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Supertwistor formulation for massless superparticle in $AdS_5 \times S^5$ superspace

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Abstract

Starting with the first-order formulation of the massless superparticle model on the $AdS_5 \times S^5$ super-background and presenting the momentum components tangent to AdS_5 and S^5 subspaces as bilinear combinations of the constrained $SU(2)$-Majorana spinors allows to bring the superparticle's Lagrangian to the form quadratic in supertwistors. The $SU(2,2|4)$ supertwistors are assembled into a pair of $SU(2)$ doublets, one of which has even $SU(2,2)$ and odd $SU(4)$ components, while the other has odd $SU(2,2)$ and even $SU(4)$ components. They are subject to the first-class constraints that generate the $psu(2|2) \oplus u(1)$ gauge algebra. This justifies previously proposed group-theoretic definition of the $AdS_5 \times S^5$ supertwistors and allows to derive the incidence relations with the (10|32) supercoordinates of the $AdS_5 \times S^5$ superspace. Whenever superparticle moves within the AdS_5 subspace of the $AdS_5 \times S^5$ space-time, twistor formulation of its Lagrangian involves just one $SU(2)$ doublet of $SU(2,2|4)$ supertwistors with even $SU(2,2)$ and odd $SU(4)$ components. If in addition particle's 5-momentum is null, four first-class constraints which are the $su(2) \oplus u(1)$ generators single out upon quantization the states of $D = 5$ $N = 8$ gauged supergravity multiplet in the superambitwistor formulation.

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1. Introduction

Incidence relations for the Penrose $SU(2,2)$ twistors [1] and Penrose–Ferber $SU(2,2|N)$ supertwistors [2][1] that come out as solutions of the twistor equation and its superextensions can also be derived from the (super)twistor formulation of the massless particle model in 4-dimensional (super)space [2], [4], [5], [6], [7], [8], [9]. (Super)twistor formulation is known to combine manifest and linearly realized (super)conformal symmetry with the simple and irreducible realization of the gauge symmetries. These features provided strong motivation to study (super)twistor formulations also for massive (super)particles [10], [11], [12], [13], [14], null and tensile (super)strings [15], [16], [17], [18] and membranes [19] in 4-dimensional Minkowski (super)space and in higher string-theoretic dimensions [20], [21], [22], [23], [24], [25], [26] [27], [28].[2] (Super)twistors also appear rather efficient in presenting scattering amplitudes of massless particles not limiting to 4-dimensional space-time (see, e.g. [31] and references therein).

Since $SU(2,2)$ is not only the covering of the conformal group of 4-dimensional Minkowski space-time but also is the isometry group of 5-dimensional anti-de Sitter space and $SU(2,2|4)$ is the superisometry of the $AdS_5 \times S^5$ superspace as well as the $N=4$ extended superconformal symmetry of the $N=4$ supersymmetric Yang–Mills theory in the $D=4$ boundary superspace that provides the crucial symmetry argument in support of the AdS_5/CFT_4 duality, it is natural to seek for relevant (super)twistors. Because AdS_5 space-time is conformally flat, the incidence relations for Penrose twistors (actually ambitwistors) admit natural extension to accommodate extra bosonic coordinate(s) [32], [33] (see also [34]). It is also quite natural to add odd coordinates for $D=4$ N-extended Poincare supersymmetry [35], [36]. The case of $AdS_5 \times S^5$ superspace isomorphic to the $PSU(2,2|4)/(SO(1,4) \times SO(5))$ supercoset manifold [37], [38] is not so easy to deal with as it is not superconformally flat [39]. In [40] there was given the group-theoretical reasoning for the definition of $AdS_5 \times S^5$ supertwistors as two $SU(2)$ doublets of $SU(2,2|4)$ supertwistors: one with even $SU(2,2)$ components and odd $SU(4)$ components and another with odd $SU(2,2)$ components and even $SU(4)$ ones. These constitute $(4|4) \times (2|2)$ rectangular block of the $PSU(2,2|4)/(SO(1,4) \times SO(5))$ supermatrix.[3] We call such supertwistors c- and a-type supertwistors by analogy with c- and a-type supermatrices (see, e.g. [42]). Their $SU(2,2|4)$-invariant products with dual supertwistors were shown to satisfy seven bosonic and eight fermionic real constraints which number matches that of $psu(2|2) \oplus u(1)$ generators.[4] However, the incidence relations with the $AdS_5 \times S^5$ supercoordinates, necessary, for instance, to elaborate on the Penrose transform, have not been obtained. Previous experience suggests that supertwistor reformulation of the point-particle model is a proper tool to find such relations. Additionally one of the advantages of the supertwistor approach is that the su-

[1] For another possibility of the definition of (super)twistors for $D=4$ Minkowski (super)space see [3].

[2] Yet another approach to the construction of (super)twistors in higher-dimensions relies on the properties of pure spinors (see, e.g. [29], [30]).

[3] Similar ideas on the definition of supertwistors for AdS_5/CFT_4 duality were also discussed in [41].

[4] Initial version [43] of the definition of $AdS_5 \times S^5$ supertwistors used four $SU(2)$ doublets two of which had even $SU(2,2)$ components and odd $SU(4)$ components and other two had odd $SU(2,2)$ components and even $SU(4)$ components and they satisfied the $s(u(2|2) \oplus u(2|2))$ constraints. We believe that both definitions are equivalent [44]. The definition given in [43] relies on the spinor Lorentz harmonics [45], [46], [47] in $D=1+4$ dimensions parametrizing the coset $Spin(1,4)/(SU(2) \times SU(2))$, that is to two $SU(2)$-Majorana spinors in $D=1+4$ dimensions, and by solving corresponding harmonicity conditions one of them can be expressed in terms of the other. Similarly the constraints on the $D=5$ spinor harmonics parametrizing the coset $Spin(5)/(SU(2) \times SU(2))$ can be solved in terms of one $SU(2)$-Majorana spinor providing an evidence for the supertwistor definition of [40].

perparticle's Lagrangian is quadratic in supertwistors that facilitates quantization and can give the supertwistor description of the $D = 10$ $N = 2$ chiral supergravity spectrum compactified on $AdS_5 \times S^5$ [48], [49].

Thus the aim of this note is to derive these $AdS_5 \times S^5$ supertwistors starting from the first-order formulation of the $D = 1 + 9$ massless superparticle model in $AdS_5 \times S^5$ superspace [50], [51], [52], [53], [54]. We note that the superparticle's momentum components tangent to AdS_5 and S^5 subspaces can be realized as the bilinears of the constrained $SU(2)$-Majorana spinors in $D = 1+4$ and $D = 5$ dimensions respectively. Contraction of their $SU(2)$ indices gives two 4×4 traceless antisymmetric matrices that, when contracted with the $(4|4) \times (4|4)$ supermatrix of the $psu(2, 2|4)$ Cartan forms, extract bosonic components of the supervielbein tangent to AdS_5 and S^5 subspaces. This allows to bring the kinetic term in the superparticle's Lagrangian to the form appropriate for the introduction of the supertwistors. The details are worked out in Section 2. Section 3 is devoted to the quantization of the superparticle propagating in the AdS_5 subspace of the $AdS_5 \times S^5$ superspace both in the oscillator and ambitwistor approaches that complement each other. In the simplest case, when the particle's momentum tangent to AdS_5 is null, we find how the $D = 5$ $N = 8$ gauged supergravity multiplet is embedded into the ambitwistor superfield of homogeneity degree zero in each of the arguments. Two appendices supply relevant details of the spinor algebra and supermatrix realization of the generators of $psu(2, 2|4)$ isometry superalgebra of the $AdS_5 \times S^5$ superspace.

2. Supertwistor mechanics of massless particle in $AdS_5 \times S^5$ superspace

2.1. *Superparticle moving on AdS_5 subspace: definition of c-type supertwistors*

Let us start with the group-theoretic consideration of the $AdS_5 \times S^5$ supervielbein components tangent to the AdS_5 space-time and $SO(1, 4)$ connection 1-form. As is well known they can be identified with the Cartan forms associated with the generators of the $su(2, 2)$ subalgebra of $psu(2, 2|4)$. If $(4|4) \times (4|4)$ supermatrix $\mathcal{G}^{\mathcal{A}}{}_{\mathcal{B}}$ is a $PSU(2, 2|4)/(SO(1, 4) \times SO(5))$ supercoset representative then the left-invariant Cartan forms can be defined as

$$\mathcal{G}^{-1}d\mathcal{G}|_{su(2,2)} = iE_{\underline{mn}}(d)\tilde{\rho}^{\underline{mn}\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}} = iE_{m'}(d)\gamma^{m'\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}} + iE_{m'n'}(d)\gamma^{m'n'\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}} \in su(2,2), \quad (2.1)$$

where restriction to the $su(2, 2)$ subalgebra amounts to focusing on the upper diagonal block of the $(4|4) \times (4|4)$ supermatrix of Cartan 1-forms, $E_{\underline{mn}}(d)$ ($\underline{m}, \underline{n} = 0', 0, 1, 2, 3, 5$) are the $su(2, 2)$ Cartan 1-forms, $E^{0'}{}_{m'}(d) = E_{m'}(d)$ ($m' = 0, 1, 2, 3, 5$) are vielbein components tangent to the anti-de Sitter space and $E_{m'n'}(d)$ is the $SO(1, 4)$ connection 1-form. $D = 1 + 4$ γ-matrices $\gamma^{m'\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}$ ($\boldsymbol{\alpha}, \boldsymbol{\beta} = 1, 2, 3, 4$) represent the $su(2, 2)$ generators from the $\mathfrak{g}_{(2)}$ eigenspace of the $\mathbb{Z}_4$ outer automorphism of the $psu(2, 2|4)$ superalgebra and their antisymmetrized products $\gamma^{m'n'\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}$ represent generators from the $\mathfrak{g}_{(0)}$ eigenspace. In the considered realization important distinction between these generators is that matrices $\gamma^{m'}_{\boldsymbol{\alpha\beta}} = C_{\boldsymbol{\alpha\gamma}}\gamma^{m'\boldsymbol{\gamma}}{}_{\boldsymbol{\beta}}$ are antisymmetric, while $\gamma^{m'n'}_{\boldsymbol{\alpha\beta}} = C_{\boldsymbol{\alpha\gamma}}\gamma^{m'n'\boldsymbol{\gamma}}{}_{\boldsymbol{\beta}}$ are symmetric in the spinor indices. Thus the product $\lambda^i_{\boldsymbol{\alpha}}\lambda^{\boldsymbol{\beta}}_i$ ($i = 1, 2$) of two $D = 1 + 4$ $SU(2)$-Majorana spinors

$$\lambda^{\boldsymbol{\alpha}}_i : \quad \overline{\lambda^{\boldsymbol{\alpha}}_i} = (\lambda^{\boldsymbol{\beta}}_i)^{\dagger}\gamma^{0\boldsymbol{\beta}}{}_{\boldsymbol{\alpha}} = (\lambda^i_{\boldsymbol{\alpha}})^{\mathrm{T}} \quad (2.2)$$

acts as a projector onto the $\mathfrak{g}_{(2)}$ eigenspace. Promoting this $SU(2)$ doublet of $Spin(1,4)$ spinors to 'superspinors'

$$\lambda^i_{\boldsymbol{\alpha}} \to \lambda^i_{\mathcal{A}} = \begin{pmatrix} \lambda^i_{\boldsymbol{\alpha}} \\ 0 \end{pmatrix}, \quad \lambda^{\boldsymbol{\beta}}_i \to \lambda^{\mathcal{B}}_i = \begin{pmatrix} \lambda^{\boldsymbol{\beta}}_i \\ 0 \end{pmatrix} \tag{2.3}$$

one can write

$$\lambda^i_{\boldsymbol{\alpha}} \gamma^{m'\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}} \lambda^{\boldsymbol{\beta}}_i E_{m'}(d) = -i\lambda^i_{\mathcal{A}} \mathcal{G}^{-1\,\mathcal{A}}{}_{\mathcal{C}} d\mathcal{G}^{\mathcal{C}}{}_{\mathcal{B}} \lambda^{\mathcal{B}}_i . \tag{2.4}$$

This relation is the key to the twistor formulation of the superparticle on the AdS_5 subspace of the $AdS_5 \times S^5$ superspace. 1-form on the r.h.s. of (2.4) can be presented in terms of c-type AdS_5 supertwistors

$$\lambda^i_{\mathcal{A}} \mathcal{G}^{-1\,\mathcal{A}}{}_{\mathcal{C}} d\mathcal{G}^{\mathcal{C}}{}_{\mathcal{B}} \lambda^{\mathcal{B}}_i = \frac{1}{2}\left(\bar{\mathcal{Z}}^i_{\mathcal{A}} d\mathcal{Z}^{\mathcal{A}}_i - d\bar{\mathcal{Z}}^i_{\mathcal{A}} \mathcal{Z}^{\mathcal{A}}_i\right), \tag{2.5}$$

where

$$\mathcal{Z}^{\mathcal{A}}_i = \mathcal{G}^{\mathcal{A}}{}_{\mathcal{B}} \lambda^{\mathcal{B}}_i, \quad \bar{\mathcal{Z}}^i_{\mathcal{A}} = \lambda^i_{\mathcal{B}} \mathcal{G}^{-1\,\mathcal{B}}{}_{\mathcal{A}} : \quad \bar{\mathcal{Z}}^i_{\mathcal{A}} = (\mathcal{Z}^{\mathcal{B}}_i)^\dagger \mathcal{H}^{\mathcal{B}}{}_{\mathcal{A}}, \tag{2.6}$$

and that on the l.h.s. of (2.4) defines the kinetic part of the superparticle's Lagrangian in the first-order form of the space-time formulation

$$\lambda^i_{\boldsymbol{\alpha}} \gamma_{m'}{}^{\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}} \lambda^{\boldsymbol{\beta}}_i E^{m'}(d) = -p_{m'} E^{m'}(d) \tag{2.7}$$

with

$$p_{m'} = -\lambda^i_{\boldsymbol{\alpha}} \gamma_{m'}{}^{\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}} \lambda^{\boldsymbol{\beta}}_i : \quad p^{m'} p_{m'} = -\Lambda^2, \quad \Lambda = \lambda^i_{\boldsymbol{\alpha}} \lambda^{\boldsymbol{\alpha}}_i . \tag{2.8}$$

Vector $p^{m'}$ can be identified with the 5-momentum of the massless or massive particle provided Λ equals 0 or m.[5] In the former case $\lambda^i_{\boldsymbol{\alpha}}$ can be identified with the 4×2 rectangular block of the $D = 1+4$ spinor Lorentz-harmonic matrix that parametrizes the coset-space $Spin(1,4)/(SO(1,1) \times ISO(3))$ and $p^{m'}$ with the light-like vector-column from the respective vector Lorentz-harmonic matrix [56]. For m non-zero $\lambda^i_{\boldsymbol{\alpha}}$ can be related to the spinor Lorentz-harmonics parametrizing the coset $Spin(1,4)/SO(4) \sim Spin(1,4)/(SU(2) \times SU(2))$ and $p^{m'}$ with the vector-column of the respective vector Lorentz-harmonic matrix [44]. The net result is

$$p_{m'} E^{m'}(d) = \frac{i}{2}\left(\bar{\mathcal{Z}}^i_{\mathcal{A}} d\mathcal{Z}^{\mathcal{A}}_i - d\bar{\mathcal{Z}}^i_{\mathcal{A}} \mathcal{Z}^{\mathcal{A}}_i\right) \tag{2.9}$$

for $p^{m'}$ satisfying (2.8) and supertwistors constrained by the relations

$$\bar{\mathcal{Z}}^i_{\mathcal{A}} \mathcal{Z}^{\mathcal{A}}_j - \frac{1}{2}\delta^i_j \Lambda = 0 \tag{2.10}$$

that provide the supertwistor realization of the generators of the $su(2) \oplus u(1)$ gauge algebra. So the Lagrangian of the superparticle moving in the AdS_5 subspace of the $AdS_5 \times S^5$ superspace

$$\mathscr{L}^{AdS_5}_{\text{first-order}} = p_{m'} E^{m'}_\tau - \frac{g}{2}(p^{m'} p_{m'} + \Lambda^2) \tag{2.11}$$

can be reformulated in terms of the $SU(2)$ doublet of c-type $SU(2,2|4)$ supertwistors

$$\mathscr{L}^{AdS_5}_{\text{supertwistor}} = \frac{i}{2}\left(\bar{\mathcal{Z}}^i_{\mathcal{A}} \dot{\mathcal{Z}}^{\mathcal{A}}_i - \dot{\bar{\mathcal{Z}}}^i_{\mathcal{A}} \mathcal{Z}^{\mathcal{A}}_i\right) + a^{ij} \bar{\mathcal{Z}}_{\mathcal{A}i} \mathcal{Z}^{\mathcal{A}}_j + t(\bar{\mathcal{Z}}^i_{\mathcal{A}} \mathcal{Z}^{\mathcal{A}}_i - \Lambda), \tag{2.12}$$

[5] $-m$ is also a possible option. For details see [55].

where g, $a^{ij}=a^{ji}$ and t are the Lagrange multipliers. It can be checked that the number of the physical degrees of freedom in both formulations is the same.

To see how the incidence relations between the supertwistor components and the coordinates of the $AdS_5\times S^5$ superspace are encoded in (2.6) consider definite $PSU(2,2|4)/(SO(1,4)\times SO(5))$ representative, e.g. that discussed in [57], [50], [58]

$$\mathcal{G}^{\mathcal{A}}{}_{\mathcal{C}}=\mathcal{G}_{AdS_5}{}^{\mathcal{A}}{}_{\mathcal{B}}\mathcal{G}_{S^5}{}^{\mathcal{B}}{}_{\mathcal{C}} \tag{2.13}$$

with

$$\mathcal{G}_{AdS_5}=\exp(ix^mP_m+i\theta^\alpha_AQ^A_\alpha+i\bar\theta^{\dot\alpha A}\bar Q_{\dot\alpha A})\exp(i\eta^B_\beta S^\beta_B+i\bar\eta_{\dot\beta B}\bar S^{\dot\beta B})\exp(i\varphi D), \tag{2.14}$$

corresponding to the isomorphic realization of the $psu(2,2|4)$ superalgebra as the $D=4$ $N=4$ superconformal algebra, and

$$\mathcal{G}_{S^5}=\exp(iy^{I'}P^{I'}). \tag{2.15}$$

Such choice of the supercoset representative implies parametrization of anti-de Sitter space-time by the Poincare coordinates x^m $(m=0,1,2,3)$, φ with the line element

$$ds^2_{AdS_5}=e^{-2\varphi}dx^mdx_m+d\varphi^2. \tag{2.16}$$

$y^{I'}$ $(I'=1,2,3,4,5)$ are the S^5 coordinates, θ^α_A, $\bar\theta^{\dot\alpha A}$ and η^B_β, $\bar\eta_{\dot\beta B}$ $(\alpha,\beta=1,2,\ \dot\alpha,\dot\beta=1,2,$ $A,B=1,2,3,4)$ are odd coordinates associated with the $N=4$ Poincare and conformal supersymmetries respectively.

In the supermatrix realization of the relevant generators of $D=4$ $N=4$ superconformal algebra given in Appendix B the first factor in (2.13) acquires the form

$$\mathcal{G}_{AdS_5}{}^{\mathcal{A}}{}_{\mathcal{B}}=\begin{pmatrix} e^{\varphi/2}(\delta^\alpha_\beta-2i\tilde x_+^{\dot\delta\alpha}\bar\eta_{\dot\delta D}\eta^D_\beta-4\theta^\alpha_D\eta^D_\beta) & ie^{-\varphi/2}\tilde x_+^{\dot\beta\alpha} & 2i(\theta^\alpha_B+i\tilde x_+^{\dot\delta\alpha}\bar\eta_{\dot\delta B})\\ -2e^{\varphi/2}\bar\eta_{\dot\alpha D}\eta^D_\beta & e^{-\varphi/2}\delta^{\dot\beta}_{\dot\alpha} & 2i\bar\eta_{\dot\alpha B}\\ 2ie^{\varphi/2}(\eta^A_\beta-2\bar\theta^{\dot\delta A}\bar\eta_{\dot\delta D}\eta^D_\beta) & 2ie^{-\varphi/2}\bar\theta^{\dot\beta A} & \delta^A_B-4\bar\theta^{\dot\delta A}\bar\eta_{\dot\delta B}\end{pmatrix}, \tag{2.17}$$

where $\tilde x_+^{\dot\beta\alpha}=x^m\tilde\sigma_m^{\dot\beta\alpha}+2i\theta^\alpha_A\bar\theta^{\dot\beta A}$, and the second is

$$\mathcal{G}_{S^5}{}^{\mathcal{B}}{}_{\mathcal{C}}=\begin{pmatrix}\delta^\beta_\gamma & 0 & 0\\ 0 & \delta^{\dot\gamma}_{\dot\beta} & 0\\ 0 & 0 & U^B_C\end{pmatrix},\quad U^B_C=\cos|y|\delta^B_C+i\frac{\sin|y|}{|y|}(\gamma\cdot y). \tag{2.18}$$

For such supercoset representative the c-type supertwistor incidence relations (2.6) acquire the form

$$\mathcal{Z}^{\mathcal{A}}_i=\begin{pmatrix}\mu^\alpha_i\\ \bar\Lambda_{\dot\alpha i}\\ \eta^A_i\end{pmatrix}=\mathcal{G}_{AdS_5}{}^{\mathcal{A}}{}_{\mathcal{B}}\begin{pmatrix}-\lambda^\beta_i\\ \bar\lambda_{\dot\beta i}\\ 0\end{pmatrix}$$
$$=\begin{pmatrix}-e^{\varphi/2}\lambda^\alpha_i+ie^{-\varphi/2}\tilde x_+^{\dot\beta\alpha}\bar\lambda_{\dot\beta i}+4e^{\varphi/2}\theta^\alpha_B\eta^B_\beta\lambda^\beta_i+2ie^{\varphi/2}\tilde x_+^{\dot\beta\alpha}\bar\eta_{\dot\beta B}\eta^B_\beta\lambda^\beta_i\\ e^{-\varphi/2}\bar\lambda_{\dot\alpha i}+2e^{\varphi/2}\bar\eta_{\dot\alpha B}\eta^B_\beta\lambda^\beta_i\\ 2ie^{-\varphi/2}\bar\theta^{\dot\beta A}\bar\lambda_{\dot\beta i}-2ie^{\varphi/2}\eta^A_\beta\lambda^\beta_i+4ie^{\varphi/2}\bar\theta^{\dot\beta A}\bar\eta_{\dot\beta B}\eta^B_\beta\lambda^\beta_i\end{pmatrix}. \tag{2.19}$$

Accordingly for the dual supertwistor we have

$$\begin{aligned}\bar{\mathcal{Z}}^i_{\mathcal{A}} = (\Lambda^i_\alpha\ \bar\mu^{\dot\alpha i}\ \bar\eta^i_A) = (\lambda^i_\beta\ \bar\lambda^{\dot\beta i}\ 0)\, \mathcal{G}^{-1}_{AdS_5}{}^{\mathcal{B}}{}_{\mathcal{A}}:\\ \Lambda^i_\alpha = e^{-\varphi/2}\lambda^i_\alpha - 2e^{\varphi/2}\bar\lambda^{\dot\beta i}\bar\eta_{\dot\beta D}\eta^D_\alpha,\\ \bar\mu^{\dot\alpha i} = e^{\varphi/2}\bar\lambda^{\dot\alpha i} - ie^{-\varphi/2}\tilde{x}_-^{\dot\alpha\beta}\lambda^i_\beta - 4e^{\varphi/2}\bar\lambda^{\dot\beta i}\bar\eta_{\dot\beta D}\bar\theta^{\dot\alpha D} + 2ie^{\varphi/2}\bar\lambda^{\dot\beta i}\bar\eta_{\dot\beta D}\eta^D_\gamma\tilde{x}_-^{\dot\alpha\gamma},\\ \bar\eta^i_A = -2ie^{-\varphi/2}\lambda^i_\beta\theta^\beta_A - 2ie^{\varphi/2}\bar\lambda^{\dot\beta i}\bar\eta_{\dot\beta A} + 4ie^{\varphi/2}\bar\lambda^{\dot\beta i}\bar\eta_{\dot\beta D}\eta^D_\gamma\theta^\gamma_A,\end{aligned} \tag{2.20}$$

where

$$\mathcal{G}^{-1}_{AdS_5}{}^{\mathcal{B}}{}_{\mathcal{A}} = \begin{pmatrix} e^{-\varphi/2}\delta^\beta_\alpha & -ie^{-\varphi/2}\tilde{x}_-^{\dot\alpha\beta} & -2ie^{-\varphi/2}\theta^\beta_A \\ -2e^{\varphi/2}\bar\eta_{\dot\beta D}\eta^D_\alpha & e^{\varphi/2}(\delta^{\dot\alpha}_{\dot\beta} + 2i\bar\eta_{\dot\beta D}\eta^D_\gamma\tilde{x}_-^{\dot\alpha\gamma} - 4\bar\eta_{\dot\beta D}\bar\theta^{\dot\alpha D}) & -2ie^{\varphi/2}(\bar\eta_{\dot\beta A} - 2\bar\eta_{\dot\beta D}\eta^D_\gamma\theta^\gamma_A) \\ -2i\eta^B_\alpha & -2i(\bar\theta^{\dot\alpha B} - i\tilde{x}_-^{\dot\alpha\gamma}\eta^B_\gamma) & \delta^B_A - 4\eta^B_\gamma\theta^\gamma_A \end{pmatrix} \tag{2.21}$$

and the conjugation rules of the supertwistor components are $(\Lambda^i_\alpha)^\dagger = \bar\Lambda_{\dot\alpha i}$, $(\mu^\alpha_i)^\dagger = \bar\mu^{\dot\alpha i}$, $(\eta^A_i)^\dagger = \bar\eta^i_A$.

In the bosonic limit introduced supertwistors reduce to those proposed in [32] modulo the overall rescaling. Penrose–Ferber supertwistors arise in the boundary limit $\varphi \to -\infty$

$$\mathcal{Z}^{\mathcal{A}}_i|_{\text{boundary}} = \lim_{\varphi\to-\infty} e^{\varphi/2}\mathcal{Z}^{\mathcal{A}}_i, \quad \bar{\mathcal{Z}}^i_{\mathcal{A}}|_{\text{boundary}} = \lim_{\varphi\to-\infty} e^{\varphi/2}\bar{\mathcal{Z}}^i_{\mathcal{A}} \tag{2.22}$$

and we also call them boundary supertwistors.

2.2. *Definition of a-type supertwistors*

Now the above discussion can be generalized to the case of superparticle moving also on the S^5 part of the superbackground. Cartan forms associated with the generators of the $su(4)$ subalgebra of $psu(2,2|4)$ decompose into the sum

$$\mathcal{G}^{-1}d\mathcal{G}|_{su(4)} = iE^{IJ}(d)\tilde\rho^{IJA}{}_B = iE^{I'}(d)\gamma^{I'A}{}_B + iE^{I'J'}(d)\gamma^{I'J'A}{}_B \in su(4), \tag{2.23}$$

where restriction to the $su(4)$ subalgebra amounts to considering the lower diagonal block of the $(4|4)\times(4|4)$ supermatrix of Cartan forms, $E^{IJ}(d)$ $(I = 1,2,3,4,5,6)$ are the $su(4)$ Cartan forms, 1-form $E^{6I'}(d) = E^{I'}(d)$ is identified with the $D = 10$ supervielbein components tangent to S^5 and $E^{I'J'}(d)$ – with the $SO(5)$ connection. $D = 5$ γ-matrices $\gamma^{I'}_{AB} = C_{AD}\gamma^{I'D}{}_B$ are antisymmetric in the spinor indices and belong to the $\mathfrak{g}_{(2)}$ eigenspace under the $\mathbb{Z}_4$ automorphism of the $psu(2,2|4)$ superalgebra, while the $so(5)$ generators realized by $\gamma^{I'J'}_{AB} = C_{AD}\gamma^{I'J'D}{}_B$ are symmetric in the spinor indices and belong to the $\mathfrak{g}_{(0)}$ eigenspace. So that similarly to the AdS_5 case taking $D = 5$ $SU(2)$-Majorana spinor $\ell^{i'}_A$ $(i' = 1,2)$:

$$(\ell^{i'}_A)^\dagger = (\ell^A_{i'})^{\mathrm{T}} \tag{2.24}$$

it is possible to construct a matrix projector onto the $\mathfrak{g}_{(2)}$ eigenspace. Presenting this spinor as a 'superspinor'

$$\ell^{i'}_{\mathcal{A}} = \begin{pmatrix} 0 \\ \ell^{i'}_A \end{pmatrix} \tag{2.25}$$

yields

$$\ell^{i'}_{A}\gamma^{I'A}{}_{B}\ell^{B}_{i'}E^{I'}(d) = -i\ell^{i'}_{\mathcal{A}}\mathcal{G}^{-1\,\mathcal{A}}{}_{\mathcal{C}}d\mathcal{G}^{\mathcal{C}}{}_{\mathcal{B}}\ell^{\mathcal{B}}_{i'}. \tag{2.26}$$

Introducing $SU(2)$ doublet of a-type $SU(2,2|4)$ supertwistors

$$\Psi^{\mathcal{A}}_{i'} = \mathcal{G}^{\mathcal{A}}{}_{\mathcal{B}}\ell^{\mathcal{B}}_{i'} \tag{2.27}$$

and its dual

$$\bar{\Psi}^{i'}_{\mathcal{A}} = \ell^{i'}_{\mathcal{B}}\mathcal{G}^{-1\,\mathcal{B}}{}_{\mathcal{A}}: \quad \bar{\Psi}^{i'}_{\mathcal{A}} = (\Psi^{\mathcal{B}}_{i'})^{\dagger}\mathcal{H}^{\mathcal{B}}{}_{\mathcal{A}}, \tag{2.28}$$

projected Cartan 1-form on the r.h.s. of (2.26) can be written as

$$\ell^{i'}_{\mathcal{A}}\mathcal{G}^{-1\,\mathcal{A}}{}_{\mathcal{C}}d\mathcal{G}^{\mathcal{C}}{}_{\mathcal{B}}\ell^{\mathcal{B}}_{i'} = \frac{1}{2}(\bar{\Psi}^{i'}_{\mathcal{A}}d\Psi^{\mathcal{A}}_{i'} - d\bar{\Psi}^{i'}_{\mathcal{A}}\Psi^{\mathcal{A}}_{i'}). \tag{2.29}$$

Further introducing 5-vector

$$p^{I'} = -\ell^{i'}_{A}\gamma^{I'A}{}_{B}\ell^{B}_{i'}, \tag{2.30}$$

(2.26) acquires the form

$$p^{I'}E^{I'}(d) = \frac{i}{2}(\bar{\Psi}^{i'}_{\mathcal{A}}d\Psi^{\mathcal{A}}_{i'} - d\bar{\Psi}^{i'}_{\mathcal{A}}\Psi^{\mathcal{A}}_{i'}). \tag{2.31}$$

The norm of this vector equals

$$p^{I'}p^{I'} = L^2, \quad L = \ell^{i'}_{A}\ell^{A}_{i'} \tag{2.32}$$

and supertwistors (2.27), (2.28) satisfy four constraints

$$\bar{\Psi}^{i'}_{\mathcal{A}}\Psi^{\mathcal{A}}_{j'} - \frac{1}{2}\delta^{i'}_{j'}L = 0. \tag{2.33}$$

Like for the c-type supertwistors we can give explicit form of the incidence relations for a-type supertwistors (2.27) and their duals (2.28) for the $PSU(2,2|4)/(SO(1,4)\times SO(5))$ supercoset representative (2.13)

$$\Psi^{\mathcal{A}}_{i'} = \begin{pmatrix} m^{\alpha}_{i'} \\ \bar{\chi}_{\dot{\alpha}i'} \\ L^{A}_{i'} \end{pmatrix} = \mathcal{G}^{\mathcal{A}}{}_{\mathcal{B}}\begin{pmatrix} 0 \\ 0 \\ \ell^{B}_{i'} \end{pmatrix} = \begin{pmatrix} 2i(\theta^{\alpha}_{B} + i\tilde{x}^{\dot{\beta}\alpha}_{+}\bar{\eta}_{\dot{\beta}B})\tilde{\ell}^{B}_{i'} \\ 2i\bar{\eta}_{\dot{\alpha}B}\tilde{\ell}^{B}_{i'} \\ (\delta^{A}_{B} - 4\bar{\theta}^{\dot{\beta}A}\bar{\eta}_{\dot{\beta}B})\tilde{\ell}^{B}_{i'} \end{pmatrix} \tag{2.34}$$

and

$$\begin{gathered} \bar{\Psi}^{i'}_{\mathcal{A}} = (\chi^{i'}_{\alpha}\ \bar{m}^{\dot{\alpha}i'}\ \bar{L}^{i'}_{A}) = (0\,0\,\ell^{i'}_{B})\mathcal{G}^{-1\mathcal{B}}{}_{\mathcal{A}}: \\ \chi^{i'}_{\alpha} = -2i\bar{\tilde{\ell}}^{i'}_{B}\eta^{B}_{\alpha}, \quad \bar{m}^{\dot{\alpha}i'} = -2i\bar{\tilde{\ell}}^{i'}_{B}(\bar{\theta}^{\dot{\alpha}B} - i\tilde{x}^{\dot{\alpha}\beta}_{-}\eta^{B}_{\beta}), \quad \bar{L}^{i'}_{A} = \bar{\tilde{\ell}}^{i'}_{B}(\delta^{B}_{A} - 4\eta^{B}_{\beta}\theta^{\beta}_{A}), \end{gathered} \tag{2.35}$$

where $\tilde{\ell}^{B}_{i'} = U^{B}{}_{C}\ell^{C}_{i'}$, $\bar{\tilde{\ell}}^{i'}_{B} = \ell^{i'}_{C}\bar{U}^{C}{}_{B}$ and the component form of the conjugation rules in (2.28) is $(m^{\alpha}_{i'})^{\dagger} = \bar{m}^{\dot{\alpha}i'}$, $(\chi^{i'}_{\alpha})^{\dagger} = \bar{\chi}_{\dot{\alpha}i'}$, $(L^{A}_{i'})^{\dagger} = \bar{L}^{i'}_{A}$. Observe that components of $\Psi^{\mathcal{A}}_{i'}$ and $\bar{\Psi}^{i'}_{\mathcal{A}}$ supertwistors depend not only on the S^5 coordinates $y^{I'}$ but also on $\tilde{x}^{\dot{\alpha}\beta}$ so that in general it is not possible to view c- and a-type supertwistors as related solely to the AdS_5 and S^5 bosonic subspaces of the $AdS_5\times S^5$ superspace. Also note that c- and a-type supertwistors are by definition mutually orthogonal

$$\bar{\Psi}^{i'}_{\mathcal{A}}\mathcal{Z}^{\mathcal{A}}_{i} = 0, \quad \bar{\mathcal{Z}}^{i}_{\mathcal{A}}\Psi^{\mathcal{A}}_{i'} = 0. \tag{2.36}$$

2.3. Classical formulation of massless particle in $AdS_5 \times S^5$ superspace in terms of c- and a-type supertwistors

The above results can be put together to describe the $D = 1 + 9$ massless superparticle on the $AdS_5 \times S^5$ superbackground. First, its null 10-momentum $p^{\hat m}$ can be assembled from 5-vectors (2.8) and (2.30)

$$p^{\hat m} = (p^{m'},\ p^{I'}):\quad p^{\hat m} p_{\hat m} = 0 \to \Lambda = \pm L. \tag{2.37}$$

The sign ambiguity is resolved by requiring the closure of the algebra of the first-class constraints. Then the first-order Lagrangian (2.11) of the superparticle propagating on the AdS_5 subspace of $AdS_5 \times S^5$ space generalizes to

$$\mathscr{L}^{AdS_5\times S^5}_{\text{first-order}} = p_{m'} E^{m'}_\tau + p^{I'} E^{I'}_\tau - \frac{g}{2}(p^{m'} p_{m'} + p^{I'} p^{I'}) \tag{2.38}$$

and its supertwistor version acquires the form

$$\begin{aligned}\mathscr{L}^{AdS_5\times S^5}_{\text{supertwistor}} = {} & \tfrac{i}{2}\left(\bar{\mathcal{Z}}^i_{\mathcal{A}}\dot{\mathcal{Z}}^{\mathcal{A}}_i - \dot{\bar{\mathcal{Z}}}{}^i_{\mathcal{A}}\mathcal{Z}^{\mathcal{A}}_i\right) + \tfrac{i}{2}(\bar\Psi^{i'}_{\mathcal{A}}\dot\Psi^{\mathcal{A}}_{i'} - \dot{\bar\Psi}{}^{i'}_{\mathcal{A}}\Psi^{\mathcal{A}}_{i'}) \\ & + a^{ij}\bar{\mathcal{Z}}_{\mathcal{A}i}\mathcal{Z}^{\mathcal{A}}_j + s^{i'j'}\bar\Psi_{\mathcal{A}i'}\Psi^{\mathcal{A}}_{j'} + t(\bar{\mathcal{Z}}^i_{\mathcal{A}}\mathcal{Z}^{\mathcal{A}}_i \pm \bar\Psi^{i'}_{\mathcal{A}}\Psi^{\mathcal{A}}_{i'}) \\ & + i\varkappa^i_{i'}\bar\Psi^{i'}_{\mathcal{A}}\mathcal{Z}^{\mathcal{A}}_i + i\bar\varkappa^{i'}_i\bar{\mathcal{Z}}^i_{\mathcal{A}}\Psi^{\mathcal{A}}_{i'}.\end{aligned} \tag{2.39}$$

Lagrange multipliers $a^{ij} = a^{ji}$, $s^{i'j'} = s^{j'i'}$ and t are even and $\varkappa^i_{i'}$, $\bar\varkappa^{i'}_i$ are odd. They introduce the first-class constraints

$$\begin{aligned} L_i{}^j &= \bar{\mathcal{Z}}^j_{\mathcal{A}}\mathcal{Z}^{\mathcal{A}}_i - \tfrac{1}{2}\delta^j_i\bar{\mathcal{Z}}^k_{\mathcal{A}}\mathcal{Z}^{\mathcal{A}}_k \approx 0, \\ M_{i'}{}^{j'} &= \bar\Psi^{j'}_{\mathcal{A}}\Psi^{\mathcal{A}}_{i'} - \tfrac{1}{2}\delta^{j'}_{i'}\bar\Psi^{k'}_{\mathcal{A}}\Psi^{\mathcal{A}}_{k'} \approx 0, \\ T_\pm &= \bar{\mathcal{Z}}^i_{\mathcal{A}}\mathcal{Z}^{\mathcal{A}}_i \pm \bar\Psi^{i'}_{\mathcal{A}}\Psi^{\mathcal{A}}_{i'} \approx 0, \\ \Phi^{i'}_i &= \bar\Psi^{i'}_{\mathcal{A}}\mathcal{Z}^{\mathcal{A}}_i \approx 0, \quad \bar\Phi^i_{i'} = \bar{\mathcal{Z}}^i_{\mathcal{A}}\Psi^{\mathcal{A}}_{i'} \approx 0.\end{aligned} \tag{2.40}$$

On the Dirac brackets (D.B.)

$$\{\mathcal{Z}^{\mathcal{A}}_i, \bar{\mathcal{Z}}^j_{\mathcal{B}}\}_{D.B.} = i\delta^{\mathcal{A}}_{\mathcal{B}}\delta^j_i, \tag{2.41}$$

$$\{\Psi^{\mathcal{A}}_{i'}, \bar\Psi^{j'}_{\mathcal{B}}\}_{D.B.} = i\delta^{\mathcal{A}}_{\mathcal{B}}\delta^{j'}_{i'} \tag{2.42}$$

constraints (2.40) satisfy the $psu(2|2) \oplus u(1)$ superalgebra relations

$$\begin{gathered} \{L_i{}^j, L_k{}^l\}_{D.B.} = i\delta^l_i L_k{}^j - i\delta^j_k L_i{}^l, \quad \{M_{i'}{}^{j'}, M_{k'}{}^{l'}\}_{D.B.} = i\delta^{l'}_{i'} M_{k'}{}^{j'} - i\delta^{j'}_{k'} M_{i'}{}^{l'}; \\ \{\Phi^{i'}_i, \bar\Phi^j_{j'}\}_{D.B.} = i\delta^{i'}_{j'} L_i{}^j + i\delta^j_i M_{j'}{}^{i'} + i\delta^j_i\delta^{i'}_{j'} T_+; \\ \{L_i{}^j, \Phi^{k'}_k\}_{D.B.} = -i\delta^j_k\Phi^{k'}_i + \tfrac{i}{2}\delta^i_j\Phi^{k'}_k, \quad \{L_i{}^j, \bar\Phi^k_{k'}\}_{D.B.} = i\delta^k_i\bar\Phi^j_{k'} - \tfrac{i}{2}\delta^j_i\bar\Phi^k_{k'}, \\ \{M_{i'}{}^{j'}, \Phi^{k'}_k\}_{D.B.} = i\delta^{k'}_{i'}\Phi^{j'}_k - \tfrac{i}{2}\delta^{j'}_{i'}\Phi^{k'}_k, \quad \{M_{i'}{}^{j'}, \bar\Phi^k_{k'}\}_{D.B.} = -i\delta^{j'}_{k'}\bar\Phi^k_{i'} + \tfrac{i}{2}\delta^{j'}_{i'}\bar\Phi^k_{k'}. \end{gathered} \tag{2.43}$$

Above D.B. relations show that $L_i{}^j$ and $M_{i'}{}^{j'}$ generate two copies of the $su(2)$ algebra and $\Phi^{i'}_i$, $\bar\Phi^i_{i'}$ are the supersymmetry generators. Altogether they span $psu(2|2)$ superalgebra. The presence of T_+ on the r.h.s. of the anticommutation relations of the supersymmetry generators necessitates its inclusion into the set of the first-class constraints and fixes the sign in (2.37) and (2.40). The real dimension of the first-class constraint set (7|8) is such that the number of the physical degrees of freedom of the superparticle matches both in the supertwistor and superspace formulations.

Let us finally discuss the gauge symmetries of the $D = 1 + 9$ massless superparticle action in the supertwistor formulation

$$S = \int d\tau \mathscr{L}^{AdS_5\times S^5}_{\text{supertwistor}} \tag{2.44}$$

that are generated by the first-class constraints (2.40). $L_i{}^j$ and $M_{i'}{}^{j'}$ are the generators of the local $SU(2)$ rotations of each doublet of the supertwistors

$$\begin{aligned}&\delta_{\mathrm{a}}\mathcal{Z}_i^{\mathcal{A}} = -i\mathrm{a}_i{}^j\mathcal{Z}_j^{\mathcal{A}}, \quad \delta_{\mathrm{a}}\bar{\mathcal{Z}}^i_{\mathcal{A}} = i\mathrm{a}_j{}^i\bar{\mathcal{Z}}^j_{\mathcal{A}}, \quad \delta_{\mathrm{a}}a_i{}^j = \dot{\mathrm{a}}_i{}^j;\\ &\delta_{\mathrm{s}}\Psi^{\mathcal{A}}_{i'} = -i\mathrm{s}_{i'}{}^{j'}\Psi^{\mathcal{A}}_{j'}, \quad \delta_{\mathrm{s}}\bar{\Psi}^{i'}_{\mathcal{A}} = i\mathrm{s}_{j'}{}^{i'}\bar{\Psi}^{j'}_{\mathcal{A}}, \quad \delta_{\mathrm{s}}s_{i'}{}^{j'} = \dot{\mathrm{s}}_{i'}{}^{j'}\end{aligned} \tag{2.45}$$

with the parameters $\mathrm{a}_i{}^j(\tau)$ and $\mathrm{s}_{i'}{}^{j'}(\tau)$. The constraint T_+ generates phase rotation of the supertwistors

$$\delta_{\mathrm{t}}\mathcal{Z}_i^{\mathcal{A}} = i\mathrm{t}\mathcal{Z}_i^{\mathcal{A}}, \quad \delta_{\mathrm{t}}\bar{\mathcal{Z}}^i_{\mathcal{A}} = -i\mathrm{t}\bar{\mathcal{Z}}^i_{\mathcal{A}}, \quad \delta_{\mathrm{t}}\Psi^{\mathcal{A}}_{i'} = i\mathrm{t}\Psi^{\mathcal{A}}_{i'}, \quad \delta_{\mathrm{t}}\bar{\Psi}^{i'}_{\mathcal{A}} = -i\bar{\Psi}^{i'}_{\mathcal{A}}, \quad \delta_{\mathrm{t}}t = \dot{\mathrm{t}}, \tag{2.46}$$

where $\mathrm{t}(\tau)$ is the local parameter. Odd constraints $\Phi_i^{i'}$ and $\bar{\Phi}^i_{i'}$ generate the 8-parametric remnant of the κ-symmetry of the superparticle action in the superspace formulation with the parameters $\kappa_i^{i'}(\tau)$ and $\bar{\kappa}^i_{i'}(\tau)$

$$\begin{aligned}&\delta_\kappa\mathcal{Z}_i^{\mathcal{A}} = (-)^{\varepsilon(\Psi)}\kappa_i^{i'}\Psi^{\mathcal{A}}_{i'}, \quad \delta_\kappa\bar{\Psi}^{i'}_{\mathcal{A}} = \kappa_i^{i'}\bar{\mathcal{Z}}^i_{\mathcal{A}}, \quad \delta_\kappa\bar{\varkappa}_i^{i'} = \kappa_i^{i'},\\ &\delta_{\bar{\kappa}}\bar{\mathcal{Z}}^i_{\mathcal{A}} = \bar{\kappa}^i_{i'}\bar{\Psi}^{i'}_{\mathcal{A}}, \quad \delta_{\bar{\kappa}}\Psi^{\mathcal{A}}_{i'} = (-)^{\varepsilon(\mathcal{Z})}\bar{\kappa}^i_{i'}\mathcal{Z}_i^{\mathcal{A}}, \quad \delta_{\bar{\kappa}}\varkappa^i_{i'} = \bar{\kappa}^i_{i'}.\end{aligned} \tag{2.47}$$

$\varepsilon(\mathcal{Z})$ and $\varepsilon(\Psi)$ equal 0 for bosonic components and 1 for fermionic components of respective supertwistors.

3. Aspects of quantum theory of massless superparticle in AdS_5 subspace of $AdS_5 \times S^5$ superspace

In this section we consider quantum theory of the superparticle moving in the AdS_5 subspace of $AdS_5 \times S^5$ superspace. Quantization is performed both in terms of the oscillators and supertwistors that yields two (complementary) views on the $D = 5$ $N = 8$ gauged supergravity multiplet.

3.1. Oscillator quantization

There is well-known intimate relation between the $SU(2)$ oscillators and (c-type) supertwistors [59], [60]. For the bosonic components of supertwistors it is based on two forms of the $SU(2,2)$ 'metric' that connects fundamental and antifundamental representations. It is off-diagonal in the (super)twistor basis and diagonal in the oscillator basis. Thus the $SU(2)$ bosonic oscillators are defined by the linear combinations of the supertwistor bosonic components

$$a^\alpha = \frac{1}{\sqrt{2}}(-\mu^\alpha + \bar{\Lambda}^\alpha), \quad a_\alpha = \frac{1}{\sqrt{2}}(-\bar{\mu}_\alpha + \Lambda_\alpha) \tag{3.1}$$

and

$$b_\alpha = \frac{1}{\sqrt{2}}(\bar{\mu}_\alpha + \Lambda_\alpha), \quad b^\alpha = \frac{1}{\sqrt{2}}(\mu^\alpha + \bar{\Lambda}^\alpha). \tag{3.2}$$

The indices of internal $SU(2)$ symmetry of the supertwistors have been suppressed for the moment. In quantum theory these oscillators satisfy the commutation relations

$$[a_\alpha, a^\beta] = \delta_\alpha^\beta, \quad [b^\alpha, b_\beta] = \delta_\beta^\alpha \tag{3.3}$$

which can be deduced from the quantum counterpart of the D.B. relations in (2.41). Relations (3.3) suggest interpretation of a^α and b_α as raising oscillators and of a_α and b^α as lowering oscillators.

Since the definition of $SU(2,2|4)$ supertwistors assumes that $SU(4)$ 'metric' is unit, fermionic $SU(2)$ oscillators can be identified with odd components of supertwistors

$$\eta^A = \begin{pmatrix} \alpha^a \\ \beta^{\dot a} \end{pmatrix}, \qquad \bar\eta_A = \begin{pmatrix} \alpha_a \\ \beta_{\dot a} \end{pmatrix}, \tag{3.4}$$

where we used the decomposition of the $SU(4)$ (anti)fundamental representation $\mathbf{4} = \mathbf{2} \oplus \tilde{\mathbf{2}}$ ($\bar{\mathbf{4}} = \bar{\mathbf{2}} \oplus \bar{\tilde{\mathbf{2}}}$). Fermionic oscillators satisfy anticommutation relations

$$\{\alpha^a, \alpha_b\} = \delta_b^a, \quad \{\beta^{\dot a}, \beta_{\dot b}\} = \delta_{\dot b}^{\dot a} \tag{3.5}$$

as follows from (2.41). It is conventional to treat α^a and $\beta_{\dot a}$ as raising oscillators, while α_a and $\beta^{\dot a}$ as lowering ones.

Now with each of the conjugate pairs of supertwistors (2.6) associate a set of the above introduced bosonic and fermionic oscillators

$$\begin{aligned} (\mathcal{Z}_1^{\mathcal{A}}, \bar{\mathcal{Z}}^1_{\mathcal{A}}) &\to (a^\alpha(1), b^\alpha(1), \alpha^a(1), \beta^{\dot a}(1); a_\alpha(1), b_\alpha(1), \alpha_a(1), \beta_{\dot a}(1)), \\ (\mathcal{Z}_2^{\mathcal{A}}, \bar{\mathcal{Z}}^2_{\mathcal{A}}) &\to (a^\alpha(2), b^\alpha(2), \alpha^a(2), \beta^{\dot a}(2); a_\alpha(2), b_\alpha(2), \alpha_a(2), \beta_{\dot a}(2)). \end{aligned} \tag{3.6}$$

Then the $su(2) \times u(1)$ constraints (2.10) can be brought to the following form in terms of the oscillators

$$\begin{aligned} -N_{(a)}(1) + N_{(b)}(1) - N_{(\alpha)}(1) + N_{(\beta)}(1) - \tfrac{1}{2}\Lambda &\approx 0, \\ -N_{(a)}(2) + N_{(b)}(2) - N_{(\alpha)}(2) + N_{(\beta)}(2) - \tfrac{1}{2}\Lambda &\approx 0, \\ -a_\alpha(2)a^\alpha(1) + b_\alpha(2)b^\alpha(1) + \alpha_a(2)\alpha^a(1) + \beta_{\dot a}(2)\beta^{\dot a}(1) &\approx 0, \\ -a_\alpha(1)a^\alpha(2) + b_\alpha(1)b^\alpha(2) + \alpha_a(1)\alpha^a(2) + \beta_{\dot a}(1)\beta^{\dot a}(2) &\approx 0, \end{aligned} \tag{3.7}$$

where $N_{(a)}(1) = a^\alpha(1)a_\alpha(1)$, $N_{(b)}(1) = b_\alpha(1)b^\alpha(1)$, $N_{(\alpha)}(1) = \alpha^a(1)\alpha_a(1)$ and $N_{(\beta)}(1) = \beta_{\dot a}(1)\beta^{\dot a}(1)$ are the oscillator number operators for bosonic and fermionic oscillators of the first set. Analogous definitions apply to the second set of oscillators. Important feature of the constraints (3.7) is that they commute with the raising supersymmetry operators

$$a^\alpha(1)\beta_{\dot a}(1) + a^\alpha(2)\beta_{\dot a}(2), \quad b_\alpha(1)\alpha^a(1) + b_\alpha(2)\alpha^a(2) \tag{3.8}$$

used to generate lowest-weight vectors corresponding to different $SU(2,2) \times SU(4)$ representations within the same $SU(2,2|4)$ supermultiplet as well as with the bosonic

$$a^\beta(1)b_\alpha(1) + a^\beta(2)b_\alpha(2) \tag{3.9}$$

and fermionic

$$\alpha^b(1)\beta_{\dot a}(1) + \alpha^b(2)\beta_{\dot a}(2) \tag{3.10}$$

raising operators which applied to a lowest-weight vector generate the whole set of basis vectors of the corresponding $SU(2,2) \times SU(4)$ representation [48], [61]. So one can consider the constraints (3.7) to act on the $SU(2,2) \times SU(4)$ lowest-weight vectors. In the simplest case $\Lambda = 0$, i.e. when the superparticle does not have non-zero momentum components in the directions tangent to S^5, it is not hard to verify that the only lowest-weight vector annihilated by the constraints is the oscillator vacuum associated with the $D = 5$ $N = 8$ gauged supergravity multiplet [48]. For $\Lambda \neq 0$ the constraints select $SU(2) \times U(1)$ invariant lowest-weight vectors which correspond to the supermultiplets discussed in [59] (see also [55]).

3.2. Ambitwistor quantization

Quantum counterpart of the D.B. relations (2.41)

$$[\mathcal{Z}_i^{\mathcal{A}}, \bar{\mathcal{Z}}^j_{\mathcal{B}}\} = \delta^{\mathcal{A}}_{\mathcal{B}} \delta_i^j \tag{3.11}$$

is consistent with the well-known in twistor theory [62], [4] realizations of quantized (super)twistors as multiplication and differentiation operators. For the ambitwistor space description of the quantized superparticle the supertwistors

$$\mathcal{Z}_1^{\mathcal{A}} = \begin{pmatrix} Z_1^{\boldsymbol{\alpha}} \\ \eta_1^A \end{pmatrix} \equiv \mathcal{Z}^{\mathcal{A}} = \begin{pmatrix} Z^{\boldsymbol{\alpha}} \\ \eta^A \end{pmatrix}, \quad \bar{\mathcal{Z}}^1_{\mathcal{A}} = (\bar{Z}^1_{\boldsymbol{\alpha}},\ \bar{\eta}^1_A) \equiv \bar{\mathcal{Z}}_{\mathcal{A}} = (\bar{Z}_{\boldsymbol{\alpha}},\ \bar{\eta}_A) \tag{3.12}$$

and

$$\mathcal{Z}_2^{\mathcal{A}} = \begin{pmatrix} Z_2^{\boldsymbol{\alpha}} \\ \eta_2^A \end{pmatrix} \equiv \mathcal{W}^{\mathcal{A}} = \begin{pmatrix} W^{\boldsymbol{\alpha}} \\ \zeta^A \end{pmatrix}, \quad \bar{\mathcal{Z}}^2_{\mathcal{A}} = (\bar{Z}^2_{\boldsymbol{\alpha}},\ \bar{\eta}^2_A) \equiv \bar{\mathcal{W}}_{\mathcal{A}} = (\bar{W}_{\boldsymbol{\alpha}},\ \bar{\zeta}_A) \tag{3.13}$$

can be conveniently realized (modulo their relabeling) as

$$\begin{aligned} &Z^{\boldsymbol{\alpha}} \to Z^{\boldsymbol{\alpha}}, \quad \bar{Z}_{\boldsymbol{\alpha}} \to -\tfrac{\partial}{\partial Z^{\boldsymbol{\alpha}}}, \quad \eta^A \to \eta^A, \quad \bar{\eta}_A \to \tfrac{\vec{\partial}}{\partial \eta^A} \\ &W^{\boldsymbol{\alpha}} \to \tfrac{\partial}{\partial \bar{W}_{\boldsymbol{\alpha}}}, \quad \bar{W}_{\boldsymbol{\alpha}} \to \bar{W}_{\boldsymbol{\alpha}}, \quad \zeta^A \to \tfrac{\vec{\partial}}{\partial \bar{\zeta}_A}, \quad \bar{\zeta}_A \to \bar{\zeta}_A, \end{aligned} \tag{3.14}$$

where odd derivatives are defined to act from the left. Then in the simplest case $\Lambda = 0$ the $su(2) \oplus u(1)$ constraints (2.10) applied to the superparticle's wave function acquire the form

$$\begin{aligned} &\mathcal{Z}^{\mathcal{A}} \frac{\partial}{\partial \mathcal{Z}^{\mathcal{A}}} F_{(0,0)}(\mathcal{Z}, \bar{\mathcal{W}}) = \left(Z^{\boldsymbol{\alpha}} \frac{\partial}{\partial Z^{\boldsymbol{\alpha}}} + \eta^A \frac{\partial}{\partial \eta^A} \right) F_{(0,0)}(\mathcal{Z}, \bar{\mathcal{W}}) = 0, \\ &\bar{\mathcal{W}}_{\mathcal{A}} \frac{\partial}{\partial \bar{\mathcal{W}}_{\mathcal{A}}} F_{(0,0)}(\mathcal{Z}, \bar{\mathcal{W}}) = \left(\bar{W}_{\boldsymbol{\alpha}} \frac{\partial}{\partial \bar{W}_{\boldsymbol{\alpha}}} + \bar{\zeta}_A \frac{\partial}{\partial \bar{\zeta}_A} \right) F_{(0,0)}(\mathcal{Z}, \bar{\mathcal{W}}) = 0 \end{aligned} \tag{3.15}$$

and

$$\bar{\mathcal{W}}_{\mathcal{A}} \mathcal{Z}^{\mathcal{A}} F_{(0,0)}(\mathcal{Z}, \bar{\mathcal{W}}) = 0. \tag{3.16}$$

Eqs. (3.15) imply that $F_{(0,0)}$ has homogeneity degree zero both in $\mathcal{Z}$ and $\bar{\mathcal{W}}$ that is indicated by its subscripts and the condition (3.16) can be taken into account by adding a δ-function factor

$$F_{(0,0)}(\mathcal{Z}, \bar{\mathcal{W}}) = \delta(\bar{\mathcal{W}} \mathcal{Z}) f_{(1,1)}(\mathcal{Z}, \bar{\mathcal{W}}). \tag{3.17}$$

The function $f_{(1,1)}(\mathcal{Z},\bar{\mathcal{W}})$ has the power series decomposition in odd supertwistor components

$$
\begin{aligned}
f_{(1,1)}(\mathcal{Z},\bar{\mathcal{W}}) = {} & h_{(1,1)}(Z,\bar{W}) + \psi_{(0,1)A}(Z,\bar{W})\eta^A + \psi_{(1,0)}{}^A(Z,\bar{W})\bar{\zeta}_A \\
& + b_{(-1,1)[AB]}(Z,\bar{W})\eta^A\eta^B + a_{(0,0)}{}^B_A(Z,\bar{W})\eta^A\bar{\zeta}_B + b_{(1,-1)}{}^{[AB]}(Z,\bar{W})\bar{\zeta}_A\bar{\zeta}_B \\
& + \lambda_{(-2,1)[ABC]}(Z,\bar{W})\eta^A\eta^B\eta^C + \lambda_{(-1,0)}{}^C_{[AB]}(Z,\bar{W})\eta^A\eta^B\bar{\zeta}_C \\
& + \lambda_{(0,-1)}{}^{[BC]}_A(Z,\bar{W})\eta^A\bar{\zeta}_B\bar{\zeta}_C + \lambda_{(1,-2)}{}^{[ABC]}(Z,\bar{W})\bar{\zeta}_A\bar{\zeta}_B\bar{\zeta}_C \\
& + \varphi_{(-3,1)[ABCD]}(Z,\bar{W})\eta^A\eta^B\eta^C\eta^D + \varphi_{(-2,0)}{}^D_{[ABC]}(Z,\bar{W})\eta^A\eta^B\eta^C\bar{\zeta}_D \\
& + \varphi_{(-1,-1)}{}^{[CD]}_{[AB]}(Z,\bar{W})\eta^A\eta^B\bar{\zeta}_C\bar{\zeta}_D + \varphi_{(0,-2)}{}^{[BCD]}_A(Z,\bar{W})\eta^A\bar{\zeta}_B\bar{\zeta}_C\bar{\zeta}_D + \\
& + \varphi_{(1,-3)}{}^{[ABCD]}(Z,\bar{W})\bar{\zeta}_A\bar{\zeta}_B\bar{\zeta}_C\bar{\zeta}_D + \dots
\end{aligned}
\tag{3.18}
$$

where dots stand for higher terms in the series that as we shall see below are irrelevant.

Since the Penrose transform between the homogeneous functions of the introduced twistors and the fields on $AdS_5 \times S^5$ superspace is yet to be elaborated,[6] we can meanwhile consider the Penrose transform of the corresponding functions of the boundary ambitwistors that should produce the fields of $D=4$ $N=4$ conformal supergravity multiplet [63].[7] According to the AdS/CFT dictionary these fields of the $D=4$ $N=4$ conformal supergravity multiplet correspond to the boundary values of non-normalizable solutions of the bulk $D=5$ $N=8$ gauged supergravity equations linearized around AdS_5 [64], [65].[8] Thus the function $h_{(1,1)}(Z,\bar{W})$ yields Minkowski space graviton field $h_{\alpha(2)\dot{\alpha}(2)}(x)$ (see, e.g. [68] and more recent [69] for the details of the ambitwistor transform). The Penrose transform for other functions in the series (3.18) deserves more detailed treatment.

First, observe that the $SU(4)$ representations spanned by monomials composed of odd supertwistor components are in general reducible and decompose into a sum of irreducible ones, e.g. as

$$
\eta^A\bar{\zeta}_B = \widetilde{\eta^A\bar{\zeta}_B} + \frac{1}{4}\delta^A_B\eta^C\bar{\zeta}_C, \quad \widetilde{\eta^A\bar{\zeta}_B} = \eta^A\bar{\zeta}_B - \frac{1}{4}\delta^A_B\eta^C\bar{\zeta}_C \tag{3.19}
$$

illustrating the rule $\mathbf{4}\times\bar{\mathbf{4}} = \mathbf{15}+\mathbf{1}$. Widetilde over a monomial indicates that its $SU(4)$ traceless part is taken. Then multiplying (3.19) by the function $a_{(0,0)}{}^B_A(Z,\bar{W})$ allows to represent corresponding term in (3.18) as the sum of two contributions that transform irreducibly under $SU(4)$

$$
a_{(0,0)}{}^B_A(Z,\bar{W})\eta^A\bar{\zeta}_B = \tilde{a}_{(0,0)}{}^B_A(Z,\bar{W})\widetilde{\eta^A\bar{\zeta}_B} + \frac{1}{4}a_{(0,0)}{}^B_B(Z,\bar{W})\eta^A\bar{\zeta}_A. \tag{3.20}
$$

In the first summand tilde over $a_{(0,0)}{}^B_A$ denotes its traceless part and in the second summand ambitwistor constraint (3.16) can be used to replace $\eta^A\bar{\zeta}_A$ by $-Z^{\alpha}\bar{W}_{\alpha}$. The Penrose

[6] See, however, Ref. [33], where, based on the realization of AdS_5 space as a projective manifold, the Penrose transform for the case of spin 0 and $1/2$ fields was considered.

[7] Hereinafter the same notation introduced above for the bulk ambitwistors is used for the boundary ones. In the Poincare coordinates taking the boundary limit of AdS_5 supertwistors is accompanied by their rescaling but since the superparticle's wave function is scale-invariant in both arguments the relations (3.17) and (3.18) are valid as they stand either for bulk or boundary supertwistors.

[8] General discussion of the correspondence between the AdS bulk gauge fields and the boundary conformal (shadow) fields not restricted to the case of low spins may be found in [66], [67].

transform of $\tilde{a}_{(0,0)}{}^{B}_{A}(Z,\bar{W})$ yields precisely 15 vector fields $\tilde{a}_{\alpha\dot{\alpha}}{}^{B}_{A}(x)$ of $N=4$ conformal supergravity multiplet and that of $Z^{\boldsymbol{\alpha}}\bar{W}_{\boldsymbol{\alpha}}\,a_{(0,0)}{}^{B}_{B}(Z,\bar{W})$ gives zero. The reasoning behind the latter statement is the following. In the oscillator approach the counterpart of $Z^{\boldsymbol{\alpha}}\bar{W}_{\boldsymbol{\alpha}}$ is the operator $-a_{\alpha}(2)a^{\alpha}(1)+b_{\alpha}(2)b^{\alpha}(1)$. The $SU(2,2)$ part of the lowest-weight vector for the $SU(2,2)\times SU(4)$ representation associated with $a_{(0,0)}{}^{B}_{B}(Z,\bar{W})$ is $a^{\alpha}(1)b_{\beta}(2)|0\rangle$ [48] and it is annihilated by $-a_{\alpha}(2)a^{\alpha}(1)+b_{\alpha}(2)b^{\alpha}(1)$. It would be interesting to find a twistor analogue of this argument. Other ambitwistor functions $b_{(1,-1)}{}^{[AB]}(Z,\bar{W})$ and $b_{(-1,1)[AB]}(Z,\bar{W})$ that appear in the second order of the decomposition (3.18) correspond via the Penrose transform to symmetric $SL(2,\mathbb{C})$ spinor fields $b_{\dot{\alpha}(2)}{}^{[AB]}(x)$ and $b_{\alpha[AB]}(x)$, i.e. (anti)selfdual antisymmetric rank-2 tensor fields, in the $SU(4)$ representation **6**.

Similarly the $\mathbf{6}\times\mathbf{6}$ representation spanned by the quartic monomial $\eta^{A}\eta^{B}\bar{\zeta}_{C}\bar{\zeta}_{D}$ is known to decompose as $\mathbf{6}\times\mathbf{6}=\mathbf{20'}+\mathbf{15}+\mathbf{1}$

$$\begin{aligned}\eta^{A}\eta^{B}\bar{\zeta}_{C}\bar{\zeta}_{D}=\ &\widetilde{\eta^{A}\eta^{B}\bar{\zeta}_{C}\bar{\zeta}_{D}}-\tfrac{1}{2}\eta^{E}\bar{\zeta}_{E}(\delta^{A}_{C}\widetilde{\eta^{B}\bar{\zeta}_{D}}-\delta^{A}_{D}\widetilde{\eta^{B}\bar{\zeta}_{C}}-\delta^{B}_{C}\widetilde{\eta^{A}\bar{\zeta}_{D}}+\delta^{B}_{D}\widetilde{\eta^{A}\bar{\zeta}_{C}})\\&-\tfrac{1}{12}(\delta^{A}_{C}\delta^{B}_{D}-\delta^{A}_{D}\delta^{B}_{C})(\eta^{E}\bar{\zeta}_{E})^{2},\end{aligned}\tag{3.21}$$

where

$$\begin{aligned}\widetilde{\eta^{A}\eta^{B}\bar{\zeta}_{C}\bar{\zeta}_{D}}=\ &\eta^{A}\eta^{B}\bar{\zeta}_{C}\bar{\zeta}_{D}+\tfrac{1}{2}\eta^{E}\bar{\zeta}_{E}(\delta^{A}_{C}\eta^{B}\bar{\zeta}_{D}-\delta^{A}_{D}\eta^{B}\bar{\zeta}_{C}-\delta^{B}_{C}\eta^{A}\bar{\zeta}_{D}+\delta^{B}_{D}\eta^{A}\bar{\zeta}_{C})\\&-\tfrac{1}{6}(\delta^{A}_{C}\delta^{B}_{D}-\delta^{A}_{D}\delta^{B}_{C})(\eta^{E}\bar{\zeta}_{E})^{2}\end{aligned}\tag{3.22}$$

is traceless and transforms according to the $\mathbf{20'}$ representation. Multiplication of (3.21) by $\varphi_{(-1,-1)}{}^{[CD]}_{[AB]}(Z,\bar{W})$ gives

$$\begin{aligned}\varphi_{(-1,-1)}{}^{[CD]}_{[AB]}(Z,\bar{W})\eta^{A}\eta^{B}\bar{\zeta}_{C}\bar{\zeta}_{D}=\ &\tilde{\varphi}_{(-1,-1)}{}^{[CD]}_{[AB]}(Z,\bar{W})\widetilde{\eta^{A}\eta^{B}\bar{\zeta}_{C}\bar{\zeta}_{D}}\\&-2\tilde{\varphi}_{(-1,-1)}{}^{CB}_{CA}(Z,\bar{W})\eta^{D}\bar{\zeta}_{D}\widetilde{\eta^{A}\bar{\zeta}_{B}}\\&-\tfrac{1}{6}\varphi_{(-1,-1)}{}^{AB}_{AB}(Z,\bar{W})(\eta^{C}\bar{\zeta}_{C})^{2},\end{aligned}\tag{3.23}$$

where $\tilde{\varphi}_{(-1,-1)}{}^{[CD]}_{[AB]}(Z,\bar{W})$ and $\tilde{\varphi}_{(-1,-1)}{}^{CB}_{CA}(Z,\bar{W})$ are traceless. The Penrose transform of $\tilde{\varphi}_{(-1,-1)}{}^{[CD]}_{[AB]}(Z,\bar{W})$ produces 20 scalar fields $\tilde{\varphi}^{[CD]}_{[AB]}(x)$ from the $N=4$ conformal supergravity multiplet, whereas the Penrose transform of other twistor functions in (3.23) should give zero by the argument similar to that of the previous paragraph.

Looking at other quartic terms in (3.18) reveals two $SU(4)$ singlets $\varphi_{(-3,1)[ABCD]}(Z,\bar{W})$ and $\varphi_{(1,-3)}{}^{[ABCD]}(Z,\bar{W})$ together with functions $\varphi_{(-2,0)}{}^{D}_{[ABC]}(Z,\bar{W})$ and $\varphi_{(0,-2)}{}^{[BCD]}_{A}(Z,\bar{W})$ transforming in the reducible $SU(4)$ representations that decompose as $\mathbf{4}\times\mathbf{4}=\mathbf{10}+\mathbf{6}$ and $\bar{\mathbf{4}}\times\bar{\mathbf{4}}=\overline{\mathbf{10}}+\mathbf{6}$ so that we can write

$$\begin{aligned}\varphi_{(-2,0)}{}^{D}_{[ABC]}(Z,\bar{W})\eta^{A}\eta^{B}\eta^{C}\bar{\zeta}_{D}=\ &\tilde{\varphi}_{(-2,0)}{}^{D}_{[ABC]}(Z,\bar{W})\widetilde{\eta^{A}\eta^{B}\eta^{C}\bar{\zeta}_{D}}\\&+\tfrac{3}{2}\varphi_{(-2,0)}{}^{D}_{D[AB]}(Z,\bar{W})\eta^{C}\bar{\zeta}_{C}\eta^{A}\eta^{B},\\ \varphi_{(0,-2)}{}^{[BCD]}_{A}(Z,\bar{W})\eta^{A}\bar{\zeta}_{B}\bar{\zeta}_{C}\bar{\zeta}_{D}=\ &\tilde{\varphi}_{(0,-2)}{}^{[BCD]}_{A}(Z,\bar{W})\widetilde{\eta^{A}\bar{\zeta}_{B}\bar{\zeta}_{C}\bar{\zeta}_{D}}\\&+\tfrac{3}{2}\varphi_{(0,-2)}{}^{D[AB]}_{D}(Z,\bar{W})\eta^{C}\bar{\zeta}_{C}\bar{\zeta}_{A}\bar{\zeta}_{B}.\end{aligned}\tag{3.24}$$

Monomials with widetilde

$$\eta^A\widetilde{\eta^B\eta^C\bar\zeta_D} = \eta^A\eta^B\eta^C\bar\zeta_D - \tfrac{1}{2}\delta^A_D\eta^E\bar\zeta_E\eta^B\eta^C + \tfrac{1}{2}\delta^B_D\eta^E\bar\zeta_E\eta^A\eta^C - \tfrac{1}{2}\delta^C_D\eta^E\bar\zeta_E\eta^A\eta^B,$$
$$\eta^A\widetilde{\bar\zeta_B\bar\zeta_C\bar\zeta_D} = \eta^A\bar\zeta_B\bar\zeta_C\bar\zeta_D - \tfrac{1}{2}\delta^A_B\eta^E\bar\zeta_E\bar\zeta_C\bar\zeta_D + \tfrac{1}{2}\delta^A_C\eta^E\bar\zeta_E\bar\zeta_B\bar\zeta_D - \tfrac{1}{2}\delta^A_D\eta^E\bar\zeta_E\bar\zeta_B\bar\zeta_C \tag{3.25}$$

are traceless and project out **10** and $\bar{\mathbf{10}}$ representations from $\varphi_{(-2,0)}{}^{D}_{[ABC]}(Z,\bar W)$ and $\varphi_{(0,-2)}{}^{[BCD]}_{A}(Z,\bar W)$. Comparison with the $N=4$ conformal supergravity multiplet shows that $\tilde\varphi_{(-2,0)}{}^{D}_{[ABC]}(Z,\bar W)$ and $\tilde\varphi_{(0,-2)}{}^{[BCD]}_{A}(Z,\bar W)$ should produce scalar fields via the Penrose transform. This can be explained in the following way. Ambitwistor functions with the above homogeneity degrees can be obtained by applying the first-order differential operators

$$\bar W_{\boldsymbol\alpha} I^{\boldsymbol{\alpha\beta}}\frac{\partial}{\partial Z^{\boldsymbol\beta}}, \qquad Z^{\boldsymbol\alpha} I_{\boldsymbol{\alpha\beta}}\frac{\partial}{\partial \bar W_{\boldsymbol\beta}} \tag{3.26}$$

to the $(-1,-1)$ ambitwistor functions that correspond to the scalar fields

$$\varphi_{(-2,0)}{}^{D}_{[ABC]}(Z,\bar W) = \left(\bar W I\tfrac{\partial}{\partial Z}\right)\varphi_{(-1,-1)}{}^{D}_{[ABC]}(Z,\bar W),$$
$$\varphi_{(0,-2)}{}^{[BCD]}_{A}(Z,\bar W) = \left(Z I\tfrac{\partial}{\partial \bar W}\right)\varphi_{(-1,-1)}{}^{[BCD]}_{A}(Z,\bar W). \tag{3.27}$$

Differential operators (3.26) involve infinity twistors

$$I^{\boldsymbol{\alpha\beta}} = \begin{pmatrix} \varepsilon^{\alpha\beta} & 0 \\ 0 & 0 \end{pmatrix}, \quad I_{\boldsymbol{\alpha\beta}} = \begin{pmatrix} 0 & 0 \\ 0 & \varepsilon^{\dot\alpha\dot\beta} \end{pmatrix} \tag{3.28}$$

and have the following form in terms of the $SL(2,\mathbb{C})$ spinor parts

$$\bar W_{\boldsymbol\alpha} I^{\boldsymbol{\alpha\beta}}\frac{\partial}{\partial Z^{\boldsymbol\beta}} = -v^{\alpha}\frac{\partial}{\partial\mu^{\alpha}}, \quad Z^{\boldsymbol\alpha} I_{\boldsymbol{\alpha\beta}}\frac{\partial}{\partial \bar W_{\boldsymbol\beta}} = -\bar u^{\dot\alpha}\frac{\partial}{\partial\bar\nu^{\dot\alpha}}. \tag{3.29}$$

In twistor theory composition of these operators is known to give the space-time d'Alambertian and plays a role in twistor description of massive fields (see, e.g. [62]). Let us note that the action of these operators does not change the sum of homogeneity degrees of the ambitwistor function. So the Penrose transform of $\varepsilon^{EABC}\tilde\varphi_{(-1,-1)}{}^{D}_{[ABC]}(Z,\bar W)$ and $\varepsilon_{EBCD}\tilde\varphi_{(-1,-1)}{}^{[BCD]}_{A}(Z,\bar W)$ yields scalar fields $\varphi^{(DE)}(x)$ and $\bar\varphi_{(AE)}(x)$ in **10** and $\bar{\mathbf{10}}$ representations respectively, whereas the Penrose transform of $Z^{\boldsymbol\alpha}\bar W_{\boldsymbol\alpha}\varphi_{(-2,0)}{}^{D}_{D[AB]}(Z,\bar W)$ and $Z^{\boldsymbol\alpha}\bar W_{\boldsymbol\alpha}\varphi_{(0,-2)}{}^{D[AB]}_{D}(Z,\bar W)$ should give zero. The argument generalizes one that has been used above for the $SU(2,2)$ representations corresponding to vector and scalar fields. Using the relation between the twistor components and $SU(2)\times SU(2)$ oscillators it is possible to find the $SU(2,2)$ lowest-weight vectors for the fields on the r.h.s. of (3.27): $(b_\alpha(2)b_\beta(1) - b_\beta(2)b_\alpha(1))|0\rangle$ and $(a^\alpha(2)a^\beta(1) - a^\beta(2)a^\alpha(1))|0\rangle$ [48] and to verify that they are annihilated by the operator $-a_\alpha(2)a^\alpha(1) + b_\alpha(2)b^\alpha(1)$. Note that each antisymmetrized product of bosonic raising oscillators does not change the $SU(2)$ labels of the $SU(2,2)$ lowest-weight vector on which it acts but increases the AdS energy by one unit [59] and so the operators (3.26) are the twistor counterparts of such antisymmetrized products of raising oscillators.

Analogously ambitwistor functions $\varphi_{(-3,1)[ABCD]}(Z,\bar W)$ and $\varphi_{(1,-3)}{}^{[ABCD]}(Z,\bar W)$ can be viewed as resulting upon the repeated application of the operators (3.26) to the functions homogeneous of degree $(-1,-1)$

$$\begin{aligned}\varphi_{(-3,1)[ABCD]}(Z,\bar W) &= \left(\bar W I \frac{\partial}{\partial Z}\right)^2 \varphi_{(-1,-1)[ABCD]}(Z,\bar W),\\ \varphi_{(1,-3)}{}^{[ABCD]}(Z,\bar W) &= \left(Z I \frac{\partial}{\partial \bar W}\right)^2 \varphi_{(-1,-1)}{}^{[ABCD]}(Z,\bar W).\end{aligned} \tag{3.30}$$

The Penrose transform of $\varepsilon_{ABCD}\varphi_{(-1,-1)}{}^{[ABCD]}(Z,\bar W)$ and $\varepsilon^{ABCD}\varphi_{(-1,-1)[ABCD]}(Z,\bar W)$ furnishes two remaining scalar fields $\varphi(x)$ and $\bar\varphi(x)$ of the $N=4$ conformal supergravity multiplet. Respective $SU(2,2)$ lowest weight vectors are $(b_\alpha(2)b_\beta(1)-b_\beta(2)b_\alpha(1))^2|0\rangle$ and $(a^\alpha(2)a^\beta(1)-a^\beta(2)a^\alpha(1))^2|0\rangle$ [48].

Turning to the discussion of the correspondence between the fermionic fields of $N=4$ conformal supergravity and odd components in the decomposition (3.18) we find that eight gravitini $\psi_{\alpha\dot\alpha(2)}{}^A(x)$ and $\psi_{\alpha(2)\dot\alpha A}(x)$ are obtained via the Penrose transform of $\psi_{(1,0)}{}^A(Z,\bar W)$ and $\psi_{(0,1)A}$ [70].

Spin 1/2 fields are obtained by performing the Penrose transform of the cubic terms in (3.18). To this end let us note that similarly to the monomials in (3.19) and (3.21), $\eta^A\eta^B\bar\zeta_C$ and $\eta^A\bar\zeta_B\bar\zeta_C$ furnish reducible representations of $SU(4)$ that decompose as $\mathbf{6}\times\bar{\mathbf{4}}=\mathbf{20}+\mathbf{4}$ and $\mathbf{6}\times\mathbf{4}=\bar{\mathbf{20}}+\bar{\mathbf{4}}$:

$$\begin{aligned}\eta^A\eta^B\bar\zeta_C &= \widetilde{\eta^A\eta^B\bar\zeta_C} + \tfrac13\delta^B_C\eta^D\bar\zeta_D\eta^A - \tfrac13\delta^A_C\eta^D\bar\zeta_D\eta^B,\\ \eta^A\bar\zeta_B\bar\zeta_C &= \widetilde{\eta^A\bar\zeta_B\bar\zeta_C} + \tfrac13\delta^A_B\eta^D\bar\zeta_D\bar\zeta_C - \tfrac13\delta^A_C\eta^D\bar\zeta_D\bar\zeta_B,\end{aligned} \tag{3.31}$$

where $\widetilde{\eta^A\eta^B\bar\zeta_C}$ and $\widetilde{\eta^A\bar\zeta_B\bar\zeta_C}$ span $\mathbf{20}$ and $\bar{\mathbf{20}}$ representations. This allows to decompose respective terms in the expansion (3.18) as

$$\begin{aligned}\lambda_{(-1,0)}{}^C_{[AB]}(Z,\bar W)\eta^A\eta^B\bar\zeta_C &= \tilde\lambda_{(-1,0)}{}^C_{[AB]}(Z,\bar W)\widetilde{\eta^A\eta^B\bar\zeta_C} - \tfrac23\lambda_{(-1,0)}{}^B_{[BA]}(Z,\bar W)\eta^C\bar\zeta_C\eta^A,\\ \lambda_{(0,-1)}{}^{[BC]}_A(Z,\bar W)\eta^A\bar\zeta_B\bar\zeta_C &= \tilde\lambda_{(0,-1)}{}^{[BC]}_A(Z,\bar W)\widetilde{\eta^A\bar\zeta_B\bar\zeta_C} + \tfrac23\lambda_{(0,-1)}{}^{[BA]}_B(Z,\bar W)\eta^C\bar\zeta_C\bar\zeta_A.\end{aligned} \tag{3.32}$$

Ambitwistor functions $\tilde\lambda_{(-1,0)}{}^C_{[AB]}(Z,\bar W)$ and $\tilde\lambda_{(0,-1)}{}^{[BC]}_A(Z,\bar W)$ give upon the Penrose transform spin 1/2 fields $\tilde\lambda_\alpha{}^C_{[AB]}(x)$ and $\tilde\lambda_{\dot\alpha}{}^{[BC]}_A(x)$ in $\bar{\mathbf{20}}$ and $\mathbf{20}$ representations of $SU(4)$. Penrose transform of other ambitwistor functions in (3.32) should give zero by extending the argument given above for the $SU(2,2)$ representations corresponding to bosonic fields. Remaining third order terms in (3.18) supply additional spin 1/2 fields in a way similar to (3.27)

$$\begin{aligned}\lambda_{(-2,1)[ABC]}(Z,\bar W) &= \left(\bar W I \frac{\partial}{\partial Z}\right)\lambda_{(-1,0)[ABC]}(Z,\bar W),\\ \lambda_{(1,-2)}{}^{[ABC]}(Z,\bar W) &= \left(Z I \frac{\partial}{\partial \bar W}\right)\lambda_{(0,-1)}{}^{[ABC]}(Z,\bar W).\end{aligned} \tag{3.33}$$

Then the Penrose transform of $\varepsilon^{DABC}\lambda_{(-1,0)[ABC]}(Z,\bar W)$ and $\varepsilon_{DABC}\lambda_{(0,-1)}{}^{[ABC]}(Z,\bar W)$ yields remaining fields $\lambda_\alpha{}^D(x)$ and $\lambda_{\dot\alpha D}(x)$ from the $N=4$ conformal supergravity multiplet.

Finally terms omitted in (3.18) do not correspond to any space-time fields since their homogeneity degrees sum up to less than -2 that is the lower bound corresponding to the scalar fields.

4. Conclusion and discussion

This note addressed the issue of the definition of supertwistors for the $AdS_5 \times S^5$ superspace, which isometry is described by the $PSU(2,2|4)$ supergroup that is also the superconformal symmetry of $D=4$ $N=4$ Minkowski superspace. Twistor reformulation of the massless particle model on the $PSU(2,2|4)/(SO(1,4) \times SO(5))$ supermanifold led us to consider $SU(2)$ doublets of c- and a-type $SU(2,2|4)$ supertwistors subject to seven bosonic and eight fermionic constraints as the $AdS_5 \times S^5$ supertwistors. This justifies proposed earlier on the group-theoretical grounds definition of $AdS_5 \times S^5$ supertwistors [40] and allows to find the incidence relations with the $AdS_5 \times S^5$ supercoordinates via the $PSU(2,2|4)/(SO(1,4) \times SO(5))$ supercoset representative.

The superparticle's Lagrangian in the supertwistor formulation is quadratic and yields only the first-class constraints that are generators of the $psu(2|2) \oplus u(1)$ gauge symmetry. This should facilitate transition to the quantum theory. Here we examined quantization of the superparticle moving in the AdS_5 subspace of the $AdS_5 \times S^5$ superspace. In this case only four bosonic constraints associated with the $su(2) \oplus u(1)$ gauge symmetry are imposed on the superparticle's wave function that we chose to depend on one c-type supertwistor and one dual c-type supertwistor. This breaks manifest $SU(2)$ symmetry but makes contact with the ambitwistor description of the massless fields on AdS_5 space-time [33], [70]. The constraints imply that the wave function is homogeneous of degree zero in its arguments and can be expanded in the supertwistor odd components. Then the Penrose transform of the component ambitwistor functions gives off-shell fields of the $D=4$ $N=4$ conformal supergravity multiplet that serve as the boundary values for on-shell fields from the $D=5$ $N=8$ gauged supergravity multiplet in the AdS_5 bulk. Quantization of the complete model with all the constraints taken into account should give supertwistor description of the whole spectrum of $D=10$ $N=2$ chiral supergravity compactified on $AdS_5 \times S^5$ space [48], [49]. To make contact with the field-theoretic description [49] it is necessary to work out the details how the Penrose transform of the homogeneous functions of c-type and a-type supertwistors produces fields in AdS_5 space-time.

The superparticle model discussed in this note can be straight-forwardly generalized to that of a tensionless string with the action

$$
\begin{aligned}
S = \int d\tau d\sigma \mathscr{L}^{AdS_5\times S^5}_{\text{stwistor T=0 string}}:& \\
\mathscr{L}^{AdS_5\times S^5}_{\text{stwistor T=0 string}} = {}& \tfrac{i}{2}\rho^{\mu}\left(\bar{\mathcal{Z}}^{i}{}_{\mathcal{A}}\partial_{\mu}\mathcal{Z}_{i}^{\mathcal{A}} - \partial_{\mu}\bar{\mathcal{Z}}^{i}{}_{\mathcal{A}}\mathcal{Z}_{i}^{\mathcal{A}}\right) + \tfrac{i}{2}\rho^{\mu}(\bar{\Psi}^{i'}{}_{\mathcal{A}}\partial_{\mu}\Psi_{i'}^{\mathcal{A}} - \partial_{\mu}\bar{\Psi}^{i'}{}_{\mathcal{A}}\Psi_{i'}^{\mathcal{A}}) \\
&+ a^{ij}\bar{\mathcal{Z}}_{\mathcal{A}i}\mathcal{Z}_{j}^{\mathcal{A}} + s^{i'j'}\bar{\Psi}_{\mathcal{A}i'}\Psi_{j'}^{\mathcal{A}} + t(\bar{\mathcal{Z}}^{i}{}_{\mathcal{A}}\mathcal{Z}_{i}^{\mathcal{A}} + \bar{\Psi}^{i'}{}_{\mathcal{A}}\Psi_{i'}^{\mathcal{A}}) \\
&+ i\varkappa^{i}_{i'}\bar{\Psi}^{i'}{}_{\mathcal{A}}\mathcal{Z}_{i}^{\mathcal{A}} + i\bar{\varkappa}^{i'}_{i}\bar{\mathcal{Z}}^{i}{}_{\mathcal{A}}\Psi_{i'}^{\mathcal{A}},
\end{aligned}
\tag{4.1}
$$

extending the ambitwistor string model [71]. Depending on the quantization prescription and the choice of the vacuum [72], [73] we expect it to provide the world-sheet CFT interpretation for the scattering amplitudes in $D=5$ $N=8$ gauged supergravity or to produce $D=10$ $N=2B$ higher-spin massless supermultiplets compactified on $AdS_5 \times S^5$. Generalization to the tensile string model is also feasible and should give an interesting reformulation of the $PSU(2,2|4)/(SO(1,4) \times SO(5))$ supercoset action [37], [38].

We anticipate that the c- and a-type supertwistors can also be used to construct the supertwistor action for $D=5$ $N=8$ gauged supergravity and possibly for the whole tower of supermultiplets arising in the compactification of $D=10$ $N=2$ chiral supergravity on $AdS_5 \times S^5$

in a way similar that of [74], where the super-Yang–Mills action was written in the supertwistor space.

Along the same lines it is possible to consider supertwistors and supertwistor formulations for point-like and extended objects on other supersymmetric supergravity backgrounds, in particular, those with the OSp-type superisometries, some of which were outlined in [40].

Appendix A. Details of the spinor algebra

In this appendix various properties of γ-matrices and spinors used in the main text are collected.

Chiral antisymmetric γ-matrices in $D = 2 + 4$ dimensions satisfy the Clifford algebra relations

$$\rho^{\underline{m}}_{\alpha\beta}\tilde{\rho}^{\underline{n}\beta\gamma} + \rho^{\underline{n}}_{\alpha\beta}\tilde{\rho}^{\underline{m}\beta\gamma} = -2\eta^{\underline{mn}}\delta_\alpha^\gamma, \quad \eta^{\underline{mn}} = \mathrm{diag}(-,-,+,+,+,+) \tag{A.1}$$

and the Hermitian conjugation rule

$$(\tilde{\rho}^{\underline{m}\alpha\beta})^\dagger = H_\beta{}^\gamma \rho^{\underline{m}}_{\gamma\delta} H^\delta{}_\alpha, \tag{A.2}$$

where matrices $H_\beta{}^\gamma$ and $H^\delta{}_\alpha$ equal

$$H_\alpha{}^\beta = i\rho^0_{\alpha\gamma}\tilde{\rho}^{0'\gamma\beta}, \quad H^\alpha{}_\beta = i\tilde{\rho}^{0'\alpha\gamma}\rho^0_{\gamma\beta}. \tag{A.3}$$

They satisfy $H_\alpha{}^\beta H_\beta{}^\gamma = \delta_\alpha^\gamma$, $H^\alpha{}_\beta H^\beta{}_\gamma = \delta_\gamma^\alpha$ and are related by the transposition $H_\alpha{}^\beta = (H^\beta{}_\alpha)^{\mathrm{T}}$.

Passing to $D = 1 + 4$ dimensions, matrices $\rho^{0'}_{\alpha\beta}$ and $\tilde{\rho}^{0'\alpha\beta}$ are identified with the charge conjugation matrix and its inverse

$$\rho^{0'}_{\alpha\beta} = C_{\alpha\beta}, \quad \tilde{\rho}^{0'\alpha\beta} = C^{\alpha\beta}: \quad C_{\alpha\beta}C^{\beta\gamma} = \delta_\alpha^\gamma \tag{A.4}$$

that are used to raise and lower $Spin(1,4)$ spinor indices

$$\lambda^\alpha = C^{\alpha\beta}\lambda_\beta \quad \lambda_\alpha = C_{\alpha\beta}\lambda^\beta. \tag{A.5}$$

$D = 1 + 4$ γ-matrices are defined as

$$\gamma^{m'}_{\alpha\beta} = i\rho^{m'}_{\alpha\beta}, \quad \gamma^{m'\alpha\beta} = i\tilde{\rho}^{m'\alpha\beta} \tag{A.6}$$

and satisfy

$$\gamma^{m'}{}_\alpha{}^\beta\gamma^{n'}{}_\beta{}^\gamma + \gamma^{n'}{}_\alpha{}^\beta\gamma^{m'}{}_\beta{}^\gamma = -2\eta^{m'n'}\delta_\alpha^\gamma, \quad \gamma^{m'}{}_\alpha{}^\beta = C_{\alpha\gamma}\gamma^{m'\gamma\beta} = -\gamma^{m'}_{\alpha\gamma}C^{\gamma\beta}. \tag{A.7}$$

For them the conjugation rule (A.2) transforms to

$$(\gamma^{m'}{}_\alpha{}^\beta)^\dagger = \gamma^0{}_\beta{}^\gamma\gamma^{m'}{}_\gamma{}^\delta\gamma^0{}_\delta{}^\alpha \tag{A.8}$$

and also $(C^{\alpha\beta})^\dagger = C_{\beta\alpha}$. Antisymmetrized products of γ-matrices

$$\gamma^{m'n'\alpha}{}_\beta = -\frac{i}{4}(\gamma^{m'\alpha}{}_\gamma\gamma^{n'\gamma}{}_\beta - \gamma^{n'\alpha}{}_\gamma\gamma^{m'\gamma}{}_\beta) \tag{A.9}$$

provide realization of the $so(1,4)$ algebra relations

$$[\gamma^{m'n'}, \gamma^{k'l'}] = i(\eta^{m'l'}\gamma^{n'k'} - \eta^{m'k'}\gamma^{n'l'} - \eta^{n'l'}\gamma^{m'k'} + \eta^{n'k'}\gamma^{m'l'}). \tag{A.10}$$

To obtain matrix form of the $so(2,4)$ generators in conformal basis $D=1+4$ γ-matrices $\gamma^{m'}{}_{\boldsymbol{\alpha}}{}^{\boldsymbol{\beta}}$ are expressed in terms of the $SL(2,\mathbb{C})$ σ-matrices $\sigma^m_{\alpha\dot\alpha}$ and $\tilde\sigma^{m\dot\alpha\alpha}$

$$\sigma^m_{\alpha\dot\alpha}\tilde\sigma^{n\dot\alpha\beta}+\sigma^n_{\alpha\dot\alpha}\tilde\sigma^{m\dot\alpha\beta}=-2\eta^{mn}\delta^\beta_\alpha,\quad \tilde\sigma^{m\dot\alpha\alpha}\sigma^n_{\alpha\dot\beta}+\tilde\sigma^{n\dot\alpha\alpha}\sigma^m_{\alpha\dot\beta}=-2\eta^{mn}\delta^{\dot\alpha}_{\dot\beta} \tag{A.11}$$

as

$$\gamma^m{}_{\boldsymbol{\alpha}}{}^{\boldsymbol{\beta}}=\begin{pmatrix}0 & \sigma^m_{\alpha\dot\beta}\\ \tilde\sigma^{m\dot\alpha\beta} & 0\end{pmatrix},\quad \gamma^5{}_{\boldsymbol{\alpha}}{}^{\boldsymbol{\beta}}=i\begin{pmatrix}\delta^\beta_\alpha & 0\\ 0 & -\delta^{\dot\alpha}_{\dot\beta}\end{pmatrix}. \tag{A.12}$$

The charge conjugation matrices

$$C_{\boldsymbol{\alpha\beta}}=\begin{pmatrix}-\varepsilon_{\alpha\beta} & 0\\ 0 & \varepsilon^{\dot\alpha\dot\beta}\end{pmatrix},\quad C^{\boldsymbol{\alpha\beta}}=\begin{pmatrix}-\varepsilon^{\alpha\beta} & 0\\ 0 & \varepsilon_{\dot\alpha\dot\beta}\end{pmatrix} \tag{A.13}$$

are realized in terms of antisymmetric unit rank 2 spinors $\varepsilon_{\alpha\beta}$, $\varepsilon^{\alpha\beta}$ and c.c. ones that are used to raise and lower $SL(2,\mathbb{C})$ spinor indices.[9] In such a realization matrices (A.3) acquire the form conventional in twistor theory

$$H_{\boldsymbol{\alpha}}{}^{\boldsymbol{\beta}}=-\gamma^0{}_{\boldsymbol{\alpha}}{}^{\boldsymbol{\beta}}=\begin{pmatrix}0 & I\\ I & 0\end{pmatrix},\quad H^{\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}=\gamma^{0\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}=\begin{pmatrix}0 & I\\ I & 0\end{pmatrix}. \tag{A.14}$$

To calculate the square of the particle's 5-momentum in directions tangent to AdS_5 it is used the completeness relation for $D=1+4$ γ-matrices

$$\gamma^{m'}{}_{\boldsymbol{\alpha}}{}^{\boldsymbol{\beta}}\gamma_{m'\boldsymbol{\gamma}}{}^{\boldsymbol{\delta}}=\delta^{\boldsymbol{\beta}}_{\boldsymbol{\alpha}}\delta^{\boldsymbol{\delta}}_{\boldsymbol{\gamma}}-2(\delta^{\boldsymbol{\delta}}_{\boldsymbol{\alpha}}\delta^{\boldsymbol{\beta}}_{\boldsymbol{\gamma}}-C_{\boldsymbol{\alpha\gamma}}C^{\boldsymbol{\beta\delta}}) \tag{A.15}$$

that follows from the completeness relation for $D=2+4$ γ-matrices

$$\rho^{\underline{m}}_{\boldsymbol{\alpha\beta}}\tilde\rho^{\boldsymbol{\gamma\delta}}_{\underline{m}}=2(\delta^{\boldsymbol{\gamma}}_{\boldsymbol{\alpha}}\delta^{\boldsymbol{\delta}}_{\boldsymbol{\beta}}-\delta^{\boldsymbol{\delta}}_{\boldsymbol{\alpha}}\delta^{\boldsymbol{\gamma}}_{\boldsymbol{\beta}}). \tag{A.16}$$

Chiral antisymmetric γ-matrices in $D=6$ dimensions satisfy

$$\rho^I_{AB}\tilde\rho^{JBC}+\rho^J_{AB}\tilde\rho^{IBC}=2\delta^{IJ}\delta^C_A. \tag{A.17}$$

They are connected by the Hermitian conjugation $(\rho^I_{AB})^\dagger=\tilde\rho^{IBA}$.

We adopt the following definition of γ- and charge conjugation matrices in $D=5$ dimensions

$$C_{AB}=\rho^6_{AB},\quad C^{AB}=\tilde\rho^{6AB},\quad \gamma^{I'}_{AB}=i\rho^{I'}_{AB},\quad \gamma^{I'AB}=i\tilde\rho^{I'AB}. \tag{A.18}$$

So that $D=5$ Clifford algebra relations read

$$\gamma^{I'}{}_A{}^B\gamma^{J'}{}_B{}^C+\gamma^{J'}{}_A{}^B\gamma^{I'}{}_B{}^C=2\delta^{I'J'}\delta^C_A,\quad \gamma^{I'}{}_A{}^B=C_{AD}\gamma^{I'DB}=-\gamma^{I'}_{AD}C^{DB}. \tag{A.19}$$

Hermitian conjugation transforms the $D=5$ γ- and charge conjugation matrices as

$$(\gamma^{I'}{}_A{}^B)^\dagger=\gamma^{I'}{}_B{}^A,\quad (C_{AB})^\dagger=C^{BA}. \tag{A.20}$$

Antisymmetrized products of γ-matrices

$$\gamma^{I'J'A}{}_B=\frac{i}{4}(\gamma^{I'A}{}_C\gamma^{J'C}{}_B-\gamma^{J'A}{}_C\gamma^{I'C}{}_B) \tag{A.21}$$

[9] We adopt the conventions of Ref. [75] for the $SL(2,\mathbb{C})$ spinor algebra unless otherwise stated.

satisfy the commutation relations of the $so(5)$ algebra

$$[\gamma^{I'J'},\gamma^{K'L'}]=i(\delta^{I'L'}\gamma^{J'K'}-\delta^{I'K'}\gamma^{J'L'}-\delta^{J'L'}\gamma^{I'K'}+\delta^{J'K'}\gamma^{I'L'}). \tag{A.22}$$

Completeness relation for $D=6$ γ-matrices

$$\rho^{I}_{AB}\tilde{\rho}^{ICD}=2(\delta^{D}_{A}\delta^{C}_{B}-\delta^{C}_{A}\delta^{D}_{B}) \tag{A.23}$$

in terms of $D=5$ γ-matrices acquires the form

$$\gamma^{I'}{}_{A}{}^{B}\gamma^{I'}{}_{C}{}^{D}=-\delta^{B}_{A}\delta^{D}_{C}+2(\delta^{D}_{A}\delta^{B}_{C}-C_{AC}C^{BD}) \tag{A.24}$$

and is used to calculate the square of the particle's momentum components in directions tangent to S^5.

Appendix B. (4|4) supermatrix realization of $psu(2,2|4)$ superalgebra generators

This Appendix contains the details of the supermatrix form of the generators of $psu(2,2|4)$ superalgebra and its realization as the $D=4$ $N=4$ superconformal algebra.

$so(2,4)\sim su(2,2)$ generators can be realized by the antisymmetrized products of $D=2+4$ γ-matrices (A.1)

$$\tilde{\rho}^{\underline{mn}\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}=-\frac{i}{4}(\tilde{\rho}^{\underline{m}\boldsymbol{\alpha\gamma}}\rho^{\underline{n}}_{\boldsymbol{\gamma\beta}}-\tilde{\rho}^{\underline{n}\boldsymbol{\alpha\gamma}}\rho^{\underline{m}}_{\boldsymbol{\gamma\beta}}), \tag{B.1}$$

which commutator equals

$$[\tilde{\rho}^{\underline{mn}},\tilde{\rho}^{\underline{kl}}]=i(\eta^{\underline{ml}}\tilde{\rho}^{\underline{nk}}-\eta^{\underline{mk}}\tilde{\rho}^{\underline{nl}}-\eta^{\underline{nl}}\tilde{\rho}^{\underline{mk}}+\eta^{\underline{nk}}\tilde{\rho}^{\underline{ml}}). \tag{B.2}$$

Retaining $so(1,4)$ or $so(1,3)$ covariance the $so(2,4)$ algebra relations (B.2) can be cast in the form of ads_5 or $conf_4$ algebras. The ads_5 algebra relations are obtained by splitting $so(2,4)$ generators into $\tilde{\rho}^{0'm'\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}=-\frac{1}{2}\gamma^{m'\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}$ and $\tilde{\rho}^{m'n'\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}=\gamma^{m'n'\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}$. Then one derives from (B.2)

$$[\gamma^{m'},\gamma^{n'}]=4i\gamma^{m'n'},\quad [\gamma^{m'n'},\gamma^{k'}]=i(\eta^{n'k'}\gamma^{m'}-\eta^{m'k'}\gamma^{n'}) \tag{B.3}$$

and relations (A.10). Generators of the $conf_4$ algebra are defined as

$$\begin{gathered}
D^{\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}=-\tilde{\rho}^{0'5\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}=\tfrac{i}{2}\begin{pmatrix}-\delta^{\alpha}_{\beta} & 0\\ 0 & \delta^{\dot\beta}_{\dot\alpha}\end{pmatrix},\quad P^{m\,\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}=\tilde{\rho}^{0'm\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}+\tilde{\rho}^{5m\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}=\begin{pmatrix}0 & \tilde{\sigma}^{\dot\beta\alpha}_{m}\\ 0 & 0\end{pmatrix},\\
K^{m\,\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}=\tilde{\rho}^{0'm\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}-\tilde{\rho}^{5m\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}=\begin{pmatrix}0 & 0\\ \sigma^{m}_{\beta\dot\alpha} & 0\end{pmatrix},\quad M^{mn\,\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}=\tilde{\rho}^{mn\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}}=i\begin{pmatrix}\sigma^{mn}{}_{\beta}{}^{\alpha} & 0\\ 0 & \tilde{\sigma}^{mn\dot\beta}{}_{\dot\alpha}\end{pmatrix},
\end{gathered} \tag{B.4}$$

where $\sigma^{mn}{}_{\beta}{}^{\alpha}=\frac{1}{4}(\sigma^{m}_{\beta\dot\beta}\tilde{\sigma}^{n\dot\beta\alpha}-\sigma^{n}_{\beta\dot\beta}\tilde{\sigma}^{m\dot\beta\alpha})$ and $\tilde{\sigma}^{mn\dot\beta}{}_{\dot\alpha}=\frac{1}{4}(\tilde{\sigma}^{m\dot\beta\beta}\sigma^{n}_{\beta\dot\alpha}-\tilde{\sigma}^{n\dot\beta\beta}\sigma^{m}_{\beta\dot\alpha})$. Then the $conf_4$ algebra relations read

$$\begin{gathered}
[D,P^m]=-iP^m,\quad [D,K^m]=iK^m,\quad [K^m,P^n]=2i(\eta^{mn}D+M^{mn}),\\
[M^{mn},P^k]=i(\eta^{nk}P^m-\eta^{mk}P^n),\quad [M^{mn},K^k]=i(\eta^{nk}K^m-\eta^{mk}K^n),\\
[M^{mn},M^{kl}]=i(\eta^{ml}M^{nk}-\eta^{mk}M^{nl}-\eta^{nl}M^{mk}+\eta^{nk}M^{ml}).
\end{gathered} \tag{B.5}$$

Similarly to the case of $so(2,4)$ algebra the generators of $so(6) \sim su(4)$ algebra can be realized by the antisymmetrized products of $D=6$ chiral γ-matrices

$$\tilde{\rho}^{IJA}{}_B = \frac{i}{4}(\tilde{\rho}^{IAC}\rho^J_{CB} - \tilde{\rho}^{JAC}\rho^I_{CB}). \tag{B.6}$$

Their commutator equals

$$[\tilde{\rho}^{IJ}, \tilde{\rho}^{KL}] = i(\delta^{IL}\tilde{\rho}^{JK} - \delta^{IK}\tilde{\rho}^{JL} - \delta^{JL}\tilde{\rho}^{IK} + \delta^{JK}\tilde{\rho}^{IL}). \tag{B.7}$$

Introducing the s^5 algebra generators

$$P^{I'A}{}_B = 2\tilde{\rho}^{6I'A}{}_B = \gamma^{I'A}{}_B, \quad M^{I'J'A}{}_B = \tilde{\rho}^{I'J'A}{}_B = \gamma^{I'J'A}{}_B \tag{B.8}$$

relations (B.7) can be written in the form

$$\begin{gathered} [P^{I'}, P^{J'}] = -4iM^{I'J'}, \quad [P^{I'}, M^{K'L'}] = i(\delta^{I'K'}P^{L'} - \delta^{I'L'}P^{K'}), \\ [M^{I'J'}, M^{K'L'}] = i(\delta^{I'L'}M^{J'K'} - \delta^{I'K'}M^{J'L'} - \delta^{J'L'}M^{I'K'} + \delta^{J'K'}M^{I'L'}). \end{gathered} \tag{B.9}$$

Generators of the Poincare supersymmetry are given by the supermatrices

$$Q^{A}_{\alpha}{}^{\mathcal{B}}{}_{\mathcal{C}} = \begin{pmatrix} 0 & 0 & 2\delta^{\beta}_{\alpha}\delta^A_C \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \bar{Q}_{\dot{\alpha}A}{}^{\mathcal{B}}{}_{\mathcal{C}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2\delta^{\dot{\gamma}}_{\dot{\alpha}}\delta^B_A & 0 \end{pmatrix} \tag{B.10}$$

and generators of the conformal supersymmetry equal

$$S^{\alpha}_{A}{}^{\mathcal{B}}{}_{\mathcal{C}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2\delta^{\alpha}_{\gamma}\delta^B_A & 0 & 0 \end{pmatrix}, \quad \bar{S}^{\dot{\alpha}A\,\mathcal{B}}{}_{\mathcal{C}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 2\delta^{\dot{\alpha}}_{\dot{\beta}}\delta^A_C \\ 0 & 0 & 0 \end{pmatrix}. \tag{B.11}$$

The reasoning behind this definition is that the $(4|4) \times (4|4)$ supermatrix $g^{\mathcal{A}}{}_{\mathcal{B}}$:

$$g^{\mathcal{A}}{}_{\mathcal{B}} = \begin{pmatrix} g^{\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}} & g^{\boldsymbol{\alpha}}{}_B \\ g^A{}_{\boldsymbol{\beta}} & g^A{}_B \end{pmatrix} \in psu(2,2|4) \tag{B.12}$$

with the 4×4 blocks

$$\begin{gathered} g^{\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}} = X^{\underline{mn}}\tilde{\rho}_{\underline{mn}}{}^{\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}} \in su(2,2), \quad g^A{}_B = Y^{IJ}\tilde{\rho}^{IJA}{}_B \in su(4), \\ g^{\boldsymbol{\alpha}}{}_B = \theta^{\lambda}_D Q^{D\,\boldsymbol{\alpha}}_{\lambda}{}_B + \bar{\eta}_{\dot{\lambda}D}\bar{S}^{\dot{\lambda}D\,\boldsymbol{\alpha}}{}_B, \quad g^A{}_{\boldsymbol{\beta}} = \bar{\theta}^{\dot{\lambda}D}\bar{Q}_{\dot{\lambda}D}{}^A{}_{\boldsymbol{\beta}} + \eta^D_{\lambda}S^{\lambda}_D{}^A{}_{\boldsymbol{\beta}} \end{gathered} \tag{B.13}$$

satisfies the Hermiticity condition

$$g^{\mathcal{A}}{}_{\mathcal{B}} = \mathcal{H}^{\mathcal{A}}{}_{\mathcal{C}}(g^{\mathcal{D}}{}_{\mathcal{C}})^{\dagger}\mathcal{H}^{\mathcal{D}}{}_{\mathcal{B}}, \tag{B.14}$$

where

$$(g^{\mathcal{A}}{}_{\mathcal{B}})^{\dagger} = \begin{pmatrix} (g^{\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}})^{\dagger} & (g^A{}_{\boldsymbol{\beta}})^{\dagger} \\ (g^{\boldsymbol{\alpha}}{}_B)^{\dagger} & (g^A{}_B)^{\dagger} \end{pmatrix} \tag{B.15}$$

and

$$\mathcal{H}^{\mathcal{A}}{}_{\mathcal{B}} = \begin{pmatrix} H^{\boldsymbol{\alpha}}{}_{\boldsymbol{\beta}} & 0 \\ 0 & \delta^A_B \end{pmatrix}. \tag{B.16}$$

Thus explicit supermatrix form of the general $psu(2,2|4)$ element in the $SL(2,\mathbb{C})$ notation is

$$g^{\mathcal{A}}{}_{\mathcal{B}} = \begin{pmatrix} -i(X^{0'5}\delta^{\alpha}_{\beta} - X^{mn}\sigma_{mn\beta}{}^{\alpha}) & (X^{5m} - X^{0'm})\tilde{\sigma}^{\dot{\beta}\alpha}_{m} & 2\theta^{\alpha}_{B} \\ -(X^{0'm} + X^{5m})\sigma_{m\beta\dot{\alpha}} & i(X^{0'5}\delta^{\dot{\beta}}_{\dot{\alpha}} + X^{mn}\tilde{\sigma}_{mn}{}^{\dot{\beta}}{}_{\dot{\alpha}}) & 2\bar{\eta}_{\dot{\alpha}B} \\ 2\eta^{A}_{\beta} & 2\bar{\theta}^{\dot{\beta}A} & Y^{IJ}\tilde{\rho}^{IJA}{}_{B} \end{pmatrix}. \quad (B.17)$$

Using the above introduced matrix form of the $psu(2,2|4)$ generators (B.4), (B.8), (B.10) and (B.11) it is possible to derive (anti)commutation relations of the $psu(2,2|4)$ superalgebra realized as the $D=4$ $N=4$ superconformal algebra. Commutation relations of the $su(2,2)\oplus su(4)$ bosonic subalgebra are given in (B.5) and (B.9). Non-zero anticommutators of odd generators equal

$$\begin{gathered} \{Q^{C}_{\lambda}, \bar{Q}_{\dot{\lambda}D}\} = -2\delta^{C}_{D}\sigma^{m}_{\lambda\dot{\lambda}}P_m, \quad \{S^{\lambda}_{C}, \bar{S}^{\dot{\lambda}D}\} = -2\delta^{D}_{C}\tilde{\sigma}^{m\dot{\lambda}\lambda}K_m, \\ \{Q^{C}_{\lambda}, S^{\rho}_{D}\} = 2i\delta^{C}_{D}(\delta^{\rho}_{\lambda}D + \sigma^{mn}{}_{\lambda}{}^{\rho}M_{mn}) + \delta^{\rho}_{\lambda}(\gamma^{I'C}{}_{D}P^{I'} + 2\gamma^{I'J'C}{}_{D}M^{I'J'}), \\ \{\bar{Q}_{\dot{\lambda}C}, \bar{S}^{\dot{\rho}D}\} = -2i\delta^{D}_{C}(\delta^{\dot{\rho}}_{\dot{\lambda}}D - \tilde{\sigma}^{mn\dot{\rho}}{}_{\dot{\lambda}}M_{mn}) + \delta^{\dot{\rho}}_{\dot{\lambda}}(\gamma^{I'D}{}_{C}P^{I'} + 2\gamma^{I'J'D}{}_{C}M^{I'J'}) \end{gathered} \quad (B.18)$$

and commutators of odd and even generators read

$$\begin{gathered} [D, Q^{C}_{\lambda}] = -\tfrac{i}{2}Q^{C}_{\lambda}, \quad [D, \bar{Q}_{\dot{\lambda}C}] = -\tfrac{i}{2}\bar{Q}_{\dot{\lambda}C}, \quad [D, S^{\lambda}_{C}] = \tfrac{i}{2}S^{\lambda}_{C}, \quad [D, \bar{S}^{\dot{\lambda}C}] = \tfrac{i}{2}\bar{S}^{\dot{\lambda}C}, \\ [P_m, S^{\lambda}_{C}] = -\tilde{\sigma}^{\dot{\lambda}\lambda}_{m}\bar{Q}_{\dot{\lambda}C}, \quad [P_m, \bar{S}^{\dot{\lambda}C}] = \tilde{\sigma}^{\dot{\lambda}\lambda}_{m}Q^{C}_{\lambda}, \\ [K_m, Q^{C}_{\lambda}] = \sigma_{m\lambda\dot{\lambda}}\bar{S}^{\dot{\lambda}C}, \quad [K_m, \bar{Q}_{\dot{\lambda}C}] = -\sigma_{m\lambda\dot{\lambda}}S^{\lambda}_{C}, \\ [M_{mn}, Q^{C}_{\lambda}] = i\sigma_{mn\lambda}{}^{\rho}Q^{C}_{\rho}, \quad [M_{mn}, \bar{Q}_{\dot{\lambda}C}] = -i\tilde{\sigma}_{mn}{}^{\dot{\rho}}{}_{\dot{\lambda}}\bar{Q}_{\dot{\rho}C}, \\ [M_{mn}, S^{\lambda}_{C}] = -i\sigma_{mn\rho}{}^{\lambda}S^{\rho}_{C}, \quad [M_{mn}, \bar{S}^{\dot{\lambda}C}] = i\tilde{\sigma}_{mn}{}^{\dot{\lambda}}{}_{\dot{\rho}}\bar{S}^{\dot{\rho}C}, \\ [P^{I'}, Q^{C}_{\lambda}] = -\gamma^{I'C}{}_{D}Q^{D}_{\lambda}, \quad [P^{I'}, \bar{Q}_{\dot{\lambda}C}] = \gamma^{I'D}{}_{C}\bar{Q}_{\dot{\lambda}D}, \\ [P^{I'}, S^{\lambda}_{C}] = \gamma^{I'D}{}_{C}S^{\lambda}_{D}, \quad [P^{I'}, \bar{S}^{\dot{\lambda}C}] = -\gamma^{I'C}{}_{D}\bar{S}^{\dot{\lambda}D}, \\ [M^{I'J'}, Q^{C}_{\lambda}] = -\gamma^{I'J'C}{}_{D}Q^{D}_{\lambda}, \quad [M^{I'J'}, \bar{Q}_{\dot{\lambda}C}] = \gamma^{I'J'D}{}_{C}\bar{Q}_{\dot{\lambda}D}, \\ [M^{I'J'}, S^{\lambda}_{C}] = \gamma^{I'J'D}{}_{C}S^{\lambda}_{D}, \quad [M^{I'J'}, \bar{S}^{\dot{\lambda}C}] = -\gamma^{I'J'C}{}_{D}\bar{S}^{\dot{\lambda}D}. \end{gathered} \quad (B.19)$$

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