



# Precision analysis of pseudoscalar interactions in neutron beta decays

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## Abstract

We analyze the contributions of the one-pion-pole (OPP) exchange, caused by strong low-energy interactions, and the pseudoscalar interaction beyond the Standard Model (BSM) to the correlation coefficients of the neutron  $\beta^-$ -decays for polarized neutrons, polarized electrons and unpolarized protons. The strength of contributions of pseudoscalar interactions is defined by the effective coupling constant  $C_{ps} = C_{ps}^{(OPP)} + C_{ps}^{(BSM)}$ . We show that the contribution of the OPP exchange is of order  $C_{ps}^{(OPP)} \sim -10^{-5}$ . The effective coupling constant  $C_{ps}^{(BSM)}$  of the pseudoscalar interaction BSM can be in principle complex. Using the results, obtained by González-Alonso et al. (2019) [34] we find that the values of the real and imaginary parts of the effective coupling constant  $C_{ps}^{(BSM)}$  are constrained by  $-3.5 \times 10^{-5} < \text{Re } C_{ps}^{(BSM)} < 0$  and  $\text{Im } C_{ps}^{(BSM)} < -2.3 \times 10^{-5}$ , respectively. The obtained results can be used as a theoretical background for experimental searches of contributions of interactions BSM in asymmetries of the neutron  $\beta^-$ -decays with a polarized neutron, a polarized electron and an unpolarized proton at the level of accuracy of a few parts of  $10^{-5}$  or even better Abele (2016) [40].

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## 1. Introduction

Nowadays the neutron lifetime and correlation coefficients of the neutron  $\beta^-$ -decays for polarized neutrons, polarized electrons and unpolarized protons are calculated within the Standard Model (SM) at the level of  $10^{-3}$  including the radiative corrections of order  $O(\alpha/\pi)$  and corrections caused by the weak magnetism and proton recoil of order  $O(E_e/M)$  [1–13], where  $\alpha$ ,  $E_e$  and  $M$  are the fine-structure constant [14], an electron energy and the nucleon mass, respectively. Such a SM theoretical background has allowed to make steps forwards investigations of contributions of interactions beyond the SM (BSM) of order  $10^{-4}$  or even smaller [40]. The analysis of interactions beyond the  $V - A$  effective theory of weak interactions [15–19] (see also [20,21]) in the neutron  $\beta^-$ -decays with different polarizations of massive fermions has a long history and started in 50th of the 20th century and is continuing at present time [22–34] (see also [8,9,11]). The most general form of the Lagrangian of interactions BSM has been written in [22–27], including non-derivative vector  $\bar{\psi}_p \gamma_\mu \psi_n$ , axial-vector  $\bar{\psi}_p \gamma_\mu \gamma^5 \psi_n$ , scalar  $\bar{\psi}_p \psi_n$ , pseudoscalar  $\bar{\psi}_p \gamma^5 \psi_n$  and tensor  $\bar{\psi}_p \sigma_{\mu\nu} \psi_n$  nucleon currents coupled to corresponding lepton currents in the form of local nucleon-lepton current-current interactions, where  $\{1, \gamma_\mu, \gamma_\mu \gamma^5, \gamma^5, \sigma_{\mu\nu}\}$  are the Dirac matrices [35], with respect to  $G$ -parity transformations [36], i.e.  $G = C e^{i\pi I_2}$ , where  $C$  and  $I_2$  are the charge conjugation and isospin operators [35], the vector, axial-vector, pseudoscalar and tensor nucleon currents are  $G$ -even and the scalar nucleon current is  $G$ -odd. According to the  $G$ -transformation properties of hadronic currents, Weinberg divided hadronic currents into two classes, which are  $G$ -even first class and  $G$ -odd second class currents [37], respectively. Thus, following Weinberg's classification the non-derivative vector, axial-vector, pseudoscalar and tensor nucleon currents in the interactions BSM, introduced in [22–27], are the first class currents, whereas the non-derivative scalar nucleon current is the second class one (see also [38]). The analysis of superallowed  $0^+ \rightarrow 0^+$  nuclear beta transitions by Hardy and Towner [39] and González-Alonso et al. [34] has shown that the phenomenological coupling constants of non-derivative scalar current-current nucleon-lepton interaction is of order  $10^{-5}$  or even smaller. This agrees well with estimates of contributions of the second class currents, caused by derivative scalar  $\partial^\mu (\bar{\psi}_p \psi_n)$  and pseudotensor  $\partial^\nu (\bar{\psi}_p \sigma_{\mu\nu} \gamma^5 \psi_n)$  nucleon currents proposed by Weinberg [37], to the neutron lifetime and correlation coefficients of the neutron  $\beta^-$ -decays carried out by Gardner and Plaster [32,33] and Ivanov et al. [11,12]. The contemporary experimental sensitivities  $10^{-4}$  or even better [40] of experimental analyses of parameters of neutron  $\beta^-$ -decays (see, for example, [41–43]) demand a theoretical background for the neutron lifetime and correlation coefficients of the neutron  $\beta^-$ -decays with different polarizations of massive fermions at the level of  $10^{-5}$  [10–13]. As has been shown in [28–31] in the linear approximation the contributions of vector and axial-vector interactions BSM can be absorbed by the matrix element  $V_{ud}$  of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix and by the axial coupling constant  $\lambda$  (see also [9–12]). As a result, taking into account the constraints on the scalar interaction [39] and [34] the contributions of interactions BSM to the neutron  $\beta^-$ -decay can be induced only by a tensor nucleon current [44,45]. As we show below the contribution of the one-pion-pole (OPP) exchange to the correlation coefficients of the neutron  $\beta^-$ -decays for a polarized neutron, a polarized electron and an unpolarized proton is of order  $10^{-5}$ . This is commensurable with the contribution of the isospin breaking correction to

the vector coupling constant of the neutron  $\beta^-$ -decay calculated by Kaiser [46] within the heavy baryon chiral perturbation theory (HB $\chi$ PT). However, unlike Kaiser's correction the contribution of the OPP exchange can be screened by the contribution of the pseudoscalar interaction BSM.

This paper is addressed to the analysis of contributions of the OPP exchange, caused by strong low-energy interactions, and the pseudoscalar interaction BSM introduced in [22–27] to the neutron lifetime and correlation coefficients of the neutron  $\beta^-$ -decays for a polarized neutron, a polarized electron and unpolarized proton. The analysis of contributions of pseudoscalar interactions to the electron-energy and angular distribution of the neutron  $\beta^-$ -decay for a polarized neutron and unpolarized electron and proton has a long history [47–54] (see also [4, 34]). For example the Fierz-like interference term [55], induced by pseudoscalar interactions, can be recognized in the electron-energy and angular distributions calculated in [47–54] (see also [4, 34]). The contributions of the pseudoscalar interactions to the correlation coefficients of the electron-energy and angular distribution of the neutron  $\beta^-$ -decay for a polarized neutron and unpolarized electron and proton can be, in principle, extracted from the electron-energy and angular distributions obtained by Harrington [47] (see Eqs. (9) – (13) of Ref. [47]) and Holstein [51] (see Appendix B of Ref. [51]) (see also section 4 of this paper). In our work in addition to the results obtained in [47–54] (see also [4, 34]) we calculate the contributions of pseudoscalar interactions to the correlation coefficients of the electron-energy and angular distribution of the neutron  $\beta^-$ -decays, caused by correlations with the electron spin. The analyze of contributions of pseudoscalar interactions to the correlation coefficients of the electron-energy and angular distribution of the neutron  $\beta^-$ -decays for a polarized neutron, a polarized electron and unpolarized proton, carried out in this paper, completes the investigations of contributions of interactions BSM to the electron-energy and angular distributions, which we have performed in [10–12], where we have calculated i) the complete set of corrections of order  $10^{-3}$ , caused by radiative corrections of order  $O(\alpha/\pi)$  and the weak magnetism and proton recoil corrections of order  $O(E_e/M)$ , and ii) contributions of vector, axial-vector, scalar and tensor interactions BSM introduced in [22–27].

The paper is organized as follows. In section 2 we write down the amplitude of the neutron  $\beta^-$ -decay by taking into account the contributions of the OPP exchange and the pseudoscalar interaction BSM only. We analyze the contributions of energy independent corrections to the pseudoscalar form factor of the nucleon defined by the Adler-Dothan-Wolfenstein (ADM) term [56, 57] and chiral corrections calculated within the HB $\chi$ PT [58–60]. We show that the ADM-term and chiral corrections, calculated in the two-loop approximation within the HB $\chi$ PT by Kaiser [60], are able in principle to induce sufficiently small real contributions to phenomenological coupling constants of the pseudoscalar interaction BSM of a neutron-proton pseudoscalar density coupled to a left-handed leptonic current. In section 3 we discuss the contributions to the correlation coefficients of the electron-energy and angular distribution of the neutron  $\beta^-$ -decays caused by the OPP exchange and the pseudoscalar interaction BSM. The distribution is calculated for a polarized neutron, a polarized electron and an unpolarized proton. Using the results, obtained in [29, 34, 53, 69] we estimate the phenomenological coupling constants of the pseudoscalar interactions BSM. We adduce the results in Table 1. In section 4 we discuss the obtained results, which can be used for experimental analyses of the neutron  $\beta^-$ -decays with experimental accuracies of about a few parts of  $10^{-5}$  [40]. Since the complete set of contributions of order  $10^{-3}$ , including the radiative corrections of order  $O(\alpha/\pi)$  and corrections of order  $O(E_0/M)$ , caused by the weak magnetism and proton recoil, are calculated at the neglect of contributions of order  $O(\alpha E_0/\pi M) \sim 10^{-6}$  and  $O(E_0^2/M^2) \sim 10^{-6}$  [9–12], the results obtained in this pa-

per should be tangible and important for a correct analysis of experimental data on searches of contributions of interactions BSM with an accuracy of a few parts of  $10^{-5}$ . We give also a comparative analysis of the results obtained in this work with those in [4,47–54]. This allows us to argue that the corrections, caused by pseudoscalar interactions, calculated for the correlation coefficients of the neutron  $\beta^-$ -decays, induced by correlations of the electron spin with the neutron spin and 3-momenta of decay fermions with standard correlation structures introduced by Jackson et al. [24], are fully new. Moreover all terms in Eq. (A.6) with correlation structures beyond the standard ones by Jackson et al. [24] and proportional to the effective coupling constants  $C'_{ps}$  and  $C''_{ps}$  were never calculated in literature. In the Appendix we give a detailed calculation of the contributions of pseudoscalar interactions caused by the OPP exchange and BSM to the correlation coefficients of the neutron  $\beta^-$ -decays for a polarized neutron, a polarized electron and an unpolarized proton, completing the analysis of contributions of interactions BSM to the correlation coefficients of the neutron  $\beta^-$ -decays carried out in [10–12].

## 2. Amplitude of the neutron $\beta^-$ -decay with contributions of OPP exchange and pseudoscalar interaction BSM

Since the expected order of contributions of pseudoscalar interactions of about  $10^{-5}$ , we take them into account in the linear approximation additively to the corrections of order  $10^{-4} - 10^{-3}$  calculated in [1–34]. In such an approximation and following [9,11,12] the amplitude of the neutron  $\beta^-$ -decay we take in the form

$$\begin{aligned} M(n \rightarrow pe^- \bar{\nu}_e) &= -\frac{G_F}{\sqrt{2}} V_{ud} \left\{ \langle p(\vec{k}_p, \sigma_p) | J_\mu^{(+)}(0) | n(\vec{k}_n, \sigma_n) \rangle [\bar{u}_e(\vec{k}_e, \sigma_e) \gamma^\mu (1 - \gamma^5) v_{\bar{\nu}}(\vec{k}_{\bar{\nu}}, +\frac{1}{2})] \right. \\ &\quad \left. + \bar{u}_p(\vec{k}_p, \sigma_p) \gamma^5 u_n(\vec{k}_n, \sigma_n) [\bar{u}_e(\vec{k}_e, \sigma_e) (C_P + \bar{C}_P \gamma^5) v_{\bar{\nu}}(\vec{k}_{\bar{\nu}}, +\frac{1}{2})] \right\}, \quad (1) \end{aligned}$$

where  $G_F$  and  $V_{ud}$  are the Fermi coupling constant and the Cabibbo–Kobayashi–Maskawa (CKM) matrix element [14]. Then,  $\langle p(\vec{k}_p, \sigma_p) | J_\mu^{(+)}(0) | n(\vec{k}_n, \sigma_n) \rangle$  is the matrix element of the charged hadronic current  $J_\mu^{(+)}(0) = V_\mu^{(+)}(0) - A_\mu^{(+)}(0)$ , where  $V_\mu^{(+)}(0)$  and  $A_\mu^{(+)}(0)$  are the charged vector and axial–vector hadronic currents [15,18,19]. The fermions in the initial and final states are described by Dirac bispinor wave functions  $u_n, u_p, u_e$  and  $v_{\bar{\nu}}$  of free fermions [9,62]. In the second term of Eq. (1) we take into account the contribution of the pseudoscalar interaction BSM [22–27] with two complex phenomenological coupling constants  $C_P$  and  $\bar{C}_P$  in the notation of [9,11,12].

For the analysis of contributions of pseudoscalar interactions to the neutron  $\beta^-$ -decays for a polarized neutron, a polarized electron and an unpolarized proton we define the matrix element  $\langle p(\vec{k}_p, \sigma_p) | J_\mu^{(+)}(0) | n(\vec{k}_n, \sigma_n) \rangle$  as follows

$$\begin{aligned} \langle p(\vec{k}_p, \sigma_p) | J_\mu^{(+)}(0) | n(\vec{k}_n, \sigma_n) \rangle &= \bar{u}_p(\vec{k}_p, \sigma_p) \left( \gamma_\mu (1 + \lambda \gamma^5) + \frac{2M\lambda q_\mu}{m_\pi^2 - q^2 - i0} \gamma^5 \right) u_n(\vec{k}_n, \sigma_n), \quad (2) \end{aligned}$$

where  $\lambda$  is the axial coupling constant with recent experimental value  $\lambda = -1.27641(45)_{\text{stat.}}(33)_{\text{syst.}}$  [41]. The first term in Eq. (1) is written in agreement with the standard  $V - A$  effective theory of weak interactions [15,18,19] (see also [20,21]). The term proportional to  $q_\mu \gamma^5$  defines the contribution of the OPP exchange, caused by strong low–energy interactions (see also [18])

with the  $\pi^-$ -meson mass  $m_\pi = 139.57061(24)$  MeV [14] and  $q = k_p - k_n = -k_e - k_{\bar{\nu}}$  is a 4-momentum transfer. The OPP contribution is required by conservation of the charged hadronic axial-vector current in the chiral limit  $m_\pi \rightarrow 0$  [18].

In the more general form the matrix element of the hadronic axial-vector current can be taken in the form accepted in the HB $\chi$ PT [58–60]. This gives

$$\begin{aligned} \langle p(\vec{k}_p, \sigma_p) | A_\mu^{(+)}(0) | n(\vec{k}_n, \sigma_n) \rangle \\ = \bar{u}_p(\vec{k}_p, \sigma_p) \left( \gamma_\mu G_A(q^2) + \frac{q_\mu}{2M} G_P(q^2) \right) \gamma^5 u_n(\vec{k}_n, \sigma_n), \end{aligned} \quad (3)$$

where  $G_A(q^2)$  and  $G_P(q^2)$  are the axial-vector form factor and the induced pseudoscalar form factor, respectively, at  $0 \leq q^2 \leq \Delta^2$  for the neutron  $\beta^-$ -decay with  $\Delta = m_n - m_p$ . The invariant 4-momentum transfer squared  $q^2$  vanishes, i.e.  $q^2 = 0$ , at the kinetic energy of the proton  $T_p = E_p - m_p = \Delta^2/2m_n$ . In the chiral limit  $m_\pi \rightarrow 0$  because of conservation of the charged hadronic axial-vector current [18] the form factors  $G_A(q^2)$  and  $G_P(q^2)$  are related by  $G_P(q^2) = -(4M^2/q^2)G_A(q^2)$ . In turn, for a finite pion mass the pseudoscalar form factor  $G_P(q^2)$  has been calculated in the two-loop approximation within HB $\chi$ PT by Kaiser [60]. A precision analysis of the induced pseudoscalar form factor in the proton weak interactions has been also carried out by Gorringer and Fearing [61].

### 2.1. Pseudoscalar interaction BSM as induced by corrections to the pseudoscalar form factor, caused by strong low-energy interactions

According to [58], the axial-vector form factor  $G_A(q^2)$  can be rather good parameterized by a dipole form (see also [63])

$$G_A(q^2) = \frac{g_A}{(1 + q^2/M_A^2)^2} = g_A \left( 1 - \frac{1}{6} \langle r_A^2 \rangle q^2 + \dots \right), \quad (4)$$

where  $g_A = -\lambda$  is the axial-coupling constant, and  $M_A$  is the cut-off mass related to the mean square axial radius of the nucleon  $\langle r_A^2 \rangle$  as  $\langle r_A^2 \rangle = 12/M_A^2 = 0.403(29)$  fm<sup>2</sup> with  $M_A = 1.077(39)$  GeV extracted from charged pion electroproduction experiments [63]. In turn, the cut-off mass  $M_A = 1.026(17)$  GeV extracted from (quasi)elastic neutrino and antineutrino scattering experiments [63] gives  $\langle r_A^2 \rangle = 12/M_A^2 = 0.440(16)$  fm<sup>2</sup>. In the approximation Eq. (4) the pseudoscalar form factor  $G_P(q^2)$  acquires the following form [58] (see also [61])

$$\frac{1}{2M} G_P(q^2) = \frac{2Mg_A}{m_\pi^2 - q^2 - i0} - \frac{1}{3} g_A M \langle r_A^2 \rangle, \quad (5)$$

where the correction to the OPP exchange is the Adler–Dothan–Wolfenstein (ADW) term [56, 57]. The ADW-term induces the BSM-like pseudoscalar interaction with the coupling constants

$$C_P^{(\text{ADW})} = -\bar{C}_P^{(\text{ADW})} = -\frac{1}{3} \lambda \langle r_A^2 \rangle m_e M = 2.1 \times 10^{-3}. \quad (6)$$

According to Eq. (11), this gives the contribution to the correlation coefficients of the neutron  $\beta^-$ -decays equal to  $\text{Re } C_{ps}^{(\text{BSM})} = C_{ps}^{(\text{ADW})} = -4.9 \times 10^{-7}$ . Using the results, obtained by Kaiser [60] (see Eq. (7) of Ref. [60]) in the two-loop approximation in the HB $\chi$ PT, the induced BSM-like pseudoscalar coupling constants are equal to

$$C_P^{(\text{K})} = -\bar{C}_P^{(\text{K})} = \frac{m_e m_\pi^2 M}{32\pi^4 f_\pi^4} \zeta_0 = 4.1 \times 10^{-5} \zeta_0, \quad (7)$$

where  $f_\pi = 92.4$  MeV is the charged pion leptonic (or PCAC) constant [58,60]. Since  $|\zeta_0| \sim 1$  [60], we get  $|C_P| = |\bar{C}_P| \sim 4.1 \times 10^{-5}$ . The contribution of  $C_P^{(K)} = -\bar{C}_P^{(K)}$  to the coupling constant  $\text{Re} C_{ps}^{(\text{BSM})}$  (see Eq. (11)) is of order  $|\text{Re} C_{ps}^{(\text{BSM})}| \sim 9.6 \times 10^{-9}$ . This means that the SM strong low-energy interactions are able to induce the BSM-like pseudoscalar interaction with real coupling constants, the contributions of which are much smaller than the current experimental sensitivity of the neutron  $\beta^-$ -decays [40]. Below we consider a more general pseudoscalar interaction BSM with complex phenomenological coupling constants  $C_P$  and  $\bar{C}_P$  such as  $C_P \neq -\bar{C}_P$ .

## 2.2. Non-relativistic approximation for the amplitude of the neutron $\beta^-$ -decay Eq. (1)

In the non-relativistic approximation for the neutron and proton the amplitude of the neutron  $\beta^-$ -decay in Eq. (1) takes the form

$$\begin{aligned} M(n \rightarrow pe^- \bar{\nu}_e) = & -\frac{G_F}{\sqrt{2}} V_{ud} 2M \left\{ [\varphi_p^\dagger \varphi_n] [\bar{u}_e \gamma^0 (1 - \gamma^5) v_{\bar{\nu}}] \right. \\ & - \lambda [\varphi_p^\dagger \vec{\sigma} \varphi_n] \cdot [\bar{u}_e \vec{\gamma} (1 - \gamma^5) v_{\bar{\nu}}] \\ & + \lambda \frac{m_e}{m_\pi^2} [\varphi_p^\dagger (\vec{\sigma} \cdot \vec{k}_p) \varphi_n] [\bar{u}_e (1 - \gamma^5) v_{\bar{\nu}}] \\ & \left. - \frac{1}{2M} [\varphi_p^\dagger (\vec{\sigma} \cdot \vec{k}_p) \varphi_n] [\bar{u}_e (C_P + \bar{C}_P \gamma^5) v_{\bar{\nu}}] \right\}, \end{aligned} \quad (8)$$

where  $\varphi_j$  for  $j = p, n$  are the Pauli spinorial wave functions of non-relativistic neutron and proton, and  $\vec{k}_p = -\vec{k}_e - \vec{k}_{\bar{\nu}}$  is a 3-momentum of the proton.

## 3. Electron-energy and angular distribution of the neutron $\beta^-$ -decay for polarized neutron, polarized electron, and unpolarized proton

The electron-energy and angular distribution of the neutron  $\beta^-$ -decays for a polarized neutron, a polarized electron and an unpolarized proton has been written by Jackson et al. [24]. It reads

$$\begin{aligned} \frac{d^5 \lambda_n(E_e, \vec{k}_e, \vec{k}_{\bar{\nu}}, \vec{\xi}_n, \vec{\xi}_e)}{dE_e d\Omega_e d\Omega_{\bar{\nu}}} = & (1 + 3\lambda^2) \frac{G_F^2 |V_{ud}|^2}{32\pi^5} (E_0 - E_e)^2 \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z = 1) \\ & \times \zeta(E_e) \left\{ 1 + b \frac{m_e}{E_e} + a(E_e) \frac{\vec{k}_e \cdot \vec{k}_{\bar{\nu}}}{E_e E_{\bar{\nu}}} + A(E_e) \frac{\vec{\xi}_n \cdot \vec{k}_e}{E_e} + B(E_e) \frac{\vec{\xi}_n \cdot \vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}} \right. \\ & + K_n(E_e) \frac{(\vec{\xi}_n \cdot \vec{k}_e)(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})}{E_e^2 E_{\bar{\nu}}} + Q_n(E_e) \frac{(\vec{\xi}_n \cdot \vec{k}_{\bar{\nu}})(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})}{E_e E_{\bar{\nu}}^2} + D(E_e) \frac{\vec{\xi}_n \cdot (\vec{k}_e \times \vec{k}_{\bar{\nu}})}{E_e E_{\bar{\nu}}} \\ & + G(E_e) \frac{\vec{\xi}_e \cdot \vec{k}_e}{E_e} + H(E_e) \frac{\vec{\xi}_e \cdot \vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}} + N(E_e) \vec{\xi}_n \cdot \vec{\xi}_e + Q_e(E_e) \frac{(\vec{\xi}_n \cdot \vec{k}_e)(\vec{k}_e \cdot \vec{\xi}_e)}{(E_e + m_e) E_e} \\ & + K_e(E_e) \frac{(\vec{\xi}_e \cdot \vec{k}_e)(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})}{(E_e + m_e) E_e E_{\bar{\nu}}} + R(E_e) \frac{\vec{\xi}_n \cdot (\vec{k}_e \times \vec{\xi}_e)}{E_e} + L(E_e) \frac{\vec{\xi}_e \cdot (\vec{k}_e \times \vec{k}_{\bar{\nu}})}{E_e E_{\bar{\nu}}} \\ & \left. - 3 \frac{E_e}{M} \frac{1 - \lambda^2}{1 + 3\lambda^2} \left( \frac{(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})^2}{E_e^2 E_{\bar{\nu}}^2} - \frac{1}{3} \frac{k_e^2}{E_e^2} \right) + 3 \frac{1 - \lambda^2}{1 + 3\lambda^2} \frac{m_e}{M} \left( \frac{(\vec{\xi}_e \cdot \vec{k}_{\bar{\nu}})(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})}{E_e E_{\bar{\nu}}^2} - \frac{1}{3} \frac{\vec{\xi}_e \cdot \vec{k}_e}{E_e} \right) \right\} \end{aligned}$$

$$+3 \frac{1 - \lambda^2}{1 + 3\lambda^2} \frac{1}{M} \left( \frac{(\vec{\xi}_e \cdot \vec{k}_e)(\vec{k}_e \cdot \vec{k}_\nu)^2}{(E_e + m_e)E_e E_\nu^2} - \frac{1}{3} (E_e - m_e) \frac{\vec{\xi}_e \cdot \vec{k}_e}{E_e} \right), \tag{9}$$

where we have followed the notation [9–12]. The last three terms in Eq. (9) are caused by the contributions of the proton recoil calculated to order  $O(E_e/M)$  [8–12]. Then,  $\vec{\xi}_n$  and  $\vec{\xi}_e$  are unit polarization vectors of the neutron and electron, respectively,  $d\Omega_e$  and  $d\Omega_{\bar{\nu}}$  are infinitesimal solid angles in the directions of electron  $\vec{k}_e$  and antineutrino  $\vec{k}_{\bar{\nu}}$  3-momenta, respectively,  $E_0 = (m_n^2 - m_p^2 + m_e^2)/2m_n = 1.2926$  MeV is the end-point energy of the electron spectrum,  $F(E_e, Z = 1)$  is the relativistic Fermi function equal to [64–66] (see also [4,9–12])

$$F(E_e, Z = 1) = \left(1 + \frac{1}{2}\gamma\right) \frac{4(2r_p m_e \beta)^{2\gamma}}{\Gamma^2(3 + 2\gamma)} \frac{e^{\pi\alpha/\beta}}{(1 - \beta^2)^\gamma} \left| \Gamma\left(1 + \gamma + i \frac{\alpha}{\beta}\right) \right|^2, \tag{10}$$

where  $\beta = k_e/E_e = \sqrt{E_e^2 - m_e^2}/E_e$  is the electron velocity,  $\gamma = \sqrt{1 - \alpha^2} - 1$ ,  $r_p$  is the electric radius of the proton. In the numerical calculations we use  $r_p = 0.841$  fm [67]. The function  $\zeta(E_e)$  contains the contributions of radiative corrections of order  $O(\alpha/\pi)$  and corrections from the weak magnetism and proton recoil of order  $O(E_e/M)$ , taken in the form used in [8–12]. Then,  $b$  is the Fierz interference term defined by the contributions of interactions beyond the SM [55]. The analytical expressions for the correlation coefficients  $a(E_e)$ ,  $A(E_e)$  and so on, calculated within the SM with the account for radiative corrections of order  $O(\alpha/\pi)$  and corrections caused by the weak magnetism and proton recoil of order  $O(E_e/M)$  together with the contributions of Wilkinson’s corrections [4], are given in [9–12].

### 3.1. Corrections to the correlation coefficients of the electron–energy and angular distribution of the neutron $\beta^-$ -decays caused by pseudoscalar interactions

In the Appendix we calculate the contributions of the OPP exchange and the pseudoscalar interaction BSM to the correlation coefficients of the electron–energy and angular distribution of the neutron  $\beta^-$ -decays for a polarized neutron, a polarized electron and an unpolarized proton. The corrections to the correlation coefficients and the correction to the electron–energy and angular distribution are given in the Appendix in Eqs. (A.5) and (A.6), respectively. The strength of these corrections (see Eq. (A.5)) is defined by the effective coupling constants  $C'_{ps}$  and  $C''_{ps}$ , which are the real and imaginary parts of the effective coupling constant  $C_{ps}$  given by

$$\begin{aligned} C_{ps} &= C_{ps}^{(\text{OPP})} + C_{ps}^{(\text{BSM})} = C'_{ps} + i C''_{ps}, \\ C_{ps}^{(\text{OPP})} &= \frac{2\lambda}{1 + 3\lambda^2} \frac{m_e}{m_\pi^2} E_0 = -1.47 \times 10^{-5}, \\ C_{ps}^{(\text{BSM})} &= -\frac{1}{1 + 3\lambda^2} \frac{E_0}{2M} (C_P - \bar{C}_P) = -1.17 \times 10^{-4} (C_P - \bar{C}_P), \\ C'_{ps} &= \text{Re } C_{ps} = C_{ps}^{(\text{OPP})} + \text{Re } C_{ps}^{(\text{BSM})}, \\ C''_{ps} &= \text{Im } C_{ps} = \text{Im } C_{ps}^{(\text{BSM})}, \end{aligned} \tag{11}$$

where  $C_{ps}^{(\text{OPP})}$  and  $C_{ps}^{(\text{BSM})}$  are the effective coupling constants caused by the OPP exchange and the pseudoscalar interaction BSM, respectively. The numerical values are calculated for  $\lambda = -1.27641$  [41],  $m_e = 0.5110$  MeV,  $m_\pi = 139.5706$  MeV [14],  $E_0 = (m_n^2 - m_p^2 + m_e^2)/2m_n = 1.2926$  MeV and  $M = (m_n + m_p)/2 = 938.9188$  MeV [14], respectively. According to our analysis (see Eqs. (6) and (7)), a real part of the phenomenological coupling constant  $C_{ps}^{(\text{BSM})}$  can be

partly induced by the SM strong low-energy interactions through the ADM-term (see Eq. (6)) and Kaiser's two-loop corrections, calculated within the HB $\chi$ PT (see Eq. (7)).

The corrections, caused by pseudoscalar interactions (see Eq. (A.5) and Eq. (A.6)), to the electron-energy and angular distribution of the neutron  $\beta^-$ -decays for a polarized neutron, a polarized electron and an unpolarized proton, taken together with the electron-energy and angular distributions calculated in [8–12] can be used as a theoretical background for experimental searches of contributions of interactions BSM of order  $10^{-4}$  or even smaller [40].

### 3.2. Estimates of the real and imaginary parts of the phenomenological coupling constant $C_P - \bar{C}_P$

According to [30], the phenomenological coupling constant  $C_P - \bar{C}_P$  can be defined as follows

$$C_P - \bar{C}_P = 2g_P \epsilon_P, \quad (12)$$

where  $\epsilon_P$  is a complex effective coupling constant of the four-fermion local weak interaction of the pseudoscalar quark current  $\bar{u}\gamma^5 d$ , where  $u$  and  $d$  are the *up* and *down* quarks, with the left-handed leptonic current  $\bar{\ell}(1 - \gamma^5)\nu_\ell$  [28–31] (see also [34,53]). Then,  $g_P$  is the matrix element  $\langle p|\bar{u}\gamma^5 d|n\rangle = g_P \bar{u}_p \gamma^5 u_n$  caused by strong low-energy interactions, where  $\bar{u}_p$  and  $u_n$  are the Dirac wave functions of a free proton and neutron, respectively. According to González-Alonso and Camalich [53], one gets  $g_P = 349(9)$  (see Eq. (13) of Ref. [53]).

Following [53] and using the constraint  $|\epsilon_P| < 5.8 \times 10^{-3}$ , obtained at 90% C.L. from the experimental data on the search for an excess of events with a charged lepton (an electron or muon) and a neutrino in the final state of the pp collision with the center-of-mass energy of  $\sqrt{s} = 8$  TeV with an integrated luminosity of  $20 \text{ fb}^{-1}$  at LHC [68], we get  $|\text{Re}(C_P - \bar{C}_P)| < 4.1$ . In this case the pseudoscalar interaction BSM can dominate in the effective coupling constant  $C'_{ps}$  in comparison to the OPP exchange, which is of order  $|C_{ps}^{(\text{OPP})}| \sim 10^{-5}$ .

In turn, the analysis of the leptonic decays of charged pions, carried out in [34] (see Eq. (113) and a discussion on p. 51 of Ref. [34]), taken together with the results, obtained in [69], gives one  $\text{Re}\epsilon_P = (0.4 \pm 1.3) \times 10^{-4}$  and, correspondingly,  $\text{Re}(C_P - \bar{C}_P) = 0.03 \pm 0.09$ . Such an analysis implies that the phenomenological coupling constants  $\text{Re}(C_P - \bar{C}_P)$  and  $\text{Re}C_{ps}^{(\text{BSM})}$  are commensurable with zero. This leads to a dominate role of the OPP exchange in the effective coupling constant  $C'_{ps}$  equal to  $C'_{ps} = -1.47 \times 10^{-5}$ .

Then, following the assumption  $\epsilon_P = 2m_e(m_u + m_d)/m_\pi^2 \sim 4 \times 10^{-4}$  [34], which is also related to the analysis of the leptonic decays of charged pions (see a discussion below Eq. (112) of Ref. [34]), we get  $\text{Re}(C_P - \bar{C}_P) \sim 0.3$  and  $\text{Re}C_{ps}^{(\text{BSM})} \sim -3.5 \times 10^{-5}$ . As a result, according to the assumption  $\epsilon_P = 2m_e(m_u + m_d)/m_\pi^2 \sim 4 \times 10^{-4}$ , the contribution of the pseudoscalar interaction BSM to the effective coupling constant  $C'_{ps}$  should be of order  $10^{-5}$ , that makes it commensurable with the contribution of the OPP exchange.

Since the constraint  $|\epsilon_P| < 5.8 \times 10^{-3}$  [53] disagrees with the constraints following from the analysis of the leptonic decays of charged pions [34,69], one may conclude that the phenomenological coupling constant  $\text{Re}(C_P - \bar{C}_P)$  should be constrained by  $0 \lesssim \text{Re}(C_P - \bar{C}_P) \lesssim 0.3$ . This leads to the effective coupling constant  $\text{Re}C_{ps}^{(\text{BSM})}$  restricted by  $-3.5 \times 10^{-5} \lesssim \text{Re}C_{ps}^{(\text{BSM})} \lesssim 0$ . This shifts the contributions of the pseudoscalar interaction BSM to the region of values  $|\text{Re}C_{ps}^{(\text{BSM})}| \sim 10^{-5}$  or even smaller.



Table 1

Estimates of the phenomenological coupling constant  $C_P - \bar{C}_P = 2g_P \epsilon_P$  for  $g_P = 349(9)$  [53] and the constraints on the parameter  $\epsilon_P$  [29,34,53,69].

$0 \lesssim \text{Re}(C_P - \bar{C}_P) \lesssim 0.3$	$-3.5 \times 10^{-5} \lesssim \text{Re} C_{ps}^{(\text{BSM})} \lesssim 0$
$\text{Im}(C_P - \bar{C}_P) < 0.2$	$\text{Im} C_{ps}^{(\text{BSM})} < -2.3 \times 10^{-5}$

The imaginary part  $\text{Im}(C_P - \bar{C}_P) = 2g_P \text{Im} \epsilon_P$  we estimate using the upper bound  $\text{Im} \epsilon_P < 2.8 \times 10^{-4}$ , obtained at 90% C.L. in [29] (see also Eq. (114) of Ref. [34]). We get  $\text{Im}(C_P - \bar{C}_P) < 0.3$ . The effective coupling constant  $C_{ps}'' = \text{Im} C_{ps}^{(\text{BSM})}$  is restricted by  $C_{ps}'' = \text{Im} C_{ps}^{(\text{BSM})} < -2.3 \times 10^{-5}$ . Since the contribution of the OPP exchange is real, the effective coupling constant  $C_{ps}''$ , constrained by  $C_{ps}'' < -2.3 \times 10^{-5}$ , is fully defined by the pseudoscalar interaction BSM.

In Table 1 we adduce the constraints on the real and imaginary parts of the phenomenological coupling constant  $C_P - \bar{C}_P$  and on the effective coupling constant  $C_{ps}^{(\text{BSM})}$ , which may follow from the results obtained in [34,53,69].

#### 4. Discussion

The corrections of order  $10^{-5}$ , calculated within the SM, are needed as a SM theoretical background for experimental searches of interactions beyond the SM in terms of asymmetries and correlation coefficients of the neutron  $\beta^-$ -decays [10–12]. An experimental accuracy of about a few parts of  $10^{-5}$  or even better, which is required for experimental analyses of interactions BSM of order  $10^{-4}$ , can be reachable at present time [40]. In this paper we have continued the analysis of corrections of order  $10^{-5}$  to the correlation coefficients of the neutron  $\beta^-$ -decays, which we have begun in [10–13]. In this paper we have taken into account the contributions of strong low-energy interactions in terms of the OPP exchange and the contributions of the pseudoscalar interaction BSM [22–27], and calculated corrections to the correlation coefficients of the electron-energy and angular distribution of the neutron  $\beta^-$ -decay for a polarized neutron, a polarized electron and an unpolarized proton.

In addition to the results, concerning the corrections caused by pseudoscalar interactions to the electron-energy and angular distributions of the neutron  $\beta^-$ -decay for a polarized neutron and unpolarized electron and proton, obtained in [47–54] and especially by Harrington [47] and Holstein [51], we have calculated corrections to the correlation coefficients, caused by correlations with the electron spin, i.e. for a polarized neutron and a polarized electron with an unpolarized proton.

We have shown that the energy independent contributions to the pseudoscalar form factor [56–60], related to the Adler-Dothan-Wolfenstein (ADM) term Eq. (6) and to the chiral corrections Eq. (7), calculated by Kaiser [60] in a two-loop approximation within the HB $\chi$ PT, are able in principle to be responsible for sufficiently small real parts of the phenomenological coupling constants  $C_P$  and  $\bar{C}_P$  and at the level of  $10^{-6} - 10^{-8}$  of the effective coupling constant  $C_{ps}^{(\text{BSM})}$ . In turn, the isospin breaking corrections of order  $10^{-5}$ , calculated by Kaiser within the HB $\chi$ PT [46] to the vector coupling constant of the neutron  $\beta^-$ -decay, should be taken into account for a correct description of the neutron lifetime at the level of  $10^{-5}$ .

As has been shown in [30] the phenomenological coupling constant  $C_P - \bar{C}_P$ , introduced at the hadronic level [22–27], can be related to the effective coupling constant  $\epsilon_P$  of the pseudoscalar interaction of the *up* and *down* quarks with left-handed leptonic current by  $C_P - \bar{C}_P =$

$2g_P\epsilon_P$ , where  $g_P = 349(9)$  [53] is the matrix element of the pseudoscalar quark current caused by strong low-energy interactions. Using the relation  $C_P - \bar{C}_P = 2g_P\epsilon_P$  [30] we have estimated the real and imaginary parts of the phenomenological coupling constant  $C_P - \bar{C}_P$ . Having summarized the results, concerning the constraints on the parameter  $\epsilon_P$ , obtained in [29,34,53,69], and taking into account that  $g_P = 349(9)$  [53], we have got  $0 \lesssim \text{Re}(C_P - \bar{C}_P) \lesssim 0.3$  and  $\text{Im}(C_P - \bar{C}_P) < 0.2$ . Such an estimate agrees well with the analysis of the contributions of the pseudoscalar interaction BSM to the lifetimes of charged pions [34].

For the effective coupling constants  $\text{Re} C_{ps}^{(\text{BSM})}$  and  $\text{Im} C_{ps}^{(\text{BSM})}$ , defining the strength of the contributions of the pseudoscalar interaction BSM to the correlation coefficients of the electron-energy and angular distribution of the neutron  $\beta^-$ -decays, we get  $-3.5 \times 10^{-5} \lesssim \text{Re} C_{ps}^{(\text{BSM})} \lesssim 0$  and  $\text{Im} C_{ps}^{(\text{BSM})} < -2.3 \times 10^{-5}$ , respectively. This implies that the effective coupling constant  $C_{ps}^{(\text{BSM})}$  is of order  $|C_{ps}^{(\text{BSM})}| \sim 10^{-5}$ .

The analysis of contributions of pseudoscalar interactions to the electron-energy and angular distributions of weak semileptonic decays of baryons has a long history [47–54] (see also [4,34]). That is why it is important to make a comparative analysis of the results obtained in our work with those in [4,34,47–54]. For the first time the contributions of pseudoscalar interactions to the correlation coefficients of electron-energy and angular distributions for weak semileptonic decays of baryons for polarized parent baryons and unpolarized decay electrons and baryons were calculated by Harrington [47]. In the notation of Jackson et al. [24] Harrington calculated the contributions of the induced pseudoscalar form factor to the Fierz interference term  $b(E_e)$  [55] and to the correlation coefficients  $a(E_e)$ ,  $A(E_e)$ ,  $B(E_e)$  and  $D(E_e)$ , caused by electron-antineutrino angular correlations and correlations of the neutron spin with electron and antineutrino 3-momenta, respectively. The corresponding contributions of pseudoscalar interactions can be obtained from Eqs. (9) – (13) of Ref. [47] keeping the leading terms in the large baryon mass expansion. They read

$$\begin{aligned} \frac{d^5\delta\lambda_n(E_e, \vec{k}_e, \vec{k}_{\bar{\nu}}, \vec{\xi}_n, \vec{\xi}_e)}{dE_e d\Omega_e d\Omega_{\bar{\nu}}} \propto & - \frac{\text{Re}(g_1 g_3^*)}{|f_1|^2 + 3|g_2|^2} \frac{m_e^2}{M^2} \frac{E_{\bar{\nu}}}{E_e} - \frac{\text{Re}(g_1 g_3^*)}{|f_1|^2 + 3|g_2|^2} \frac{m_e^2}{M^2} \frac{\vec{k}_e \cdot \vec{k}_{\bar{\nu}}}{E_e E_{\bar{\nu}}} \\ & - \frac{\text{Re}(f_1 g_3^*)}{|f_1|^2 + 3|g_2|^2} \frac{m_e^2}{M^2} \frac{\vec{\xi}_n \cdot \vec{k}_e}{E_e} - \frac{\text{Re}(f_1 g_3^*)}{|f_1|^2 + 3|g_2|^2} \frac{m_e^2}{M^2} \frac{E_{\bar{\nu}}}{E_e} \frac{\vec{\xi}_n \cdot \vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}} \\ & + \frac{\text{Im}(g_1 g_3^*)}{|f_1|^2 + 3|g_2|^2} \frac{m_e^2}{M^2} \frac{\vec{\xi}_n \cdot (\vec{k}_e \times \vec{k}_{\bar{\nu}})}{E_e E_{\bar{\nu}}}, \end{aligned} \quad (13)$$

where the first term describes the contribution of pseudoscalar interactions to the Fierz-like interference term [55]. The analogous corrections can be extracted from the expressions, calculated by Holstein [51] (see Appendix B of Ref. [51]). The corrections of pseudoscalar interactions to the Fierz-like interference term  $\delta b_{ps}(E_e)$  and correlation coefficients  $\delta a_{ps}(E_e)$ ,  $\delta A_{ps}(E_e)$ ,  $\delta B_{ps}(E_e)$  and  $\delta D_{ps}(E_e)$ , calculated in Eqs. (A.5) and (A.6), agree well with those calculated by Harrington [47] (see Eq. (13)). Since in [4,34,48–54] the electron-energy and angular distributions were analyzed for weak semileptonic decays either for polarized parent baryons and unpolarized decay electrons and baryons or for unpolarized parent baryons and unpolarized decay electrons and baryons the overlap of our results with those obtained in [4,48–54] is at the level of the corrections shown in Eq. (13). Indeed, the contribution of the Fierz-like interference term  $\delta b_{ps}(E_e)$  in Eq. (A.4) agrees well with the result, obtained by Wilkinson [4] and by González-Alonso and Camalich [53]

$$\begin{aligned} \frac{d^5 \delta \lambda_n(E_e, \vec{k}_e, \vec{k}_{\bar{\nu}}, \vec{\xi}_n, \vec{\xi}_e)}{dE_e d\Omega_e d\Omega_{\bar{\nu}}} \propto C'_{ps} \lambda \frac{E_0 - E_e}{E_0} \frac{m_e}{E_e} + \dots \rightarrow -\frac{g_A g_{IP}}{g_V^2 + 3g_A^2} \frac{E_0 - E_e}{M} \frac{m_e}{E_e} \\ - \frac{\lambda}{1 + 3\lambda^2} g_P \text{Re} \epsilon_P \frac{E_0 - E_e}{M} \frac{m_e}{E_e} + \dots, \end{aligned} \quad (14)$$

where the term proportional to  $g_A g_{IP}$ , describing the contribution of the OPP exchange with  $g_{IP} = 2g_A M/m_\pi^2$ , was calculated by Wilkinson (see Table 1 and a definition of  $g_{IP}$  on p. 479 of Ref. [4]), whereas the second term, caused by the contribution of the pseudoscalar interaction BSM and where we have taken into account the relation  $C_P - \bar{C}_P = 2g_P \epsilon_P$  [30], was calculated by González-Alonso and Camalich [53] (see Eqs. (16) and (17) of Ref. [53]).

In turn, the contributions of pseudoscalar interactions to the correlation coefficients, induced by correlations with the electron spin, were not calculated in [4,34,47–54]. Thus, the calculation of contributions of pseudoscalar interactions to the correlation coefficients, induced by correlations with the electron spin, distinguishes our results from those obtained in [4,34,47–54]. However, we would like to notice that in the book by Behrens and Bühring [52] there is a capture entitled “Electron polarization”, concerning an analysis of a polarization of decay electrons in beta decays. In this capture the authors propose a most general density matrix, which can be applied to a description of energy and angular distributions for beta decays by taking into account a polarization of decay electrons (see Eq. (7.6) and Eq. (7.7) of Ref. [52]). Of course, by using such a general density matrix and the technique, developed by Biedenharn and Rose [70], one can, in principle, calculate contributions of pseudoscalar interactions to the correlation coefficients induced by correlations with the electron spin. Nevertheless, the calculation of these corrections were not performed in [52]. The authors applied such a general density matrix to a calculation of a general formula for a value of a longitudinal polarization of decay electrons in beta decays only (see Eq. (7.151) of Ref. [52]). Thus, we may assert that all corrections of pseudoscalar interactions to the correlation coefficients, induced by correlations with the electron spin (see Eq. (A.5)), and also other terms proportional to the coupling constants  $C'_{ps}$  and  $C''_{ps}$  in Eq. (A.6) are new in comparison to the results, obtained in [4,34,47–54] and were never calculated in literature. Moreover, a theoretical accuracy  $O(\alpha E_0/\pi M) \sim 10^{-6}$  and  $O(E_0^2/M^2) \sim 10^{-6}$  of the calculation of a complete set of corrections of order  $10^{-3}$  [9–12] including radiative corrections of order  $O(\alpha/\pi)$  and corrections of order  $O(E_0/M)$ , caused by the weak magnetism and proton recoil, makes the contributions of corrections of order  $10^{-5}$ , induced by pseudoscalar interactions, observable in principle and important as a part of theoretical background for experimental searches of contributions of interactions BSM in asymmetries of the neutron  $\beta^-$ -decays with a polarized neutron, a polarized electron and an unpolarized proton [40].

Thus, in this work we have calculated the contributions of pseudoscalar interactions, induced by the OPP exchange and BSM, to the complete set of correlation coefficients of the electron–energy and angular distribution of the neutron  $\beta^-$ -decays for a polarized neutron, a polarized electron and an unpolarized proton. The corrections to the Fierz interference term  $b(E_e)$ , the correlation coefficients  $a(E_e)$ ,  $A(E_e)$ ,  $B(E_e)$  and  $D(E_e)$ , caused by electron–antineutrino angular correlations and correlations of the neutron spin with electron and antineutrino 3–momenta, respectively, and as well as the correlation coefficients, induced by correlations with the electron spin such as  $G(E_e)$ ,  $N(E_e)$  and so on, and also corrections, given by the terms proportional to the effective coupling constants  $C'_{ps}$  and  $C''_{ps}$  in Eq. (A.6), are calculated by using one of the same theoretical technique. The agreement of the corrections to the Fierz interference term  $b(E_e)$  and the correlation coefficients  $a(E_e)$ ,  $A(E_e)$ ,  $B(E_e)$  and  $D(E_e)$  with the results obtained in [4,34,47–54] may only confirm a correctness of our results.

The obtained corrections (see Eq. (A.5) and Eq. (A.6)), caused by the OPP exchange and the pseudoscalar interaction BSM, complete the analysis of contributions of interactions BSM to the correlation coefficients of the neutron  $\beta^-$ -decays for a polarized neutron, a polarized electron and an unpolarized proton carried out in [9–12]. For experimental accuracies of about a few parts of  $10^{-5}$  or even better [40] the exact analytical expressions of these corrections can be practically distinguished from the contributions of order  $10^{-5}$ , caused by the second class hadronic currents or  $G$ -odd correlations, calculated by Gardner and Plaster [33] and Ivanov et al. [11,12].

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## Appendix A. Calculation of corrections caused by pseudoscalar interactions to the electron–energy and angular distribution of the neutron $\beta^-$ -decays for a polarized neutron, a polarized electron and an unpolarized proton

A direct calculation of the corrections, caused by the OPP exchange and the pseudoscalar interaction BSM [9], to the electron–energy and angular distribution of the neutron  $\beta^-$ -decays for a polarized neutron, a polarized electron and an unpolarized proton yields

$$\begin{aligned} \frac{d^5\delta\lambda_n(E_e, \vec{k}_e, \vec{k}_{\bar{\nu}}, \vec{\xi}_n, \vec{\xi}_e)}{dE_e d\Omega_e d\Omega_{\bar{\nu}}} &= (1 + 3\lambda^2) \frac{G_F^2 |V_{ud}|^2}{32\pi^5} (E_0 - E_e)^2 \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z = 1) \\ &\times \frac{1}{E_0 E_e E_{\bar{\nu}}} \left\{ C'_{ps} \left[ \lambda \left( -m_e (\vec{k}_p \cdot \vec{k}_{\bar{\nu}}) - (\vec{k}_p \cdot \vec{k}_e) (\zeta_e \cdot k_{\bar{\nu}}) + (\vec{k}_p \cdot \vec{\xi}_e) (k_e \cdot k_{\bar{\nu}}) \right) \right. \right. \\ &+ (\vec{\xi}_n \cdot \vec{k}_p) (m_e E_{\bar{\nu}} + E_e (\zeta_e \cdot k_{\bar{\nu}}) - \zeta_e^0 (k_e \cdot k_{\bar{\nu}})) \\ &+ \lambda (\vec{\xi}_n \times \vec{k}_p) \cdot \left( -E_e (\vec{\zeta}_e \times \vec{k}_{\bar{\nu}}) + \zeta_e^0 (\vec{k}_e \times \vec{k}_{\bar{\nu}}) + E_{\bar{\nu}} (\vec{\zeta}_e \times \vec{k}_e) \right) \\ &+ C''_{ps} \left[ \vec{\zeta}_e \cdot (\vec{k}_e \times \vec{k}_{\bar{\nu}}) (\vec{\xi}_n \cdot \vec{k}_p) + \lambda \left( E_e \vec{k}_p \cdot (\vec{\zeta}_e \times \vec{k}_{\bar{\nu}}) - E_{\bar{\nu}} \vec{k}_p \cdot (\vec{\zeta}_e \times \vec{k}_e) \right. \right. \\ &\left. \left. - m_e \vec{k}_{\bar{\nu}} \cdot (\vec{\xi}_n \times \vec{k}_p) - \vec{k}_e \cdot (\vec{\xi}_n \times \vec{k}_p) (\zeta_e \cdot k_{\bar{\nu}}) + \vec{\zeta}_e \cdot (\vec{\xi}_n \times \vec{k}_p) (k_e \cdot k_{\bar{\nu}}) \right) \right] \left. \right\}. \quad (\text{A.1}) \end{aligned}$$

The strength of the contributions of pseudoscalar interactions is defined by the effective coupling constants  $C'_{ps}$  and  $C''_{ps}$ , which are the real and imaginary parts of the effective coupling constant  $C_{ps}$  given by

$$C_{ps} = C_{ps}^{(\text{OPP})} + C_{ps}^{(\text{BSM})},$$

$$\begin{aligned}
 C_{ps}^{(\text{OPP})} &= \frac{2\lambda}{1+3\lambda^2} \frac{m_e}{m_\pi^2} E_0 = -1.47 \times 10^{-5}, \\
 C_{ps}^{(\text{BSM})} &= -\frac{1}{1+3\lambda^2} \frac{E_0}{2M} (C_P - \bar{C}_P) = -1.17 \times 10^{-4} (C_P - \bar{C}_P), \\
 C'_{ps} &= \text{Re } C_{ps} = C_{ps}^{(\text{OPP})} + \text{Re } C_{ps}^{(\text{BSM})}, \\
 C''_{ps} &= \text{Im } C_{ps} = \text{Im } C_{ps}^{(\text{BSM})}.
 \end{aligned} \tag{A.2}$$

The numerical values are obtained at  $\lambda = -1.27641$ ,  $E_0 = (m_n^2 - m_p^2 + m_e^2)/2m_n = 1.2926$  MeV,  $m_e = 0.511$  MeV and  $M = (m_n + m_p)/2 = 938.918$  MeV [14]. Then,  $\zeta_e$  is a 4-polarization vector of the electron [35]

$$\zeta_e = (\zeta_e^0, \vec{\zeta}_e) = \left( \frac{\vec{\xi}_e \cdot \vec{k}_e}{m_e}, \vec{\xi}_e + \frac{\vec{k}_e(\vec{\xi}_e \cdot \vec{k}_e)}{m_e(E_e + m_e)} \right) \tag{A.3}$$

obeying the constraints  $\zeta_e^2 = -\vec{\xi}_e^2 = -1$  and  $k_e \cdot \zeta_e = 0$ . The right-hand-side (r.h.s.) of Eq. (A.1) can be transcribed into the form

$$\begin{aligned}
 \frac{d^5 \delta \lambda_n(E_e, \vec{k}_e, \vec{k}_{\bar{\nu}}, \vec{\xi}_n, \vec{\xi}_e)}{dE_e d\Omega_e d\Omega_{\bar{\nu}}} &= (1+3\lambda^2) \frac{G_F^2 |V_{ud}|^2}{32\pi^5} (E_0 - E_e)^2 \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z=1) \\
 &\times \left\{ C'_{ps} \left[ \lambda \left( \frac{E_{\bar{\nu}}}{E_0} \frac{m_e}{E_e} + \frac{m_e}{E_0} \frac{\vec{k}_e \cdot \vec{k}_{\bar{\nu}}}{E_e E_{\bar{\nu}}} - \frac{m_e}{E_0} \frac{\vec{\xi}_e \cdot \vec{k}_e}{E_e} - \left( 1 - \frac{m_e^2}{E_0 E_e} \right) \frac{\vec{\xi}_e \cdot \vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}} \right. \right. \\
 &+ \left( 1 + \frac{m_e}{E_0} \right) \frac{(\vec{\xi}_e \cdot \vec{k}_e)(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})}{(E_e + m_e) E_e E_{\bar{\nu}}} + \left( -\frac{m_e}{E_0} \frac{\vec{\xi}_n \cdot \vec{k}_e}{E_e} - \frac{m_e}{E_0} \frac{E_{\bar{\nu}}}{E_e} \frac{\vec{\xi}_n \cdot \vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}} + \frac{E_e}{E_0} \frac{(\vec{\xi}_n \cdot \vec{k}_e)}{E_e} \frac{(\vec{\xi}_e \cdot \vec{k}_{\bar{\nu}})}{E_{\bar{\nu}}} \right. \\
 &+ \left. \left. \frac{E_{\bar{\nu}}}{E_0} \frac{(\vec{\xi}_n \cdot \vec{k}_{\bar{\nu}})(\vec{\xi}_e \cdot \vec{k}_{\bar{\nu}})}{E_{\bar{\nu}}^2} - \frac{E_e}{E_0} \frac{(\vec{\xi}_n \cdot \vec{k}_e)(\vec{\xi}_e \cdot \vec{k}_e)(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})}{(E_e + m_e) E_e^2 E_{\bar{\nu}}} - \frac{E_{\bar{\nu}}}{E_0} \frac{(\vec{\xi}_n \cdot \vec{k}_{\bar{\nu}})(\vec{\xi}_e \cdot \vec{k}_e)(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})}{(E_e + m_e) E_e E_{\bar{\nu}}^2} \right) \right. \\
 &+ \lambda \left( \frac{E_e E_{\bar{\nu}} - k_e^2}{E_e E_0} (\vec{\xi}_n \cdot \vec{\xi}_e) + \frac{E_e - E_{\bar{\nu}}}{E_0} \frac{(\vec{\xi}_n \cdot \vec{\xi}_e)(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})}{E_e E_{\bar{\nu}}} + \frac{E_e - E_{\bar{\nu}} + m_e}{E_0} \frac{(\vec{\xi}_n \cdot \vec{k}_e)(\vec{\xi}_e \cdot \vec{k}_e)}{(E_e + m_e) E_e} \right. \\
 &- \frac{m_e}{E_0} \frac{(\vec{\xi}_n \cdot \vec{k}_{\bar{\nu}})(\vec{\xi}_e \cdot \vec{k}_e)}{E_e E_{\bar{\nu}}} + \frac{E_{\bar{\nu}}}{E_0} \frac{(\vec{\xi}_n \cdot \vec{k}_e)(\vec{\xi}_e \cdot \vec{k}_{\bar{\nu}})}{E_e E_{\bar{\nu}}} - \frac{E_{\bar{\nu}}}{E_0} \frac{(\vec{\xi}_n \cdot \vec{k}_{\bar{\nu}})(\vec{\xi}_e \cdot \vec{k}_{\bar{\nu}})}{E_{\bar{\nu}}^2} \\
 &- \left. \left. \frac{E_e}{E_0} \frac{(\vec{\xi}_n \cdot \vec{k}_e)(\vec{\xi}_e \cdot \vec{k}_e)(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})}{(E_e + m_e) E_e^2 E_{\bar{\nu}}} + \frac{E_{\bar{\nu}}}{E_0} \frac{(\vec{\xi}_n \cdot \vec{k}_{\bar{\nu}})(\vec{\xi}_e \cdot \vec{k}_e)(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})}{(E_e + m_e) E_e E_{\bar{\nu}}^2} \right) \right] \\
 &+ C''_{ps} \left[ -\frac{E_e}{E_0} \frac{\vec{\xi}_e \cdot (\vec{k}_e \times \vec{k}_{\bar{\nu}})(\vec{\xi}_n \cdot \vec{k}_e)}{E_e^2 E_{\bar{\nu}}} - \frac{E_{\bar{\nu}}}{E_0} \frac{\vec{\xi}_e \cdot (\vec{k}_e \times \vec{k}_{\bar{\nu}})(\vec{\xi}_n \cdot \vec{k}_{\bar{\nu}})}{E_e E_{\bar{\nu}}^2} + \lambda \left( \frac{\vec{\xi}_e \cdot (\vec{k}_e \times \vec{k}_{\bar{\nu}})}{E_e E_{\bar{\nu}}} \right. \right. \\
 &+ \frac{m_e}{E_0} \frac{\vec{\xi}_n \cdot (\vec{k}_e \times \vec{k}_{\bar{\nu}})}{E_e E_{\bar{\nu}}} + \frac{E_e}{E_0} \frac{\vec{\xi}_n \cdot (\vec{\xi}_e \times \vec{k}_e)}{E_e} + \frac{E_{\bar{\nu}}}{E_0} \frac{\vec{\xi}_n \cdot (\vec{\xi}_e \times \vec{k}_{\bar{\nu}})}{E_{\bar{\nu}}} - \frac{E_e}{E_0} \frac{\vec{\xi}_n \cdot (\vec{\xi}_e \times \vec{k}_e)(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})}{E_e^2 E_{\bar{\nu}}} \\
 &- \left. \left. \frac{E_{\bar{\nu}}}{E_0} \frac{\vec{\xi}_n \cdot (\vec{\xi}_e \times \vec{k}_{\bar{\nu}})(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})}{E_e E_{\bar{\nu}}^2} + \frac{E_{\bar{\nu}}}{E_0} \frac{\vec{\xi}_n \cdot (\vec{k}_e \times \vec{k}_{\bar{\nu}})(\vec{\xi}_e \cdot \vec{k}_{\bar{\nu}})}{E_e E_{\bar{\nu}}^2} - \frac{E_{\bar{\nu}}}{E_0} \frac{\vec{\xi}_n \cdot (\vec{k}_e \times \vec{k}_{\bar{\nu}})(\vec{\xi}_e \cdot \vec{k}_e)}{(E_e + m_e) E_e E_{\bar{\nu}}} \right) \right] \Bigg\}.
 \end{aligned} \tag{A.4}$$

We obtain the following contributions to the correlation coefficients

$$\delta \zeta_{ps}(E_e) = 0, \quad \delta b_{ps}(E_e) = C'_{ps} \lambda \frac{E_0 - E_e}{E_0}, \quad \delta a_{ps}(E_e) = C'_{ps} \lambda \frac{m_e}{E_0},$$

$$\begin{aligned}
\delta A_{ps}(E_e) &= -C'_{ps} \frac{m_e}{E_0}, \delta B_{ps}(E_e) = -C'_{ps} \frac{m_e}{E_0} \frac{E_0 - E_e}{E_e}, \delta K_{nps}(E_e) = \delta Q_{nps}(E_e) = 0, \\
\delta G_{ps}(E_e) &= -C'_{ps} \lambda \frac{m_e}{E_0}, \delta H_{ps}(E_e) = -C'_{ps} \lambda \left(1 - \frac{m_e^2}{E_0 E_e}\right), \\
\delta Q_{eps}(E_e) &= C'_{ps} \left(\lambda \frac{2E_e - E_0 + m_e}{E_0} + (\lambda - 1) \frac{1}{3} \frac{E_0 - E_e}{E_0}\right), \\
\delta K_{eps}(E_e) &= C'_{ps} \lambda \left(1 + \frac{m_e}{E_0}\right), \\
\delta N_{ps}(E_e) &= C'_{ps} \left(\lambda \frac{-2E_e^2 + E_0 E_e + m_e^2}{E_0 E_e} + (1 - \lambda) \frac{1}{3} \frac{E_0 - E_e}{E_0}\right), \\
\delta D_{ps}(E_e) &= C''_{ps} \lambda \frac{m_e}{E_0}, \delta R_{ps}(E_e) = C''_{ps} \left(-\lambda \frac{E_e}{E_0} + (1 + 2\lambda) \frac{1}{3} \frac{E_0 - E_e}{E_0}\right), \\
\delta L_{ps}(E_e) &= C''_{ps} \lambda.
\end{aligned} \tag{A.5}$$

In terms of corrections to the correlation coefficients Eq. (A.5) the correction to the electron-energy and angular distribution Eq. (A.4) is given by

$$\begin{aligned}
\frac{d^5 \delta \lambda_n(E_e, \vec{k}_e, \vec{k}_{\bar{\nu}}, \vec{\xi}_n, \vec{\xi}_e)}{dE_e d\Omega_e d\Omega_{\bar{\nu}}} &= (1 + 3\lambda^2) \frac{G_F^2 |V_{ud}|^2}{32\pi^5} (E_0 - E_e)^2 \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z = 1) \\
&\times \left\{ \delta b_{ps}(E_e) \frac{m_e}{E_e} + \delta a_{ps}(E_e) \frac{\vec{k}_e \cdot \vec{k}_{\bar{\nu}}}{E_e E_{\bar{\nu}}} + \delta A_{ps}(E_e) \frac{\vec{\xi}_n \cdot \vec{k}_e}{E_e} + \delta B_{ps}(E_e) \frac{\vec{\xi}_n \cdot \vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}} \right. \\
&+ \delta G_{ps}(E_e) \frac{\vec{\xi}_e \cdot \vec{k}_e}{E_e} + \delta H_{ps}(E_e) \frac{\vec{\xi}_e \cdot \vec{k}_{\bar{\nu}}}{E_{\bar{\nu}}} + \delta Q_{eps}(E_e) \frac{(\vec{\xi}_n \cdot \vec{k}_e)(\vec{k}_e \cdot \vec{\xi}_e)}{(E_e + m_e)E_e} \\
&+ \delta K_{eps}(E_e) \frac{(\vec{\xi}_e \cdot \vec{k}_e)(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})}{(E_e + m_e)E_e E_{\bar{\nu}}} + \delta N_{ps}(E_e) (\vec{\xi}_n \cdot \vec{\xi}_e) + \delta R_{ps} \frac{\vec{\xi}_n \cdot (\vec{k}_e \times \vec{\xi}_e)}{E_e} \\
&+ \delta L_{ps} \frac{\vec{\xi}_n \cdot (\vec{k}_e \times \vec{k}_{\bar{\nu}})}{E_e E_{\bar{\nu}}} + C'_{ps} \left[ \lambda \frac{E_0 - E_e}{E_0} \frac{(\vec{\xi}_n \cdot \vec{\xi}_e)(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})}{E_e E_{\bar{\nu}}} \right. \\
&+ \frac{(1 - \lambda)E_e + \lambda E_0}{E_0} \frac{(\vec{\xi}_n \cdot \vec{k}_e)(\vec{\xi}_e \cdot \vec{k}_{\bar{\nu}})}{E_e E_{\bar{\nu}}} - \lambda \frac{m_e}{E_0} \frac{(\vec{\xi}_n \cdot \vec{k}_{\bar{\nu}})(\vec{\xi}_e \cdot \vec{k}_e)}{E_e E_{\bar{\nu}}} \\
&- (1 + \lambda) \frac{E_e}{E_0} \frac{(\vec{\xi}_n \cdot \vec{k}_e)(\vec{\xi}_e \cdot \vec{k}_e)(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})}{(E_e + m_e)E_e^2 E_{\bar{\nu}}} + (1 - \lambda) \frac{E_0 - E_e}{E_0} \left( \frac{(\vec{\xi}_n \cdot \vec{k}_{\bar{\nu}})(\vec{\xi}_e \cdot \vec{k}_{\bar{\nu}})}{E_{\bar{\nu}}^2} - \frac{1}{3} \vec{\xi}_n \cdot \vec{\xi}_e \right) \\
&+ (\lambda - 1) \frac{E_0 - E_e}{E_0} \left( \frac{(\vec{\xi}_n \cdot \vec{k}_{\bar{\nu}})(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})}{E_{\bar{\nu}}^2} - \frac{1}{3} \vec{\xi}_n \cdot \vec{k}_e \right) \frac{(\vec{\xi}_e \cdot \vec{k}_e)}{(E_e + m_e)E_e} \left. \right] \\
&+ C''_{ps} \left[ - \frac{E_e}{E_0} \frac{\vec{\xi}_e \cdot (\vec{k}_e \times \vec{k}_{\bar{\nu}})(\vec{\xi}_n \cdot \vec{k}_e)}{E_e^2 E_{\bar{\nu}}} + \lambda \frac{E_0 - E_e}{E_0} \frac{\vec{\xi}_n \cdot (\vec{\xi}_e \times \vec{k}_{\bar{\nu}})}{E_{\bar{\nu}}} \right. \\
&- \lambda \frac{E_e}{E_0} \frac{\vec{\xi}_n \cdot (\vec{\xi}_e \times \vec{k}_e)(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})}{E_e^2 E_{\bar{\nu}}} - \lambda \frac{E_0 - E_e}{E_0} \frac{\vec{\xi}_n \cdot (\vec{k}_e \times \vec{k}_{\bar{\nu}})(\vec{\xi}_e \cdot \vec{k}_e)}{(E_e + m_e)E_e E_{\bar{\nu}}} \\
&- \frac{E_0 - E_e}{E_0} \left( \frac{\vec{\xi}_e \cdot (\vec{k}_e \times \vec{k}_{\bar{\nu}})(\vec{\xi}_n \cdot \vec{k}_{\bar{\nu}})}{E_e E_{\bar{\nu}}^2} - \frac{1}{3} \frac{\vec{\xi}_e \cdot (\vec{k}_e \times \vec{\xi}_n)}{E_e} \right) \\
&- \lambda \frac{E_0 - E_e}{E_0} \left( \frac{\vec{\xi}_n \cdot (\vec{\xi}_e \times \vec{k}_{\bar{\nu}})(\vec{k}_e \cdot \vec{k}_{\bar{\nu}})}{E_e E_{\bar{\nu}}^2} - \frac{1}{3} \frac{\vec{\xi}_n \cdot (\vec{\xi}_e \times \vec{k}_e)}{E_e} \right)
\end{aligned}$$

$$+\lambda \frac{E_0 - E_e}{E_0} \left( \frac{\vec{\xi}_n \cdot (\vec{k}_e \times \vec{k}_{\bar{\nu}})(\vec{\xi}_e \cdot \vec{k}_{\bar{\nu}})}{E_e E_{\bar{\nu}}^2} - \frac{1}{3} \frac{\vec{\xi}_n \cdot (\vec{k}_e \times \vec{\xi}_e)}{E_e} \right) \Bigg\}. \quad (\text{A.6})$$

This correction to the electron–energy and angular distribution together with the results obtained in [8–12], can be used for experimental analyses of asymmetries and correlation coefficients of the neutron  $\beta^-$ -decays for a polarized neutron, a polarized electron and an unpolarised proton with experimental uncertainties of a few parts of  $10^{-5}$  [40].

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