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Eta-meson couplings with low-lying nucleon resonances

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ABSTRACT

Using QCD sum rules, supplemented with projected correlation function approach, we have calculated coupling constants of an eta-meson with the two lowest mass nucleon resonances. This includes both the diagonal as well as non-diagonal coupling constants involving a nucleon resonance and a nucleon. For this, we first calculate vacuum-to-eta correlation function of the interpolating fields of two nucleons and then take its matrix elements with respect to a nucleon spinor and/or a nucleon resonance spinor(s). Desired coupling constants are obtained by solving different equations obtained from different matrix elements. Stability of the results with respect to variation of different QCD and phenomenological input parameters has been checked. We have also analyzed the effect of the changes in the coupling constants under the hypothetical situation when the contribution of the hidden strangeness is removed and when that of the anomalous glue is removed from the correlation function.

1. Introduction

The study of the structure and interactions of nucleon resonances has been getting increasing attention in last few decades. In a recent study the role of nucleon resonances in spin-dependent observables of nucleons has been emphasized [1]. The meson-nucleon interaction has played a crucial role in the investigation of the nucleon and its resonances. The interaction of the isoscalar η-η' mesons with the nucleon and its resonances are of particular interest, since these pseudoscalar mesons carry the footprints of chiral symmetry beaking and gluon dynamics, contain hidden strangeness and their phenomenology is characterized by substantial OZI-violation [2]. A reliable determination of the coupling constants $g_{N^*\eta N}$ and $g_{N^*\eta N^*}$, where N* stands for a nucleon resonance, will be useful for construction of realistic NN potential, and will have an essential role in analysing photoproduction, electroproduction and hadroproduction of η-mesons and $\pi\eta$- mesons on nucleon targets. It has been observed that the N(1535) resonance strongly dominates the photoproduction of η-mesons on a single nucleon [3–5]. On the other hand, for the Roper resonance N(1440), which is about 50 MeV below η N threshold, an excitation in η photoproduction can be considered due to its large width of 350 MeV [6]. In general, for describing NN observables, it has been found that the N(1535) resonance has a tendency to counterbalance the contributions of the N(1440) resonance [7].

In early studies, it has been found that the N(1440) coupling with the η-meson is rather small indicating that this resonance may not be significant player in (γ, η) process [8]. On the other hand, some authors maintain that ηNN(1440) coupling strongly contributes to the near threshold production of the η-meson [9]. Among the possible coupling constants of the η-meson with the nucleon resonances, in the literature we have found the estimates of only ηNN(1535) and ηNN(1650) coupling constants. These coupling constants have been determined mostly by fitting the data on η-meson production on a proton via electromagnetic probes [10–13,4,14,6,15–20,5,21,22,8], and some via hadronic probes [23,24,3,25–27,7]. Theoretical analyses of determination of these coupling constants are based on effective Lagrangian approach [3,25,19,8,7,28], chiral quark model [17,20,18,22], coupled-channels approach [14,16,26,27,29–31], isobar model [12,6,5], dispersion relation [12], Regge model [20], K-matrix approach [3], non-local interaction model [4], etc. Some authors have emphasized, using phenomenological Lagrangian, the role of axial $U(1)_A$ anomaly on the decay N(1535)→Nη [28]. In recent past, QCD-based methods have been used to determine coupling constants of a pion with the low-lying nucleon resonances [32]. These methods have advantage of

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giving a microscopic view of hadronic interactions and providing a test of QCD at low energies. Furthermore, these methods can be applied to determine even those coupling constants of η-meson with nucleon resonances which have not been estimated by any experimental analysis so far.

2. The sum rules

In order to calculate the coupling constants of eta-meson with the nucleon resonances, we are going to follow the method developed in Ref. [33]. We first consider a correlation function.

$$\Pi(q,p) = i\int d^4x\, e^{iqx}\langle 0|T\{J_p(x)\bar{J}_p(0)\}|\eta(p)\rangle \tag{1}$$

where $J_p(x)$ is the most general form of the interpolating field of a proton and its resonances and is made up of three quark fields without any derivative:

$$J_p = 2\epsilon^{abc}[(u^{aT}Cd^b)\gamma_5 u^c + t(u^{aT}C\gamma_5 d^b)u^c] \tag{2}$$

In the above equation a, b, c are color indices and t is an arbitrary real parameter. Depending upon the problem, different values of t have been used in the literature, though $t = -1$ is used generally where only proton is involved [34]. A general requirement for the choice of a specific value of t should be the stability of the result for the physical quantity under consideration upon changes in t around its chosen value. We parameterize the couplings of J_p with the nucleon and its resonance states as

$$\langle 0|J_p(0)|N(k,s)\rangle = \lambda\, u_N(k,s) \tag{3}$$

$$\langle 0|J_p(0)|N_+(k,s)\rangle = \lambda_1\, u_{N_+}(k,s) \tag{4}$$

$$\langle 0|J_p(0)|N_-(k,s)\rangle = \lambda_2\, i\gamma_5\, u_{N_-}(k,s) \tag{5}$$

where N, N_+ and N_- stand for the nucleon, N(1440) and N(1535) states and + and − denote the parity of the state. The correlation function $\Pi(q,p)$ can be evaluated phenomenologically as well as using operator product expansion (OPE). For the former, we will saturate the $\Pi(q,p)$ with the nucleon and its resonance states and use the following Lagrangian:

$$\begin{aligned}\mathcal{L} = {} & g_1\bar{N}i\gamma_5\eta N + g_2\bar{N_+}i\gamma_5\eta N_+ + g_3\bar{N_-}i\gamma_5\eta N_- + g_4(\bar{N}i\gamma_5\eta N_+ \\ & + \bar{N_+}i\gamma_5\eta N) + g_5(\bar{N}\eta N_- + \bar{N_-}\eta N) + g_6(\bar{N_+}\eta N_- + \bar{N_-}\eta N_+)\end{aligned} \tag{6}$$

The phenomenological expression for the correlation function $\Pi(q,p)$ is as given in the Appendix of Ref. [32]. The Wilson coefficients of the OPE are calculated in two steps: the light-cone expansion of the correlation function is performed first. The short-distance expansion of the vacuum-to-eta matrix elements of the light-cone operators is performed next. The parameterization of vacuum-to-eta matrix elements of the light-cone operators has been taken from Ref. [35].

We define the projected correlation function [33,36] as

$$\Pi^{ij}_{+}(q,p) = \bar{u}_i(q,r)\gamma_0\Pi(q,p)\gamma_0 u_j(q-p,s) \tag{7}$$

where each of (i, j) stand for N, N_+ or N_-. $\Pi^{ij}_{+}(q,p)$ can be regarded as a function of q_0 in the reference frame in which **q**=0. The odd and even parts of $\Pi^{ij}_{+}(q_0)$ satisfy dispersion relation which can be Borel transformed with respect to q_0^2; this gives the following equations:

$$B[\Pi^{ij}_{+o}(q_0^2)] = \frac{1}{\pi}\int dq_0'\; e^{-q_0'^2/M^2}\, Im\Pi^{ij}_{+}(q_0') \tag{8}$$

$$B[\Pi^{ij}_{+e}(q_0^2)] = \frac{1}{\pi}\int dq_0' q_0'\; e^{-q_0'^2/M^2} Im\Pi^{ij}_{+}(q_0') \tag{9}$$

For calculating the correlation function using OPE, we have used quark-gluon basis [37,35]. Following is the result of our OPE evaluation of the correlation function $\Pi(q,p)$:

$$
\begin{aligned}
\Pi(q,p) = \frac{ih_q C\gamma_5}{16\sqrt{2}\pi^2 m_q}\Big[&-\frac{t_3}{2}\Big\{ln(-q^2)\big(p.q-q^2-\frac{p^2}{3}\big)-\frac{2}{3}\frac{(q.p)^2}{q^2}+\frac{p^2q.p}{2q^2}-\frac{(q.p)^3}{3q^4}\Big\}\\
&+\frac{t_7}{36}\langle\frac{\alpha_s}{\pi}G^2\rangle\frac{\pi^2}{q^2}\Big\{1+\frac{q.p}{q^2}-\frac{1}{3}\big(\frac{p^2}{q^2}-\frac{4(p.q)^2}{q^4}\big)\Big\}\Big]+\frac{iC\gamma_5}{4\sqrt{2}\pi^2}f_{3\pi}t_6\times\\
&\Big[-p^2(ln(-q^2)-\frac{q.p}{q^2})-\frac{1}{4}\big(p^2 ln(-q^2)+\frac{2(q.p)^2-3p^2q.p}{q^2}+\frac{2(q.p)^3}{q^4}\big)\Big]\\
&-i\gamma_5\not{q}\frac{a_\eta}{24\pi^2}t_5 ln\big(-\frac{q^2}{\mu^2}\big)\Big[\gamma_E+ln\big(\frac{\pi}{4}\big)-\frac{5}{4}+\frac{1}{2}ln\big(-\frac{q^2}{\mu^2}\big)\Big]\\
&+i\gamma_5\not{q}C\Big[-\frac{f_q}{16\sqrt{2}\pi^2}\Big\{\frac{t_2}{6}\big[ln(-q^2)(p^2-2q.p)+\frac{2(q.p)^2}{q^2}+\frac{\delta^2}{q^2}\big(2q.p-p^2\\
&+\frac{2(q.p)^2}{q^2}\big)\big]+\frac{t_1}{9}\frac{\delta^2}{q^2}\big(p^2-2q.p-\frac{2(q.p)^2}{q^2}\big)-\frac{\pi^2}{18}\langle\frac{\alpha_s}{\pi}G^2\rangle\frac{t_1}{q^4}\times\\
&\big(2q.p-p^2+\frac{4(q.p)^2}{q^2}+\frac{32\delta^2 q.p}{9q^2}\big)-\frac{4}{3}t_{10}\delta^2\big(\frac{2q.p-p^2}{q^2}+\frac{2(q.p)^2}{q^4}\big)\Big\}\\
&+\frac{h_q}{72\sqrt{2}m_q}\frac{1}{q^4}\Big\{2t_6\langle\bar{q}q\rangle\big(p^2-2q.p-\frac{4(q.p)^2}{q^2}+\frac{18}{5}\frac{p^2q.p}{q^2}-\frac{36}{5}\frac{(q.p)^3}{q^4}\big)\\
&-t_9\langle\bar{q}g\sigma.Gq\rangle\frac{q.p}{q^2}\Big\}\Big]\\
&+i\gamma_5\not{p}C\Big[-\frac{f_q}{16\sqrt{2}\pi^2}\Big\{t_1\big[ln(-q^2)(q.p-q^2+\delta^2)-\frac{8}{9}\frac{\delta^2 q.p}{q^2}\big]+\frac{t_2}{6}\times\\
&\big[ln(-q^2)(2q.p-q^2+\delta^2)-2\delta^2\frac{q.p}{q^2}\big]-\frac{\delta^2}{6}\big[8t_{10}\big(ln(-q^2)-\frac{2q.p}{q^2}\big)\\
&-4t_1\big(ln(-q^2)-\frac{q.p}{q^2}\big)\big]-\frac{\pi^2}{18}\langle\frac{\alpha_s}{\pi}G^2\rangle\frac{t_1}{q^4}\big(4q^2+3q.p-\frac{8}{9}\delta^2\big)\Big\}\\
&+\frac{h_q}{72\sqrt{2}m_q}\frac{1}{q^2}\Big\{t_6\langle\bar{q}q\rangle\big(-2-\frac{3p^2}{5q^2}+\frac{12}{5}\frac{(q.p)^2}{q^4}\big)+\frac{t_9}{4q^2}\langle\bar{q}g\sigma.Gq\rangle\Big\}\Big]\\
&+\gamma_5\sigma^{\mu\nu}q_\mu p_\nu C\Big[\frac{f_q}{6\sqrt{2}}\Big\{\langle\bar{q}q\rangle\frac{2t_6}{q^2}\big[1+\frac{q.p}{q^2}+\frac{5\delta^2}{9q^2}\big(1+\frac{2q.p}{q^2}\big)\big]+\langle\bar{q}g\sigma.Gq\rangle\\
&\times\frac{t_9}{4q^4}\big(1+\frac{2q.p}{q^2}\big)+\frac{t_6}{3}\delta^2\langle\bar{q}q\rangle\frac{1}{q^4}\big(1+\frac{2q.p}{q^2}\big)\Big\}+\frac{h_q}{96\sqrt{2}\pi^2 m_q}\\
&\times\Big\{t_4\big(ln(-q^2)-\frac{q.p}{q^2}+\frac{3p^2}{10q^2}-\frac{3}{5}\frac{(q.p)^2}{q^4}+\frac{2}{5}\frac{p^2q.p}{q^4}-\frac{8}{15}\frac{(q.p)^3}{q^6}\big)\\
&-\frac{t_8\pi^2}{18}\langle\frac{\alpha_s}{\pi}G^2\rangle\frac{1}{q^4}\big(1+\frac{2q.p}{q^2}\big)\Big\}\Big]
\end{aligned}
\tag{10}
$$

In the above equation, C=cosϕ where ϕ is the mixing angle in quark-flavor scheme. f_q and f_s are decay constants of isoscalar vector quark currents in quark-flavor basis. The constant h_q/m_q appears in parameterization of the matrix elements of isoscalar pseudoscalar as well as tensor quark currents between vacuum and one-meson state in quark-flavor basis and is parameterized as: h_q/m_q=$-4f_q\langle\bar{q}q\rangle/f_\pi^2$. Details of the notation can be found in Ref. [35] and references therein. As explained in [35], the light-cone expansion of the quark propagator gives rise to topological charge density $\sim\alpha_s G\tilde{G}$ whose vacuum-to-η-meson matrix element gives rise to a_η-term, where

$$
a_\eta=-\frac{m_{\eta'}^2-m_\eta^2}{\sqrt{2}}sin\phi cos\phi(-f_q sin\phi+\sqrt{2}f_s cos\phi). \tag{11}
$$

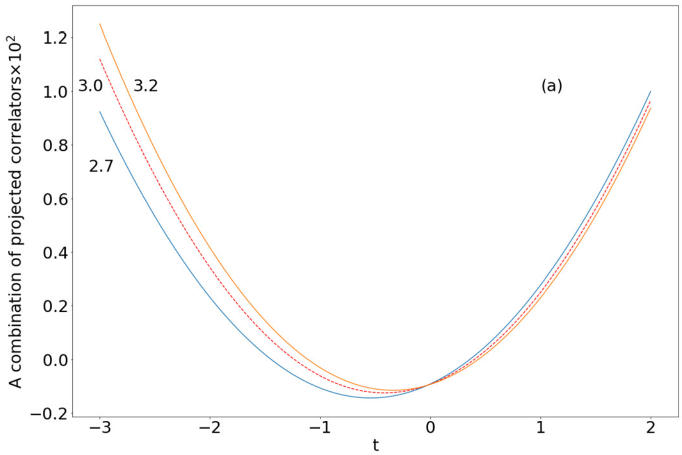
$f_{3\pi}$ appears in two-particle wave function of twist-3 as well as in three-particle pion wave function [38] and is also generally used for η-meson wave function [39,35]. t-dependence of $\Pi(q,p)$ appears in form of $t_1=5+2t+5t^2$, $t_2=2+20t+2t^2$, $t_3=t_4=7-2t-5t^2$, $t_5=7+22t+7t^2$, $t_6=(1-t)^2$, $t_7=(1-t)(15+21t)$, $t_8=(1-t)(19+17t)$, $t_9=3-2t-t^2$ and $t_{10}=1+4t+t^2$. Expression (10) has some differences from our previous result for $\Pi(q,p)$ in [35] in the limit $t=-1$ and the above result should supersede that.

The coefficients of $\bar{u}_N i\gamma_5 u_N$ in the phenomenological as well as in the OPE expressions of the projected correlation function have been Borel transformed. The expression for the phenomenological form is as given in Ref. [32]. The OPE expression has the following form:

$$
\begin{aligned}
B[\Pi^{NN}_{+o}]_{OPE} = M^2\Big[&\frac{-Cf_q}{8\sqrt{2}\pi^2}\big(t_1+\frac{t_2}{3}\big)E_{k1}\omega_{p1}L^{-4/9}+\frac{t_5}{24\pi^2}a_\eta\big(2\gamma_E+ln\big(\frac{\pi}{4}\big)-\frac{5}{4}\\
&-ln(M^2\mu^2)\big)L^{-4/9}+\frac{h_qC}{32\sqrt{2}\pi^2 m_q}\big(t_3\omega_{p1}-\frac{t_4}{3}(m+E_{k1})\big)\Big]E_0(s_\eta/M^2)\\
&-\frac{Cf_q\delta^2}{16\sqrt{2}\pi^2}\Big[\frac{t_1}{9}(p^2-2\omega_{p1}^2-16E_{k1}\omega_{p1})-\frac{t_2}{6}(2\omega_{p1}^2-p^2+4E_{k1}\omega_{p1})\\
&+\frac{4}{3}t_{10}(4E_{k1}\omega_{p1}+2\omega_{p1}^2-p^2)-\frac{4}{3}t_1E_{k1}\omega_{p1}\Big]\\
&+\frac{h_qC}{32\sqrt{2}\pi^2 m_q}t_3\left(\frac{p^2\omega_{p1}}{2}-\frac{\omega_{p1}^3}{3}\right)-\frac{Ct_6}{4\sqrt{2}\pi^2}f_{3\pi}\left(\frac{7}{4}p^2\omega_{p1}-2\omega_{p1}^3\right)\\
&+\frac{h_qCt_4}{160\sqrt{2}\pi^2 m_q}(m+E_{k1})\big(\omega_{p1}^2-p^2/2\big)-\frac{f_qC}{3\sqrt{2}}t_6\langle\bar{q}q\rangle(m+E_{k1})\\
&+\frac{1}{M^2}\Big[\langle\frac{\alpha_s}{\pi}G^2\rangle\Big\{\frac{-t_1Cf_q}{288\sqrt{2}}(6E_{k1}\omega_{p1}-4\omega_{p1}^2+p^2)+\frac{h_qC}{288\sqrt{2}m_q}\times\\
&\big(\frac{t_7}{2}\omega_{p1}-\frac{t_8}{6}(m+E_{k1})\big)\Big\}+\frac{1}{18}\langle\bar{q}q\rangle\Big\{\frac{h_qCt_6}{2\sqrt{2}m_q}L^{4/9}(p^2-4\omega_{p1}^2)\\
&+\frac{13f_qC}{3\sqrt{2}}t_6\delta^2L^{32/75}(m+E_{k1})\Big\}+\frac{t_9f_qC}{24\sqrt{2}}(m+E_{k1})\times\\
&\langle\bar{q}g\sigma.Gq\rangle L^{-14/27}\Big]
\end{aligned}
\tag{12}
$$

In the above equation m, m_1 and m_2 are masses of N, N_+ and N_- states. The continuum contribution of $B[\Pi^{NN}_{+o}]_{ph}$ is modelled by $\Pi(q,p)_{OPE}$ with $q_0^2 > s_\eta$, a threshold parameter and only terms with positive powers of q_0 are taken into account [34,33]. This continuum contribution has been transferred and combined with $B[\Pi^{NN}_{+o}]_{OPE}$ in form of E_0, where $E_0(x) = 1 - e^{-x}$ as is generally done [34]. $L = \alpha_s(\mu^2)/\alpha_s(M^2)$ and it appears in the OPE expression of the correlation function for renormalization group improvement; the anomalous dimensions of various operators have been taken from Refs. [34,40]. E_{k1} and ω_{p1} are the final state energies of the nucleon and η-meson when the initial state nucleon is at rest. We have done a similar exercise with $\Pi^{N_+N_+}_{+o}$, $\Pi^{N_-N_-}_{+o}$, $\Pi^{N_+N}_{+o}$ and $\Pi^{N_-N}_{+o}$ also getting five equations in all. The emitted η-meson has significant energies for $N_+ \to N$ and $N_- \to N$ processes. We are giving below the expression for $B[\Pi^{N_-N}_{+o}]_{OPE}$ to demonstrate its characteristic difference.

$$
\begin{aligned}
B[\Pi^{N_-N}_{+o}(q_0^2)]_{OPE} = M^2\Big[&\frac{Cf_q}{16\sqrt{2}\pi^2}\Big\{-\big(t_1+\frac{t_2}{3}\big)(m+E_{k5}-\omega_{p5})\omega_{p5}+\frac{t_2}{6}p^2\Big\}L^{-4/9}\\
&+\frac{t_5}{24\pi^2}a_\eta\big(2\gamma_E+ln\big(\frac{\pi}{4}\big)-\frac{5}{4}-ln(M^2\mu^2)\big)L^{-4/9}\\
&+\frac{h_qC}{32\sqrt{2}\pi^2 m_q}\big(t_3\omega_{p5}-\frac{t_4}{3}(m+E_{k5})\big)\Big]E_0(s_\eta/M^2)\\
&-\frac{Cf_q\delta^2}{16\sqrt{2}\pi^2}\Big[\frac{t_1}{9}(p^2-2\omega_{p5}^2-8\omega_{p5}(m+E_{k5}-\omega_{p5}))\\
&-\frac{t_2}{6}(2\omega_{p5}^2-p^2+2\omega_{p5}(m+E_{k5}-\omega_{p5}))+\frac{4}{3}t_{10}(2\omega_{p5}(m\\
&+E_{k5}-\omega_{p5})+2\omega_{p5}^2-p^2)-\frac{2}{3}t_1(m+E_{k5}-\omega_{p5})\omega_{p5}\Big]\\
&+\frac{h_qC}{32\sqrt{2}\pi^2 m_q}t_3\left(\frac{p^2\omega_{p5}}{2}-\frac{\omega_{p5}^3}{3}\right)-\frac{Ct_6}{4\sqrt{2}\pi^2}f_{3\pi}\left(\frac{7}{4}p^2\omega_{p5}-2\omega_{p5}^3\right)\\
&+\frac{h_qCt_4}{160\sqrt{2}\pi^2 m_q}(m+E_{k5})\big(\omega_{p5}^2-p^2/2\big)-\frac{f_qC}{3\sqrt{2}}t_6\langle\bar{q}q\rangle(m+E_{k5})\\
&+\frac{1}{M^2}\Big[\langle\frac{\alpha_s}{\pi}G^2\rangle\Big\{\frac{-t_1Cf_q}{288\sqrt{2}}L^{-4/9}(3\omega_{p5}(m+E_{k5}-\omega_{p5})-4\omega_{p5}^2\\
&+p^2)+\frac{h_qC}{576\sqrt{2}m_q}\big(t_7\omega_{p5}-\frac{t_8}{3}(m+E_{k5})\big)\Big\}+\frac{1}{18}\langle\bar{q}q\rangle\times\\
&\Big\{\frac{h_qCt_6}{2\sqrt{2}m_q}L^{4/9}(p^2-4\omega_{p5}^2)+\frac{13f_qC}{3\sqrt{2}}t_6\delta^2L^{32/75}(m+E_{k5})\Big\}\\
&+\frac{t_9f_qC}{24\sqrt{2}}(m+E_{k5})\langle\bar{q}g\sigma.Gq\rangle L^{-14/27}\Big]
\end{aligned}
\tag{13}
$$



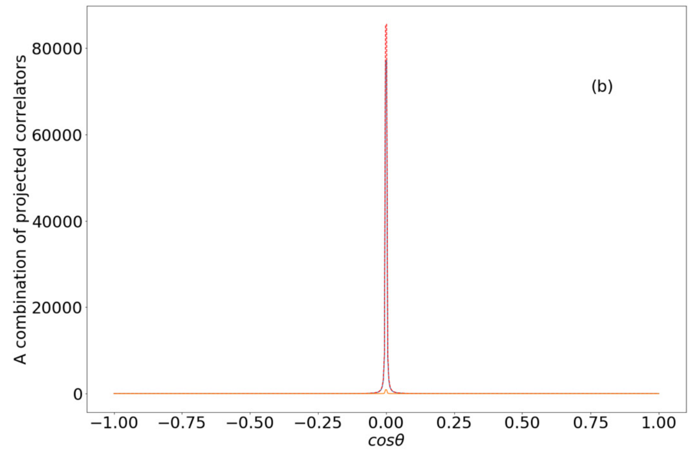


Fig. 1. Plots of a combination of projected correlation functions, obtained from OPE, as a function of t for a typical case with s_η=2.7 GeV^2, $\langle\bar{q}q\rangle = -0.0117$ GeV^3 and other QCD parameters as given in the text for M^2= (2.7, 3.0, 3.2) GeV^2 have been shown in Fig. (a). Plots of the same functions as functions of $cos\theta$ ($t = tan\theta$) have been shown in Fig. (b).

where E_{k5} and ω_{p5} are the final state energies of the nucleon and η-meson when the initial state is N_--resonance at rest.

3. Numerical estimates

We have nine coupling constants g_i's including six non-diagonal ones involving nucleon and its resonances. The phenomenological form of the correlation function also contains derivatives of these coupling constants g_i''s. The coupling constant of η-meson with the nucleon only, g_1, can be determined only with Π_{+o}^{NN} as done in Ref. [35] with a smaller continuum threshold s_η=2.57 GeV^2. Here, we have proceeded somewhat differently from Ref. [32] to determine the four coupling constants involving nucleon and its resonances. In the present work, we have approximated $g_6(m_2) = g_6(m_1) + (m_2 - m_1)g_6'(m_1)$ and assumed $g_6'(m_2) = g_6'(m_1)$. The three coupling constants $g_4(m)$, $g_5(m)$ and $g_6(m_1)$ and their derivatives can be eliminated using four equations. Each one of the coupling constants $g_4(m)$ and $g_5(m)$, and its respective derivative appears in a specific combination; each pair can be eliminated using one equation whereas $g_6(m_1)$ and $g_6'(m_1)$ require one equation each for elimination. Thus, we are left with the four coupling constants, $g_2(m_1)$, $g_3(m_2)$, $g_4(m_1)$ and $g_5(m_2)$ (hereafter denoted simply as g_2, g_3, g_4 and g_5), and their derivatives only to be determined. We have tried to fit the combination of the projected correlation functions containing these coupling constants and their derivatives with its OPE counterpart over a suitable range of Borel mass squared M^2. Among the many sets of possible values of g_i's and g_i''s, we have tried to retain a set which has the lowest possible values of g_i's and g_i''s, more so for g_i's. To the best of our knowledge, in the literature there exist estimates of only g_5 out of the four coupling constants. We have tried to keep values of g_5 close to these values. It may be noted that a set of g_i's ($i = 2, 3, 4, 5$) with small values can be made to fit over the desired range of M^2, but that will require, in general, large values of g_i' ($i = 2, 3, 4, 5$). Hence, we choose only those sets where both g_i and g_i' are small.

Following values of parameters have been used in this work (all quantities, unspecified, are in GeV unit): Λ_{QCD}=0.355, $\langle\bar{q}q\rangle$= (−0.0117, −0.0145), $\langle\frac{\alpha_s}{\pi}G^2\rangle$=0.012, $\langle\bar{q}g\sigma.Gq\rangle$=$m_0^2\langle\bar{q}q\rangle$ with m_0^2=0.8, δ^2=0.2, f_π=0.093, $f_{3\pi}$=0.0045, $f_q = 1.07f_\pi$, $f_s = 1.34f_\pi$, s_η=(2.7, 3.0), s_η'=2.57, m_1=1.44, m_2=1.54, ϕ=40°.

First, we check the stability of our results with respect to variation of the parameter t. For this we first plot the combination of the OPE expressions of the projected correlation functions, which will be matched with the corresponding phenomenological expression, with respect to t. In Fig. 1a we have shown such a plot for a typical case of $\langle\bar{q}q\rangle$=−0.0117 GeV^3, s_η=2.7 GeV^2 and M^2=(2.7, 3.0, 3.2) GeV^2 to show its behavior close to the origin. In Fig. 1b, the same functions have been plotted with respect to cosθ (t=tanθ) for displaying their large t behavior. For M^2=3.0 GeV^2, the minimum occurs for t=−0.436. The coupling constants λ_i's of the interpolating field J_p with the nucleon and its resonances have been determined from the mass sum rule [41,32]. A plot of the chirality conserving part of the polarization operator, constructed from J_p and including radiative corrections, as a function of t was found to have a minimum at $t \simeq -0.2$ [32]. Here, we have also found that the ratio of the combination of the projected correlation functions to the chirality conserving part of the polarization operator has a plateau for $t \lesssim -20$ and for $t \gtrsim 20$. Based on these observations, we have shown in Table 1 our exploratory results of fit of g_i and g_i' over a chosen range of Borel mass parameter 2.6 GeV$^2 \lesssim M^2 \lesssim$ 3.2 GeV^2. At the upper limit, the continuum plus resonance contribution is up to 40% of the perturbative contribution whereas at the lower limit the last term in the expansion, namely $1/M^2$-term is up to 25% of the total contribution. We emphasize that we determine $\lambda(t)$, $\lambda_1(t)$ and $\lambda_2(t)$ for each value of t separately to use them in Π_{ph}.

It is clear from Table 1 that the results for g_i's and their derivatives are relatively stable around $t = -0.436$ but start deviating beyond a range of Δt= ±1. In addition to the parameter t, we have also varied the continuum threshold s_η and the quark condensate $\langle\bar{q}q\rangle$. We have varied s_η between (2.7 − 3.0) GeV^2 which covers the mass of the next nucleon resonance N(1650) along with its width. In view of different values of $\langle\bar{q}q\rangle$ used in the literature, we have also found results for $\langle\bar{q}q\rangle = -0.0145$ GeV^3. It is to be noted that the minimum of the combination of the projected correlation functions for this case occurs at $t = -0.55$. A sample of the results for all these parameter variations has been shown in Table 2. Variations in g_i's and g_i''s due to changes in $\langle\frac{\alpha_s}{\pi}G^2\rangle$, δ^2, m_0^2 and $f_{3\pi}$, ϕ, f_q and f_s are insignificant.

A similar analysis with the even projected correlation function Π_{+e}^{ij} gives larger continuum contribution and larger contribution of $1/M^2$ terms, and hence we have not considered it.

Table 1
Our results on coupling constants of an η-meson with low-lying nucleon resonances for various parameter values, $\langle\bar{q}q\rangle$ =$-$0.0117 GeV^3, s_η=2.7 GeV^2 and other QCD parameters as given earlier in the text.

t	g_2	g_3	g_4	g_5	g'_2	g'_3	g'_4	g'_5	matching range of M^2
0	0.4	29.54	-8.85	6.73	26.0	-22.74	-32.68	-0.03	2.3 - 3.5
-0.2	2.75	4.42	-28.77	-2.24	-30.58	27.83	-64.87	34.38	2.5 - 3.25
-0.32	1.35	-9.38	-13.43	-0.59	112.32	101.0	2.57	-47.19	2.6 - 3.25
-0.436	1.87	-16.63	-7.35	-1.15	26.84	68.99	-15.36	40.58	2.55 - 3.2
-0.54	1.15	-19.85	-17.85	-0.5	-17.65	56.77	5.25	59.16	2.6 - 3.2
-0.7	1.0	-29.75	-7.52	-6.12	-123.5	137.29	-207.99	58.09	2.65 - 3.2
-1.0	4.8	-33.89	-14.09	-11.28	-23.81	-21.72	137.03	116.9	2.6 - 3.2
-20.0	-2.96	31.46	0.44	31.17	-60.82	-114.41	-47.75	-120.7	2.4 - 3.4
20.0	44.77	41.12	56.27	5.34	-73.31	96.65	-76.73	-120.3	2.5 - 3.4

Table 2
Our results on coupling constants of a η-meson with low-lying nucleon resonances for various parameter values, and other QCD parameters as given earlier in the text.

t	$\langle\bar{q}q\rangle$ (GeV^3)	s_η (GeV^2)	g_2	g_3	g_4	g_5	g'_2	g'_3	g'_4	g'_5	matching range of M^2
-0.32	-0.0117	2.7	1.35	-9.38	-13.43	-0.59	112.32	101.0	2.57	-47.19	2.6 - 3.25
-0.32	-0.0117	3.0	1.21	-5.14	-17.56	-1.41	106.96	91.35	86.84	-25.63	2.55 - 3.2
-0.436	-0.0117	2.7	1.87	-16.63	-7.35	-1.15	26.84	68.99	-15.36	40.58	2.55 - 3.2
-0.436	-0.0117	3.0	2.43	-14.96	-4.76	-1.59	63.96	93.21	-118.25	45.28	2.55 - 3.2
-0.54	-0.0117	2.7	1.15	-19.85	-17.85	-0.5	-17.65	56.77	5.25	59.16	2.6 - 3.2
-0.54	-0.0117	3.0	2.69	-15.04	-13.28	-5.69	-0.3	36.7	-63.05	108.37	2.6 - 3.15
-0.436	-0.0145	2.7	2.71	-29.83	-23.95	-5.65	43.05	10.44	55.56	184.34	2.68 - 3.3
-0.436	-0.0145	3.0	5.63	-28.58	-27.96	-6..45	62.15	93.21	-87.84	163.91	2.6 - 3.2
-0.55	-0.0145	2.7	4.99	-30.67	-10.46	-7.65	63.79	34.71	116.53	147.44	2.6 - 3.2
					f_s=0						
-0.436	-0.0117	2.7	2.09	-14.77	-13.22	-1.2	49.82	65.42	29.77	47.5	2.55 - 3.2
					$a_\eta = 0$						
-0.436	-0.0117	2.7	1.73	-17.18	-5.97	-0.25	40.95	89.72	22.81	18.36	2.55 - 3.2

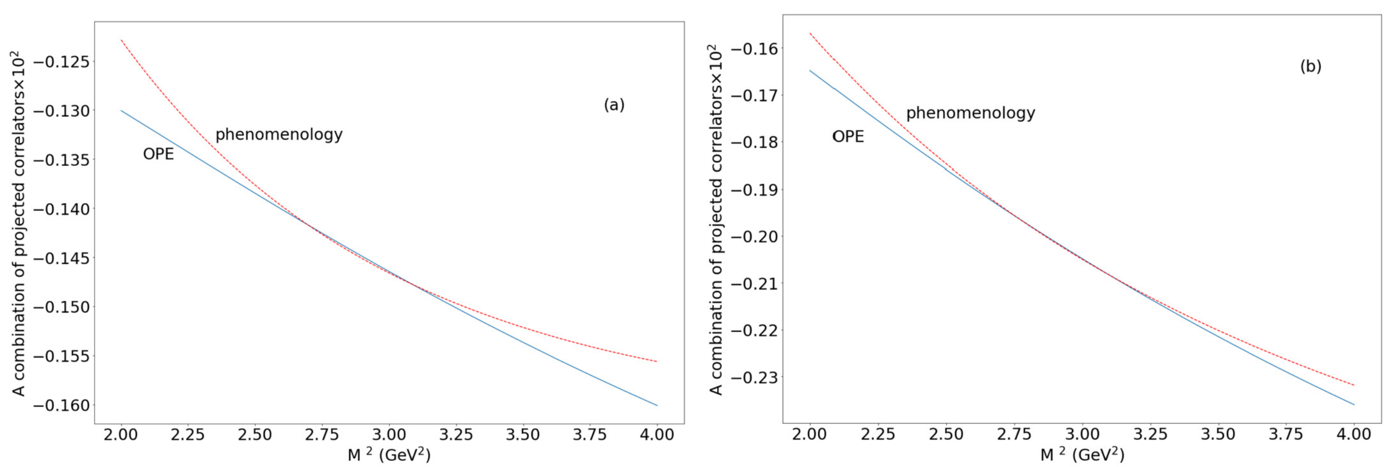


Fig. 2. A combination of projected correlation functions plotted as a function of Borel mass squared, M^2. The values of parameters used for Fig. (a) are: $t=-0.436$, s_η=2.7 GeV^2, $\langle\bar{q}q\rangle$=$-$0.0117 GeV^3, and g_2=1.87, $g_3=-16.63$, $g_4=-7.35$, $g_5=-1.15$, g'_2=26.84, g'_3=68.99, $g'_4=-15.36$, g'_5=40.58; for Fig. (b) the values of the parameters are: $t=-0.436$, s_η=3.0 GeV^2, $\langle\bar{q}q\rangle$=$-$0.0117 GeV^3, and g_2=2.43, $g_3=-14.96$, $g_4=-4.76$, $g_5=-1.59$, g'_2=63.96, g'_3=93.21, $g'_4=-118.25$, g'_5=45.28.

In general, numerical values of the coupling constants g_i's increase with the increase in numerical value of $\langle\bar{q}q\rangle$. This was also observed for the coupling constants of a pion with the nucleon resonances [32]. It is also noticed that there are relatively larger changes in the derivatives of the coupling constants g'_i's on variation of the input parameters. We believe this is an artifact of the method. In Figs. 2–4 we have shown some of the fits of a combination of $[\Pi^{ij}_{+0}]_{ph}$'s with the same combination of $[\Pi^{ij}_{+0}]_{OPE}$'s over a range of M^2. On setting $f_s=0$, g_4 gets almost doubled whereas other coupling constants have undergone marginal changes. On the other hand, for $a_\eta=0$, it is g_5 which undergoes a dramatic reduction while other coupling constants get marginally changed. This is in accordance with the claim of Ref. [28] that the axial $U(1)_A$ anomaly is responsible for enhanced coupling g_5. Based on the analysis of our results, we find the following values of the coupling constants of the η-meson with the nucleon and its resonances:

$$\begin{aligned} g_2 &\equiv g_{N_+\eta N_+} = 3.39 \pm 2.24,\ g_3 \equiv g_{N_-\eta N_-} = -17.91 \pm 12.77 \\ g_4 &\equiv g_{N_+\eta N} = -16.36 \pm 11.60,\ g_5 \equiv g_{N_-\eta N} = -4.08 \pm 3.58 \end{aligned} \quad (14)$$

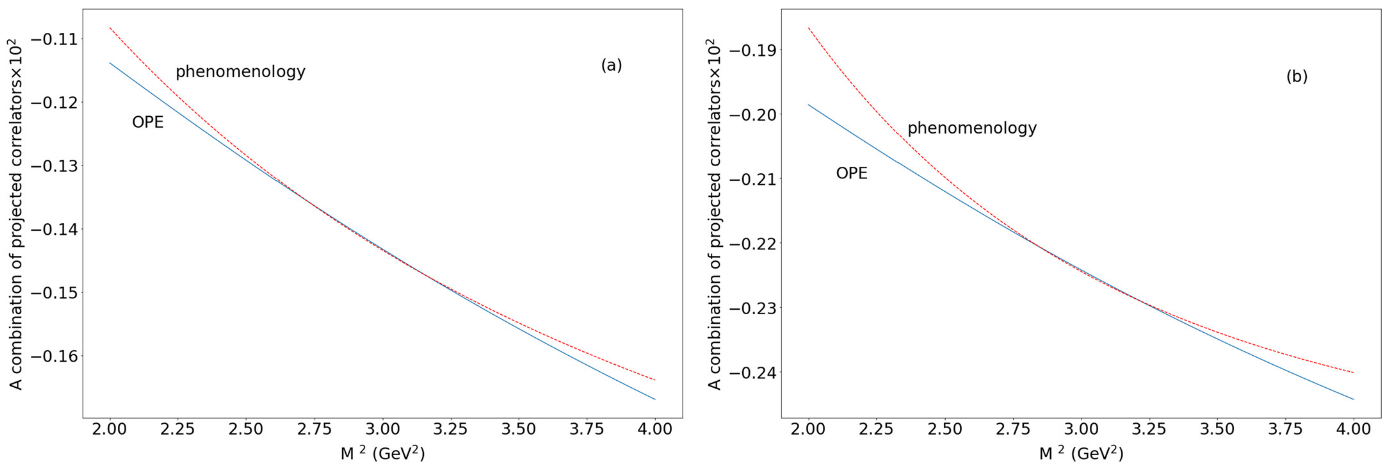


Fig. 3. A combination of projected correlation functions plotted as a function of Borel mass squared, M^2. The values of parameters used for Fig. (a) are: $t=-0.32$, s_η=2.7 GeV^2, $\langle\bar{q}q\rangle$=−0.0117 GeV^3, and g_2=1.35, $g_3=-9.38$, $g_4=-13.43$, $g_5=-0.59$, g_2'=112.32, g_3'=101.0, g_4'=82.57, g_5'=-47.19; for Fig. (b) the values of the parameters are: $t=-0.436$, s_η=2.7 GeV^2, $\langle\bar{q}q\rangle$=−0.0145 GeV^3, and g_2=2.71, $g_3=-29.83$, $g_4=-23.95$, $g_5=-5.65$, g_2'=43.05, g_3'=10.44, g_4'=55.56, g_5'=184.34.

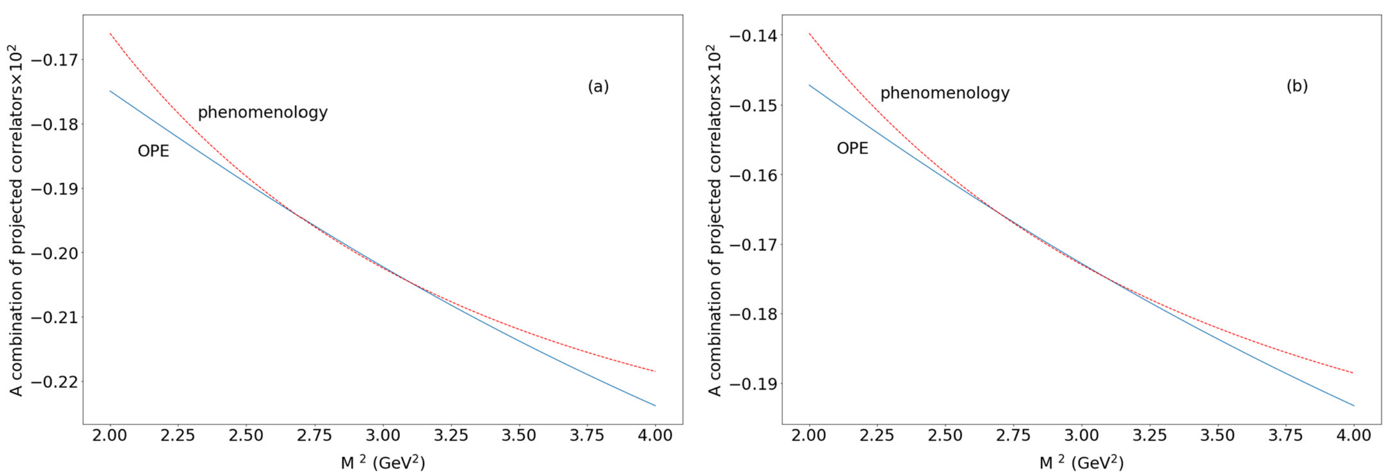


Fig. 4. A combination of projected correlation functions plotted as a function of Borel mass squared, M^2. The values of parameters used for Fig. (a) are: $t=-0.54$, s_η=3.0 GeV^2, $\langle\bar{q}q\rangle$=−0.0117 GeV^3, and g_2=2.69, $g_3=-15.04$, $g_4=-13.28$, $g_5=-5.69$, $g_2'=-0.3$, g_3'=36.7, $g_4'=-63.05$, g_5'=108.37; for Fig. (b) the values of the parameters are: $t=-0.436$, s_η=2.7 GeV^2, $\langle\bar{q}q\rangle$=−0.0117 GeV^3, $a_\eta=0$ and g_2=1.73, $g_3=-17.18$, $g_4=-5.97$, $g_5=-0.25$, g_2'=40.95, g_3'=89.72, $g_4'=22.81$, g_5'=18.36.

Table 3
List of some results on coupling constants of a η-meson with nucleons (g_1), and with nucleon and N(1535) resonance (g_5) from recent literature.

Ref.	g_1	g_5	Comment
[10]		1.58 - 2.14	Effective Lagrangian + Regge approach
[23]		-1.86	(p, p') reaction on spin-isospin saturated nucleus
[24]	0.68	1.88	Effective Lagrangian approach
[42]		-2.57 ±0.17, -2.07±0.15	Effective Lagrangian approach
[43]		±1.84	Coupled-channel unitary approach
[44]		±(0.97 ± 0.45)	Photoproduction
[11]	0.07, 0.088	-2.61, 1.92, 2.7	Photoproduction
[12]	0.92, 0.39	-3.97, -3.19	Dispersion relation + isobar model
[3]		-2.10 − (-2.16)	K-matrix approach
[25]		- 2.2	Effective Lagrangian approach
[13]		-2.08, -2.1	Relat. calculation incoherent photoproduction
[4]	-2.24	- 2.1	Relativistic non-local model
[35]	0.96 ± 0.3		QCD sum rule + projected correl. function
[31]		± 2.1	Coupled-channel unitary approach

where we have introduced conventional notation for the sake of clarity. Since N_+ has the same parity as N, though the masses are different, we expect g_2 should not be very different from g_1 which has been estimated by different authors (see Table 3). Also, according to some authors η-meson strongly couples to the Roper resonance [9] which will make g_4 rather large.

4. Summary and conclusion

η production offers an appropriate frame to study the properties of the nucleon resonances [20] and this, in turn, requires the knowledge of the coupling constants of η-meson with the nucleon resonances. In this work we have been able to determine even those coupling constants of η-meson with the nucleon resonances which are hard to extract from the available experimental data. Our approach, based on projected correlation function, has given us a handle to quantify the contributions of the axial anomaly and those of the hidden strangeness to these coupling constants.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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