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Inclusive J/ψ photoproduction at the ILC within the framework of non-relativistic QCD

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Abstract: Based on the non-relativistic quantum chromodynamics factorization framework, we study the inclusive J/ψ photoproduction at the future high energy e^+e^- collider, International Linear Collider(ILC), where the initial photons come from the back-scattering of laser and electron (positron). The intermediate states, $c\bar{c}({}^3S_1^{[\mathbf{1}]}, {}^1S_0^{[\mathbf{8}]}, {}^3S_1^{[\mathbf{8}]}, {}^3P_{0,1,2}^{[\mathbf{8}]})$, and resolved photoproduction processes are considered. Numerical results show that the cross section of J/ψ photoproduction at the ILC could be very large, and the single resolved process via $c\bar{c}({}^1S_0^{[\mathbf{8}]})$ intermediate state dominates primarily the production. We also present various kinematic distributions for J/ψ photoproduction. Combining the high luminosity of the collision, J/ψ photoproduction at the ILC shall provide a good platform to test the NRQCD factorization.

Keywords: Quarkonium, Effective Field Theories of QCD

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Contents

1 Introduction

Heavy quarkonium consist of two quarks with heavy mass. Since its discovery, it has been well probed to investigate the strong interaction and test quantum chromodynamics (QCD) due to its characteristic scales in the system. Non-relativistic QCD(NRQCD) factorization framework [1] was proposed to describe the production and decay of heavy quarkonium. This effective theory introduces the color-octet (CO) mechanism and surpasses the traditional color-singlet (CS) model [2–6]. NRQCD factorization has achieved great successes in the production [7–10] and polarization [11–15] of heavy quarkonium at the hadron colliders. The high-energy and high-luminosity electron-positron collider is considered as one of the main next generation colliders, such as the CEPC [16, 17], FCC-ee [18], ILC [19, 20] and so on. According to their designs, these colliders can run at various collision energies with high luminosity. Therefore it could be expected that future high-energy e^+e^- colliders will provide excellent experimental platforms for the study of heavy quarkonium. In contrast to hadron colliders, the e^+e^- collider has less background for the production of heavy quarkonium and theoretical calculation is simpler. At the e^+e^- collider, the best way to produce heavy quarkonium via the electron-positron annihilation, especially when the collision energy is around Z-boson mass; and due to the resonance effect, high yield of heavy quarkonium is expected [21].

In addition to the annihilation mode, photoproduction at the e^+e^- collider is also an important production channel of heavy quarkonium. There are two main sources of photons. For example, at the CEPC, photons can come from bremsstrahlung of initial electron and positron, which is described by Weizäcker-Williams approximation (WWA) [22]. At the ILC, photons can be scattered out by external laser and the electron beam. In the eyes of QCD, the photon could be in a hadronic state besides the bare photon state [23]. Due to quantum fluctuation, the photon can undergo a transition for a short period of time into a light quark pair or gluon. As a result, the photon as a whole particle, can participate directly the hard interaction to produce heavy quarkonium, which is called as the direct photoproduction. Alternatively, the photon resolves and the light quarks or gluons in the

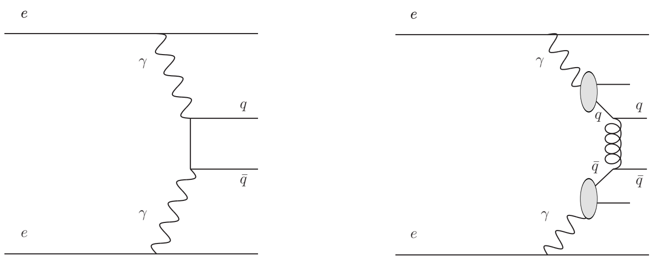

Figure 1. Diagrams for example of direct(left) and resolved(right) photoproduction processes at e^+e^- collider. The diagrams are drawn by JaxoDraw [24].

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photon also get into the interaction. This is called as the resolved photoproduction, as shown in figure 1.

The J/ψ photoproduction has been studied in many literature [25–41]. The inclusive J/ψ photoproduction was measured in 2001 by the DELPHI Collaboration at CERN LEP II [42]. Theoretical calculation of the p_t distribution based on the CS model is smaller by one order of magnitude than the experimental measurements. After considering the CO mechanism, NRQCD gave nice description of the measurements [43] and this has been viewed as one of the earliest evidences of the existence of color-octet processes in nature. With the CO LDMEs extracted by a global fit of worldwide data [44], however, the next-to-leading order (NLO) NRQCD prediction of J/ψ photoproduction systematically underestimate the DELPHI data. It is noteworthy that only a few J/ψ events were reconstructed at LEP II and the measurements have large uncertainties [45]. At present there are no other experiments yet to verify the results of LEP II. As for the hadron collider, NRQCD factorization has achieved great success in explaining the experimental measurements of heavy quarkonium, but it still does not give a unified description of those observables, such as the total yield, kinematic distribution, and polarization [46, 47]. Consequently it is necessary to measure the production of heavy quarkonium at other platforms such as the high-energy e^+e^- collider so as to further test the NRQCD factorization framework.

In this work, we study the inclusive J/ψ photoproduction at the ILC, including both the color-octet channels and the resolved photoproduction processes. In section 2, we give the basic theoretical framework of the calculation. The numerical results and discussions are presented in section 3 and a brief summary is in section 4.

2 Formulation and calculation

At the International Linear Collider, initial photons can achieve high energy and high luminosity and its spectrum is described as [48],

$$f_{\gamma/e}(x) = \frac{1}{N}\left[1 - x + \frac{1}{1-x} - 4r(1-r)\right], \tag{2.1}$$

where $x = E_\gamma/E_e$, $r = x/\left[x_m(1-x)\right]$, and the normalization factor,

$$N = \left(1 - \frac{4}{x_m} - \frac{8}{x_m^2}\right)\log(1+x_m) + \frac{1}{2} + \frac{8}{x_m} - \frac{1}{2(1+x_m)^2}. \tag{2.2}$$

Here $x_m = 4E_e E_l \cos^2\frac{\theta}{2}$, E_e and E_l are the energies of incident electron and laser beams, respectively, and θ is the angle between them. The energy of the laser backscattering (LBS) photon is restricted by

$$0 \leq x \leq \frac{x_m}{1+x_m}, \tag{2.3}$$

with optimal value of x_m being 4.83 [49].

Within the NRQCD factorization framework, the production cross section of heavy quarkonium is separated into a product of the long distance matrix elements (LDMEs) and the short distance coefficients (SDCs). The SDC describes the production of an intermediate $Q\bar{Q}(n)$-pair, where the quantum number $n = {}^{2S+1}L_J^{[c]}$ with c being color multiplicity. The LDME describes the hadronization of the $Q\bar{Q}(n)$-pair into heavy quarkonium. The SDCs can be calculated perturbatively while the LDMEs are assumed to be process-independent and can be extracted by global fitting of experimental data. The differential cross section for heavy quarkonium (H) photoproduction is then formulated as double convolution of the cross section of parton-parton (or photon) processes and the corresponding parton distribution functions,

$$\begin{aligned} &\mathrm{d}\sigma\left(e^+e^- \to e^+e^- H + X\right) \\ &= \int \mathrm{d}x_1 f_{\gamma/e}(x_1) \int \mathrm{d}x_2 f_{\gamma/e}(x_2) \sum_{i,j,k} \int \mathrm{d}x_i f_{i/\gamma}(x_i) \int \mathrm{d}x_j f_{j/\gamma}(x_j) \\ &\quad \times \sum_n \mathrm{d}\sigma(ij \to c\bar{c}[n] + k) \left\langle \mathcal{O}^H[n] \right\rangle, \end{aligned} \tag{2.4}$$

here, $f_{i/\gamma}$ is the Glück-Reya-Schienbein(GRS) parton distribution functions of the light quarks and gluon in photon [50]. $d\sigma(ij \to c\bar{c}[n] + k)$ represents the differential partonic cross section, $i, j = \gamma, g, q, \bar{q}$ and $k = g, q, \bar{q}$ with $q = u, d, s$. $c\bar{c}[n]$ are the intermediate $c\bar{c}$ pair with states $n = {}^3S_1^{[\mathbf{1}]}, {}^1S_0^{[\mathbf{8}]}, {}^3S_1^{[\mathbf{8}]}, {}^3P_J^{[\mathbf{8}]}$ for $H = J/\psi, \psi(2S)$ and $n = {}^3P_J^{[\mathbf{1}]}, {}^3S_1^{[\mathbf{8}]}$ for $H = \chi_{cJ}(J = 0, 1, 2)$, respectively. $\langle \mathcal{O}^H[n] \rangle$ are the LDMEs of H.

Heavier charmonia, such as $\psi(2S)$ and $\chi_{cJ}(J = 0, 1, 2)$, can decay into J/ψ. These feed-down contributions are taken into account by multiplying their direct-production cross sections with corresponding decay branching ratios to J/ψ,

$$\begin{aligned} d\sigma^{\mathrm{prompt}J/\psi} &= d\sigma^{J/\psi} + d\sigma^{\psi(2S)} Br(\psi(2S) \to J/\psi + X) \\ &\quad + \sum_J d\sigma^{\chi_{cJ}} Br\left(\chi_{cJ} \to J/\psi + \gamma\right). \end{aligned} \tag{2.5}$$

Following are all the sub-processes to be calculated, which are for three production mechanisms. As for the direct photoproduction, we have

$$\begin{aligned} &\gamma + \gamma \to c\bar{c}[{}^3S_1^{[\mathbf{1}]}, {}^1S_0^{[\mathbf{8}]}, {}^3S_1^{[\mathbf{8}]}, {}^3P_J^{[\mathbf{8}]}] + c + \bar{c} \to J/\psi(\psi(2S)) + X, \\ &\gamma + \gamma \to c\bar{c}[{}^3P_J^{[\mathbf{1}]}, {}^3S_1^{[\mathbf{8}]}] + c + \bar{c} \to \chi_{cJ} + X, \\ &\gamma + \gamma \to c\bar{c}[{}^3S_1^{[\mathbf{8}]}] + g \to J/\psi(\psi(2S), \chi_{cJ}) + X. \end{aligned} \tag{2.6}$$

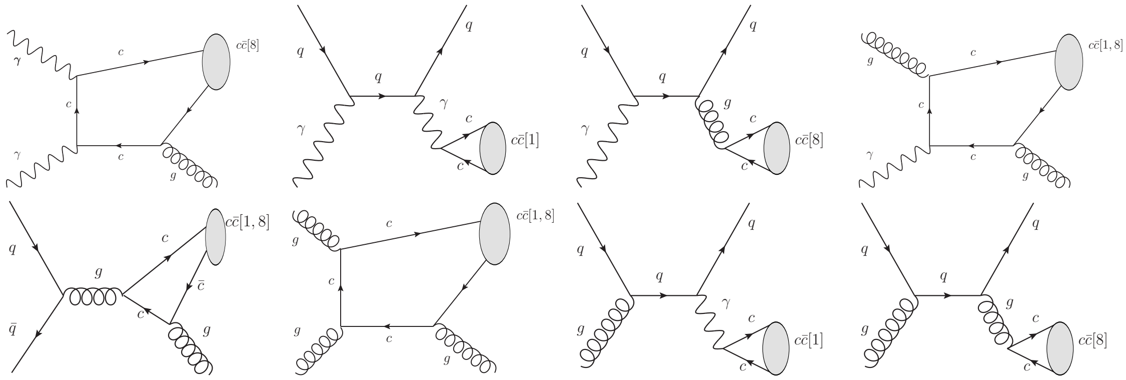

Figure 2. Some Feynman diagrams of the photoproduction. The diagrams are drawn by Jaxo-Draw [24].

As for single resolved photoproduction, we have

$$
\begin{aligned}
\gamma + q(\bar{q}, q = u, d, s) &\rightarrow c\bar{c}[{}^3S_1^{[\mathbf{1}]}, {}^1S_0^{[\mathbf{8}]}, {}^3S_1^{[\mathbf{8}]}, {}^3P_J^{[\mathbf{8}]}] + q(\bar{q}) \rightarrow J/\psi(\psi(2S)) + X, \\
\gamma + g &\rightarrow c\bar{c}[{}^3S_1^{[\mathbf{1}]}, {}^1S_0^{[\mathbf{8}]}, {}^3S_1^{[\mathbf{8}]}, {}^3P_J^{[\mathbf{8}]}] + c + \bar{c} \rightarrow J/\psi(\psi(2S)) + X, \\
\gamma + g &\rightarrow c\bar{c}[{}^3S_1^{[\mathbf{1}]}, {}^1S_0^{[\mathbf{8}]}, {}^3S_1^{[\mathbf{8}]}, {}^3P_J^{[\mathbf{8}]}] + g \rightarrow J/\psi(\psi(2S)) + X, \\
\gamma + q(\bar{q}, q = u, d, s) &\rightarrow c\bar{c}[{}^3P_J^{[\mathbf{1}]}, {}^3S_1^{[\mathbf{8}]}] + q(\bar{q}) \rightarrow \chi_{cJ} + X, \\
\gamma + g &\rightarrow c\bar{c}[{}^3P_J^{[\mathbf{1}]}, {}^3S_1^{[\mathbf{8}]}] + c + \bar{c} \rightarrow \chi_{cJ} + X, \\
\gamma + g &\rightarrow c\bar{c}[{}^3S_1^{[\mathbf{8}]}] + g \rightarrow \chi_{cJ} + X.
\end{aligned}
\tag{2.7}
$$

As for double resolved photoproduction, we have

$$
\begin{aligned}
q(q = u, d, s) + \bar{q} &\rightarrow c\bar{c}[{}^3S_1^{[\mathbf{1}]}, {}^1S_0^{[\mathbf{8}]}, {}^3S_1^{[\mathbf{8}]}, {}^3P_J^{[\mathbf{8}]}] + g \rightarrow J/\psi(\psi(2S)) + X, \\
q(q = u, d, s) + \bar{q} &\rightarrow c\bar{c}[{}^3S_1^{[\mathbf{1}]}, {}^1S_0^{[\mathbf{8}]}, {}^3S_1^{[\mathbf{8}]}, {}^3P_J^{[\mathbf{8}]}] + c + \bar{c} \rightarrow J/\psi(\psi(2S)) + X, \\
g + g &\rightarrow c\bar{c}[{}^3S_1^{[\mathbf{1}]}, {}^1S_0^{[\mathbf{8}]}, {}^3S_1^{[\mathbf{8}]}, {}^3P_J^{[\mathbf{8}]}] + g \rightarrow J/\psi(\psi(2S)) + X, \\
g + g &\rightarrow c\bar{c}[{}^3S_1^{[\mathbf{1}]}, {}^1S_0^{[\mathbf{8}]}, {}^3S_1^{[\mathbf{8}]}, {}^3P_J^{[\mathbf{8}]}] + c + \bar{c} \rightarrow J/\psi(\psi(2S)) + X, \\
g + q(\bar{q}, q = u, d, s) &\rightarrow c\bar{c}[{}^3S_1^{[\mathbf{1}]}, {}^1S_0^{[\mathbf{8}]}, {}^3S_1^{[\mathbf{8}]}, {}^3P_J^{[\mathbf{8}]}] + q(\bar{q}) \rightarrow J/\psi(\psi(2S)) + X, \\
q(q = u, d, s) + \bar{q} &\rightarrow c\bar{c}[{}^3P_J^{[\mathbf{1}]}, {}^3S_1^{[\mathbf{8}]}] + g \rightarrow \chi_{cJ} + X, \\
q(q = u, d, s) + \bar{q} &\rightarrow c\bar{c}[{}^3P_J^{[\mathbf{1}]}, {}^3S_1^{[\mathbf{8}]}] + c + \bar{c} \rightarrow \chi_{cJ} + X, \\
g + g &\rightarrow c\bar{c}[{}^3P_J^{[\mathbf{1}]}, {}^3S_1^{[\mathbf{8}]}] + g \rightarrow \chi_{cJ} + X, \\
g + g &\rightarrow c\bar{c}[{}^3P_J^{[\mathbf{1}]}, {}^3S_1^{[\mathbf{8}]}] + c + \bar{c} \rightarrow \chi_{cJ} + X, \\
g + q(\bar{q}, q = u, d, s) &\rightarrow c\bar{c}[{}^3P_J^{[\mathbf{1}]}, {}^3S_1^{[\mathbf{8}]}] + q(\bar{q}) \rightarrow \chi_{cJ} + X.
\end{aligned}
\tag{2.8}
$$

Some Feynman diagrams of these photoproduction processes are presented in figure 2. The well-established package, Feynman Diagram Calculation (FDC) [51], is used to do the analytical and numerical calculations. In FDC, the standard projection method [52] is employed to deal with the hard process. After dealing with the squared amplitudes analytically, FDC generates FORTRAN codes for numerical integration of phase space.

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$\sqrt{S}$(GeV)	$\sigma_{\text{direct}J/\psi}$	$\sigma_{\psi(2S)\to J/\psi}$	$\sigma_{\chi_{cJ}\to J/\psi}$	$\sigma_{\text{prompt}J/\psi}$
250	7.04	0.54	0.65	7.64
500	11.91	0.89	0.14	12.94
1000	19.63	1.48	0.29	21.41

Table 1. The integrated cross sections (in unit of pb) for prompt J/ψ photoproduction under different collision energies at the ILC. The cut $p_t > 5\,\text{GeV}$ is set for J/ψ. Both the CS and the CO channels have been summed up.

3 Numerical results and discussions

To do the numerical calculation, we choose the electromagnetic fine structure constant $\alpha = 1/137$ and the one-loop running strong coupling constant $\alpha_s(\mu_r)$. To conserve the gauge invariant of the hard scattering amplitude, the charm quark mass, m_c, is set approximately as $m_c = m_H/2$, where the charmonia masses $m_H = 3.097, 3.415, 3.511, 3.556, 3.686\,\text{GeV}$ [53] for $H = J/\psi, \chi_{cJ}(J = 0, 1, 2)$ and $\psi(2S)$, respectively. The branching ratios are taken as $Br(\psi(2S) \to J/\psi) = 0.61$ and $Br\,(\chi_{cJ} \to J/\psi) = 0.014, 0.343, 0.19$ for $J = 0, 1, 2$ [53]. In dealing with the feed-down contributions, a shift of the transverse momentum of charmonium, $p_t^H \approx p_t^{H'} \times (m_H/m_{H'})$, is used. The renormalization scale μ_r is set to be $\mu_r = m_T = \sqrt{m_H^2 + (p_t^H)^2}$. Taking $\mu_r = m_T/2$, the cross section ($\sqrt{s} = 500\,\text{GeV}$) shall be increased by about 80%, and taking $\mu_r = 2m_T$, the cross section shall decreased by about 40%. Such a large scale dependence could be tamed by higher order calculation or a proper scale setting, cf. ref. [54]. As for the non-perturbative LDMEs, we take [15],

$$
\begin{aligned}
\langle \mathcal{O}^{\psi}(^3S_1^{[\mathbf{1}]})\rangle &= \frac{3N_c}{2\pi}|R_\psi(0)|^2, \\
\langle O^{\chi_{cJ}}(^3P_J^{[\mathbf{1}]})\rangle &= \frac{3}{4\pi}(2J+1)|R'_{\chi_c}(0)|^2, \\
\langle O^{J/\psi}(^1S_0^{[\mathbf{8}]})\rangle &= 5.66 \times 10^{-2}\,\text{GeV}^3, \\
\langle O^{J/\psi}(^3S_1^{[\mathbf{8}]})\rangle &= 1.77 \times 10^{-2}\,\text{GeV}^3, \\
\langle O^{J/\psi}(^3P_0^{[\mathbf{8}]})\rangle/m_c^2 &= 3.42 \times 10^{-3}\,\text{GeV}^3, \\
\langle O^{\psi(2S)}(^1S_0^{[\mathbf{8}]})\rangle &= -1.20 \times 10^{-4}\,\text{GeV}^3, \\
\langle O^{\psi(2S)}(^3S_1^{[\mathbf{8}]})\rangle &= 3.40 \times 10^{-3}\,\text{GeV}^3, \\
\langle O^{\psi(2S)}(^3P_0^{[\mathbf{8}]})\rangle/m_c^2 &= 4.20 \times 10^{-3}\,\text{GeV}^3, \\
\langle O^{\chi_{c0}}(^3S_1^{[\mathbf{8}]})\rangle &= 2.21 \times 10^{-3}\,\text{GeV}^3,
\end{aligned}
\tag{3.1}
$$

where the wave functions at the origin are given as $|R_{J/\psi}(0)|^2 = 0.81\,\text{GeV}^3$, $|R_{\psi(2S)}(0)|^2 = 0.53\,\text{GeV}^3$ and $|R'_{\chi_c}(0)|^2 = 0.075\,\text{GeV}^5$ [55].

In table 1, the integrated cross sections of prompt J/ψ photoproduction at the ILC under different energies are listed.[1] It can be seen that the integrated cross section becomes

[1]NRQCD factorization may hold only for transverse momenta that are substantially large than the J/ψ mass [56, 57], so we adopt a relatively high low-p_t cut as $p_t > 5\,\text{GeV}$ to do the calculations.

$\sqrt{S}$(GeV)	$\sigma^{\text{direct}J/\psi}_{^3S_1^{[\mathbf{1}]}}$	$\sigma^{\psi(2S)\to J/\psi}_{^3S_1^{[\mathbf{1}]}}$	$\sigma^{\chi_{cJ}\to J/\psi}_{^3P_J^{[\mathbf{1}]}}$	$\sigma^{\text{direct}J/\psi}_{^1S_0^{[\mathbf{8}]}}$	$\sigma^{\text{direct}J/\psi}_{^3S_1^{[\mathbf{8}]}}$	$\sigma^{\text{direct}J/\psi}_{^3P_J^{[\mathbf{8}]}}$
250	0.30	0.15	0.046	6.14	0.034	0.56
500	0.38	0.20	0.10	10.49	0.075	0.96
1000	0.56	0.29	0.21	17.32	0.16	1.59

Table 2. The integrated cross sections (in unit of pb) from various channels for J/ψ photoproduction under different collision energies at the ILC. The cut $p_t > 5\,\text{GeV}$ is set for J/ψ.

larger with the increment of the collision energy, while the ratios of feed-down contributions are not sensitive to the energy, which are about 8%. The contribution of direct J/ψ photoproduction dominates over those from the feed-down. Due to the large cross section, a sizable number of J/ψ events are expected to be generated via the photoproduction at the ILC.

Table 2 presents the integrated cross sections for different channels of J/ψ photoproduction. The color-octet channels are dominant and two channels, $^1S_0^{[\mathbf{8}]}$ and $^3P_J^{[\mathbf{8}]}$, provide about 95% contributions to the direct J/ψ production. For the J/ψ photoproduction at the ILC, the NRQCD factorization framework and the CS model therefore give predictions that differ by one order of magnitude. In the color-singlet channel, the feed-down contributions from $\psi(2S)$ and χ_{cJ} are significant, where the situation is very different from that in the color-octet channels. Although the integrated cross section for the CS channel is very small when compared with that of the CO, it itself is still a sizable cross section. For example, if setting the integrated luminosity of the ILC as $\mathcal{O}(10^4)\,\text{fb}^{-1}$, there would be $\mathcal{O}(10^6)$ J/ψ mesons to be produced via only the CS channels. Consequently, the measurement of J/ψ photoproduction at the ILC could be done precisely and it shall be very helpful to test NRQCD factorization and to further study physics of heavy quarkonium. In the following we take $\sqrt{S} = 500\,\text{GeV}$ for more discussions.

Figure 3 shows the prompt J/ψ photoproduction in terms of transverse momentum distributions, where the direct and feed-down channels are displayed separately, both for the NRQCD and the CSM predictions. We can see that the direct production dominate in the whole p_t region.

Figure 4 shows the kinematic distributions for different intermediate $c\bar{c}$ Fock states. Here three kind of kinematic distributions of transverse momentum(p_t), rapidity(y) and angular ($\cos\theta$), are presented. θ is the angle between J/ψ and the collision beams. The CO channels dominate in most p_t regions while the contribution from the CS channel become relatively important in very large p_t region. The $^1S_0^{[\mathbf{8}]}$ channels are always primarily dominant in almost whole kinematic regions shown in figure 4. The curves of these channels also have different p_t and y behaviors. Such different kinematic distributions of these channels can be used to constrain the LDMEs. Curves of the latter two kinematic distributions change gently in middle regions, which means there would be enough events in the whole region to make well measurement. Taking the integrated luminosity of the ILC to be $\mathcal{O}(10^4)\,\text{fb}^{-4}$ as reference, in the lowest region of y and $\cos\theta$, e.g., $-0.5 < y < 0.5$ and $-0.05 < \cos\theta < 0.05$, there would produce in both places about $\mathcal{O}(10^4)$ J/ψ mesons

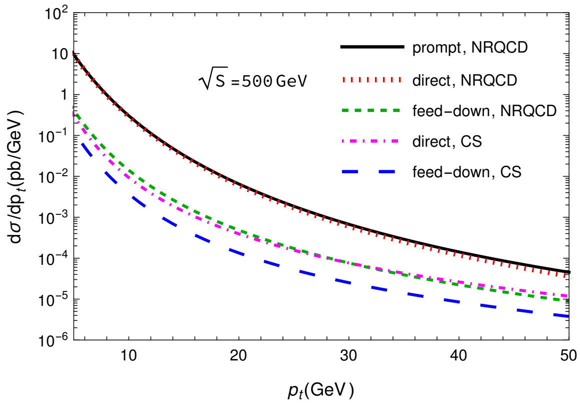

Figure 3. The p_t distributions for prompt J/ψ photoproduction at the ILC ($\sqrt{S} = 500\,\text{GeV}$).

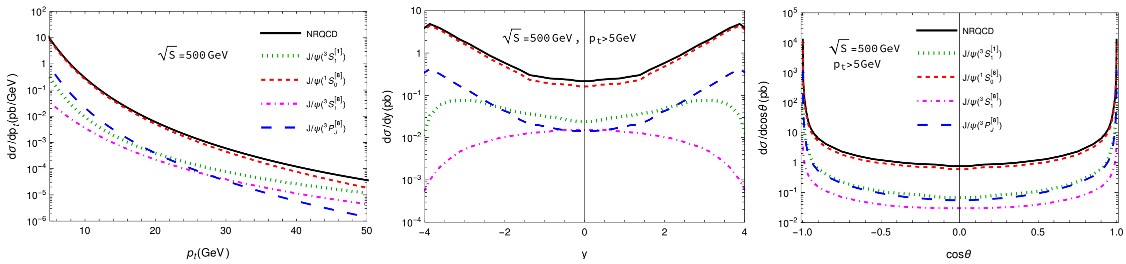

Figure 4. Kinematic distributions for different intermediate $c\bar{c}$ states of J/ψ photoproduction at the ILC ($\sqrt{S} = 500\,\text{GeV}$). The y and $\cos\theta$ distributions are plotted under the cut $p_t > 5\,\text{GeV}$. y curves use same legends as those of p_t.

by the NRQCD prediction, and about $\mathcal{O}(10^3)$ J/ψ by the CS model. Due to the high luminosity and the large cross section of J/ψ photoproduction, precise measurements of these kinematic distributions at the ILC are very promising. In there three kinds of distributions, the NRQCD prediction are larger than those of the CS model by about one order of magnitude in the most kinematic regions, which indicates the photoproduction of J/ψ at the ILC could be well probe to test the CO mechanism.

In figure 5, kinematic distributions of the direct photoproduction, the single resolved photoproduction and the double resolved photoproduction are presented. In the region of $5\,\text{GeV} < p_t < 50\,\text{GeV}$, the single resolved photoproduction contributes 90% the integrated cross section. Although in large p_t region, the direct photoproduction give more contributions than the single resolved photoproduction, there may be not enough events to make precise measurement. In the small middle regions of both y and $\cos\theta$, the double resolved channels provide relatively sizable contributions, while the single resolved ones are always dominant.

The single resolved sub-processes $\gamma + g \to c\bar{c}({}^1S_0^{[\mathbf{8}]}, {}^3P_{J=0,1,2}^{[\mathbf{8}]}) + g \to J/\psi + X$ provide the most contributions for J/ψ photoproduction at the ILC. This can be explained by looking insight into their Feynman diagrams. The diagrams shown in figure 6 are absent for other sub-processes due to parity and color conservation, and they provide the main contributions of ${}^1S_0^{[\mathbf{8}]}, {}^3P_J^{[\mathbf{8}]}$ channels. The squared invariant mass of the gluon propagator

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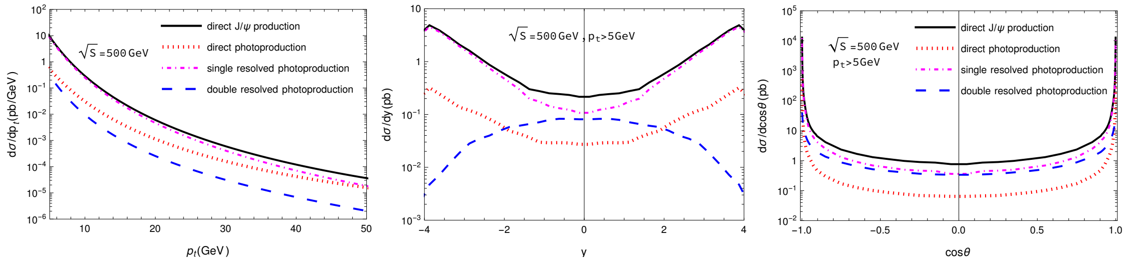

Figure 5. Kinematic distributions for the resolved photoproduction of J/ψ at the ILC ($\sqrt{S} =$ 500 GeV). The y and $\cos\theta$ distributions are plotted under the cut $p_t > 5\,\text{GeV}$. All the distributions are plotted for the NRQCD predictions and y curves use same legends as those of p_t.

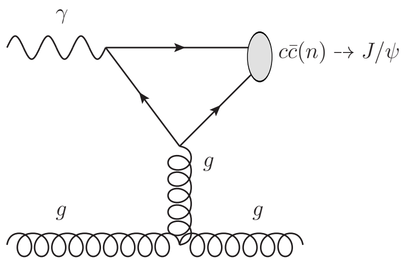

Figure 6. Feynman diagram of single resolved photoproduction $\gamma + g \to c\bar{c}(n = {}^1S_0^{[\mathbf{8}]}, {}^3P_{J=0,1,2}^{[\mathbf{8}]}) + g \to J/\psi + X$.

in figure 6 can be expressed as,

$$
\begin{aligned}
k^2 &= 4m_c^2 - x\sqrt{S}M_t e^{-y} \\
&= 4m_c^2 - 2\left(E_{J/\psi} + E_g\right) M_t e^{-y} \\
&= -4m_c^2 e^{-2y} - \left(1 + e^{-2y}\right)\left(p_t^{J/\psi}\right)^2 - 2E_g M_t e^{-y},
\end{aligned}
\tag{3.2}
$$

from which it can be seen that $1/k^2$ and hence the cross section could be very large in small p_t and large y regions. This feature is also illustrated in the p_t and y distributions of figure 4.

4 Summary

In this work, inclusive J/ψ photoproduction at the future ILC was preliminarily studied at the leading order (LO) in α_s within the NRQCD factorization framework. Both the color-octet channels and the resolved photoproduction processes were considered. The numerical results show that the production has sizable cross section, where the color-octet channels and the single resolved processes dominate primarily. At the ILC with high luminosity, it could be expected that there will be numerous J/ψ mesons to be produced and various kinematic distributions will be measured precisely, which provides a good laboratory to test the NRQCD factorization framework. When confronted with real measurements, however, more precise and high-order calculations are definitely needed. It is well known that the

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NLO corrections in α_s can enhance cross sections significantly. At the NLO in α_s, graphs of new topologies appear and their harder p_t behaviour can easily compensate the suppression in α_s. In the first complete photoproduction analyses at the NLO [35, 36], the NRQCD cross sections are enhanced by up to 115% in the considered kinematic range and the polarisation also has very different distributions from those of the LO predictions. Such enhancements are thus also expected in the NLO analysis of J/ψ photoproduction at the ILC. As an exploratory work, we did not carry the NLO corrections out and left it for future study.

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