charm tetraquark made of $nc\bar{n}\bar{c}$ with $J^{PC} = 1^{+-}$.

Compared to two-meson thresholds determined via the final states in details, the most-likely strong decay channels are noted. Our mass computation shows that all of these hidden heavy-flavor tetraquarks are above the thresholds of the lowest two-mesons final states and unstable against strong decay to these final states. For the doubly heavy systems of the tetraquarks $sb\bar{s}\bar{b}$, $sc\bar{s}\bar{c}$, $nb\bar{n}\bar{b}$ and $nc\bar{n}\bar{c}$, there are a few states below thresholds except for their lowest final states, indicating that they may have longer lifetime compared to the fully heavy tetraquarks. We also find some near-threshold states for which coupled channel effects are possible. We hope that upcoming LHCb experiments with increased data can test the prediction in this work.

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**Data Availability Statement** This manuscript has associated data in a data repository. [Authors' comment: All data included in this manuscript are available up request by contacting the corresponding authors, group (e.g., PDG group) of collaboration or looking into the cited references.]


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## Appendix A

Based on the color $SU(3)_c$ symmetry, one can obtain two components of color singlets $6_c \otimes \bar{6}_c$ and $\bar{3}_c \otimes 3_c$ for the hidden-flavor tetraquarks,

$$\begin{aligned}\phi_1^T &= \frac{1}{\sqrt{6}}\left(rr\bar{r}\bar{r} + gg\bar{g}\bar{g} + bb\bar{b}\bar{b}\right)\\ &+\frac{1}{2\sqrt{6}}\left(rb\bar{b}\bar{r} + br\bar{b}\bar{r} + gr\bar{g}\bar{r} + rg\bar{g}\bar{r} + gb\bar{b}\bar{g} + bg\bar{b}\bar{g}\right.\\ &\left.+gr\bar{r}\bar{g} + rg\bar{r}\bar{g} + gb\bar{g}\bar{b} + bg\bar{g}\bar{b} + rb\bar{r}\bar{b} + br\bar{r}\bar{b}\right), \qquad (30)\\ \phi_2^T &= \frac{1}{2\sqrt{3}}\left(rb\bar{b}\bar{r} - br\bar{b}\bar{r} - gr\bar{g}\bar{r} + rg\bar{g}\bar{r} + gb\bar{b}\bar{g} - bg\bar{b}\bar{g}\right.\\ &\left.+gr\bar{r}\bar{g} - rg\bar{r}\bar{g} - gb\bar{g}\bar{b} + bg\bar{g}\bar{b} - rb\bar{r}\bar{b} + br\bar{r}\bar{b}\right), \qquad (31)\end{aligned}$$

which corresponds to two color configurations in Eq. (1).

For six states $\chi^T_{1\sim6}$ (2) of heavy tetraquarks, one can construct their spin wave functions via writing the $CG$ coefficients explicitly:

$$\begin{aligned}\chi_1^T &= \uparrow\uparrow\uparrow\uparrow,\\ \chi_2^T &= \frac{1}{2}\left(\uparrow\uparrow\uparrow\downarrow + \uparrow\uparrow\downarrow\uparrow - \uparrow\downarrow\uparrow\uparrow - \downarrow\uparrow\uparrow\uparrow\right),\\ \chi_3^T &= \frac{1}{\sqrt{3}}\left(\uparrow\uparrow\downarrow\downarrow + \downarrow\downarrow\uparrow\uparrow\right),\\ &-\frac{1}{2\sqrt{3}}\left(\uparrow\downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow\uparrow + \downarrow\uparrow\uparrow\downarrow + \downarrow\uparrow\downarrow\uparrow\right),\\ \chi_4^T &= \frac{1}{\sqrt{2}}\left(\uparrow\uparrow\uparrow\downarrow - \uparrow\uparrow\downarrow\uparrow\right),\\ \chi_5^T &= \frac{1}{\sqrt{2}}\left(\uparrow\downarrow\uparrow\uparrow - \downarrow\uparrow\uparrow\uparrow\right),\\ \chi_6^T &= \frac{1}{2}\left(\uparrow\downarrow\uparrow\downarrow - \uparrow\downarrow\downarrow\uparrow - \downarrow\uparrow\uparrow\downarrow + \downarrow\uparrow\downarrow\uparrow\right), \qquad (32)\end{aligned}$$

in which notations $\uparrow$ and $\downarrow$ represent the third component of the quark's spin. Alternatively, one can also use the $CG$ coefficients given in Ref. [16] to examine Eq. (32) for the different spin states. Note that the results of spin factors in Ref. [16] are shown as matrix form in the spanned space of the states $\chi^T_{1\sim6}$ as the spin multiplets (2) indicated. Combining with two color configurations $\phi^T_{1\sim2}$ in Eq. (1) and six spin configurations $\chi^T_{1\sim6}$ in Eq. (2), one can then construct their color-spin wavefunctions (3). The allowed states of the hidden-flavor tetraquarks to be mix due to chromomagnetic interaction are listed in Table 1.

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