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The formal seesaw mechanism of Majorana neutrinos with unbroken gauge symmetry

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Abstract

We reformulate the canonical seesaw mechanism in the case that the electroweak gauge symmetry is unbroken, and show that it can *formally* work and allow us to derive an exact seesaw formula for the light and heavy Majorana neutrinos. We elucidate the reason why there is a mismatch between the mass eigenstates of heavy Majorana neutrinos associated with thermal leptogenesis and those associated with the seesaw framework, and establish the exact and explicit relations between the *original* and *derivational* seesaw parameters by using an Euler-like parametrization of the 6×6 active-sterile flavor mixing matrix.

1. Motivation

Among all the proposed mechanisms toward deeply understanding the true origin of tiny masses of the three known neutrinos ν_i (for $i = 1, 2, 3$), whose flavor eigenstates are commonly denoted as ν_α (for $\alpha = e, \mu, \tau$), the canonical seesaw mechanism [1–5] stands out as being most economical and most natural. The simplicity of this mechanism lies in two aspects: (a) it just takes into account the right-handed neutrino fields $N_{\alpha\mathrm{R}}$, the chiral counterparts of the left-handed

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neutrino fields $\nu_{\alpha \rm L}$ (for $\alpha = e, \mu, \tau$), which were originally ignored from the particle content of the standard model (SM) [6]; (b) it simply allows for lepton number violation or the Majorana nature of massive neutrinos [7], which is completely harmless to the theoretical framework of the SM itself. The naturalness of this mechanism is reflected in its attributing the small masses of ν_i to the existence of three heavy Majorana neutrinos N_i (for $i = 1, 2, 3$), whose masses are expected to be far above the fulcrum of the seesaw — presumably the electroweak symmetry breaking scale of the SM characterized by the vacuum expectation value of the Higgs field. On the other hand, the seesaw mechanism offers a big bonus to cosmology: the CP-violating and out-of-equilibrium decays of heavy Majorana neutrinos may give rise to a net lepton-antilepton asymmetry in the early Universe, and such a *leptogenesis* mechanism [8] can finally lead to *baryogenesis* as a natural interpretation of the observed baryon-antibaryon asymmetry in today's Universe [9]. In this sense the seesaw mechanism is the very *stone* that can kill two fundamental *birds* in particle physics and cosmology.

Note that the seesaw mechanism is expected to take effect at a superhigh energy scale Λ which is essentially of the order of the heavy Majorana neutrino masses. But the $\rm SU(2)_L \times U(1)_Y$ electroweak gauge symmetry has been unbroken until the Higgs field develops a nonzero vacuum expectation value v of $\mathcal{O}\big(10^2\big)$ GeV. In this situation the three active neutrinos are actually impossible to acquire their *true* masses of $\mathcal{O}(v^2/\Lambda)$ at the seesaw scale Λ due to the absence of a *real* fulcrum of the seesaw. On the other hand, thermal leptogenesis can be realized via the lepton-number-violating decays of heavy Majorana neutrinos into the leptonic and Higgs doublets at Λ. So we are well motivated to ask a conceptually important question: how can the seesaw mechanism *formally* survive with the unbroken electroweak gauge symmetry and work together with the leptogenesis mechanism? If the answer to this question is affirmative, we wonder whether the mass eigenstates of heavy Majorana neutrinos associated with thermal leptogenesis are exactly the same as those associated with the seesaw mechanism itself.[1] In case that there exists a mismatch between these two sets of mass bases, then the question becomes how small this mismatch is likely to be.

To answer the above questions and clarify some conceptual ambiguities that have never been taken seriously, we are going to study how to make the seesaw mechanism formally work before spontaneous electroweak symmetry breaking. We show that an exact seesaw relation between the light and heavy Majorana neutrinos can be established far above the electroweak scale, and it becomes the realistic seesaw relation after the Higgs field develops its vacuum expectation value. In this way it is straightforward to elucidate the reason why there is a mismatch between the mass eigenstates of heavy Majorana neutrinos associated with thermal leptogenesis and those associated with the seesaw mechanism. With the help of a full Euler-like parametrization of the flavor structure in the seesaw framework, we illuminate such a mismatch in a more specific way. The exact and explicit relations between the *original* and *derivational* parameters of massive Majorana neutrinos are obtained as a by-product, and they are expected to be useful in determining or constraining some of the original seesaw parameters from the low-energy neutrino experiments.

[1] A mismatch of this kind has been observed and discussed in the seesaw framework *after* spontaneous electroweak symmetry breaking and in an *approximate* way (see, e.g., Refs. [10–14]). Here we shall take a new look at it *before* electroweak symmetry breaking and in an *exact* way at the tree level.

2. A formal seesaw mechanism?

2.1. The leptonic Yukawa interactions

Let us begin with the gauge-invariant leptonic Yukawa interactions and the $SU(2)_L$-singlet Majorana neutrino mass term of the canonical seesaw mechanism at Λ[2]

$$-\mathcal{L}_\Lambda = \overline{\ell_L} Y_l H l_R + \overline{\ell_L} Y_\nu \widetilde{H} N_R + \frac{1}{2}\overline{(N_R)^c} M_R N_R + \text{h.c.}\,, \tag{1}$$

where $\ell_L = \begin{pmatrix} \nu_L & l_L \end{pmatrix}^T$ denotes the leptonic $SU(2)_L$ doublet of the SM with $\nu_L = \begin{pmatrix} \nu_{eL} & \nu_{\mu L} & \nu_{\tau L} \end{pmatrix}^T$ and $l_L = \begin{pmatrix} l_{eL} & l_{\mu L} & l_{\tau L} \end{pmatrix}^T$ standing respectively for the column vectors of the left-handed neutrino and charged lepton fields, $\widetilde{H} \equiv \mathrm{i}\sigma_2 H^*$ with $H = \begin{pmatrix} \phi^+ & \phi^0 \end{pmatrix}^T$ being the Higgs doublet of the SM and σ_2 being the second Pauli matrix, $l_R = \begin{pmatrix} l_{eR} & l_{\mu R} & l_{\tau R} \end{pmatrix}^T$ and $N_R = \begin{pmatrix} N_{eR} & N_{\mu R} & N_{\tau R} \end{pmatrix}^T$ stand respectively for the column vectors of the right-handed charged lepton and neutrino fields which are the $SU(2)_L$ singlets, $(N_R)^c \equiv \mathcal{C}\overline{N_R}^T$ with $\mathcal{C}$ being the charge-conjugation matrix and satisfying $\mathcal{C}^{-1} = \mathcal{C}^\dagger = \mathcal{C}^T = -\mathcal{C}$, Y_l and Y_ν represent the respective Yukawa coupling matrices of charged leptons and neutrinos, and M_R is the symmetric right-handed neutrino mass matrix. In Eq. (1) the hypercharges of ℓ_L, l_R, N_R, H and $\widetilde{H}$ are $-1/2$, -1, 0, $+1/2$ and $-1/2$, respectively. Since $\overline{\nu_L} Y_\nu N_R$ is a Lorentz scalar and can be transformed into

$$\overline{\nu_L} Y_\nu N_R = [\overline{\nu_L} Y_\nu N_R]^T = \overline{(N_R)^c} Y_\nu^T (\nu_L)^c\,, \tag{2}$$

where $(\nu_L)^c \equiv \mathcal{C}\overline{\nu_L}^T$ is the charge-conjugated counterpart of ν_L, one may easily rewrite Eq. (1) as

$$-\mathcal{L}_\Lambda = \overline{l_L} Y_l l_R \phi^0 + \frac{1}{2}\overline{\begin{bmatrix} \nu_L & (N_R)^c \end{bmatrix}} \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_R \end{pmatrix} \begin{bmatrix} (\nu_L)^c \\ N_R \end{bmatrix} + \overline{\nu_L} Y_l l_R \phi^+ - \overline{l_L} Y_\nu N_R \phi^- + \text{h.c.}\,. \tag{3}$$

This expression is highly nontrivial in the sense that it clearly shows a direct correlation between the left- and right-handed neutrino fields via their Yukawa couplings to the neutral component of the Higgs doublet even though the $SU(2)_L \times U(1)_Y$ gauge symmetry is perfect at the seesaw scale Λ. In this situation the 3×3 Yukawa coupling matrix Y_ν can be regarded as a "virtual" fulcrum of the seesaw before spontaneous electroweak symmetry breaking.

Note that both the scalar field ϕ^0 and its charge-conjugated counterpart ϕ^{0*} have the mass dimension and act like two complex numbers in Eq. (3). But of course they possess the respective hypercharges $+1/2$ and $-1/2$ as $\phi^\pm$ do. After spontaneous symmetry breaking ϕ^0 and ϕ^{0*} will acquire the same vacuum expectation value $\langle\phi^0\rangle = \langle\phi^{0*}\rangle = v/\sqrt{2}$ with $v \simeq 246$ GeV, together with $\langle\phi^-\rangle = \langle\phi^+\rangle = 0$, as in the SM. Then the formal seesaw will acquire a real fulcrum which allows one to naturally attribute the smallness of three active Majorana neutrino masses to the existence of three heavy Majorana neutrinos, as can be seen later on.

[2] Throughout this paper, our discussions are subject to the minimal extension of the SM with three right-handed neutrino fields and lepton number violation at *zero* temperature, so as to make our key point clear and avoid possible complications (e.g., thermal corrections to the masses of heavy Majorana neutrinos [14]).

2.2. The leptogenesis-associated basis

Now that all the SM particles are exactly massless in the early Universe when the temperature is far above the electroweak scale, a realization of thermal leptogenesis at $\Lambda \gg v$ only needs to calculate the lepton-number-violating decays of heavy Majorana neutrinos into the leptonic doublet and the Higgs doublet at the one-loop level by simply starting from Eq. (1) instead of Eq. (3) (see, e.g., Refs. [8,15–18]). In this case the column vector of the mass eigenstates of three heavy Majorana neutrinos, denoted as $\mathcal{N}' = \begin{pmatrix}\mathcal{N}_1 & \mathcal{N}_2 & \mathcal{N}_3\end{pmatrix}^T$, can easily be obtained by making the Autonne-Takagi transformation [19,20] as follows:

$$U_0^{\prime\dagger} M_{\rm R} U_0^{\prime *} = \mathcal{D}_{\mathcal{N}} \; , \quad \mathcal{N}_{\rm R}' = U_0^{\prime T} N_{\rm R} \; , \tag{4}$$

where U_0' is a unitary matrix, and $\mathcal{D}_{\mathcal{N}} \equiv {\rm Diag}\{\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3\}$ with $\mathcal{M}_i$ being the masses of $\mathcal{N}_i$ (for $i = 1, 2, 3$). As a result, the Lagrangian $\mathcal{L}_\Lambda$ in Eq. (1) becomes

$$-\mathcal{L}_\Lambda = \overline{\ell_{\rm L}} Y_l H l_{\rm R} + \overline{\ell_{\rm L}} \mathcal{Y}_\nu \widetilde{H} \mathcal{N}_{\rm R}' + \frac{1}{2} \overline{(\mathcal{N}_{\rm R}')^c} \mathcal{D}_{\mathcal{N}} \mathcal{N}_{\rm R}' + {\rm h.c.} \; , \tag{5}$$

where $\mathcal{Y}_\nu \equiv Y_\nu U_0^{\prime *}$ is defined for the sake of simplicity. The rates of $\mathcal{N}_i$ decaying into $\ell_{\rm L}$ and H or their CP-conjugated states are therefore determined by $\mathcal{M}_i$ and $\mathcal{Y}_\nu$, so are the corresponding CP-violating asymmetries associated closely with thermal leptogenesis [15–18].[3] To be more specific, the flavor-dependent CP-violating asymmetries of $\mathcal{N}_i$ decays are given by

$$\begin{aligned} \varepsilon_{i\alpha} \equiv & \frac{\Gamma(\mathcal{N}_i \to \ell_\alpha + H) - \Gamma(\mathcal{N}_i \to \overline{\ell_\alpha} + \overline{H})}{\sum\limits_\alpha [\Gamma(\mathcal{N}_i \to \ell_\alpha + H) + \Gamma(\mathcal{N}_i \to \overline{\ell_\alpha} + \overline{H})]} \\ = & \frac{1}{8\pi (\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{ii}} \sum_{j \neq i} \left\{ {\rm Im}\left[(\mathcal{Y}_\nu^*)_{\alpha i} (\mathcal{Y}_\nu)_{\alpha j} (\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{ij} \xi(x_{ji}) + (\mathcal{Y}_\nu^*)_{\alpha i} (\mathcal{Y}_\nu)_{\alpha j} (\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{ij}^* \zeta(x_{ji}) \right] \right\} , \end{aligned} \tag{6}$$

where the Latin and Greek subscripts run respectively over $(1, 2, 3)$ and (e, μ, τ), $x_{ji} \equiv \mathcal{M}_j^2 / \mathcal{M}_i^2$ are defined, $\xi(x_{ji}) = \sqrt{x_{ji}} \left\{ 1 + 1/\left(1 - x_{ji}\right) + \left(1 + x_{ji}\right) \ln\left[x_{ji}/\left(1 + x_{ji}\right)\right] \right\}$ and $\zeta(x_{ji}) = 1/\left(1 - x_{ji}\right)$ are the loop functions. A net lepton-antilepton asymmetry can therefore result from $\varepsilon_{i\alpha}$ in the early Universe, and later on it can be partly converted into a net baryon-antibaryon asymmetry via the sphaleron interactions (see Ref. [21] for a recent review).

At this point it is worth remarking that the right-handed neutrino fields $N_{\alpha {\rm R}}$ have zero weak isospin and hypercharge, and hence they have no coupling with the charged and neutral gauge bosons of the SM. As a consequence, the mass eigenstates of heavy Majorana neutrinos obtained from Eq. (4) do not participate in the weak charged-current interactions of the SM,

$$-\mathcal{L}_{\rm cc} = \frac{g}{2} \overline{\ell_{\rm L}} \gamma^\mu \left(\sigma_1 W_\mu^1 + \sigma_2 W_\mu^2\right) \ell_{\rm L} = \frac{g}{\sqrt{2}} \overline{l_{\rm L}} \gamma^\mu W_\mu^- \nu_{\rm L} + {\rm h.c.} \; , \tag{7}$$

where g denotes the weak gauge coupling constant, $\sigma_{1,2}$ represent the first and second Pauli matrices, $W_{1,2}^\mu$ are two of the original ${\rm SU(2)_L}$ gauge fields, and $W_\mu^\pm \equiv \left(W_\mu^1 \mp {\rm i} W_\mu^2\right)/\sqrt{2}$ stand for the fields of the physical charged gauge bosons $W^\pm$. But in the seesaw framework we shall see that the expression of $\mathcal{L}_{\rm cc}$ in Eq. (7) will get modified, and the corresponding mass eigenstates of three heavy Majorana neutrinos can definitely take part in the weak charged-current interactions.

[3] Here we have used some *calligraphic* characters to denote the relevant physical quantities in the basis where $M_{\rm R}$ is diagonalized by the unitary transformation made in Eq. (4). This basis is associated with $\mathcal{N}_i$ decays and thermal leptogenesis, and it is conceptually different from the basis taken for the seesaw mechanism as can be seen below.

2.3. The seesaw-associated basis

We proceed to show that the canonical seesaw mechanism can "formally" work before spontaneous electroweak symmetry breaking but the corresponding mass eigenstates of three heavy Majorana neutrinos are not exactly the same as $\mathcal{N}_i$ (for $i = 1, 2, 3$) obtained above for the neutrino decays and thermal leptogenesis. To clarify this important point, let us diagonalize the symmetric 6×6 matrix in Eq. (3) in the following Autonne-Takagi way:

$$\mathbb{U}^\dagger \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_{\rm R} \end{pmatrix} \mathbb{U}^* = \begin{pmatrix} D_\nu & \mathbf{0} \\ \mathbf{0} & D_N \end{pmatrix} , \tag{8}$$

where $\mathbb{U}$ is a 6×6 unitary matrix, and the diagonal and real matrices D_ν and D_N are defined as $D_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\}$ and $D_N \equiv {\rm Diag}\{M_1, M_2, M_3\}$. Meanwhile, the column vectors of left- and right-handed neutrino fields $[\nu_{\rm L} \quad (N_{\rm R})^c]^T$ and $[(\nu_{\rm L})^c \quad N_{\rm R}]^T$ undergo the transformations

$$\begin{bmatrix} \nu_{\rm L} \\ (N_{\rm R})^c \end{bmatrix} \longrightarrow \mathbb{U}^\dagger \begin{bmatrix} \nu_{\rm L} \\ (N_{\rm R})^c \end{bmatrix} , \quad \begin{bmatrix} (\nu_{\rm L})^c \\ N_{\rm R} \end{bmatrix} \longrightarrow \mathbb{U}^T \begin{bmatrix} (\nu_{\rm L})^c \\ N_{\rm R} \end{bmatrix} , \tag{9}$$

such that the Lagrangian $\mathcal{L}_\Lambda$ in Eq. (3) keeps unchanged and thus its *overall* gauge symmetry is unbroken. Now that Y_ν is dimensionless and ϕ^0 has the same mass dimension as $M_{\rm R}$, one may argue that m_i should be the "working" or "virtual" mass parameters of three light Majorana neutrinos as the electroweak gauge symmetry is unbroken at the seesaw scale Λ. In comparison, M_i are essentially the true masses of three heavy Majorana neutrinos in the existence of the $\phi^{0(*)}$-mediated neutrino Yukawa interactions. Along this line of thought, we find that it is useful to decompose $\mathbb{U}$ into the product of three matrices,

$$\mathbb{U} = \begin{pmatrix} I & \mathbf{0} \\ \mathbf{0} & U_0^\prime \end{pmatrix} \begin{pmatrix} A & R \\ S & B \end{pmatrix} \begin{pmatrix} U_0 & \mathbf{0} \\ \mathbf{0} & I \end{pmatrix} , \tag{10}$$

where the 3×3 unitary matrix $U_0^\prime$ has been defined in Eq. (4) to primarily describe flavor mixing in the sterile (heavy) neutrino sector, U_0 denotes the other 3×3 unitary matrix that is mainly responsible for flavor mixing in the active (light) neutrino sector, while the 3×3 matrices A, B, R and S signify the interplay between these two sectors [22–24]. The unitarity of $\mathbb{U}$ assures

$$\begin{aligned} & AA^\dagger + RR^\dagger = BB^\dagger + SS^\dagger = I , \\ & AS^\dagger + RB^\dagger = A^\dagger R + S^\dagger B = \mathbf{0} , \\ & A^\dagger A + S^\dagger S = B^\dagger B + R^\dagger R = I . \end{aligned} \tag{11}$$

On the other hand, the arbitrary charged-lepton Yukawa coupling matrix Y_l in Eq. (3) can be diagonalized by a bi-unitary transformation:

$$U_l^\dagger \left(Y_l \phi^0\right) V_l = D_l , \quad l_{\rm L}^\prime = U_l^\dagger l_{\rm L} , \quad l_{\rm R}^\prime = V_l l_{\rm R} , \tag{12}$$

where U_l and V_l are unitary, $D_l \equiv {\rm Diag}\{m_e, m_\mu, m_\tau\}$ stands for the "working" or "virtual" masses of three charged leptons *before* spontaneous electroweak symmetry breaking,[4] and

[4] Note that the scalar field ϕ^0 in Eq. (12) carries a hypercharge, and hence D_l cannot be simply understood as a diagonal "mass" matrix. The physical meaning of D_l is actually vague in our calculations which are mathematically exact and clear, so is the physical meaning of D_ν in Eq. (8). But this vagueness will automatically disappear after spontaneous electroweak symmetry breaking, as can be subsequently seen.

$l' = \begin{pmatrix} e & \mu & \tau \end{pmatrix}^T$ is defined as the column vector of the mass eigenstates of three charged leptons versus the column vector of their flavor eigenstates $l = \begin{pmatrix} l_e & l_\mu & l_\tau \end{pmatrix}^T$. Substituting Eqs. (8)—(10) and (12) into Eq. (3), we immediately arrive at

$$-\mathcal{L}_\Lambda = \overline{l'_{\rm L}} D_l l'_{\rm R} + \frac{1}{2}\overline{\nu'_{\rm L}} D_\nu (\nu'_{\rm L})^c + \frac{1}{2}\overline{(N'_{\rm R})^c} D_N N'_{\rm R} + \overline{\nu_{\rm L}} Y_l l_{\rm R} \phi^+ - \overline{l_{\rm L}} Y_\nu N_{\rm R} \phi^- + {\rm h.c.} \,, \quad (13)$$

where $\nu' = \begin{pmatrix} \nu_1 & \nu_2 & \nu_3 \end{pmatrix}^T$ denotes the column vector of the *working* mass eigenstates of three light Majorana neutrinos far above the electroweak scale, and $N' = \begin{pmatrix} N_1 & N_2 & N_3 \end{pmatrix}^T$ stands for the column vectors of the mass eigenstates of three heavy Majorana neutrinos relevant to the seesaw mechanism at $\Lambda \gg v$. In this case the flavor eigenstates $\nu_{\rm L}$ and $N_{\rm R}$ can be expressed in terms of the mass eigenstates $\nu'_{\rm L}$ and $N'_{\rm R}$ or their charge-conjugated states as follows:

$$\nu_{\rm L} = U \nu'_{\rm L} + R (N'_{\rm R})^c \,, \quad N_{\rm R} = S'^* (\nu'_{\rm L})^c + U'^* N'_{\rm R} \,, \quad (14)$$

where $U \equiv A U_0$, $U' \equiv U'_0 B$ and $S' \equiv U'_0 S U_0$ are defined. Taking account of the Majorana property of ν_i and N_i (i.e., $\nu_i^c = \nu_i$ and $N_i^c = N_i$ [7] for $i = 1, 2, 3$), one simply obtains $(N'_{\rm R})^c = (N'^c)_{\rm L} = N'_{\rm L}$ and $(\nu'_{\rm L})^c = (\nu'^c)_{\rm R} = \nu'_{\rm R}$. One may then substitute the expression of $l_{\rm L}$ in Eq. (12) and that of $\nu_{\rm L}$ in Eq. (14) into the standard form of $\mathcal{L}_{\rm cc}$ in Eq. (7) and get at

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{\begin{pmatrix} e & \mu & \tau \end{pmatrix}_{\rm L}} \gamma^\mu \left[U_{\rm PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R_{\rm PMNS} \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L} \right] W^-_\mu + {\rm h.c.} \,, \quad (15)$$

where $U_{\rm PMNS} = U_l^\dagger U$ is just the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) lepton flavor mixing matrix [25–27] used to describe the flavor oscillations of three active neutrinos, and $R_{\rm PMNS} = U_l^\dagger R$ is an analogue of $U_{\rm PMNS}$ in the seesaw mechanism which characterizes the strengths of weak charged-current interactions for three heavy Majorana neutrinos.

Without loss of generality, one may choose a convenient flavor basis in which the mass eigenstates of three charged leptons are identified with their corresponding flavor eigenstates (i.e., $l_{\rm L} = l'_{\rm L}$, or equivalently $U_l = I$). In this case we are simply left with $U_{\rm PMNS} = U$ and $R_{\rm PMNS} = R$, namely the effects of lepton flavor mixing originate purely from the active and sterile Majorana neutrino sectors and from the interplay between these two sectors. We shall take advantage of this flavor basis in the following discussions unless otherwise specified.

2.4. *Mismatch between the two bases*

Before discussing a mismatch between the mass eigenstates of heavy Majorana neutrinos associated with thermal leptogenesis and those associated with the seesaw mechanism, let us take a look at the flavor structures of active and sterile neutrinos in the case that the electroweak gauge symmetry is unbroken at Λ. First of all, a combination of Eqs. (8) and (10) allows us to immediately derive the exact seesaw relation between the working masses of three light Majorana neutrinos and the real masses of three heavy Majorana neutrinos:

$$U D_\nu U^T + R D_N R^T = \mathbf{0} \,, \quad (16)$$

in which U and R are also correlated with each other via the unitarity condition $U U^\dagger + R R^\dagger = I$. Note that $U = A U_0$ holds, where the unitary matrix U_0 is primarily responsible for flavor mixing of the three active neutrinos. So we find it useful to rewrite Eq. (16) as

$$U_0 D_\nu U_0^T = \left(\mathrm{i}A^{-1}R\right) D_N \left(\mathrm{i}A^{-1}R\right)^T \ , \tag{17}$$

whose left- and right-hand sides are composed of the *derivational* and *original* seesaw parameters, respectively. This point will become more obvious when a complete Euler-like parametrization of the 6×6 unitary matrix $\mathbb{U}$ in Eq. (10) is adopted, as can be seen in section 3. Needless to say, the active-sterile flavor mixing matrix R essentially plays the role of the neutrino Yukawa coupling matrix Y_ν in the canonical seesaw framework,

$$Y_\nu \phi^{0*} = R D_N \left[I - \left(B^{-1}SA^{-1}R\right)^T\right] U^{\prime T} \ ; \tag{18}$$

and the right-handed Majorana neutrino mass matrix $M_{\rm R}$ can be reconstructed into the form

$$M_{\rm R} = U^\prime \left[D_N - \left(B^{-1}SA^{-1}R\right) D_N \left(B^{-1}SA^{-1}R\right)^T\right] U^{\prime T} \ . \tag{19}$$

Note that all the quantities in Eqs. (18) and (19) belong to the *original* seesaw parameters in the sense that they have nothing to do with D_ν and U_0 — the working masses and the primary flavor mixing matrix of three light Majorana neutrinos which are *derived* from the seesaw mechanism.

Now we turn to an unavoidable mismatch between the mass eigenstates of three heavy Majorana neutrinos associated with the seesaw and leptogenesis mechanisms. Eq. (14) tells us that the mass eigenstates $N^\prime_{\rm R}$ in the seesaw basis can be expressed as

$$N^\prime_{\rm R} = (U^{\prime *})^{-1}\left[N_{\rm R} - S^{\prime *}(\nu^\prime_{\rm L})^c\right] = (B^*)^{-1}\left[\mathcal{N}^\prime_{\rm R} - U_0^{\prime T} S^{\prime *}(\nu^\prime_{\rm L})^c\right] , \tag{20}$$

where Eq. (4) has been used to link $N^\prime_{\rm R}$ to $\mathcal{N}^\prime_{\rm R}$. To be more explicit, Eq. (20) means

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = (B^*)^{-1}\left[\begin{pmatrix} \mathcal{N}_1 \\ \mathcal{N}_2 \\ \mathcal{N}_3 \end{pmatrix} - U_0^{\prime T} S^{\prime *}\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}\right] , \tag{21}$$

from which the differences between N_i in the seesaw basis and $\mathcal{N}_i$ (for $i = 1, 2, 3$) in the thermal leptogenesis basis can be clearly seen. Similarly, a combination of Eqs. (4) and (19) leads us to

$$\mathcal{D}_\mathcal{N} = B\left[D_N - \left(B^{-1}SA^{-1}R\right) D_N \left(B^{-1}SA^{-1}R\right)^T\right] B^T \ , \tag{22}$$

from which one may easily see the difference between $\mathcal{D}_N$ and D_N. Although $N^\prime_{\rm R}$ (or D_N) and $\mathcal{N}^\prime_{\rm R}$ (or $\mathcal{D}_\mathcal{N}$) would exactly coincide with each other if the Yukawa coupling matrix Y_ν (or equivalently, R or S) were switched off, such a coincidence would make no sense because both the seesaw and leptogenesis mechanisms would fail in this special case. In the presence of the neutrino Yukawa interactions, thermal leptogenesis may take effect via the CP-violating and out-of-equilibrium decays of heavy Majorana neutrinos into the leptonic and Higgs doublets, while the seesaw mechanism can "formally" work with the help of an interplay between the active and sterile neutrino fields coupled only to the neutral component of the Higgs doublet. That is the key reason why there is an inevitable mismatch between the seesaw- and leptogenesis-associated bases for heavy Majorana neutrinos before spontaneous electroweak symmetry breaking.

2.5. After gauge symmetry breaking

So far we have made some proper transformations of the charged lepton and neutrino fields in the flavor space to obtain their respective working or true mass eigenstates. All such unitary flavor basis transformations are completely reversible, and hence they do not affect the gauge invariance of $\mathcal{L}_\Lambda$ at the seesaw scale. As already shown in Eqs. (20) and (22), a seeable mismatch between $N^\prime_{\rm R}$ and $\mathcal{N}^\prime_{\rm R}$ or between D_N and $\mathcal{D}_N$ results from the fact that the working seesaw mechanism itself is only associated with the neutral component of the Higgs doublet while the heavy Majorana neutrino decays and thermal leptogenesis at the seesaw scale Λ are associated with the whole Higgs doublet. This unavoidable mismatch deserves to be conceptually clarified as we have done, because it is an intrinsic issue of the seesaw and leptogenesis mechanisms.

It is now straightforward to prove that the *formal* seesaw mechanism far above the electroweak scale will become *real* after the Higgs potential of the SM is minimized at $\langle H\rangle \equiv \langle 0|H|0\rangle = v/\sqrt{2}$ with a special direction characterized by $\langle\phi^\pm\rangle = 0$ and $\langle\phi^0\rangle = v/\sqrt{2}$, by which the electroweak gauge symmetry is spontaneously broken and thus all the particles coupled to the Higgs field acquire their nonzero masses. In this case the Lagrangian in Eq. (3) can be simplified to a more popular form,

$$-\mathcal{L}^\prime_\Lambda = \overline{l_{\rm L}} M_l l_{\rm R} + \frac{1}{2}\overline{\begin{bmatrix}\nu_{\rm L} & (N_{\rm R})^c\end{bmatrix}} \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M^T_{\rm D} & M_{\rm R}\end{pmatrix} \begin{bmatrix} (\nu_{\rm L})^c \\ N_{\rm R}\end{bmatrix} + {\rm h.c.} \,, \tag{23}$$

where $M_l \equiv Y_l\langle\phi^0\rangle = Y_l v/\sqrt{2}$ denotes the charged lepton mass matrix, and $M_{\rm D} \equiv Y_\nu\langle\phi^0\rangle = Y_\nu v/\sqrt{2}$ is usually referred to as the Dirac neutrino mass matrix. The expression of $M_{\rm D}$ in terms of the seesaw parameters can be directly read off from Eq. (18), namely

$$M_{\rm D} = R D_N \left[I - \left(B^{-1} S A^{-1} R\right)^T \right] U^{\prime T} \,. \tag{24}$$

We find that the exact seesaw formula obtained in Eq. (16) and the analytical results obtained in Eqs. (19)—(22) formally keep unchanged after spontaneous gauge symmetry breaking, but they are now subject to the electroweak scale. In other words, the electroweak symmetry breaking itself does not really affect the flavor structures of the seesaw mechanism. This observation implies that it is possible to determine or constrain some of the original seesaw-associated flavor parameters in some low-energy neutrino experiments, after the radiative corrections to such parameters are properly taken into account with the help of the relevant renormaliztion-group equations (RGEs) between a superhigh seesaw scale and the electroweak scale [28].

Note that the exact seesaw formula obtained in Eq. (16) can be simplified to the more popular form in the leading-order approximations of Eqs. (19) and (24). That is, $M_{\rm R} \simeq U^\prime D_N U^{\prime T}$ and $M_{\rm D} \simeq R D_N U^{\prime T}$, so the *effective* mass matrix for three active Majorana neutrinos is given by

$$M_\nu \equiv U_0 D_\nu U^T_0 \simeq -R D_N R^T \simeq -M_{\rm D} M^{-1}_{\rm R} M^T_{\rm D} \,, \tag{25}$$

where $A \simeq B \simeq I$ has been assumed (i.e., $U \simeq U_0$ holds in the neglect of the non-unitary effects characterized by $A \neq I$). In this approximation the effective Majorana mass term for three active neutrinos at low energies turns out to be

$$-\mathcal{L}_\nu = \frac{1}{2}\overline{\nu_{\rm L}} M_\nu (\nu_{\rm L})^c + {\rm h.c.} = \frac{1}{2}\overline{\nu^\prime_{\rm L}} D_\nu (\nu^\prime_{\rm L})^c + {\rm h.c.} \,, \tag{26}$$

where the column vector of the light neutrino mass eigenstates $\nu^\prime_{\rm L}$ has already been defined below Eq. (13), and the physical meaning of D_ν as the diagonal Majorana neutrino mass matrix becomes definite and obvious.

3. How small is the mismatch?

3.1. An Euler-like parametrization

To clearly see how small the difference between N_R' (or D_N) and $\mathcal{N}_R'$ (or $\mathcal{D}_\mathcal{N}$) is expected to be, let us follow Refs. [22–24] to make an Euler-like parametrization of the 6×6 unitary matrix $\mathbb{U}$ in Eq. (10). First of all we introduce fifteen 6×6 Euler-like unitary matrices of the form O_{ij} (for $1 \leq i < j \leq 6$): its (i, i) and (j, j) entries are both identical to $c_{ij} \equiv \cos\theta_{ij}$ with θ_{ij} being a flavor mixing angle and lying in the first quadrant, its other four diagonal elements are all equal to one, its (i, j) and (j, i) entries are given respectively by $\hat{s}^*_{ij} \equiv e^{-{\rm i}\delta_{ij}} \sin\theta_{ij}$ and $-\hat{s}_{ij} \equiv -e^{{\rm i}\delta_{ij}} \sin\theta_{ij}$ with δ_{ij} being a CP-violating phase, and its other off-diagonal elements are all equal to zero. These matrices are then grouped in the following way to respectively describe the *active* flavor sector, the *sterile* flavor sector and the *interplay* between these two sectors:

$$\begin{aligned} \begin{pmatrix} U_0 & 0 \\ 0 & I \end{pmatrix} &= O_{23} O_{13} O_{12} \, , \\ \begin{pmatrix} I & 0 \\ 0 & U_0' \end{pmatrix} &= O_{56} O_{46} O_{45} \, , \\ \begin{pmatrix} A & R \\ S & B \end{pmatrix} &= O_{36} O_{26} O_{16} O_{35} O_{25} O_{15} O_{34} O_{24} O_{14} \, , \end{aligned} \tag{27}$$

where the pattern of U_0 is quite similar to the standard parametrization of a unitary PMNS matrix as advocated by the Particle Data Group [9],[5]

$$U_0 = \begin{pmatrix} c_{12} c_{13} & \hat{s}^*_{12} c_{13} & \hat{s}^*_{13} \\ -\hat{s}_{12} c_{23} - c_{12} \hat{s}_{13} \hat{s}^*_{23} & c_{12} c_{23} - \hat{s}^*_{12} \hat{s}_{13} \hat{s}^*_{23} & c_{13} \hat{s}^*_{23} \\ \hat{s}_{12} \hat{s}_{23} - c_{12} \hat{s}_{13} c_{23} & -c_{12} \hat{s}_{23} - \hat{s}^*_{12} \hat{s}_{13} c_{23} & c_{13} c_{23} \end{pmatrix} , \tag{28}$$

and the expression of U_0' can be directly read off from that of U_0 with the subscript replacements $12 \leftrightarrow 45$, $13 \leftrightarrow 46$ and $23 \leftrightarrow 56$ for the three rotation angles and three CP-violating phases. The explicit expressions of A, B, R and S in terms of c_{ij} and $\hat{s}_{ij}$ (for $i = 1, 2, 3$ and $j = 4, 5, 6$) are rather lengthy, and hence they are listed in Eqs. (A.1) and (A.2) in Appendix A for the same of simplicity. Among the four active-sterile flavor mixing matrices, only A and R affect the physical processes in which the light and heavy Majorana neutrinos take part, as can be seen from Eq. (15). As both $U = AU_0$ and R appear in $\mathcal{L}_{\rm cc}$ in the chosen flavor basis (i.e., $U_l = I$), three of the nice CP-violating phases (or their combinations) of A and R can always be rotated away by properly redefining the phases of three charged lepton fields [29,30].

The PMNS matrix U is obviously non-unitary because of $UU^\dagger = AA^\dagger = I - RR^\dagger \neq I$, but its deviation from exact unitarity (i.e., from U_0) is found to be very small. A detailed and careful analysis of currently available electroweak precision measurements and neutrino oscillation data has put a stringent constraint on the non-unitarity of U — the latter is below or far below $\mathcal{O}(10^{-2})$ [31–35]. This result implies that the deviation of $AA^\dagger$ from I ought to be smaller than $\mathcal{O}(10^{-2})$, and thus the nine active-sterile flavor mixing angles in R should be smaller than $\mathcal{O}(10^{-1})$. The advantage of such a phenomenological observation is that $U \simeq U_0$ can be a quite

[5] When U_0 is applied to the phenomenology of neutrino physics in the basis of $U_l = I$, it is the phase parameter $\delta \equiv \delta_{13} - \delta_{12} - \delta_{23}$ that characterizes the strength of CP violation in neutrino oscillations.

reliable approximation in most cases, but its disadvantage is that an experimental exploration of the seesaw-induced non-unitary effects of U at low energies will be rather challenging.

3.2. Smallness of the mismatch

Eq. (20) tells us that a difference between the mass eigenstates of three heavy Majorana neutrinos associated with the seesaw mechanism (i.e., $N'_{\rm R}$) and those associated with thermal leptogenesis (i.e., $\mathcal{N}'_{\rm R}$) is mainly characterized by the deviation of $(B^*)^{-1}$ from the identity matrix I. With the help of Eq. (A.2), we arrive at

$$
\begin{aligned}
(B^*)^{-1} &= \begin{pmatrix} c_{14}^{-1} c_{24}^{-1} c_{34}^{-1} & 0 & 0 \\ \begin{matrix} +\hat{t}_{14} c_{24}^{-1} c_{34}^{-1} \hat{t}_{15}^* + \hat{t}_{24} c_{34}^{-1} c_{24}^{-1} \hat{t}_{25}^* \\ +\hat{t}_{34} c_{15}^{-1} c_{25}^{-1} \hat{t}_{35}^* \end{matrix} & c_{15}^{-1} c_{25}^{-1} c_{35}^{-1} & 0 \\ \begin{matrix} +\hat{t}_{14} c_{24}^{-1} c_{34}^{-1} c_{24}^{-1} \hat{t}_{16}^* + \hat{t}_{24} c_{34}^{-1} \hat{t}_{15} \hat{t}_{25}^* \hat{t}_{16}^* \\ +\hat{t}_{24} c_{34}^{-1} c_{25}^{-1} c_{16}^{-1} \hat{t}_{26}^* + \hat{t}_{34} \hat{t}_{15} c_{25}^{-1} \hat{t}_{35}^* \hat{t}_{16}^* \\ +\hat{t}_{34} c_{35}^{-1} c_{16}^{-1} c_{26}^{-1} \hat{t}_{36}^* + \hat{t}_{34} \hat{t}_{25} \hat{t}_{35}^* c_{16}^{-1} \hat{t}_{26}^* \end{matrix} & \begin{matrix} +\hat{t}_{15} c_{25}^{-1} c_{35}^{-1} \hat{t}_{16}^* + \hat{t}_{25} c_{35}^{-1} c_{16}^{-1} \hat{t}_{26}^* \\ +\hat{t}_{35} c_{16}^{-1} c_{26}^{-1} \hat{t}_{36}^* \end{matrix} & c_{16}^{-1} c_{26}^{-1} c_{36}^{-1} \end{pmatrix} \\
&\simeq I + \begin{pmatrix} \frac{1}{2}\left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) & 0 & 0 \\ \hat{s}_{14}\hat{s}_{15}^* + \hat{s}_{24}\hat{s}_{25}^* + \hat{s}_{34}\hat{s}_{35}^* & \frac{1}{2}\left(s_{15}^2 + s_{25}^2 + s_{35}^2\right) & 0 \\ \hat{s}_{14}\hat{s}_{16}^* + \hat{s}_{24}\hat{s}_{26}^* + \hat{s}_{34}\hat{s}_{36}^* & \hat{s}_{15}\hat{s}_{16}^* + \hat{s}_{25}\hat{s}_{26}^* + \hat{s}_{35}\hat{s}_{36}^* & \frac{1}{2}\left(s_{16}^2 + s_{26}^2 + s_{36}^2\right) \end{pmatrix} ,
\end{aligned} \tag{29}
$$

where $\hat{t}_{ij} \equiv e^{{\rm i}\delta_{ij}} \tan\theta_{ij}$ is defined, and all the terms of $\mathcal{O}(s_{ij}^4)$ or smaller have been omitted from the second equation as an excellent approximation due to the smallness of θ_{ij} (for $i = 1, 2, 3$ and $j = 4, 5, 6$). We see that $(B^*)^{-1}$ is also a lower triangular matrix like B itself. On the other hand, the factor $U_0^{\prime T} S^{\prime *}$ appearing in Eq. (20) can be explicitly expressed as follows:

$$
U_0^{\prime T} S^{\prime *} = B^T S^* U_0^* \simeq - \begin{pmatrix} \hat{s}_{14}^* & \hat{s}_{24}^* & \hat{s}_{34}^* \\ \hat{s}_{15}^* & \hat{s}_{25}^* & \hat{s}_{35}^* \\ \hat{s}_{16}^* & \hat{s}_{26}^* & \hat{s}_{36}^* \end{pmatrix} U_0^* , \tag{30}
$$

where Eq. (A.2) has been used, and the terms of $\mathcal{O}(s_{ij}^3)$ or smaller have been omitted from the second equation as a very good approximation. Now we conclude that the heavy Majorana neutrino mass basis $N'_{\rm R}$ is identical to $\mathcal{N}'_{\rm R}$ up to the accuracy of $\mathcal{O}(s_{ij}^2)$, but it contains a small contribution of $\mathcal{O}(s_{ij})$ from the light Majorana neutrino mass basis $(\nu'_{\rm L})^c$ in the seesaw framework. Since the magnitudes of θ_{ij} (for $i = 1, 2, 3$ and $j = 4, 5, 6$) are highly suppressed in a realistic seesaw model with little fine-tuning, the mismatch between $N'_{\rm R}$ and $\mathcal{N}'_{\rm R}$ is expected to be negligible in most cases.

Let us proceed to examine how small the difference between D_N and $\mathcal{D}_{\mathcal{N}}$ in Eq. (21) can be. First of all, Eq. (A.2) allows us to make the approximation

$$
B \simeq I - \begin{pmatrix} \frac{1}{2}\left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) & 0 & 0 \\ \hat{s}_{14}^*\hat{s}_{15} + \hat{s}_{24}^*\hat{s}_{25} + \hat{s}_{34}^*\hat{s}_{35} & \frac{1}{2}\left(s_{15}^2 + s_{25}^2 + s_{35}^2\right) & 0 \\ \hat{s}_{14}^*\hat{s}_{16} + \hat{s}_{24}^*\hat{s}_{26} + \hat{s}_{34}^*\hat{s}_{36} & \hat{s}_{15}^*\hat{s}_{16} + \hat{s}_{25}^*\hat{s}_{26} + \hat{s}_{35}^*\hat{s}_{36} & \frac{1}{2}\left(s_{16}^2 + s_{26}^2 + s_{36}^2\right) \end{pmatrix} , \tag{31}
$$

where the terms of $\mathcal{O}(s_{ij}^4)$ or smaller have been omitted. Secondly, we obtain

$$\begin{aligned} & A^{-1}R \\ & = \begin{pmatrix} \hat{t}_{14}^* & c_{14}^{-1}\hat{t}_{15}^* & c_{14}^{-1}c_{15}^{-1}\hat{t}_{16}^* \\ c_{14}^{-1}\hat{t}_{24}^* & \hat{t}_{14}\hat{t}_{15}^*\hat{t}_{24}^* + c_{15}^{-1}c_{24}^{-1}\hat{t}_{25}^* & \begin{matrix} +\hat{t}_{14}c_{15}^{-1}\hat{t}_{16}^*\hat{t}_{24}^* + \hat{t}_{15}\hat{t}_{16}^*c_{24}^{-1}\hat{t}_{25}^* \\ +c_{16}^{-1}c_{24}^{-1}c_{25}^{-1}\hat{t}_{26}^* \end{matrix} \\ c_{14}^{-1}c_{24}^{-1}\hat{t}_{34}^* & \begin{matrix} +\hat{t}_{14}\hat{t}_{15}^*c_{24}^{-1}\hat{t}_{34}^* + c_{15}^{-1}\hat{t}_{24}\hat{t}_{25}^*\hat{t}_{34}^* \\ +c_{15}^{-1}c_{25}^{-1}c_{34}^{-1}\hat{t}_{35}^* \end{matrix} & \begin{matrix} +\hat{t}_{14}c_{15}^{-1}\hat{t}_{16}^*c_{24}^{-1}\hat{t}_{34}^* + \hat{t}_{15}\hat{t}_{16}^*\hat{t}_{24}\hat{t}_{25}^*\hat{t}_{34}^* \\ +\hat{t}_{15}\hat{t}_{16}^*c_{25}^{-1}c_{34}^{-1}\hat{t}_{35}^* + c_{16}^{-1}\hat{t}_{24}c_{25}^{-1}\hat{t}_{26}^*\hat{t}_{34}^* \\ +c_{16}^{-1}\hat{t}_{25}\hat{t}_{26}^*c_{34}^{-1}\hat{t}_{35}^* + c_{16}^{-1}c_{26}^{-1}c_{34}^{-1}c_{35}^{-1}\hat{t}_{36}^* \end{matrix} \end{pmatrix} \\ & \simeq \begin{pmatrix} \hat{s}_{14}^* & \hat{s}_{15}^* & \hat{s}_{16}^* \\ \hat{s}_{24}^* & \hat{s}_{25}^* & \hat{s}_{26}^* \\ \hat{s}_{34}^* & \hat{s}_{35}^* & \hat{s}_{36}^* \end{pmatrix} \end{aligned} \tag{32}$$

from Eq. (A.1), where the terms of $\mathcal{O}(s_{ij}^3)$ or smaller have been neglected in the second equation as a reasonably good approximation. The exact expression of $B^{-1}S$ can be directly read off from that of $-\left(A^{-1}R\right)^*$ with the help of Eq. (32) by making the subscript replacements $15 \leftrightarrow 24$, $16 \leftrightarrow 34$ and $26 \leftrightarrow 35$, so can its approximate expression. As a consequence,

$$B^{-1}SA^{-1}R \simeq -\begin{pmatrix} s_{14}^2+s_{24}^2+s_{34}^2 & \hat{s}_{14}\hat{s}_{15}^*+\hat{s}_{24}\hat{s}_{25}^*+\hat{s}_{34}\hat{s}_{35}^* & \hat{s}_{14}\hat{s}_{16}^*+\hat{s}_{24}\hat{s}_{26}^*+\hat{s}_{34}\hat{s}_{36}^* \\ \hat{s}_{14}^*\hat{s}_{15}+\hat{s}_{24}^*\hat{s}_{25}+\hat{s}_{34}^*\hat{s}_{35} & s_{15}^2+s_{25}^2+s_{35}^2 & \hat{s}_{15}\hat{s}_{16}^*+\hat{s}_{25}\hat{s}_{26}^*+\hat{s}_{35}\hat{s}_{36}^* \\ \hat{s}_{14}^*\hat{s}_{16}+\hat{s}_{24}^*\hat{s}_{26}+\hat{s}_{34}^*\hat{s}_{36} & \hat{s}_{15}^*\hat{s}_{16}+\hat{s}_{25}^*\hat{s}_{26}+\hat{s}_{35}^*\hat{s}_{36} & s_{16}^2+s_{26}^2+s_{36}^2 \end{pmatrix} \tag{33}$$

holds in the same approximation as made above. This result implies that $\mathcal{D}_{\mathcal{N}}$ and D_N are identical to each other up to the accuracy of $\mathcal{O}(s_{ij}^2)$, simply because on the right-hand side of Eq. (22) the second term is suppressed in magnitude to $\mathcal{O}(s_{ij}^4)$ as compared with the first term.

It is worth remarking that our above analytical approximations are more or less subject to the canonical seesaw mechanism at an energy scale far above the electroweak scale, and thus the mismatch between $N_{\rm R}^\prime$ (or D_N) and $\mathcal{N}_{\rm R}^\prime$ (or $\mathcal{D}_{\mathcal{N}}$) is very small. This situation will change when the low-scale seesaw and leptogenesis scenarios, in which a mismatch between the two sets of mass bases for heavy Majorana neutrinos is crucial, are taken into account (see, e.g., Refs. [11,12]).

3.3. Determination of D_ν and U_0

As already shown in Eq. (17), the nine effective flavor parameters of three light Majorana neutrinos in D_ν and U_0 (i.e., three effective masses, three flavor mixing angles and three CP-violating phases) can be expressed in terms of the eighteen seesaw parameters hidden in A, R and D_N (i.e., three heavy Majorana neutrino masses, nine active-sterile flavor mixing angles and six CP-violating phases). It is obvious that all the derivational seesaw parameters on the left-hand side of Eq. (17) would vanish if $R \propto Y_\nu$ were switched off. So this equation provides an unambiguous way to determine the light degrees of freedom from the heavy degrees of freedom in the seesaw framework.

To be more specific, the six independent elements of the effective Majorana neutrino mass matrix $M_\nu \equiv U_0 D_\nu U_0^T$ are given as follows:

$$\begin{aligned} \left(M_\nu\right)_{11} &= m_1 c_{12}^2 c_{13}^2 + m_2 \hat{s}_{12}^{*2} c_{13}^2 + m_3 \hat{s}_{13}^{*2} \, , \\ \left(M_\nu\right)_{12} &= -m_1 c_{12} c_{13} \left(\hat{s}_{12} c_{23} + c_{12} \hat{s}_{13} \hat{s}_{23}^*\right) \end{aligned}$$

$$
\begin{aligned}
& \quad +m_2 \hat{s}_{12}^* c_{13} \left(c_{12} c_{23} - \hat{s}_{12}^* \hat{s}_{13} \hat{s}_{23}^*\right) + m_3 c_{13} \hat{s}_{13}^* \hat{s}_{23}^* , \\
\left(M_\nu\right)_{13} &= m_1 c_{12} c_{13} \left(\hat{s}_{12} \hat{s}_{23} - c_{12} \hat{s}_{13} c_{23}\right) \\
& \quad -m_2 \hat{s}_{12}^* c_{13} \left(c_{12} \hat{s}_{23} + \hat{s}_{12}^* \hat{s}_{13} c_{23}\right) + m_3 c_{13} \hat{s}_{13}^* c_{23} , \\
\left(M_\nu\right)_{22} &= m_1 \left(\hat{s}_{12} c_{23} + c_{12} \hat{s}_{13} \hat{s}_{23}^*\right)^2 + m_2 \left(c_{12} c_{23} - \hat{s}_{12}^* \hat{s}_{13} \hat{s}_{23}^*\right)^2 + m_3 c_{13}^2 \hat{s}_{23}^{*2} , \\
\left(M_\nu\right)_{23} &= -m_1 \left(\hat{s}_{12} c_{23} + c_{12} \hat{s}_{13} \hat{s}_{23}^*\right) \left(\hat{s}_{12} \hat{s}_{23} - c_{12} \hat{s}_{13} c_{23}\right) \\
& \quad -m_2 \left(c_{12} c_{23} - \hat{s}_{12}^* \hat{s}_{13} \hat{s}_{23}^*\right) \left(c_{12} \hat{s}_{23} + \hat{s}_{12}^* \hat{s}_{13} c_{23}\right) + m_3 c_{13}^2 c_{23} \hat{s}_{23}^* , \\
\left(M_\nu\right)_{33} &= m_1 \left(\hat{s}_{12} \hat{s}_{23} - c_{12} \hat{s}_{13} c_{23}\right)^2 + m_2 \left(c_{12} \hat{s}_{23} + \hat{s}_{12}^* \hat{s}_{13} c_{23}\right)^2 + m_3 c_{13}^2 c_{23}^2 . \qquad (34)
\end{aligned}
$$

On the other hand, Eq. (17) tells us that these six matrix elements can originally be determined by $M_\nu = -\left(A^{-1} R\right) D_N \left(A^{-1} R\right)^T$ thanks to the exact seesaw relation bridging the big gap between the light and heavy Majorana neutrinos. With the help of the explicit expression of $A^{-1} R$ given in Eq. (32), it is straightforward to obtain the expressions for the elements of M_ν in terms of M_i, θ_{ij} and δ_{ij} (for $i = 1, 2, 3$ and $j = 4, 5, 6$). Instead of presenting the exact analytical results, which are rather lengthy and hence less instructive, here we make the leading-order approximations for the expressions of A and R given in Eq. (A.1) and then arrive at

$$
\begin{aligned}
\left(M_\nu\right)_{11} &\simeq -\left[M_1 \hat{s}_{14}^{*2} + M_2 \hat{s}_{15}^{*2} + M_3 \hat{s}_{16}^{*2}\right] , \\
\left(M_\nu\right)_{12} &\simeq -\left[M_1 \hat{s}_{14}^* \hat{s}_{24}^* + M_2 \hat{s}_{15}^* \hat{s}_{25}^* + M_3 \hat{s}_{16}^* \hat{s}_{26}^*\right] , \\
\left(M_\nu\right)_{13} &\simeq -\left[M_1 \hat{s}_{14}^* \hat{s}_{34}^* + M_2 \hat{s}_{15}^* \hat{s}_{35}^* + M_3 \hat{s}_{16}^* \hat{s}_{36}^*\right] , \\
\left(M_\nu\right)_{22} &\simeq -\left[M_1 \hat{s}_{24}^{*2} + M_2 \hat{s}_{25}^{*2} + M_3 \hat{s}_{26}^{*2}\right] , \\
\left(M_\nu\right)_{23} &\simeq -\left[M_1 \hat{s}_{24}^* \hat{s}_{34}^* + M_2 \hat{s}_{25}^* \hat{s}_{35}^* + M_3 \hat{s}_{26}^* \hat{s}_{36}^*\right] , \\
\left(M_\nu\right)_{33} &\simeq -\left[M_1 \hat{s}_{34}^{*2} + M_2 \hat{s}_{35}^{*2} + M_3 \hat{s}_{36}^{*2}\right] . \qquad (35)
\end{aligned}
$$

Let us emphasize that there appear nine CP-violating phases in Eq. (35), but three of them (or their combinations) are redundant and can always be removed by rephasing the charged lepton fields in a proper way.[6] A combination of Eqs. (34) and (35) allows us to establish the direct relations between the nine derivational and eighteen original seesaw parameters. So the former can in principle be determined from the latter for a given seesaw model (a top-down approach), and the latter may be partly probed or constrained from the former with the help of some low-energy neutrino experiments (a bottom-up approach). A careful and detailed analysis of the parameter space along this line of thought will be made elsewhere.

4. Summary

We have reformulated the canonical seesaw mechanism by considering the fact that the electroweak gauge symmetry is unbroken at the seesaw scale characterized by the masses of heavy Majorana neutrinos, and shown that it can *formally* work and allow us to derive an exact seesaw relation between the active (light) and sterile (heavy) Majorana neutrinos. In this way we have

[6] A straightforward way to remove the three redundant phase parameters of A and R is just to switch off three of the nine phases in the nine active-sterile flavor mixing matrices O_{ij} (for $i = 1, 2, 3$ and $j = 4, 5, 6$) in Eq. (27) from the very beginning. As there are many options in doing so, we do not go into details here.

elucidated the reason why there is an unavoidable mismatch between the mass eigenstates of heavy Majorana neutrinos associated with the seesaw and thermal leptogenesis mechanisms. The smallness of this mismatch has been discussed with the help of a complete Euler-like parametrization of the flavor structure in the seesaw framework, and the exact and explicit relations between the *original* and *derivational* seesaw parameters have been established as a by-product.

We hope that this work may help clarify some conceptual ambiguities associated with the validity of the seesaw mechanism before and after spontaneous electroweak symmetry breaking, because such ambiguities have never been taken serious in the literature. It should also be helpful to clarify the ambiguities associated with the RGE evolution between the "virtual" flavor parameters of Majorana neutrinos at the seesaw scale and those "real" ones at the electroweak scale, which is crucial to bridge the gap between a well-motivated UV-complete flavor theory including the seesaw mechanism and all the possible low-energy flavor experiments.

CRediT authorship contribution statement

Zhi-zhong Xing: Conceptualization, Investigation, Methodology, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix A. The expressions of A, B, R and S

Given the Euler-like parametrization of the 6×6 unitary flavor mixing matrix $\mathbb{U}$ decomposed in Eq. (27), the 3×3 active-sterile flavor mixing matrices A, B, R and S depend on the same nine rotation angles θ_{ij} and nine phase angles δ_{ij} (for $i = 1, 2, 3$ and $j = 4, 5, 6$). To be explicit [22, 23],

$$
\begin{aligned}
A &= \begin{pmatrix}
c_{14}c_{15}c_{16} & 0 & 0 \\
\begin{array}{l} -c_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - c_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} \\ -\hat{s}_{14}\hat{s}^*_{24}c_{25}c_{26} \end{array} & c_{24}c_{25}c_{26} & 0 \\
\begin{array}{l} -c_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + c_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -c_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} + \hat{s}_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} \\ +\hat{s}_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} - \hat{s}_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & \begin{array}{l} -c_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} - c_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} \\ -\hat{s}_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & c_{34}c_{35}c_{36}
\end{pmatrix} , \\
R &= \begin{pmatrix}
\hat{s}^*_{14}c_{15}c_{16} & \hat{s}^*_{15}c_{16} & \hat{s}^*_{16} \\
\begin{array}{l} -\hat{s}^*_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - \hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} \\ +c_{14}\hat{s}^*_{24}c_{25}c_{26} \end{array} & -\hat{s}^*_{15}\hat{s}_{16}\hat{s}^*_{26} + c_{15}\hat{s}^*_{25}c_{26} & c_{16}\hat{s}^*_{26} \\
\begin{array}{l} -\hat{s}^*_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + \hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -\hat{s}^*_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} - c_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -c_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} + c_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & \begin{array}{l} -\hat{s}^*_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} - c_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ +c_{15}c_{25}\hat{s}^*_{35}c_{36} \end{array} & c_{16}c_{26}\hat{s}^*_{36}
\end{pmatrix} ;
\end{aligned}
\tag{A.1}
$$

and

$$
\begin{aligned}
B &= \begin{pmatrix}
c_{14}c_{24}c_{34} & 0 & 0 \\
\begin{array}{l} -c_{14}c_{24}\hat{s}^*_{34}\hat{s}_{35} - c_{14}\hat{s}^*_{24}\hat{s}_{25}c_{35} \\ -\hat{s}^*_{14}\hat{s}_{15}c_{25}c_{35} \end{array} & c_{15}c_{25}c_{35} & 0 \\
\begin{array}{l} -c_{14}c_{24}\hat{s}^*_{34}c_{35}\hat{s}_{36} + c_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}\hat{s}_{36} \\ -c_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}c_{36} + \hat{s}^*_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}\hat{s}_{36} \\ +\hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}c_{36} - \hat{s}^*_{14}c_{15}\hat{s}_{16}c_{26}c_{36} \end{array} & \begin{array}{l} -c_{15}c_{25}\hat{s}^*_{35}\hat{s}_{36} - c_{15}\hat{s}^*_{25}\hat{s}_{26}c_{36} \\ -\hat{s}^*_{15}\hat{s}_{16}c_{26}c_{36} \end{array} & c_{16}c_{26}c_{36}
\end{pmatrix} , \\
S &= \begin{pmatrix}
-\hat{s}_{14}c_{24}c_{34} & -\hat{s}_{24}c_{34} & -\hat{s}_{34} \\
\begin{array}{l} \hat{s}_{14}c_{24}\hat{s}^*_{34}\hat{s}_{35} + \hat{s}_{14}\hat{s}^*_{24}\hat{s}_{25}c_{35} \\ -c_{14}\hat{s}_{15}c_{25}c_{35} \end{array} & \hat{s}_{24}\hat{s}^*_{34}\hat{s}_{35} - c_{24}\hat{s}_{25}c_{35} & -c_{34}\hat{s}_{35} \\
\begin{array}{l} \hat{s}_{14}c_{24}\hat{s}^*_{34}c_{35}\hat{s}_{36} - \hat{s}_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}\hat{s}_{36} \\ +\hat{s}_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}c_{36} + c_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}\hat{s}_{36} \\ +c_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}c_{36} - c_{14}c_{15}\hat{s}_{16}c_{26}c_{36} \end{array} & \begin{array}{l} \hat{s}_{24}\hat{s}^*_{34}c_{35}\hat{s}_{36} + c_{24}\hat{s}_{25}\hat{s}^*_{35}\hat{s}_{36} \\ -c_{24}c_{25}\hat{s}_{26}c_{36} \end{array} & -c_{34}c_{35}\hat{s}_{36}
\end{pmatrix} .
\end{aligned}
\tag{A.2}
$$

We see that both A and B are the lower triangular matrices, and the expression of B can be read off from that of A^* with the subscript replacements $15 \leftrightarrow 24$, $16 \leftrightarrow 34$ and $26 \leftrightarrow 35$. The expression of S can be similarly obtained from that of $-R^*$ with the same subscript replacements [24]. Note, however, that B and S do not affect any physical processes in the seesaw mechanism.

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