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FRB 121102 casts new light on the photon mass

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ABSTRACT

The photon mass, m_γ, can in principle be constrained using measurements of the dispersion measures (DMs) of fast radio bursts (FRBs), once the FRB redshifts are known. The DM of the repeating FRB 121102 is known to $< 1\%$, a host galaxy has now been identified with high confidence, and its redshift, z, has now been determined with high accuracy: $z = 0.19273(8)$. Taking into account the plasma contributions to the DM from the Intergalactic medium (IGM) and the Milky Way, we use the data on FRB 121102 to derive the constraint $m_\gamma \lesssim 2.2 \times 10^{-14}$ eV c^{-2} (3.9×10^{-50} kg). Since the plasma and photon mass contributions to DMs have different redshift dependences, they could in principle be distinguished by measurements of more FRB redshifts, enabling the sensitivity to m_γ to be improved.

The photon is generally expected to be massless, but a number of theorists have challenged this assumption, starting from de Broglie and nowadays considering models with massive photons for dark energy and dark matter. Examples of mechanisms for providing mass include Standard Model Extensions with supersymmetry and Lorentz invariance breaking [1] and Higgs mechanisms [2]. In view of these possibilities and the fundamental importance of the photon mass, it is important to constrain the magnitude of the photon mass as robustly as possible. The most robust limits available are those from laboratory experiments [3] – see [4,5] for reviews – but these are much weaker than those derived from astrophysical observations. The Particle Data Group (PDG) [6] cites the upper limit $m_\gamma < 8.4 \times 10^{-19}$ eV c^{-2} ($= 1.5 \times 10^{-54}$ kg) [7] obtained by modelling the magnetic field of the solar system [7,8]. However, this limit relies on assumptions about the form of the magnetic field and does not discuss measurement accuracy and errors. Another limit ($m_\gamma < 4 \times 10^{-52}$ kg) which has been derived from atmospheric radio waves has been reported in [9]. A more conservative approach was followed in an analysis of Cluster data [10], leading to an upper limit between 7.9×10^{-14} and 1.9×10^{-15} eV c^{-2} (1.4×10^{-49} and 3.4×10^{-51} kg). It is clearly desirable to explore more direct and robust astrophysical constraints on a possible photon mass.

This was the motivation for a study we made [11] (see also [12]) showing how data from fast radio bursts (FRBs) could be used to constrain m_γ. These have durations in the millisecond range, and their signals are known to arrive with a frequency-dependent dispersion in time of the $1/\nu^2$ form. This is the dependence expected from plasma effects, but a similar dispersion $\propto m_\gamma^2/\nu^2$ could also arise from a photon mass. The dispersions induced by plasma effects and m_γ both increase with distance (redshift z), but with different dependences on z. We note in this connection that the lower frequencies of FRB emissions give a distinct advantage over gamma-ray bursters and other sources of high-energy γ rays for constraining m_γ, since mass effects are suppressed for higher-energy photons.[1] Moreover, using FRB emissions to constrain m_γ

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[1] In contrast, sources of high-energy photons are better suited for probing models of Lorentz violation [13].

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is much more direct and involves fewer uncertainties than using the properties of astrophysical magnetic fields.[2]

That said, although the large dispersion measures (DMs) and other arguments led to the general belief that FRBs occur at cosmological distances, until recently no FRB redshift had been measured. The first claim to measure a redshift was made for FRB 150418 [20], and this was the example we used in [11] to show how FRB measurements could in principle be used to constrain m_γ. However, the identification of the host galaxy of FRB 150418 has subsequently been challenged [21], and is now generally not accepted [22].

Our interest in the possibility of using FRBs to constrain m_γ has recently been revived, however, by the observation of repeated emissions from FRB 121102 [22]. These have permitted precise localisation of its host galaxy, which has made possible a precise determination of its redshift, $z = 0.19273(8)$ [23]. This redshift determination makes it possible, in turn, to use data on FRB 121102 to provide a robust constraint on m_γ, as we discuss in this paper.

The dispersion measure (DM) is related to the frequency-dependent time lag of an FRB by

$$\Delta t_{\rm DM} = 415 \left(\frac{\nu}{1\,{\rm GHz}}\right)^{-2} \frac{\rm DM}{10^5\,{\rm pc\,cm^{-3}}}\,{\rm s}\,. \tag{1}$$

In the absence of a photon mass, the time-lag of an FRB is given by integrating the column density n_e of free electrons along the line of flight of its radio signal

$$\Delta t_{\rm DM} = \int \frac{dl}{c}\frac{\nu_p^2}{2\nu^2}\,, \tag{2}$$

where $\nu_p = (n_e e^2/\pi m_e)^{1/2} = 8.98 \cdot 10^3 n_e^{1/2}$ Hz. Several sources contribute to this integrated column density of free electrons, notably the Milky Way galaxy, the intergalactic medium (IGM) and the host galaxy. The contribution to the DM (1) of an FRB at redshift z from the IGM is given by the cosmological density fraction $\Omega_{\rm IGM}$ of ionised baryons [24,25]:

$${\rm DM_{IGM}} = \frac{3cH_0\Omega_{\rm IGM}}{8\pi G m_p} H_e(z)\,, \tag{3}$$

where $H_0 = 67.8(9)$ km/s/Mpc [6] is the present Hubble expansion rate,[3] G is the Newton constant, m_p is the proton mass, and the redshift-dependent factor

$$H_e(z) \equiv \int_0^z \frac{(1+z')dz'}{\sqrt{\Omega_\Lambda + (1+z')^3\Omega_m}}\,, \tag{4}$$

where $\Omega_\Lambda = 0.692(12)$ and $\Omega_m = 0.308(12)$ [6]. For comparison, the difference in time lags between photons of energies $E_{1,2}$ due to a non-zero photon mass has the form:

$$\Delta t_{m_\gamma} = \frac{m_\gamma^2}{2H_0}\cdot\left(\frac{1}{E_1^2} - \frac{1}{E_2^2}\right)\cdot H_\gamma(z)\,, \tag{5}$$

where we use natural units $h = c = 1$, and [27,28]

$$H_\gamma(z) \equiv \int_0^z \frac{dz'}{(1+z')^2\sqrt{\Omega_\Lambda + (1+z')^3\Omega_m}}\,. \tag{6}$$

As already commented, the time lags due to the IGM and a possible photon mass have different dependences (4) and (6) on the redshift. The uncertainties in the cosmological parameters and the measurement of the redshift measurement of FRB 121102 are taken into account in our analysis, with the uncertainties in the former being much more important, as we will see later.

The top (green) band in Fig. 1 shows the total DM $= 558.1 \pm 3.3$ pc cm^{-3} measured for FRB 121102 [22]. The most conservative approach to constraining m_γ would be to set to zero the other contributions, and assign this total DM to a possible photon mass. However, this is surely over conservative, and a reasonable approach is to subtract from the total DM the expected contribution from the Milky Way [22], DM$_{\rm MW}$, which is the sum of contributions from the disk [29]: DM$_{\rm NE2001} \simeq 188$ pc cm^{-3} and the halo: DM$_{\rm halo} \simeq 30$ pc cm^{-3} [23], to which we assign an overall uncertainty of 20%, namely 44 pc cm^{-3} [23], leaving the middle (blue) band in Fig. 1 that is centred at 340 pc cm^{-3}. From this we may also subtract the contribution from the IGM, which is estimated within the ΛCDM model to be $\simeq 200$ pc cm^{-3} [23–25]. To this an uncertainty of 85 pc cm^{-3} associated with inhomogeneities in the IGM [23,30] is assigned, which is much larger than the 1.2% variation associated with uncertainties in the cosmological parameters (shown as the narrow magenta band). The bottom (pink) band in Fig. 1, centred at 140 pc cm^{-3}, shows the effect of subtracting these contributions from the measured DM for FRB 121102.

After subtracting these contributions, we are left with a residual DM $= 140$ pc cm^{-3} with a total uncertainty of ± 96 pc cm^{-3}, shown as the outer pink band, where the error is calculated by combining in quadrature the uncertainties in the experimental measurement of the total DM, the uncertainty in DM$_{\rm MW}$, and the uncertainties in DM$_{\rm IGM}$ associated with the cosmological parameters H_0, Ω_Λ and Ω_m and possible inhomogeneities. One cannot exclude the possibility that all the residual DM of FRB 121102 is due to the host galaxy, which is estimated to lie within the range $55 \lesssim {\rm DM_{Host}} \lesssim 225$ pc cm^{-3} [23]. However, in the absence of detailed information about the host galaxy, when constraining the photon mass we allow conservatively for the possibility that all the residual DM is due to $m_\gamma \neq 0$.

The curved band in Fig. 1 shows the possible contribution to the DM of FRB 121102 of a photon mass, as a function of m_γ: DM $= 10^5 m_\gamma^2 H_\gamma/(415 A^2 h_0)$, where H_γ is given in (6), $A = 1.05 \cdot 10^{-14}$ eV s$^{-1/2}$ and $h_0 \equiv H_0/100$ km/s/Mpc. The width of this band is due to the uncertainties in the cosmological parameters H_0, Ω_Λ and Ω_m [6], and the uncertainty in the determination of the redshift of FRB 121102. Assuming that the photon mass contribution to the total DM of FRB 121102 lies within the range allowed for the residual DM, after subtraction of the Milky Way and IGM contributions and taking their uncertainties into account, we find $m_\gamma \lesssim 2.2 \times 10^{-14}$ eV c^{-2} (3.9×10^{-50} kg).[4] This limit is similar to, though slightly weaker than, that obtained from similar considerations of FRB 150418 [11,12], whose redshift is now contested, as discussed earlier [21]. If FRB 150418 was indeed at a cosmological distance, using its DM value determined in [20] and the same values of $H_e(z)$ and $H_\gamma(z)$ as in the present analysis, we

[2] For an early consideration of possible astrophysical photon propagation delays, see [14]. For pioneering studies using astrophysical sources, see [15] (flare stars) and [16] (Crab nebula), and for an analogous subsequent study with greater sensitivity to the photon mass, see [17] (GRB 980703, $m_\gamma < 4.2 \times 10^{-47}$ kg). The most recent such studies are those in [18] (GRB 050416A, $m_\gamma < 1.1 \times 10^{-47}$ kg) and [19] (radio pulsars in the Magellanic clouds, $m_\gamma < 2 \times 10^{-48}$ kg). Our limit on m_γ is significantly stronger.

[3] We discuss later the impact of assuming a broader range $H_0 = 70(4)$ km/s/Mpc [26].

[4] This limit would increase to $m_\gamma \lesssim 2.3 \times 10^{-14}$ eV c^{-2} (4.1×10^{-50} kg) if the more relaxed range $H_0 = 70(4)$ km/s/Mpc [26] were used for H_0. On the other hand, it would decrease to $m_\gamma \lesssim 1.8 \times 10^{-14}$ eV c^{-2} (3.2×10^{-50} kg) if the minimum estimate of DM$_{\rm Host}$ [23] were taken into account.

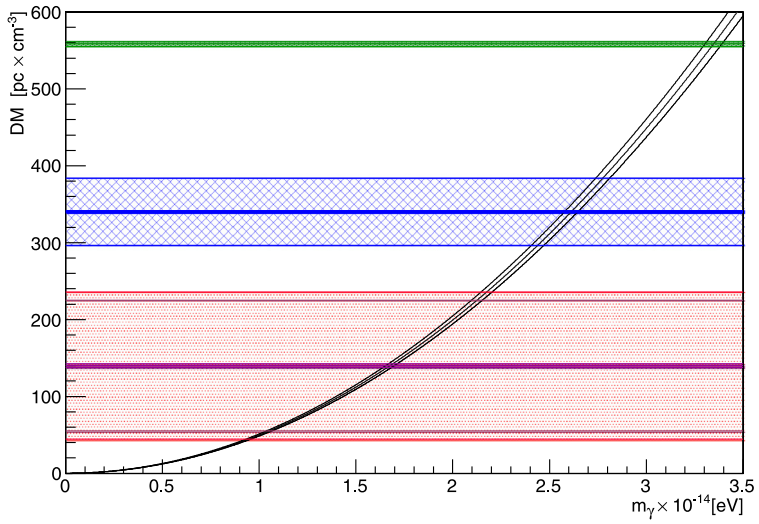

Fig. 1. Contributions to the dispersion measure (DM) budget for FRB 121102. The top (green) band represents the experimental measurement of the total DM $= 558.1 \pm 3.3$ pc cm^{-3}. The middle (blue) band shows the extragalactic contribution, as obtained by subtracting from the central value of the total DM the galactic contribution $\mathrm{DM_{MW}} \equiv \mathrm{DM_{NE2001}} + \mathrm{DM_{halo}} = 218$ pc cm^{-3} [23], to which is assigned an estimated uncertainty of 20%. The bottom (pink) band is obtained by subtracting also the estimated contribution from the intergalactic medium (IGM): $\mathrm{DM_{IGM}} \simeq 200$ pc cm^{-3} [23], which has an uncertainty of 85 pc cm^{-3} associated with inhomogeneities in the IGM, and a smaller uncertainty associated with the cosmological parameters (indicated by the narrow magenta band). The outer pink band shows the total uncertainty in the residual DM after subtraction of $\mathrm{DM_{MW}}$ and $\mathrm{DM_{IGM}}$, with errors added in quadrature. The curved (black) band shows the possible contribution of a non-zero photon mass, m_γ, also including the uncertainties in cosmological parameters and the redshift of FRB 121102. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

find that the inferred limits on m_γ would coincide if FRB 150418 had a redshift $z = 0.38$, instead of the value $z = 0.492$ reported in [20] and challenged in [21].

How could this constraint be improved in the future? Clearly it is desirable to reduce the uncertainties in the modelling of the Milky Way and IGM contributions. Also, the limit could be strengthened by a redshift measurement for an FRB at higher z, if the uncertainty in the IGM contribution can be controlled. Finally, as remarked in [11], comparing the DMs for FRBs with different redshifts could enable the IGM and m_γ contributions to be disentangled, in view of their different dependences (4) and (6) on z.

A hitherto unexplored window at very low frequencies in the MHz–KHz region could be opened by a space mission consisting of a swarm of nanosatellites [31]. One possible configuration would be orbiting the Moon, where it would be sufficiently away from the ionosphere to avoid terrestrial interference, and would have stable conditions for calibration during observations. Such low frequencies would offer a sensitive probe of any delays due to a non-zero photon mass.

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