

PROBING SPACE-TIME NONCOMMUTATIVITY IN THE BHABHA SCATTERING*

LINDA GHEGAL

Mentouri Constantine 1 University, Route Ain El Bey 25017, Constantine, Algeria

(Received March 22, 2020; accepted October 15, 2020)

We investigate the Bhabha scattering with the Seiberg–Witten extended noncommutative Standard Model scenario to the first order of the noncommutativity parameter $\Theta_{\mu\nu}$. This study is based on the definition of the noncommutativity parameter that we have assumed. We explore the noncommutative scale $A_{\text{NC}} \geq 0.8$ TeV considering different machine energy ranging from 0.5 TeV to 1.5 TeV.

DOI:10.5506/APhysPolB.51.2031

1. Introduction and motivation

The noncommutative Standard Model is based on the noncommutativity of the space and time variables. The noncommutative space-time is deformation of the ordinary one that can be realized by representing ordinary space-time coordinates x^μ by Hermitian operators \hat{x}^ν

$$[\hat{x}^\mu, \hat{x}^\nu] = i\Theta^{\mu\nu} = i\frac{c^{\mu\nu}}{A_{\text{NC}}}, \quad (1)$$

where $c^{\mu\nu}$ are dimensionless parameters and A_{NC} is the energy scale where the noncommutative effects of the space-time will be relevant. In the present work, we are especially interested in the spacetime noncommutative Standard Model based on the Moyal–Weyl (WM) product [1] and Seiberg–Witten (SW) maps [2]. In the WM product formalism, to define a field theory on noncommutative spacetime, we replace the ordinary product by the WM \star -product

$$(f \star g)(x) = \exp\left(\frac{1}{2}\Theta_{\mu\nu}\partial_{x^\mu}\partial_{y^\nu}\right) f(x)g(y) \Big|_{y=x} \quad (2)$$

and the SW maps express noncommutative fields and parameters as local functions of the commutative fields and parameters.

* Funded by SCOAP³ under Creative Commons License, CC-BY 4.0.

The existence of the Seiberg–Witten maps to all orders leads to expansions for matter field ψ , gauge fields V_α and gauge parameters λ_α as follows:

$$\hat{\psi} = \hat{\psi}[\psi, V] = \psi + \frac{1}{2}\Theta^{\mu\nu}V_\mu\partial_\nu\psi + \frac{i}{8}\Theta^{\mu\nu}[V_\mu, V_\nu]\psi + O(\Theta^2), \quad (3)$$

$$\hat{V}_\alpha = \hat{V}_\alpha[V] = V_\alpha + \frac{1}{4}\Theta^{\mu\nu}\{\partial_\mu V_\alpha + F_{\mu\alpha}, V_\nu\} + O(\Theta^2), \quad (4)$$

$$\hat{\lambda}_\alpha = \hat{\lambda}_\alpha[\lambda, V] = \lambda_\alpha + \frac{1}{4}\Theta^{\mu\nu}\{\partial_\mu\lambda_\alpha, V_\nu\} + O(\Theta^2). \quad (5)$$

The full description of the minimal noncommutative Standard Model (mNCSM), built on the $U(1)_Y \otimes SU(2)_L \otimes SU(3)_C$ group, and the complete Feynman rules derived from NCSM, including new interactions between the gauge bosons in the nonminimal NCSM version, are presented in [3]. The extension of SM to noncommutative space-time with motivations coming from the string theory and quantum gravity provides interesting phenomenological implications since the scale of noncommutativity could be as low as a few TeV, which can be explored at the present or future colliders. Due to the breaking of Lorentz invariance for fixed $\Theta_{\mu\nu}$ background, noncommutativity of space-time leads to dependence of cross section on azimuthal angle which is absent in the Standard Model (SM) as well as in other models beyond the SM. Thus, the azimuthal dependence of the cross section is a typical signature of noncommutativity and can be used in order to discriminate it against other New Physics effects. We have found this dependence to be best suited for deriving the sensitivity bounds on the noncommutative scale A_{NC} , and the main purpose of this work is to derive the bounds on the noncommutative scale A_{NC} in the Bhabha scattering in the context of mNCSM, by using the framework introduced in [3].

The present manuscript will be organised as follows. In the next section, we will present some bounds for the noncommutativity parameter $\Theta_{\mu\nu}$ provided by the most recent papers. We will then conclude with the results that we have obtained for noncommutative scale A_{NC} in the Bhabha scattering, in the framework of the mNCSM, using the SW maps to the first order of the NC parameter $\Theta_{\mu\nu}$.

2. Overview of bounds on noncommutative scale

We will dedicate this section to the bounds which can presently be studied in several processes and systems, including estimates for some experiments [4–16]. Most of them used noncommutative antisymmetric constant matrix $\Theta_{\mu\nu}$ analogous to the electromagnetic field strength tensor; denote the time-like components c^{oi} by \vec{E} and the space-like components c^{ij} by \vec{B} .

However, they often use two different parameterizations for the noncommutative parameter $\Theta_{\mu\nu}$. The first one is considered as an elementary constant in nature, its direction is fixed in the specific celestial Cartesian reference, so one should take into account these rotation effects on $\Theta_{\mu\nu}$ in this frame, before moving towards the phenomenological investigations. In the second parameterizations, the values of the two vectors are fixed as $\vec{E} = \frac{1}{\sqrt{3}}(\vec{i} + \vec{j} + \vec{k})$ and $\vec{B} = \frac{1}{\sqrt{3}}(\vec{i} + \vec{j} + \vec{k})$. The phenomenological consequences of the noncommutative space-time have been explored in several studies for giving bounds on noncommutative parameter. From the $e^+e^- \rightarrow q\bar{q}$ subprocess and by comparing with the ALEPH and OPAL data of LEP in [4], there were found two constraints on the noncommutative space-time scale, $215 \text{ GeV} \leq \Lambda_{\text{NC}} \leq 240 \text{ GeV}$ and $270 \text{ GeV} \leq \Lambda_{\text{NC}} \leq 310 \text{ GeV}$ respectively. In [5], the authors investigate the TeV scale signatures of NC space-time in the $e^+e^- \rightarrow \gamma\gamma$, $e^-\gamma \rightarrow e^-\gamma$ and $\gamma\gamma \rightarrow e^+e^-$ using polarized beams in the mNCSM. The process $e^+e^- \rightarrow \gamma\gamma$ at the International Linear Collider (ILC), in the framework of nonminimal NCSM, with anomalous triple gauge boson couplings is studied in [6]; knowing that, all these bounds deal with the first parameterization. For the constant parameterization case, the bound $\Lambda_{\text{NC}} \geq 400 \text{ GeV}$ was found from the Drell–Yan process $pp \rightarrow \gamma, Z \rightarrow l^+l^-$ at the Large Hadron Collider (LHC) in the framework of the nonminimal NCSM [7]. The Higgs boson pair production $e^+e^- \rightarrow HH$ in the linear collider (LC) gives the range of $\Lambda_{\text{NC}} = 0.5\text{--}1.0 \text{ TeV}$ [8]. The study of the Lorentz violation in the Higgs sector in the NCSM [9] leads to a bound on the noncommutative parameter as large as $\Lambda_{\text{NC}} \sim 10^6 \text{ TeV}$. The Higgs boson production process $e^+e^- \rightarrow ZH$ and the SM forbidden process $e^+e^- \rightarrow HH$ were investigated in the framework of the mNCSM [10], with Feynman rules involving all orders of the noncommutative parameter which are derived using recursive formation of SW map. By applying the same Feynman rules, the authors in [11] have studied the Higgs plus Z -boson production at a future electron–positron collider to explore the sensitivity of future accelerator experiments to noncommutativity, and obtained as a lower limit of $\Lambda_{\text{NC}} = 1.1 \text{ TeV}$. The bound $\Lambda_{\text{NC}} \geq 0.5 \text{ TeV}$ was determined in [12] from the same process at the future Linear Collider. An experimental constraint on Λ_{NC} and projected sensitivities for the future collider with neutrino–electron scattering was summarized in [13]. The authors in [14] have also explored an experiment to probe the effects of noncommutative structure in quantum optical.

Since the models considered by the various authors are different, and since also the experimental setups probe different energy and length scales, one must be careful in the comparison of the various results of the noncommutative scale Λ_{NC} .

3. Results and conclusion

In this section, we investigate the effect of space-time noncommutativity on the Bhabha scattering, with the Ansatz for noncommutative parameter $\Theta_{\mu\nu}$ that we have assumed, and we provide the numerical results of our investigation. We are restricting ourselves only to the first order in Θ ; thus, the interference between SM and NC term can provide required corrections to cross section. In order to check the sensitivity of Bhabha scattering in the context of noncommutative space-time and especially the resulting phenomenological effects, we have defined the noncommutative parameter with the help of the gamma matrices, by which the noncommutative structure is determined by some spinor background on which the gamma-dependent $\Theta_{\mu\nu}$ acts

$$c^{\mu\nu} = \frac{1}{2} (\sigma^{\mu\nu} + (\sigma^{\mu\nu})^+) \tag{6}$$

knowing that

$$\sigma^{\mu\nu} = \frac{i}{2} (\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu), \tag{7}$$

where γ^μ are Dirac matrices.

We study now, how the space-time noncommutativity affects the $e^-(p_1)e^+(p_2) \rightarrow e^-(p_3)e^+(p_4)$ scattering process, through the exchange of γ and Z bosons at tree level (via the s and t channel). The Feynman rules to the first order in Θ are given in [3].

The Feynman rule for $e(p_{\text{in}}) - e(p_{\text{out}}) - \gamma(k)$ vertex is

$$= ieQ_f \left[\gamma_\mu - \frac{i}{2}k^\nu (\Theta_{\mu\nu\rho}p_{\text{in}}^\rho - \Theta_{\mu\nu}m_f) \right] \tag{8}$$

and for $e(p_{\text{in}}) - e(p_{\text{out}}) - Z(k)$ vertex,

$$= \frac{ie}{\sin 2\theta_W} \left\{ \left(\gamma_\mu - \frac{i}{2}k^\nu \Theta_{\mu\nu\rho}p_{\text{in}}^\rho \right) \left(C_V^f - C_A^f\gamma_5 \right) - \frac{i}{2}\Theta_{\mu\nu}m_f \left[p_{\text{in}}^\nu \left(C_V^f - C_A^f\gamma_5 \right) - p_{\text{out}}^\nu \left(C_V^f + C_A^f\gamma_5 \right) \right] \right\}, \tag{9}$$

where $\Theta_{\mu\nu\rho} = \Theta_{\mu\nu}\gamma_\rho + \Theta_{\nu\rho}\gamma_\mu + \Theta_{\rho\mu}\gamma_\nu$, $C_V^f = T_3^f - 2Q_f \sin^2 \theta_W$ and $C_A^f = T_3^f$ with θ_W is the Weinberg angle and $Q_f = \mp 1$ for e^\mp . Also $p_{\text{out}}\Theta p_{\text{in}} = p_{\text{out}}^\mu \Theta_{\mu\nu} p_{\text{in}}^\nu = -p_{\text{in}}\Theta p_{\text{out}}$. The momentum conservation reads as $p_{\text{in}} + k = p_{\text{out}}$.

The corresponding Feynman diagrams are shown in Fig. 1.

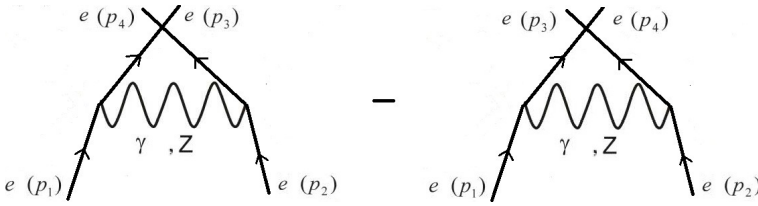


Fig. 1. Feynman diagrams for the $e^-e^+ \rightarrow (\gamma, Z) \rightarrow e^-e^+$ process in the NCSM.

The scattering amplitude to the first order in Θ can be written as:

— For the γ -mediated diagram

$$A^\gamma = A_s^\gamma + A_t^\gamma, \tag{10}$$

where

$$A_s^\gamma = e^2 \left[1 + \frac{i}{2} (p_2 \Theta p_1 + p_4 \Theta p_3) \right] \times \left[\bar{v}(p_2, s_2) \gamma^\mu u(p_1, s_1) \bar{u}(p_3, s_3) \gamma_\mu v(p_4, s_4) \left(\frac{i}{s} \right) \right] \tag{11}$$

and

$$A_t^\gamma = -e^2 \left[1 - \frac{i}{2} (p_3 \Theta p_1 + p_4 \Theta p_2) \right] \times \left[\bar{u}(p_3, s_3) \gamma^\mu u(p_1, s_1) \bar{v}(p_2, s_2) \gamma_\mu v(p_4, s_4) \left(\frac{i}{t} \right) \right]. \tag{12}$$

— For the Z -mediated diagram

$$A^Z = A_s^Z + A_t^Z, \tag{13}$$

where

$$A_s^Z = \frac{e^2}{\sin^2 2\theta_W} \left[1 + \frac{i}{2} (p_2 \Theta p_1 + p_4 \Theta p_3) \right] \times \left[\bar{v}(p_2, s_2) \gamma^\mu \Gamma_A^- u(p_1, s_1) \bar{u}(p_3, s_3) \gamma_\mu \Gamma_A^- v(p_4, s_4) \left(\frac{i}{s - m_Z^2} \right) \right] \tag{14}$$

and

$$A_t^Z = -\frac{e^2}{\sin^2 2\theta_W} \left[1 - \frac{i}{2} (p_3 \Theta p_1 + p_4 \Theta p_2) \right] \times \left[\bar{u}(p_3, s_3) \gamma^\mu \Gamma_A^- u(p_1, s_1) \bar{v}(p_2, s_2) \gamma_\mu \Gamma_A^- v(p_4, s_4) \left(\frac{i}{t - m_Z^2} \right) \right], \tag{15}$$

where $\Gamma_A^\pm = (C_V^e \pm C_A^e \gamma_5)$, with $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_1 - p_4)^2$.

The spin-averaged squared amplitude is given by (see Appendix A for more details)

$$|\bar{A}|^2 = \frac{1}{4} \sum_{\text{spins}} |A^\gamma + A^Z|^2. \quad (16)$$

The differential cross section can be written as

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |\bar{A}|^2, \quad (17)$$

where θ and ϕ are polar and azimuthal angles, respectively, with $\frac{d\sigma}{d\Omega} = d \cos \theta d\phi$.

We can obtain the cross section $\sigma = \sigma(\sqrt{s}, \Lambda_{\text{NC}}, \theta, \phi)$ as

$$\sigma = \int_{-1}^1 d(\cos \theta) \int_0^{2\pi} d\phi \frac{d\sigma}{d\Omega}. \quad (18)$$

Our results are based on Feynman rules for NCSM given in [3]. We analyze the total cross section in the presence of space-time noncommutativity. The results are shown in Fig. 2.

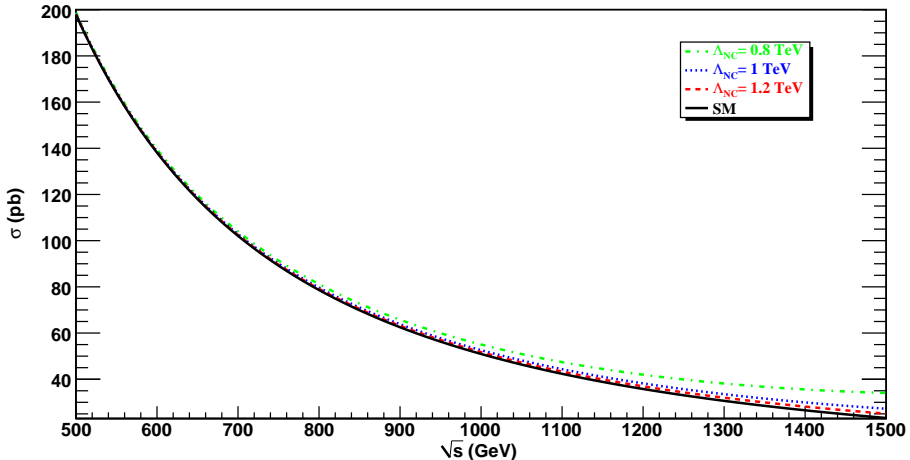


Fig. 2. (Colour on-line) The total cross section for the $e^-e^+ \rightarrow (\gamma, Z) \rightarrow e^-e^+$ [pb] process as a function of the center-of-mass energy $E_{\text{com}} = \sqrt{s}$ [GeV].

The ordinary SM is presented by the solid black curve and the NCSM with different curves: dash-dotted green, dotted blue and dashed red, with the corresponding $\Lambda_{\text{NC}} = 0.8, 1.0$ and 1.2 TeV, respectively. As can be seen, the noncommutative correction increases on increasing the center-of-mass energy of the collisions, and we found that the total cross section departs significantly from the Standard Model value as the machine energy starts getting larger than 1.0 TeV. Due to the significant sensitivity of the total cross section, it can be used to set lower limits on the noncommutative scale.

The NCSM is one of the extensions for physics beyond SM; Its phenomenological implications are quite interesting since the scale of noncommutativity could be as low as a few TeV, which can be explored at present or future colliders. In the present work, we have examined the testable nature of noncommutative Standard Model by analyzing the fundamental Bhabha scattering process at high-energy positron–electron linear collider. We have parameterized the noncommutative relationship in terms of noncommutative scale Λ_{NC} and antisymmetric matrix $c^{\mu\nu}$, which is defined with the help of the gamma matrices, whose noncommutative structure is determined by some spinor background on which the gamma-dependent $\Theta_{\mu\nu}$ acts, and we have shown how the positron–electron scattering process at the tree level is affected by space-time noncommutativity. The NC effects are found to be significant for $\Lambda_{\text{NC}} = 0.8, 1.0$ and 1.2 TeV for the center-of-mass energy $E_{\text{com}} = 1.5$ TeV. We got the same results that have been obtained by [16], *i.e.* the asymmetry at $\Lambda_{\text{NC}} = 0.8$ TeV is greater than that obtained at $\Lambda_{\text{NC}} = 1.2$ TeV.

Linda Ghegal would like to express her gratitude to Prof. Fedele Lizzi for his useful discussions during the period of her stay at the University of Naples Federico II, Italy.

Appendix A

The spin squared-amplitude can be written as [16]

$$\begin{aligned} |\overline{\mathcal{A}}|^2 = & \overline{|\mathcal{A}_s^\gamma|^2} + \overline{|\mathcal{A}_t^\gamma|^2} + \overline{|\mathcal{A}_s^Z|^2} + \overline{|\mathcal{A}_t^Z|^2} - 2 \text{Re} (\overline{\mathcal{A}_s^\gamma \mathcal{A}_t^{\gamma*}}) - 2 \text{Re} (\overline{\mathcal{A}_s^Z \mathcal{A}_t^{Z*}}) \\ & + 2 \text{Re} (\overline{\mathcal{A}_s^\gamma \mathcal{A}_s^{Z*}}) - 2 \text{Re} (\overline{\mathcal{A}_t^\gamma \mathcal{A}_s^{Z*}}) + 2 \text{Re} (\overline{\mathcal{A}_t^\gamma \mathcal{A}_t^{Z*}}) \end{aligned} \quad (\text{A.1})$$

with the several terms

$$\begin{aligned} \overline{|\mathcal{A}_s^\gamma|^2} = & \frac{e^4}{4s^2} \left(1 + \frac{1}{4} C^2 \right) \\ & \times \text{Tr} \left[\not{p}_2 \gamma_\mu \not{p}_1 \gamma_\nu \right] \text{Tr} \left[\not{p}_3 \gamma^\mu \not{p}_4 \gamma^\nu \right], \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} \overline{|\mathcal{A}_t^\gamma|^2} &= \frac{e^4}{4t^2} \left(1 + \frac{1}{4} D^2 \right) \\ &\quad \times \text{Tr} \left[\not{p}_1 \gamma_\nu \not{p}_3 \gamma_\mu \right] \text{Tr} \left[\not{p}_4 \gamma^\nu \not{p}_2 \gamma^\mu \right], \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} \overline{|\mathcal{A}_s^Z|^2} &= \frac{e^4}{4(\sin 2\theta_W)^4 (s - m_Z^2)^2} \left(1 + \frac{1}{4} C^2 \right) \\ &\quad \times \text{Tr} \left[\not{p}_1 \gamma_\nu \Gamma_A^- \not{p}_2 \gamma_\mu \Gamma_A^- \right] \text{Tr} \left[\not{p}_4 \gamma^\nu \Gamma_A^- \not{p}_3 \gamma^\mu \Gamma_A^- \right], \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned} \overline{|\mathcal{A}_t^Z|^2} &= \frac{e^4}{4(\sin 2\theta_W)^4 (t - m_Z^2)^2} \left(1 + \frac{1}{4} D^2 \right) \\ &\quad \times \text{Tr} \left[\not{p}_1 \gamma_\nu \Gamma_A^- \not{p}_3 \gamma_\mu \Gamma_A^- \right] \text{Tr} \left[\not{p}_4 \gamma^\nu \Gamma_A^- \not{p}_2 \gamma^\mu \Gamma_A^- \right], \end{aligned} \quad (\text{A.5})$$

$$\begin{aligned} -2 \overline{\text{Re}(\mathcal{A}_s^\gamma \mathcal{A}_t^{\gamma*})} &= -\frac{e^4}{2st} \text{Re} \left[\left(1 + \frac{i}{2} C \right) \left(1 + \frac{i}{2} D \right) \right] \\ &\quad \times \text{Tr} \left[\not{p}_1 \gamma_\nu \not{p}_3 \gamma^\mu \not{p}_4 \gamma^\nu \not{p}_2 \gamma_\mu \right], \end{aligned} \quad (\text{A.6})$$

$$\begin{aligned} +2 \overline{\text{Re}(\mathcal{A}_s^\gamma \mathcal{A}_s^{Z*})} &= \frac{e^4}{2(\sin 2\theta_W)^2 s (t - m_Z^2)} \\ &\quad \times \text{Re} \left[\left(1 + \frac{1}{4} C^2 \right) \text{Tr} \left[\not{p}_1 \gamma_\nu \not{p}_2 \gamma_\mu \right] \text{Tr} \left[\not{p}_4 \gamma^\nu \not{p}_3 \gamma^\mu \right] \right], \end{aligned} \quad (\text{A.7})$$

$$\begin{aligned} -2 \overline{\text{Re}(\mathcal{A}_s^\gamma \mathcal{A}_t^{Z*})} &= -\frac{e^4}{2(\sin 2\theta_W)^2 s (t - m_Z^2)} \\ &\quad \times \text{Re} \left[\left(1 + \frac{i}{2} C \right) \left(1 + \frac{i}{2} D \right) \text{Tr} \left[\not{p}_1 \gamma_\nu \Gamma_A^- \not{p}_3 \gamma_\mu \not{p}_4 \gamma^\nu \Gamma_A^- \not{p}_2 \gamma^\mu \right] \right], \end{aligned} \quad (\text{A.8})$$

$$\begin{aligned} -2 \overline{\text{Re}(\mathcal{A}_t^\gamma \mathcal{A}_s^{Z*})} &= -\frac{e^4}{2(\sin 2\theta_W)^2 t (s - m_Z^2)} \\ &\quad \times \text{Re} \left[\left(1 - \frac{i}{2} C \right) \left(1 - \frac{i}{2} D \right) \text{Tr} \left[\not{p}_1 \gamma_\nu \Gamma_A^- \not{p}_2 \gamma^\mu \not{p}_4 \gamma^\nu \Gamma_A^- \not{p}_3 \gamma_\mu \right] \right], \end{aligned} \quad (\text{A.9})$$

$$\begin{aligned} +2 \overline{\text{Re}(\mathcal{A}_t^\gamma \mathcal{A}_t^{Z*})} &= \frac{e^4}{2(\sin 2\theta_W)^2 u (u - m_Z^2)} \\ &\quad \times \text{Re} \left[\left(1 + \frac{1}{4} D^2 \right) \text{Tr} \left[\not{p}_1 \gamma_\nu \not{p}_4 \gamma_\mu \right] \text{Tr} \left[\not{p}_2 \gamma^\nu \not{p}_3 \gamma^\mu \right] \right], \end{aligned} \quad (\text{A.10})$$

$$\begin{aligned} -2 \overline{\text{Re}(\mathcal{A}_s^Z \mathcal{A}_t^{Z*})} &= -\frac{e^4}{2(\sin 2\theta_W)^4 (u - m_Z^2) (t - m_Z^2)} \\ &\quad \times \text{Re} \left[\left(1 + \frac{i}{2} C \right) \left(1 + \frac{i}{2} D \right) \text{Tr} \left[\not{p}_1 \gamma_\nu \Gamma_A^- \not{p}_4 \gamma_\mu \Gamma_A^- \not{p}_2 \gamma^\nu \Gamma_A^- \not{p}_3 \gamma_\mu \Gamma_A^- \right] \right]. \end{aligned} \quad (\text{A.11})$$

The factors C and D are given by

$$\begin{aligned} C &= p_2\Theta p_1 + p_4\Theta p_3, \\ D &= p_3\Theta p_1 + p_4\Theta p_2. \end{aligned}$$

The differential cross section is calculated using the center-of-mass frame for the Bhabha $e^-(p_1)e^+(p_2) \rightarrow (\gamma, Z) \rightarrow e^-(p_3)e^+(p_4)$ scattering process, in which the four-momenta of the incoming and outgoing particles are given by

$$\begin{aligned} p_1 &= p_{e^-} = \frac{\sqrt{s}}{2}(1, 0, 0, 1) = (E_1, \vec{P}_1), \\ p_2 &= p_{e^+} = \frac{\sqrt{s}}{2}(1, 0, 0, -1) = (E_2, \vec{P}_2), \\ p_3 &= p_{e^-} = \frac{\sqrt{s}}{2}(1, \sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta) = (E_3, \vec{P}_3), \\ p_4 &= p_{e^+} = \frac{\sqrt{s}}{2}(1, -\sin\theta \cos\phi, -\sin\theta \sin\phi, -\cos\theta) = (E_4, \vec{P}_4). \end{aligned} \quad (\text{A.12})$$

In evaluating the matrix element square, we have ignored the mass of ingoing particles ($m_e \simeq 0$), with $\vec{P}_1 + \vec{P}_2 = 0 = \vec{P}_3 + \vec{P}_4$. In the relativistic limit $s \gg 4m^2$, we get $s = (E_1 + E_2)^2 = 4E^2$ (with $E_1 = E_2 = E$), $u = -\frac{s}{2}(1 + \cos\theta)$ and $t = -\frac{s}{2}(1 - \cos\theta)$.

In our analysis, we have assumed an Ansatz for $\Theta_{\mu,\nu}$ (see Eq. (6)). The NC antisymmetric tensor $\Theta_{\mu,\nu}$ is defined with the help of the gamma matrices. Using this definition, we may write C and D as follows:

$$C = p_2\Theta p_1 + p_4\Theta p_3 = \frac{\sqrt{s}}{4\Lambda_{\text{NC}}^2}[1 + \sin\theta(\cos\phi + \sin\phi)], \quad (\text{A.13})$$

$$D = p_3\Theta p_1 + p_4\Theta p_2 = \frac{\sqrt{s}}{4\Lambda_{\text{NC}}^2}[\sin\theta(\cos\phi + \sin\phi)]. \quad (\text{A.14})$$

REFERENCES

- [1] J.E. Moyal, «Quantum mechanics as a statistical theory», *Proc. Camb. Phil. Soc.* **45**, 99 (1949).
- [2] N. Seiberg, E. Witten, «String theory and noncommutative geometry», *J. High Energy Phys.* **09**, 032 (1999).
- [3] X. Calmet *et al.*, «The Standard Model on non-commutative space-time», *Eur. Phys. J. C* **23**, 363 (2002); B. Melic *et al.*, «The Standard Model on non-commutative space-time: electroweak currents and the Higgs sector», *Eur. Phys. J. C* **42**, 483 (2005); «The Standard Model on non-commutative space-time: strong interactions included», *Eur. Phys. J. C* **42**, 499 (2005).

- [4] M. Haghghat *et al.*, «Location and direction dependent effects in collider physics from noncommutativity», *Phys. Rev. D* **82**, 016007 (2010).
- [5] S.K. Garg *et al.*, «TeV scale implications of non-commutative space time in laboratory frame with polarized beams», *J. High Energy Phys.* **1107**, 024 (2011).
- [6] N.G. Deshpande *et al.*, «Anomalous triple gauge boson couplings in $e^+e^- \rightarrow \gamma\gamma$ for non-commutative Standard Model», *Phys. Lett. B* **708**, 150 (2012).
- [7] J. Selvaganapathy *et al.*, «Drell–Yan process as an avenue to test a noncommutative Standard Model at the Large Hadron Collider», *Phys. Rev. D* **93**, 116003 (2016).
- [8] P.K. Das *et al.*, «Neutral Higgs boson pair production at the linear collider in the noncommutative Standard Model», *Phys. Rev. D* **83**, 056002 (2011).
- [9] S. Aghababaei *et al.*, «Lorentz violation in the Higgs sector and the noncommutative Standard Model», *Phys. Rev. D* **87**, 047703 (2013).
- [10] W. Wang *et al.*, «Higgs-strahlung and pair production in the collision in the noncommutative Standard Model», *Phys. Rev. D* **84**, 045012 (2011).
- [11] M. Ghasemkhani *et al.*, «Higgs production in e^-e^+ collisions as a probe of noncommutativity», *Prog. Theor. Exp. Phys.* **2014**, 081B01 (2014).
- [12] J. Selvaganapathy *et al.*, «Search for associated production of Higgs with Z boson in the noncommutative Standard Model at linear colliders», *Int. J. Mod. Phys. A* **30**, 1550159 (2015).
- [13] S. Bilmiş *et al.*, «Constraints on a noncommutative physics scale with neutrino–electron scattering», *Phys. Rev. D* **85**, 073011 (2012).
- [14] S. Dey *et al.*, «Probing noncommutative theories with quantum optical experiments», *Nucl. Phys. B* **924**, 578 (2017).
- [15] L. Ghegal, A. Benslama, «New limit for the noncommutativity parameter of the noncommutative Standard Model», *Int. J. Mod. Phys. A* **29**, 1450199 (2014).
- [16] P.K. Das *et al.*, «Möller and Bhabha scattering in the noncommutative Standard Model», *Phys. Rev. D* **77**, 035010 (2008).