Letter

Discovery of the doubly charmed T_{cc}^+ state implies a triply charmed H_{ccc} hexaquark state

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The doubly charmed exotic state T_{cc} recently discovered by the LHCb Collaboration could well be a DD^* molecular state long predicted in various theoretical models, in particular, the DD^* isoscalar axial vector molecular state predicted in the one-boson-exchange model. In this work, we study the DDD^* system in the Gaussian expansion method with the DD^* interaction derived from the one-boson-exchange model and constrained by the precise binding energy of 273 ± 63 keV of T_{cc} with respect to the $D^{*+}D^0$ threshold. We show the existence of a DDD^* state with a binding energy of a few hundred keV, isospin $1/2$, and spin-parity 1^-. Its main decay modes are $DDD\pi$ and $DDD\gamma$. The existence of such a state could in principle be confirmed with the upcoming LHC data and will unambiguously determine the nature of the T_{cc}^+ state and of the many exotic states of similar kind, thus deepening our understanding of the nonperturbative strong interaction.

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I. INTRODUCTION

Starting from the discovery of $D_{s0}^*(2317)$ [1] and $X(3872)$ [2] in 2003, a large number of the so-called exotic states that do not fit into the conventional quark model have been observed, which have led to intensive studies both theoretically and experimentally [3–8]. The latest addition to this long list is the T_{cc}^+ state reported by the LHCb Collaboration at the European Physical Society conference on high energy physics 2021 [9,10]. This state has a minimum quark content of $cc\bar{u}\,\bar{d}$ with a binding energy of $\delta = 273 \pm 61 \pm 5^{+11}_{-14}$ keV with respect to the $D^{*+}D^0$ threshold and a decay width of $\Gamma = 410 \pm 165 \pm 43^{+18}_{-38}$ keV determined from a Breit-Wigner parametrization [11], while the mass and decay width change to $\delta = 360 \pm 40^{+4}_{-0}$ keV and $\Gamma = 48 \pm 2^{+0}_{-14}$ keV in a unitarized three-body Breit-Wigner amplitude model [12]. Although such a doubly charmed state has long been anticipated theoretically [13–24], it has remained elusive experimentally until now. Being the first doubly charmed tetraquark state, its discovery will undoubtedly usher in a new era in hadron spectroscopy studies and advance our understanding of the nonperturbative strong interaction.

The measured binding energy and preferred quantum numbers of the T_{cc} state are in very good agreement with our predictions based on the one-boson-exchange (OBE) model [18,22], thus qualifies as a DD^* molecule with $I(J^P) = 0(1^+)$. It should be noted that the nature of T_{cc} is yet unsettled, and one could find many works that advocated the compact tetraquark picture, for example, Refs. [21,25–28]. As a result, an urgent question of high relevance is to understand the nature of this state, to distinguish the various interpretations, and to study the consequences.

Being close to some certain two-hadron thresholds is only a necessary but not sufficient condition for a

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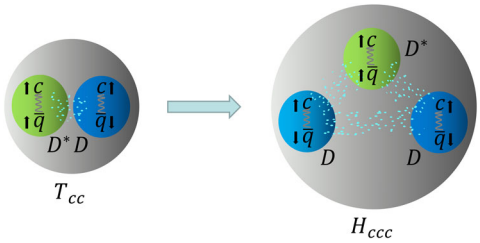

FIG. 1. From T_{cc} (as a DD^* molecule) to H_{ccc} (as a DDD^* molecule).

particular hadron to be of molecular nature. Taking $X(3872)$ as one example, after almost 20 years of extensive studies, there is still ongoing heated debate about its true nature being either a conventional $c\bar{c}$ charmonium, a $\bar{D}D^*$ molecule, a compact tetraquark state, or a combination of them. In a series of recent studies [29–38], we argued that one way to check the molecular nature of certain exotic hadrons is to search for existence of multihadron molecules built from the same constituents, in the way that atomic nuclei are bound states of multinucleons.[1] More specifically, it was shown that if $D_{s0}^*(2317)$ is dominantly a DK bound state, then DDK, $D\bar{D}K$, and $D\bar{D}^*K$ states should exist [31,32,36].[2] Similarly, if the latest T_{cc}^+ state is indeed a DD^* molecule, then it is very likely that a DDD^* bound state exists (see Fig. 1). Given the capacity of the LHCb experiment, such a state could very well be discovered in the near future, which will not only provide a highly nontrivial check on the molecular nature of the T_{cc}^+ state but also deepen our understanding of the nonperturbative strong interaction.

In this work, with the latest experimental measurements [11,12], we fix the DD^* interaction provided by the time-honored OBE model, and study the DDD^* system using the Gaussian expansion method.

II. THEORETICAL FORMALISM

The Gaussian expansion method has been widely used to solve three-, four- and even five-body problems, because of its high precision and rapid convergence [41]. In this framework, the three-body DDD^* system is described by the following Schrödinger equation

$$\hat{H}\Psi = E\Psi, \tag{1}$$

where the Hamiltonian $\hat{H}$ includes the kinetic term and three two-body interaction terms

[1]For a concrete demonstration that one can confidently deduce the existence of triton from that of deuteron using either the OBE model or a phenomenological model to describe the nucleon-nucleon interaction, see Ref. [39].

[2]The predicted DDK state has a minimum quark content of $cc\bar{s}\,\bar{u}\,/\bar{d}$, isospin $1/2$, and spin-parity 0^-. Such a state has recently been searched for by the Belle Collaboration [40].

TABLE I. Couplings of the light mesons of the OBE model (π, σ, ρ, ω) to the heavy D/D^* mesons. For the magnetic-type coupling of the ρ and ω vector mesons we have used the decomposition $f_{\rho(\omega)} = \kappa_{\rho(\omega)} g_{\rho(\omega)}$. M (in units of MeV) refers to the mass scale involved in the magnetic-type couplings [22].

Coupling	Value for D/D^*
g	0.60
g_σ	3.4
g_ρ	2.6
g_ω	2.6
κ_ρ	4.5
κ_ω	4.5
M	1867

$$\hat{H} = T + V_{DD} + V_{DD^*} + V_{DD^*}. \tag{2}$$

In order to solve the Schrödinger equation, we have to first specify the two-body interactions. In our present work, both the DD interaction and the DD^* interaction are derived from the OBE model. In Ref. [32], the DD OBE potential has been derived with the exchange of σ, ρ, and ω mesons. For the DD^* interaction, one can also exchange a π meson in addition to the σ, ρ and ω exchanges [22]. It should be noted that the DD^* interaction of Ref. [22] generates a molecular DD^* state with a cutoff of 1.01 GeV, which was fixed by reproducing the binding energy 4.0 MeV of $X(3872)$ with respect to the $D\bar{D}^*$ threshold. A detailed description of the OBE potential used can be found in Refs. [22,32]. With the relevant couplings between $DD^{(*)}$ and the exchanged mesons fixed (as shown in Table I), the only free parameter is the cutoff related to the regulator function needed to take into account the finite size of exchanged mesons. More specifically, we use a regulator function of the following form

$$F(q, m, \Lambda) = \left(\frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}\right), \tag{3}$$

where m is the mass of the exchanged meson (see Table II) and Λ the cutoff.

First, we slightly fine-tune the cutoff (from the value of 1.01 GeV fixed by reproducing a binding energy of 4 MeV for the $D\bar{D}^*$ bound state assigned to be $X(3872)$ [22]) taking advantage of the latest experimental data [11,12]. As the T_{cc} state is found about 0.3 MeV below the $D^{*+}D^0$ threshold and the difference between the thresholds of D^+D^{*0} and $D^{*+}D^0$ is 1.41 MeV, we study three different binding energy scenarios for the DD^* binding energy, i.e., 0.3 MeV, 1.0 MeV, and 1.7 MeV. The so-determined cutoffs for these three scenarios considering only S-wave interactions and $S-D$ mixings are

TABLE II. Masses and quantum numbers of the light mesons of the OBE model (π, σ, ρ, ω) and the heavy mesons D and D^* [42].

Light Meson	$I^G(J^{PC})$	M (MeV)
π	1^- (0^{-+})	138
σ	0^+ (0^{++})	600
ρ	1^+ (1^{--})	770
ω	0^- (1^{--})	780
Heavy Meson	$I(J^P)$	**M (MeV)**
D	$\frac{1}{2}(0^-)$	1867.24
D^*	$\frac{1}{2}(1^-)$	2008.56

TABLE III. Cutoffs for three different binding energy scenarios with/without considering $S-D$ mixing. The binding energies (B) are in units of MeV and RMS radii r in units of fm.

Λ (Only S)	B	r_{DD^*}	Λ ($S-D$)	B	r_{DD^*}
976	0.3	5.94	945	0.3	6.09
998	1.0	3.47	970	1.0	3.55
1013	1.7	2.72	986	1.7	2.81

given in Table III.[3] In the same table, we also provide the corresponding root-mean-square (RMS) radius of the T_{cc}^{+} state. Two things are noteworthy. First, the RMS radius ranges from 3 to 6 fm, consistent with the expectation for a molecular state whose size should be larger than the sum of its constituents. Second, the impact of $S-D$ mixing is small at the two-body level, consistent with the analysis of Ref. [43]. Based on this observation, we only consider S-wave interactions among the $D^{(*)}$ mesons in the following study of the DDD^* system. For the DD OBE potential, we relate it to the DD^* OBE potential via heavy quark spin symmetry (HQSS) and choose the same cutoff. To estimate the uncertainties caused by the breaking of HQSS, following Ref. [44], we consider a 15% uncertainty for the DD OBE potential (which means that we multiply the DD OBE potential by a factor of 0.85-1.15 in the specific calculations).

As all the two-body interactions have been specified, we employ the GEM to solve the Schrödinger equation. The three-body wave functions can be constructed in Jacobi coordinates as

[3]It is clear that the binding energy is sensitive to the cutoff value. This is often the case for OBE models. Conventionally, a cutoff value of 0.8 to 1.2 GeV is preferred when only light meson exchanges are considered. As a result, without concrete experimental constraints, it is difficult for OBE models to make precise predictions. Of course, in the present work, as the T_{cc} state has been observed, its binding energy fixes the cutoff of the OBE model.

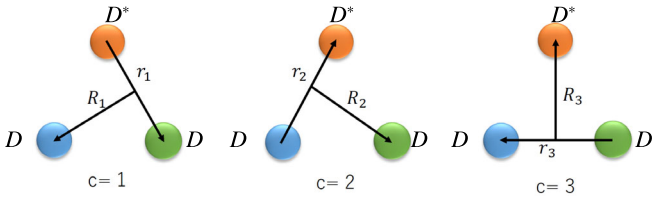

FIG. 2. Three permutations of the Jacobi coordinates for the DDD^* system.

$$\Psi = \sum_{c=1}^{3} \Psi(\mathbf{r}_c, \mathbf{R}_c), \tag{4}$$

where $c = 1$–3 is the label of the Jacobi channels shown in Fig. 2. In each Jacobi channel the wave function $\Psi(\mathbf{r}_c, \mathbf{R}_c)$ reads

$$\Psi(\mathbf{r}_c, \mathbf{R}_c) = C_{c,\alpha} H^c_{t,T} \Phi_{lL,\lambda}(\mathbf{r}_c, \mathbf{R}_c) \tag{5}$$

where $C_{c,\alpha}$ is the expansion coefficient and the $\alpha = \{nN, tT, lL\lambda\}$ labels the basis number with the configuration sets of the Jacobi channels. $H^c_{t,T}$ is the three-body isospin wave function, where t is the isospin of the subsystem in Jacobi channel c and T is the total isospin.

The three-body spatial wave function $\Phi(\mathbf{r}_c, \mathbf{R}_c)$ is constructed by two two-body wave functions as

$$\begin{aligned} \Phi_{lL,\lambda}(\mathbf{r}_c, \mathbf{R}_c) &= [\phi^G_{n_c l_c}(\mathbf{r}_c)\psi^G_{N_c L_c}(\mathbf{R}_c)]_\lambda, \\ \phi^G_{nlm}(\mathbf{r}_c) &= N_{nl} r_c^l e^{-\nu_n r_c^2} Y_{lm}(\hat{r}_c), \\ \psi^G_{NLM}(\mathbf{R}_c) &= N_{NL} R_c^L e^{-\lambda_n R_c^2} Y_{LM}(\hat{R}_c). \end{aligned} \tag{6}$$

Here, $N_{nl}(N_{NL})$ is the normalization constant of the Gaussian basis, $n(N)$ is the number of Gaussian basis used, $l(L)$ is the orbital angular momentum corresponding to the Jacobi coordinates $r(R)$, and λ is the total orbital angular momentum.

With the constructed wave functions, the Schrödinger equation can be transformed into a generalized matrix eigenvalue problem with the Gaussian basis functions

$$[T^{ab}_{\alpha\alpha'} + V^{ab}_{\alpha\alpha'} - EN^{ab}_{\alpha\alpha'}]C_{b,\alpha'} = 0, \tag{7}$$

where $T^{ab}_{\alpha\alpha'}$ is the matrix element of kinetic energy, $V^{ab}_{\alpha\alpha'}$ is the matrix element of potential energy, and $N^{ab}_{\alpha\alpha'}$ is the normalization matrix element.

III. RESULTS AND DISCUSSIONS

Considering only S-wave interactions, the corresponding configurations of the three Jacobi channels are given in Table IV. With these configurations and the OBE potentials specified above, we solve the Schrödinger equation in the GEM method and obtain the results shown in Table V, of which the uncertainties are from the breaking of HQSS in

TABLE IV. Quantum numbers of different Jacobi coordinate channels ($c = 1$–3) of the DDD^* $I(J^P) = \frac{1}{2}(1^-)$ state, considering only S-wave interactions.

c	l	L	λ	t	T	J	P	$n_{\max}$	$N_{\max}$
1	0	0	0	0	1/2	1	⋯	10	10
1	0	0	0	1	1/2	1	⋯	10	10
2	0	0	0	0	1/2	1	⋯	10	10
2	0	0	0	1	1/2	1	⋯	10	10
3	0	0	0	1	1/2	1	⋯	10	10

determining the DD OBE potential as mentioned above. It is interesting to note that for all the three scenarios studied, the DDD^* system is bound. Compared to the DD^* system, the addition of a second D meson increases the binding energy by about 87%, reflecting the fact that the DD interaction is less attractive than the DD^* interaction. This is corroborated by the observation that for all the three scenarios r_{DD} is larger than r_{DD^*} and $|\langle V_{DD^*}\rangle|$ is much larger than $|\langle V_{DD}\rangle|$.

In the above study, we have considered three binding energy assignments for the T_{cc} state, and obtained three corresponding cutoff values and three-body binding energies. As the two-body binding energy increases from 0.3 to 1.0 and 1.7 MeV, the three-body binding energy increases from 0.37, to 1.29 and 2.27 MeV. It is interesting to note that the binding energy increase in the two-body system of 3.33 and 1.7 times translate into 3.49 and 1.76 times in the three-body results. Clearly the three-body binding energy is proportional to the two-body binding energy (the deviation is at the 5% level). As a result, the prediction on the existence of a DDD^* state is rather robust.

Since the isospin of the studied DDD^* system is 1/2, this system consists of two charged states, i.e.,

$$I(1/2,1/2)\colon \sqrt{\frac{2}{3}}D^+D^+D^{*0} - \sqrt{\frac{1}{6}}(D^+D^0 + D^0D^+)D^{*+},$$

$$I(1/2,-1/2)\colon -\sqrt{\frac{2}{3}}D^0D^0D^{*+} + \sqrt{\frac{1}{6}}(D^+D^0 + D^0D^+)D^{*0}.$$

The Coulomb interaction may play a role for the doubly charged state which corresponds to the $I_3 = 1/2$ component. We include the Coulomb interaction for this state and find that the binding energies are 0.16, 1.09, and 2.22 MeV corresponding to the cutoff of 0.967, 0.998, and 1.013 GeV, respectively. The results are shown in Table VI. The Coulomb interaction makes the binding energy of the doubly charged state smaller compared to the singly charged one, but is not strong enough to break it up. The main reason is that the DD pair is widely separated at a distance of about 4–10 fm.

In principle the predicted triply charmed H_{ccc} state can decay into $DDD\pi$ because the DDD^* system could be viewed as a weakly bound DT_{cc} state, in which the T_{cc} state decays into $DD\pi$ as observed by the LHCb Collaboration [11,12]. Such a process is schematically shown in Fig. 3. Theoretically, as D^* can also decay into $D\gamma$, the H_{ccc} state can also be observed in the $DDD\gamma$ mode. According to the LHCb measurements, the estimated yield of $T_{cc} \to DD\pi$ with respect to that of $X(3872) \to D\bar{D}\pi$ is about 1/20 [12]. Naively the yield of $H_{ccc} \to DDD\pi$ with respect to that of $T_{cc} \to DD\pi$ might only be one to two order of magnitude smaller, thus accessible to future LHCb experiments.

IV. SUMMARY AND OUTLOOK

The recently discovered doubly charmed T_{cc} state is consistent with a DD^* molecule predicted in the OBE model. The precisely measured binding energy with respect to the $D^{*+}D^0$ threshold allows one to fix the

TABLE V. Binding energies, RMS radii and Hamiltonian expectation values of the DDD^* system with $I(J^P) = \frac{1}{2}(1^-)$ and S-wave OBE interactions. The uncertainties originate from the breaking of HQSS as explained in the main text.

Λ (MeV)	B (MeV)	r_{DD^*}	r_{DD}	$\langle T\rangle$	$\langle V_{DD^*}\rangle$	$\langle V_{DD}\rangle$
976	$0.56^{+0.10}_{-0.08}$	$5.64^{+0.80}_{-0.74}$	$7.17^{+1.07}_{-1.02}$	$15.95^{+2.37}_{-2.02}$	$-15.90^{+1.86}_{-2.13}$	$-0.61^{+0.24}_{-0.34}$
998	$1.87^{+0.25}_{-0.20}$	$3.28^{+0.36}_{-0.31}$	$4.13^{+0.52}_{-0.43}$	$29.18^{+2.69}_{-2.49}$	$-29.57^{+2.24}_{-2.36}$	$-1.49^{+0.46}_{-0.57}$
1013	$3.18^{+0.34}_{-0.30}$	$2.60^{+0.24}_{-0.20}$	$3.27^{+0.33}_{-0.29}$	$38.05^{+2.82}_{-2.68}$	$-39.10^{+2.39}_{-2.45}$	$-2.13^{+0.59}_{-0.61}$

TABLE VI. Binding energies, RMS radii and Hamiltonian expectation values of the doubly charged $I(J^P) = \frac{1}{2}(1^-)$ DDD^* state with S-wave OBE and Coulomb interactions. The uncertainties originate from the breaking of HQSS as explained in the main text.

Λ (MeV)	B (MeV)	r_{DD^*}	r_{DD}	$\langle T\rangle$	$\langle V_{DD^*}\rangle$	$\langle V_{DD}\rangle$
976	$0.16^{+0.01}_{-0.01}$	$8.83^{+0.01}_{-0.23}$	$10.74^{+0.09}_{-0.08}$	$7.65^{+1.32}_{-0.52}$	$-7.81^{+0.47}_{-1.27}$	$-0.00^{+0.02}_{-0.07}$
998	$1.09^{+0.17}_{-0.13}$	$4.50^{+0.83}_{-0.65}$	$5.86^{+1.19}_{-0.92}$	$23.65^{+2.85}_{-2.03}$	$-24.14^{+2.45}_{-2.58}$	$-0.60^{+0.31}_{-0.43}$
1013	$2.22^{+0.27}_{-0.23}$	$3.15^{+0.41}_{-0.33}$	$4.04^{+0.59}_{-0.47}$	$33.34^{+2.93}_{-2.76}$	$-34.40^{+2.52}_{-2.62}$	$-1.16^{+0.46}_{-0.56}$

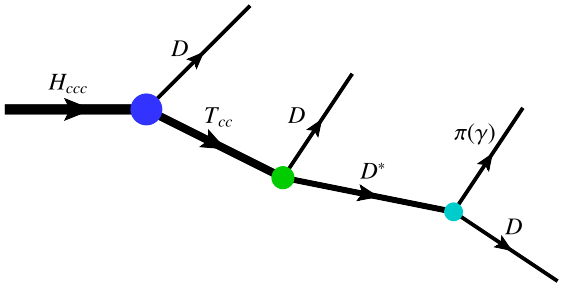

FIG. 3. Decay chain of H_{ccc}.

DD^* interaction. We utilized this valuable information and studied the three-body DDD^* system with the Gaussian expansion method. Our studies showed that the DDD^* system is bound even taking into account the Coulomb interaction. We discussed the possible decay modes, where the DDD^* states can be discovered. We strongly encourage that this state be directly searched for at present and future experiments.

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