Physics Letters B 793 (2019) 234–239

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Thermal fluctuations of AdS black holes in three-dimensional rainbow gravity

M. Dehghani

Department of Physics, Razi University, Kermanshah, Iran

ARTICLE INFO

Article history:
Received 12 February 2019
Received in revised form 19 April 2019
Accepted 23 April 2019
Available online 30 April 2019
Editor: N. Lambert

Keywords:
Three-dimensional black hole
Nonlinear electrodynamics
Thermal fluctuations

ABSTRACT

In this work, we explore the charged black holes with the power-law modified electromagnetic theory in a three-dimensional energy dependent space-time. Through exact solution of the field equations, we introduce a new class of nonlinearly charged black holes which are asymptotically anti-de Sitter (AdS). The black hole entropy, temperature and electric potential are calculated from the geometrical approaches. The counterterm method and Gauss's electric law are utilized for calculating the black hole mass and electric charge, respectively. By use of the Smarr formula, which states the black hole mass as the function of thermodynamic extensive parameters, we prove the validity of the first law of thermodynamics for the new AdS black holes. By use of the canonical ensemble method, the black hole remnant or phase transitions are investigated regarding the signature of black hole heat capacity. We show that the AdS black hole solutions, we just obtained, are thermodynamically stable if their horizon radii are greater than a minimum value. Then, by considering the black hole thermal fluctuations, we examine the quantum gravitational effects on the thermodynamic properties of the new AdS black holes. We prove that, when the thermal fluctuations are taken into account, the thermodynamical first law is no longer valid. Also, the thermal stability of the black holes gets some corrections.

1. Introduction

Gravity's rainbow just like the Horava-Lifshitz gravity theory is the ultraviolet completion of general relativity. Both of them, as the attempts for constructing the quantum theory of gravity, are based on promoting the usual dispersion relation to the so-called modified dispersion relation. This modification is proposed by almost all of the quantum gravity models [1–3]. The modified dispersion relation is written as [4,5]

$$E^2 f^2(\varepsilon) - p^2 g^2(\varepsilon) = m^2, \tag{1.1}$$

where $f(\varepsilon)$ and $g(\varepsilon)$, in order, are known as the temporal and spatial rainbow functions and ε is a dimensionless quantity defined as the ratio of the energy of the test particle E to the Planck energy E_P. The infrared version of dispersion relation is recovered by the requirements

$$\lim_{\varepsilon\to 0} f(\varepsilon) = 1, \quad \text{and} \quad \lim_{\varepsilon\to 0} g(\varepsilon) = 1. \tag{1.2}$$

There are several proposed functional forms for the rainbow functions which are obtained from different motivations, among them are [6]

$$f(\varepsilon) = 1, \quad \text{and} \quad g(\varepsilon) = \sqrt{1-\eta\varepsilon^n}, \tag{1.3}$$

$$f(\varepsilon) = \frac{e^{\zeta\varepsilon}-1}{\zeta\varepsilon}, \quad \text{and} \quad g(\varepsilon) = 1, \tag{1.4}$$

$$f(\varepsilon) = g(\varepsilon) = \frac{1}{1-\beta\varepsilon}. \tag{1.5}$$

The coefficients η, ζ and β, known as the rainbow parameters, are of the order of unity, $\varepsilon \le 1$ and the power n is a positive integer. In general, the rainbow functions have a magnitude equal to or slightly different from unity [7,8].

Obviously, modified dispersion relation (1.1) is not Lorentz invariant. Doubly/or deformed special relativity which, instead of usual Lorentz transformations, is based on the nonlinear Lorentz transformations that preserve Lorentz invariance of modified dispersion relation. In the doubly special relativity, in addition to the speed of light Planck energy is another constant quantity, which are the upper bound of the speed and energy that a particle can attain. In 2004 this theory was extended to the curved manifolds

E-mail address: m.dehghani@razi.ac.ir.

https://doi.org/10.1016/j.physletb.2019.04.058
0370-2693/

by Magueijo and Smolin. This doubly general relativity is named as rainbow gravity (or gravity's rainbow) [9,10]. The name rainbow gravity comes from the fact that in this theory the space-time geometry depends on the energy of the test particle which probes the gravity. Therefore, particles with different energies identify different space-time metrics. In other words there are a family of metrics which are parameterized by the ratio $\varepsilon = E/E_p$ this is why this doubly general relativity is called as rainbow gravity (or gravity's rainbow). Study of black holes physical and thermodynamical properties in gravity's rainbow have provided new and interesting results such as black hole remnant [11] and nonsingular universe [8]. Here, we explore the three-dimensional charged AdS black holes in rainbow gravity. Among the reasons that study of physics in the lower dimensional space-times can be interesting are that: black holes in lower dimensions are easier to study and this can essentially lead to a deeper insight into the fundamental ideas in comparison to higher dimensional black holes. Also, according to the AdS/CFT duality there is a connection between quantum gravity on AdS space and a Euclidean conformal field theory on the lower dimensional space-times [12,13]. Therefore, three-dimensional AdS black holes are more realized compared with the four and higher dimensional ones, and they are more useful objects for understanding of quantum field theory on AdS space-times.

In addition, Maxwell's classical electrodynamics is confronted with the divergence of electric field and self-energy at the position of the point-like charged particles. With the purpose of solving this problem and other issues related to this theory, the idea of nonlinear electrodynamics was proposed. The lagrangian of various models of nonlinear electrodynamics are nonlinear functions of Maxwell's invariant $F^{ab}F_{ab}$. The first attempts in this line was made by Born and Infeld in 1934 [14–16]. After that some other models of nonlinear electrodynamics such as logarithmic, exponential and power-law models were established [17,18]. Maxwell's theory of electrodynamics, as the special case of nonlinear electromagnetic theory, is valid only for the case of weak electromagnetic fields when the interaction between photons can be ignored [19]. Nowadays, study of the exact black hole solutions have been extended by use of these models of nonlinear electrodynamics and they have provided many interesting results [20–22]. Here, we are interested on considering the power-law model of nonlinear electrodynamics. Despite the Lagrangian of Maxwell's electrodynamics, power-law nonlinear electrodynamics preserves its conformal invariance symmetry in all the space-time dimensions, provided that the power is chosen properly [23].

On the other hand, based on the outstanding results of Bekenstein, Bardeen, Carter and Hawking, it is well-known that black holes are thermodynamic systems with the well-defined pure geometrical temperature and entropy. Here, we analyze the thermodynamic properties of the three-dimensional AdS black holes in gravity's rainbow with the power-Maxwell electromagnetic field and making use of the canonical ensemble method [24,25]. It is a common to believe that black holes, as the extreme quantum gravity regimes, can not be completely described without consideration of quantum gravitational effects. Thus, with the purpose of finding the quantum gravitational corrections on the black hole thermodynamics and thermal stability, we consider the black hole thermal fluctuations [26,27]. It has been shown that the black hole thermal fluctuations are corresponded to the quantum fluctuations of the space-time geometry [28]. The effects of quantum gravity theory, through consideration of the thermal fluctuations, have been explored for a variety of black holes [29,30].

This paper is structured as follows: In Sec. 2, by use of the variational principle, we obtain the electromagnetic and gravitational field equations in an energy dependent space-time. By considering the power-law model of nonlinear electrodynamics we solve the field equations and introduce a new class of asymptotically AdS black hole solutions. Sec. 3, is devoted to study of the thermodynamic properties and thermal stability analysis of the new black hole solutions we just obtained. We calculate the temperature, entropy, electric potential, conserved mass and charge of the black holes and show that they satisfy the first law of black hole thermodynamics. Then, making use of the canonical ensemble method and regarding the black hole heat capacity, we analyze the thermodynamic stability or phase transitions of the new AdS black holes. Sec. 4 is dedicated to study of the quantum gravitational corrections on the thermodynamic properties and thermal stability of the new AdS black holes by considering the black hole thermal fluctuations. The results are summarized and discussed in section 5.

2. The field equations in gravity's rainbow

The Lagrangian density for three-dimensional Einstein gravity with the cosmological constant Λ coupled to a nonlinear electrodynamics can be written in the following general form [31]

$$L = \mathcal{R} - 2\Lambda + \mathcal{L}(\mathcal{F}), \tag{2.1}$$

where $\mathcal{R}$ is the Ricci scalar, $\mathcal{L}(\mathcal{F})$ being the lagrangian of matter field in the form of nonlinear electrodynamics which is expressed as a function of Maxwell's invariant $\mathcal{F} = F^{ab}F_{ab}$. In terms of potential four-vector A_a, $F_{ab} = \partial_a A_b - \partial_b A_a$ is the Faraday tensor. Here, we are interesting on the power-law nonlinear electrodynamics with the following form of Lagrangian density [32]

$$\mathcal{L}(\mathcal{F}) = (-\mathcal{F})^p, \tag{2.2}$$

with p as the power which may be named as nonlinearity parameter. The case corresponding to the choice of $p = 1$ is nothing but the Lagrangian of Maxwell's classical electrodynamics. Noting Eq. (2.1), we get the equation of motion corresponding to the metric tensor as

$$\mathcal{R}_{ab} - \frac{1}{2}\mathcal{R}g_{ab} + \Lambda g_{ab} = T_{ab}, \tag{2.3}$$

and the stress-energy tensor T_{ab} takes the following form

$$T_{ab} = \frac{1}{2}\mathcal{L}(\mathcal{F})g_{ab} - 2\mathcal{L}'(\mathcal{F})F_{ac}F_b{}^c, \tag{2.4}$$

where prime means derivative with respect to the argument. Also, the equation of motion for the electromagnetic tensor is written as

$$\nabla_a\left[\mathcal{L}'(\mathcal{F})F^{ab}\right] = 0. \tag{2.5}$$

Now, we solve the gravitational and electromagnetic field equations (2.3) and (2.5) in a spherically symmetric and energy dependent space-time identified by the following ansatz [4,33]

$$ds^2 = -\frac{V(r)}{f^2(\varepsilon)}dt^2 + \frac{1}{g^2(\varepsilon)}\left[\frac{dr^2}{V(r)} + r^2 d\theta^2\right], \tag{2.6}$$

where, $V(r)$ is an unknown function of radial component r, named as the metric function, to be determined. $f(\varepsilon)$ and $g(\varepsilon)$ are known as temporal and spatial rainbow functions, respectively.

Also, the only nonzero component of the Faraday tensor is $F_{tr} = -F_{rt}$. Taking it as a function of r, we have $F_{tr}(r) = -A_t'(r)$ and the Maxwell invariant $\mathcal{F}$ can be written in the following explicit form

$$\mathcal{F} = -2f^2(\varepsilon)g^2(\varepsilon)F_{tr}^2. \tag{2.7}$$

Regarding Eqs. (2.5), (2.6) and (2.7) we can write

$$(A_t'(r))^{2p-2}\left[A_t'(r)+(2p-1)rA_t''(r)\right]=0, \qquad p\neq\frac{1}{2}, \tag{2.8}$$

and its solution can be obtained as

$$A_t(r)=q\left(\frac{2p-1}{2-2p}\right)r^{\frac{2p-2}{2p-1}} \qquad \text{for} \quad p\neq\frac{1}{2},\ 1, \tag{2.9}$$

where q is the constant of integration and is related to the black hole electric charge. Also the nonzero component of the electromagnetic field is given by

$$F_{tr}=q\,r^{\frac{-1}{2p-1}} \qquad \text{for} \quad p\neq\frac{1}{2},\ 1. \tag{2.10}$$

Now, one can determine the range of the nonlinear parameter p at which our solutions are physically reasonable. It is evident that the electric potential $A_t(r)$ should be finite at infinity. Thus, the following condition must be satisfied

$$\frac{2p-2}{2p-1}<0 \qquad \text{or equivalently} \qquad \frac{1}{2}<p<1. \tag{2.11}$$

In the geometry, introduced by (2.6), the gravitational field equations (2.3) lead to the following differential equations

$$V'(r)+\frac{r}{g^2(\varepsilon)}\left[2\Lambda+(2p-1)(2q_\varepsilon)^p r^{-\frac{2p}{2p-1}}\right], \tag{2.12}$$

$$V''(r)+\frac{1}{g^2(\varepsilon)}\left[2\Lambda-(2q_\varepsilon)^p r^{-\frac{2p}{2p-1}}\right], \tag{2.13}$$

for the tt (rr) and $\theta\theta$ components, respectively. Here, q_ε is defined as $q_\varepsilon=f^2(\varepsilon)g^2(\varepsilon)q^2$. By taking derivative with respect to r from Eq. (2.12), one is able to show that the result is just the second order differential equation given by Eq. (2.13). It means that these two differential equations are not independent. Thus one can solve (2.12) and ensure that the solution satisfies Eq. (2.13).

Now, the metric function $V(r)$, as the solution to the differential equation (2.12), can be calculated as

$$V(r)=-m-\frac{1}{g^2(\varepsilon)}\left[\Lambda r^2+\frac{(2p-1)^2}{2p-2}(2q_\varepsilon)^p r^{\frac{2p-2}{2p-1}}\right], \quad \text{for} \quad \frac{1}{2}<p<1, \tag{2.14}$$

where m is another integration constant which is related to the black hole mass. It is worth mentioning that in the case $p=\frac{3}{4}$, which is corresponding to the case of conformally invariant (CI) electromagnetic Lagrangian, the metric function takes the following form

$$V^{(CI)}(r)=-m-\frac{1}{g^2(\varepsilon)}\left[\Lambda r^2-\frac{(2q_\varepsilon)^{\frac{3}{4}}}{2r}\right], \quad \text{for} \quad p=\frac{3}{4}. \tag{2.15}$$

In order to determine the asymptotic behavior of the solutions, we notice the behavior of $V(r)$ in the limiting case $r\to\infty$. As the p dependent power of r (i.e. $\frac{2p-2}{2p-1}$) is negative in the interval $\frac{1}{2}<p<1$, one can write

$$\lim_{r\to\infty}V(r)=-m-\frac{\Lambda}{g^2(\varepsilon)}r^2 \quad \text{for} \quad \frac{1}{2}<p<1, \tag{2.16}$$

which confirms that the metric function $V(r)$ describes an asymptotically AdS space-time for the p is in its allowed range. Also the space-time must be pure AdS for the case $p=0$ with the following effective cosmological constant

$$\Lambda_{eff}=\frac{1}{g^2(\varepsilon)}\left(\Lambda-\frac{1}{2}\right). \tag{2.17}$$

The space-time singularities can be examined by calculating the curvature scalars. The Ricci scalar and Riemann invariant are the important scalars from which one can extract the information about space-time singularities. After some manipulations, they can be calculated in the following forms

$$\mathcal{R}=6\Lambda+(4p-3)\,(2q_\varepsilon)^p\,r^{\frac{-2p}{2p-1}}, \tag{2.18}$$

$$\mathcal{R}^{\mu\nu\rho\lambda}\mathcal{R}_{\mu\nu\rho\lambda}=12\Lambda^2+4\Lambda(4p-3)\,(2q_\varepsilon)^p\,r^{\frac{-2p}{2p-1}} +\left(8p^2-8p+3\right)(2q_\varepsilon)^{2p}\,r^{\frac{-4p}{2p-1}}. \tag{2.19}$$

From Eqs. (2.18) and (2.19) one can argue that the Ricci scalar and Riemann invariant are finite for finite values of r and diverge as r goes to infinity for the p-values in the range $\frac{1}{2}<p<1$. There is a singularity located at the origin (i.e. $r=0$ is an essential singularity) for the new AdS black holes, we just obtained. Noting this fact and appearance of the horizons as the real roots of $V(r=r_+)=0$, as it is displayed by Fig. 1, ensure one to interpret the solutions as black holes. The plots show that the black holes with two horizons, extreme black holes and naked singularity black holes can be obtained from the new AdS solutions in the presence of rainbow functions.

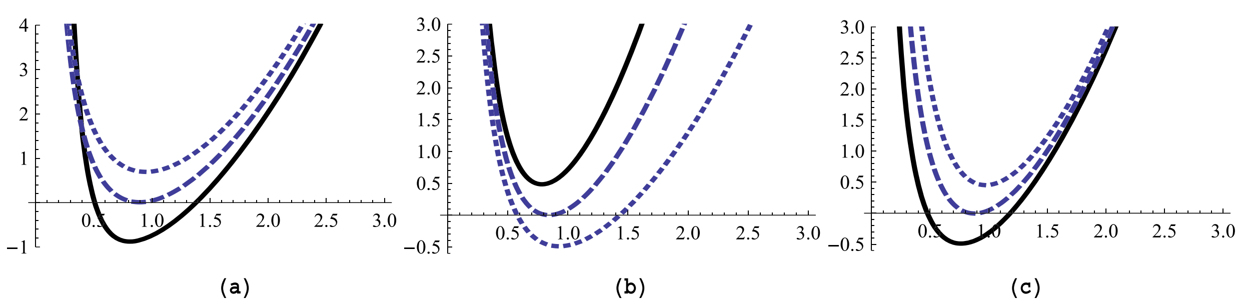

Fig. 1. $V(r)$ versus r for $\Lambda=-1$, $q=2$, $m=2.5$ Eq. (2.14). (a) $g(\varepsilon)=1$, $f(\varepsilon)=0.8$ and $p=0.63$ (continues), 0.715 (dashed), 0.75 (dotted). (b) $p=0.7$, $f(\varepsilon)=0.8$ and $g(\varepsilon)=0.76$ (continues), 0.91 (dashed), 1.15 (dotted). (c) $p=0.7$, $g(\varepsilon)=0.95$ and $f(\varepsilon)=0.6$ (continues), 0.85 (dashed), 1, 1 (dotted).

3. Black hole thermodynamics and thermal stability

In this section we focus on the thermodynamic properties of the three-dimensional nonlinearly charged AdS black hole solutions we have obtained in the previous section. We seek for satisfaction of the first law of thermodynamics and investigate the thermal stability or phase transition regarding the black hole heat capacity in the canonical ensemble method. To do these, we need to calculate the conserved and thermodynamic quantities of the black holes.

Let's start with the calculation of the black hole temperature T. It can be obtained by using the concept of surface gravity κ. The Hawking temperature associated with the black hole horizon is given by $T=\frac{\kappa}{2\pi}$, with $\kappa=\sqrt{-\frac{1}{2}(\nabla_\mu\chi_\nu)(\nabla^\mu\chi^\nu)}$. Taking $\chi^\mu=(-1,\ 0,\ 0)$, as a matter of calculation, one is able to show that $T=\frac{1}{4\pi}V'(r_+)$. r_+ is the black hole horizon radius which can be determined as the real root(s) of equation $V(r_+)=0$. Thus, making use of Eq. (2.14), we have

$$T=-\frac{1}{4\pi f(\varepsilon)g(\varepsilon)}\left[2\Lambda r_+ + (2p-1)(2q_\varepsilon)^p r_+^{\frac{-1}{2p-1}}\right]. \tag{3.1}$$

It must be noted that the extreme black holes (i.e. black holes with zero temperature) can occur provided that the black hole charge and size are fixed such that $T(r_{ext},q_{ext})=0$. After some algebraic calculation we arrive at

$$r_{ext}=\left\{(2p-1)\ell^2 2^{p-1}\left[q_{ext}f(\varepsilon)g(\varepsilon)\right]^{2p}\right\}^{\frac{2p-1}{2p}}, \tag{3.2}$$

which exist always for $p>\frac{1}{2}$. It must be noted that the physical black holes, having positive temperature, are those with $r_+>r_{ext}$. Otherwise the black holes have negative temperature and are not physically reasonable, which we call unphysical black holes throughout the paper.

The black hole entropy, as a pure geometrical quantity, is obtained by use of the well-known entropy-area law. It can be written as

$$S=\frac{\pi r_+}{2g(\varepsilon)}. \tag{3.3}$$

The electric potential Φ, measured with respect to a reference point at a large distance from the horizon, is defined by the following standard relation [21,32,34]

$$\Phi=A_\mu\chi^\mu|_{\text{reference}}-A_\mu\chi^\mu|_{r=r_+}, \tag{3.4}$$

where, A_t is given by Eq. (2.9) and χ^μ is the null generator of the horizon. Therefore, we obtain

$$\Phi(r_+)=q\left(\frac{2p-1}{2-2p}\right)r_+^{\frac{2p-2}{2p-1}}. \tag{3.5}$$

Now, we obtain the black hole electric charge, as a conserved quantity, by calculating the flux of the electric field at infinity (i.e. $r\to\infty$). For this purpose we must use the Gauss's electric law. It yields [35]

$$Q=\frac{p}{g(\varepsilon)}(2)^{p-2}(q_\varepsilon)^{\frac{2p-1}{2}}. \tag{3.6}$$

Therefore, the obtained total charge depends on rainbow functions, which shows that due to the contribution of rainbow gravity, the total electric charge is modified. It reduces to the result of ref. [4] when $p=1$ is chosen. Also, similar results have been obtained by many authors (for example see refs. [5,30,33] and [36–38]).

The space-time under consideration is an asymptotically AdS one, thus we can obtain the conserved mass by utilizing the counterterm method [39,40]. As a matter of calculation one is able to show that [4]

$$M=\frac{m}{8f(\varepsilon)}. \tag{3.7}$$

Note that the integration constant m is obtained by use of the condition $V(r_+)=0$.

Here, we check the first law of thermodynamics for the quantities obtained in this subsection. At first we obtain the mass as a function of the extensive quantities S and Q as

$$M(Q,S)=-\frac{\Lambda S^2}{2\pi^2 f(\varepsilon)}-\frac{(2p-1)^2 2^p}{16(p-1)f(\varepsilon)}\left(\frac{Q}{p2^{p-2}}\right)^{\frac{2p}{2p-1}}\times\left(\frac{2S}{\pi}\right)^{\frac{2p-2}{2p-1}}, \tag{3.8}$$

where, Eqs. (2.14), (3.1), (3.3) and (3.7) have been used. By treating Q and S as the thermodynamical extensive variables one can calculate

$$\Phi=\left(\frac{\partial M}{\partial Q}\right)_S, \qquad T=\left(\frac{\partial M}{\partial S}\right)_Q, \tag{3.9}$$

and show that the results are compatible with those of Eqs. (3.5) and (3.1). It confirms the validity of the thermodynamical first law in the form of

$$dM=TdS+\Phi dQ. \tag{3.10}$$

At this stage we explore the thermal stability or thermodynamic phase transition of the new AdS black holes identified here. It is well-known that the type-one and type-two phase transition points and the ranges at which the black holes remain stable can be extracted regarding the signature of the black hole heat capacity. In the canonical ensemble method the black hole heat capacity, with the black hole charge as a constant, can be calculated via the following relation [41]

$$\mathcal{H}_Q=T\left(\frac{\partial S}{\partial T}\right)_Q. \tag{3.11}$$

Regarding this definition and making use of Eqs. (3.1) and (3.3), after some algebraic calculations, one is able to show that

$$\mathcal{H}_Q=\frac{\pi r_+\left[2\Lambda+(2p-1)\eta_+\right]}{2g(\varepsilon)(2\Lambda-\eta_+)}, \qquad \eta_+=(2q_\varepsilon)^p\, r_+^{\frac{-2p}{2p-1}}. \tag{3.12}$$

According to the basis of the canonical ensemble method, a physically reasonable black hole (i.e. black holes having positive temperature) is thermally stable if its heat capacity is positive. Otherwise, it is unstable and experience thermodynamic phase transition to be stabilized. The real root(s) of $\mathcal{H}_Q=0$ (if any) determine the points at which the thermodynamic type-one phase transition occurs. Type-two phase transition takes place at the divergent points of the black hole heat capacity. In order to determine the points of type-one and type-two phase transition and to characterize the ranges at which the black holes are stable, we have plotted $\mathcal{H}_Q$ and T versus r_+ with and without rainbow functions in Fig. 2. The plots show that there is no point of type-two phase transition. The black holes undergo type-one phase transition at the vanishing point of the black hole heat capacity, which is just the horizon radius of the extreme black holes $r_+=r_{ext}$. Also, the black holes with the horizon radii in the range $r_+>r_{ext}$ have positive heat capacity and they are locally stable.

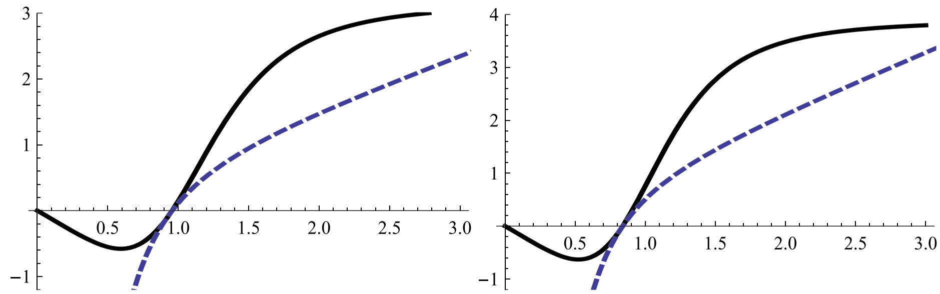

Fig. 2. $\mathcal{H}_Q$(continues) and $5T$(dashed) versus r_+ for $\Lambda = -1$, $q = 2$, $p = 0.7$ Eqs. (3.1) and (3.6). Left: $f(\varepsilon) = 1$, $g(\varepsilon) = 1$ and Right: $f(\varepsilon) = 0.8$, $g(\varepsilon) = 0.9$.

4. Corrected thermodynamics in the presence of thermal fluctuations

The aim of this section is to find the corrections arisen from consideration of thermal fluctuations (TF) on the thermodynamic quantities and thermal stability of the new AdS black holes (more details on black hole TF can be found in refs. [42–45]). It is well-known that entropy is the only thermodynamic quantity which is affected when the first-order corrections are taken into account. It means that, even in the presence of the black hole TF, the black hole temperature [Eq. (3.1)] and electric potential [Eq. (3.5)] remain unchanged [46,47]. By considering the leading order corrections, the black hole entropy gets logarithmic correction which can be written as [48–50]

$$S^{(TF)} = S - \frac{\xi}{2} \ln\left(ST^2\right). \tag{4.1}$$

Note that S is the uncorrected black hole entropy given by Eq. (3.3) and T being the black hole temperature [Eq. (3.1)] and ξ is the parameter of TF or correction parameter with the dimension of Length.

Now, we check the validity of the first law of black hole thermodynamics by considering the impacts of TF. For this purpose, we proceed with two following alternatives:

• If we take the black hole mass unchanged and equal to that is given by Eq. (3.8), it is evident that

$$\left(\frac{\partial M}{\partial Q}\right)_{S^{(TF)}} = \Phi, \tag{4.2}$$

and after some manipulations we arrive at

$$\left(\frac{\partial M}{\partial S^{(TF)}}\right)_Q = \frac{T}{1 - \frac{\xi}{2S}\left[1 + \frac{2(2\Lambda - \eta_+)}{2\Lambda + (2p-1)\eta_+}\right]}. \tag{4.3}$$

Noting Eqs. (4.2) and (4.3), the first law of black hole thermodynamics remains valid provided that the following relation is satisfied

$$1 + \frac{2(2\Lambda - \eta_+)}{2\Lambda + (2p-1)\eta_+} = 0, \tag{4.4}$$

or equivalently, the first law of black hole thermodynamics is valid if the real root of Eq. (4.4) exist. That is

$$r_+ = \left[\frac{(2p-3)(2q_\varepsilon)^p}{-6\Lambda}\right]^{\frac{2p-1}{2p}}. \tag{4.5}$$

The horizon radius r_+ given by Eq. (4.5), if exist, identifies the radius of AdS black holes for which the first law of black hole thermodynamics is valid [26]. But in the AdS space-time with $\Lambda < 0$ and with the allowed p-values in the range $\frac{1}{2} < p < 1$ the horizon radius (4.5) does not exist. As the result, the first law of black hole thermodynamics, with these assumptions, is not valid.

• Following the works of Refs. [27,30], we define the corrected black hole mass as $M^{(TF)} = F^{(TF)} + TS^{(TF)}$ where $F^{(TF)} = -\int S^{(TF)}\, dT$ is the Helmholtz free energy. In this case, after some calculations, we obtain

$$\left(\frac{\partial M^{(TF)}}{\partial S^{(TF)}}\right)_Q = T^{(TF)} = T, \tag{4.6}$$

$$\left(\frac{\partial M^{(TF)}}{\partial Q}\right)_{S^{(TF)}} = \Phi - \frac{\xi(2p-3)}{\pi f(\varepsilon)}(q_\varepsilon)^{\frac{1}{2}} r_+^{-\frac{1}{2p-1}}. \tag{4.7}$$

Regarding Eqs. (4.6) and (4.7), one can argue that the first law of black hole thermodynamics is valid provided that $p = \frac{3}{2}$ is chosen. But $p = \frac{3}{2}$ is not in the allowed range and the first law of black hole thermodynamics, with this new approach, is not valid too.

Now, in order to investigate the effects of TF on the stability of the black holes, starting from the modified version of Eq. (3.11) in the form of $\mathcal{H}_Q^{(TF)} = T\frac{\partial S^{(TF)}}{\partial T}$, we obtained

$$\mathcal{H}_Q^{(TF)} = \frac{\pi r_+ - 3\xi g(\varepsilon)}{g(\varepsilon)(2\Lambda - \eta_+)}\Lambda + \frac{(2p-1)\pi r_+ - (2p-3)\xi g(\varepsilon)}{g(\varepsilon)(2\Lambda - \eta_+)}\eta_+. \tag{4.8}$$

Eq. (4.8) indicates the heat capacity of the new AdS black holes in which the quantum fluctuations are taken into account via consideration of the TF. The plots of $\mathcal{H}_Q^{(TF)}$ and T versus r_+ in the absence and presence of the rainbow functions are displayed in the left and right panels of Fig. 3, respectively. They show that, just like the previous case, there is no point of type-two phase transition. There is only one point of type-one phase transition where the black hole heat capacity vanishes and we label it by $r_+ = R$. Despite the previous case, this point of type-two phase transition does not coincide with the vanishing point of the black hole temperature, it is clear that $R > r_{ext}$. This kind of black holes are locally stable if the horizon radii are in the range $r_+ > R$.

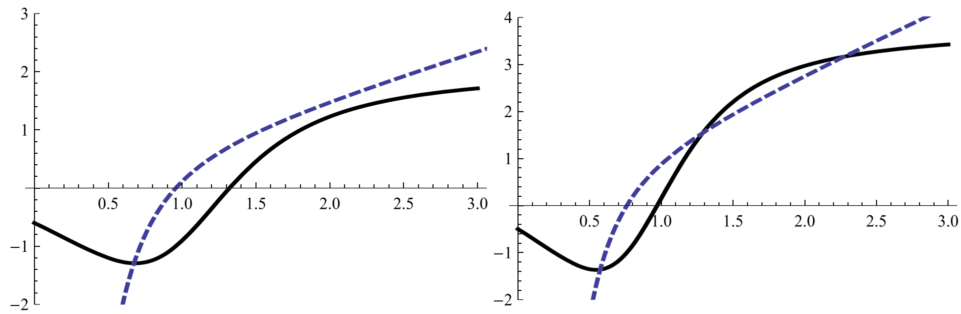

Fig. 3. $\mathcal{H}_Q^{(TF)}$ (continues) and $5T$ (dashed) versus r_+ for $\Lambda=-1,\ q=2,\ p=0.7$ Eqs. (3.1) and (4.8). Left: $f(\varepsilon)=1,\ g(\varepsilon)=1$ and Right: $f(\varepsilon)=0.7,\ g(\varepsilon)=0.8$.

5. Conclusion

This work considers the Einstein-Λ-power-Maxwell three-dimensional charged black hole solutions in rainbow gravity, as the extension of the charged Einstein black holes to the case of nonlinear electrodynamics. By considering a static and circularly symmetric energy dependent geometry, we solved the equations of motion and found that the solutions are asymptotically AdS. As it shown in Fig. 1 the solutions can produce black holes with two horizons, extreme and naked singularity black holes for the suitably fixed parameters. Calculation of the curvature scalars show that there is an essential space-time singularity located at $r=0$. Existence of the horizons and appearance of the singularity in the curvature scalars are sufficient to ensure that the solutions can be interpreted as black holes.

We considered the thermodynamic behavior of the obtained black hole solutions and discussed the thermal stability or phase transitions through the canonical ensemble approach. By calculating the black hole conserved and thermodynamic quantities at first we proved that, although some of these quantities get modified in the presence of rainbow functions, the first law of black hole thermodynamics is valid in its standard form. Then, regarding the signature of the black hole heat capacity, we analyzed the black hole thermal stability or thermodynamic phase transition. We found that no type-two phase transition takes place. There is only one point of type-one phase transition located at the $r_+=r_{ext}$ where the block hole temperature vanishes and extreme black holes occur. Also, the AdS black holes with the radii greater than r_{ext} are locally stable (Fig. 2).

Finally, with the purpose of finding the quantum gravitational effects on the thermodynamic properties and their thermal stability, we considered black hole TF. It is well-known that in the presence of the black hole TF the black hole entropy gets logarithmic corrections. Through alternative approaches we showed that the thermodynamical first law is no longer valid. Also, by analyzing the thermodynamic stability we found that, up to the first order corrections, the black hole heat capacity gets modified. As the result there is a point of type-one phase transition located at $r_+=R>r_{ext}$, which no longer coincides with the vanishing point of the black hole temperature. Also, the black holes with horizon radii in the range $r_+>R$ are locally stable (Fig. 3).

Acknowledgement

The author thanks to the Razi University Research Council for official supports of this work.

References

[1] J. Alfaro, H.A. Morales-Técotl, L.F. Urrutia, Phys. Rev. Lett. 84 (2000) 2318.
[2] G. Amelino-Camelia, M. Arzano, A. Procaccini, Phys. Rev. D 70 (2004) 107501.
[3] G. Amelino-Camelia, M. Arzano, Y. Ling, G. Mandanici, Class. Quantum Gravity 23 (2006) 2585.
[4] M. Dehghani, Phys. Lett. B 777 (2018) 351.
[5] S.H. Hendi, M. Faizal, B. Eslam Panah, S. Panahiyan, M. Momennia, Eur. Phys. J. C 76 (2016) 296.
[6] M. Dehghani, Phys. Lett. B 785 (2018) 274.
[7] S.H. Hendi, M. Faizal, Phys. Rev. D 92 (2015) 044027.
[8] S.H. Hendi, M. momennia, B. Eslam Panah, M. Faizal, Astrophys. J. 827 (2016) 153.
[9] J. Magueijo, L. Smolin, Phys. Rev. Lett. 88 (2002) 190403.
[10] J. Magueijo, L. Smolin, Class. Quantum Gravity 21 (2004) 1725.
[11] A.F. Ali, Phys. Rev. D 89 (2014) 104040.
[12] R.G. Cai, Nucl. Phys. B 628 (2002) 375.
[13] P.O. Mazur, E. Mottola, Phys. Rev. D 64 (2001) 104022.
[14] M. Born, L. Infeld, Proc. R. Soc. A 143 (1934) 410.
[15] M. Born, L. Infeld, Proc. R. Soc. A 144 (1934) 425.
[16] M. Dehghani, Phys. Rev. D 99 (2019) 024001.
[17] H.H. Soleng, Phys. Rev. D 52 (1995) 6178.
[18] S.H. Hendi, J. High Energy Phys. 03 (2012) 065.
[19] S.I. Kruglov, Phys. Rev. D 92 (2015) 123523.
[20] M. Dehghani, S.F. Hamidi, Phys. Rev. D 96 (2017) 044025.
[21] A. Sheykhi, S. Hajkhalili, Phys. Rev. D 89 (2014) 104019.
[22] M. Dehghani, Phys. Rev. D 98 (2018) 044008.
[23] M. Kord Zangeneh, M.H. Dehghani, A. Sheykhi, Phys. Rev. D 92 (2015) 104035.
[24] M. Dehghani, Int. J. Mod. Phys. D 27 (2018) 1850073.
[25] M. Dehghani, Eur. Phys. J. Plus 133 (2018) 474.
[26] B. Pourhassan, M. Faizal, Z. Zaz, A. Bhat, Phys. Lett. B 773 (2017) 325.
[27] M. Dehghani, Phys. Lett. B 781 (2018) 553.
[28] B. Pourhassan, M. Faizal, S. Capozziello, Ann. Phys. 377 (2017) 108.
[29] B. Pourhassan, M. Faizal, Nucl. Phys. B 913 (2016) 834.
[30] S.H. Hendi, B. Eslam Panah, S. Panahiyan, M. Momennia, Eur. Phys. J. C 77 (2017) 647.
[31] M. Dehghani, Phys. Rev. D 94 (2016) 104071.
[32] S.H. Hendi, B. Eslam Panah, S. Panahiyan, A. Sheykhi, Phys. Lett. B 67 (2017) 214.
[33] S. Panahiyan, S.H. Hendi, N. Riazi, Nucl. Phys. B 938 (2019) 388.
[34] M. Dehghani, Phys. Rev. D 96 (2017) 044014.
[35] M. Dehghani, Phys. Lett. B 773 (2017) 105.
[36] B. Eslam Panah, Phys. Lett. B 787 (2018) 45.
[37] S.H. Hendi, S. Panahiyan, S. Upadhyay, B. Eslam Panah, Phys. Rev. D 95 (2017) 084036.
[38] S. Upadhyay, S.H. Hendi, S. Panahiyan, B. Eslam Panah, Prog. Theor. Exp. Phys. 2018 (9) (2018) 093E01.
[39] V. Balasubramanian, P. Kraus, Commun. Math. Phys. 28 (1999) 413.
[40] J. Brown, J. York, Phys. Rev. D 47 (1993) 1407.
[41] M. Dehghani, Phys. Rev. D 97 (2018) 044030.
[42] S. Upadhyay, S. Soroushfar, R. Saffari, arXiv:1801.09574.
[43] B. Pourhassan, S. Upadhyay, H. Farahani, arXiv:1701.08650.
[44] N. Islam, P.A. Ganai, S. Upadhyay, arXiv:1811.05313.
[45] S. Upadhyay, B. Pourhassan, Prog. Theor. Exp. Phys. 2019 (1) (2019) 013B03.
[46] S. Upadhyay, Gen. Relativ. Gravit. 50 (2018) 128.
[47] B. Pourhassan, S. Upadhyay, H. Saadat, H. Farahani, Nucl. Phys. B 928 (2018) 451.
[48] S. Upadhyay, Phys. Lett. B 775 (2017) 130.
[49] B. Pourhassan, M. Faizal, S. Upadhyay, L. Al Asfar, Eur. Phys. J. C 77 (2017) 555.
[50] S. Upadhyay, B. Pourhassan, H. Farahani, Phys. Rev. D 95 (2017) 106014.