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Charged lepton flavor violating radiative decays $l_i \rightarrow l_j \gamma$ in G2HDM

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ABSTRACT: We compute the electromagnetic form factors of the $l_i l_j \gamma$ vertex at one-loop level in the minimal G2HDM which has a sub-GeV vector dark matter candidate. The results are applied to the radiative decay rates for the charged lepton flavor violating processes $l_i \rightarrow l_j \gamma$, and the anomalous magnetic dipole moment and the electric dipole moment of the charged lepton. To numerically compute the branching ratio for $\mu \rightarrow e \gamma$ and compare with the latest experimental limit from MEG, we adapt our previous parameter space scan that is consistent with the relic density and constraints from direct searches of dark matter, W and Z mass measurements, as well as the LHC Higgs signal strengths and invisible width. While the extra contributions are at least an order of magnitude smaller than required to explain the $\sim 4.2\sigma$ discrepancy in the muon anomaly, the existing MEG limit imposes stringent constraint on the parameter space. The remaining viable parameter space can be further probed by the MEG II sensitivity for $\mu \rightarrow e \gamma$ as well as from the direct searches of sub-GeV dark matter in foreseeable future. Higher loop contributions may be significant to resolve the discrepancy in the muon anomaly and generate a non-vanishing electric dipole moments for the standard model quarks and leptons in G2HDM.

KEYWORDS: Lepton Flavour Violation (charged), Models for Dark Matter, Electric Dipole Moments, Gauge Symmetry

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1 Introduction

Charged lepton flavor violating processes like $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$ or $\mu - e$ conversion in nuclei, etc. without any neutrino in the final states are absent at tree level in the standard model (SM) of particle physics. However they are not strictly forbidden by symmetry and can be induced by one-loop diagram with the W boson exchange. Thus their branching ratios are vanishingly small as they are proportional to the neutrino masses [1–3]. The most stringent experimental constraint is for $\mu \rightarrow e\gamma$, with the following limit on its branching ratio published in 2016 by the MEG collaboration [4],

$$\mathcal{B}(\mu^+ \rightarrow e^+ \gamma) < 4.2 \times 10^{-13} \text{ (90\%C.L.)} \quad (1.1)$$

and the projected future sensitivity is expected to improve about an order of magnitude $\sim 6 \times 10^{-14}$ by MEG II [5]. For reviews on the charged lepton flavor violation, see for example refs. [6–9].

The process $\mu \rightarrow e\gamma$ (or in general $l_i \rightarrow l_j\gamma$ with the Latin indices $i, j, k = 1, 2, 3$ labeling the generation (or flavor) here and henceforth) has been widely studied beyond the standard model (bSM) [6–8, 10, 11] with predictions on the branching ratios that are more reachable experimentally than the SM one in foreseeable future. Here we will study this process in the context of minimal gauged two-Higgs-doublet model (G2HDM) [12–14] which has a hidden SM-like dark sector of $SU(2)_H \times U(1)_X$ with a sub-GeV $\mathcal{W}^{(p,m)}$ dark matter candidate. The stability of the dark matter in the model is due to a hidden h -parity which emerges naturally without introducing it on *ad hoc* basis. Under the h -parity, all the SM particles and extra neutral gauge bosons are even while other new particles in G2HDM are odd.

The new contributions to the one-loop process $l_i \rightarrow l_j\gamma$ in G2HDM involve the new gauge or Yukawa couplings between a h -parity odd particle like the dark gauge boson $\mathcal{W}^{(p,m)}$, complex scalar \mathcal{D} or charged Higgs \mathcal{H}^\pm couple to another h -parity odd heavy hidden leptons l_k^H or ν_k^H and the external SM charged leptons l_i and l_j . These new couplings are in general off-diagonal in the generation space and hence can give rise to $l_i \rightarrow l_j$ transition with $i > j$ (in particular muon \rightarrow electron) at one-loop. While the contribution from the dark charged Higgs \mathcal{H}^\pm is suppressed by the neutrino masses like the SM W^\pm , the other new contributions are not and therefore can give rise to a branching ratio that is more accessible experimentally. Turning the argument around, one can use the present and future experimental limits on the charged lepton violating processes to constrain our model parameters in G2HDM.

As a byproduct of our computation of the form factors for $l_i \rightarrow l_j\gamma$, we can also extract the anomalous magnetic dipole moment a_{l_i} and the electric dipole moment d_{l_i} easily by setting $i = j$ in our results. The muon anomalous magnetic dipole moment,

$$a_\mu \equiv \left(\frac{g_\mu - 2}{2} \right), \tag{1.2}$$

where g_μ is the g -factor of the muon, is the most precise measured quantity in SM, with a value measured at the Brookhaven National Laboratory (BNL) E821 experiment (1997–2001) [15–17],

$$a_\mu(\text{BNL}) = (11\,659\,208.9 \pm 5.4_{\text{stat}} \pm 3.3_{\text{sys}}) \times 10^{-10}. \tag{1.3}$$

Recently, the Fermilab (FNAL) Muon $g - 2$ Collaboration, based on the analysis of data set from Run 1 and Run 2, announced the first result on the measurement [18]

$$a_\mu(\text{FNAL}) = (11\,659\,204.0 \pm 5.4) \times 10^{-10}. \tag{1.4}$$

The average value of a_μ from the two experiments is given by [18]

$$a_\mu(\text{BNL} + \text{FNAL}) = (11\,659\,206.1 \pm 4.1) \times 10^{-10}. \tag{1.5}$$

For recent reviews of the muon $g - 2$, see for example refs. [8, 19, 20]. The recommended value for the SM prediction of the muon $g - 2$ is [19]

$$a_\mu^{\text{SM}} = (11\,659\,181.0 \pm 4.3) \times 10^{-10}. \tag{1.6}$$

Observable	Experimental Result/Limit	Future Goal
$a_\mu(\text{BNL})$	$(11\,659\,208.9 \pm 5.4_{\text{stat}} \pm 3.3_{\text{sys}}) \times 10^{-10}$ [15–17]	–
$a_\mu(\text{FNAL})$	$(11\,659\,204.0 \pm 5.4) \times 10^{-10}$ [18]	Uncertainty $\sim 1/4$ of BNL
$a_\mu(\text{BNL} + \text{FNAL})$	$(11\,659\,206.1 \pm 4.1) \times 10^{-10}$ [18]	Uncertainty $\sim 1/4$ of BNL
$\mathcal{B}(\mu^+ \rightarrow e^+\gamma)$ (MEG)	$< 4.2 \times 10^{-13}$ (90% C.L.) [4]	$\sim 6 \times 10^{-14}$ (MEG II [5])
$ \frac{d_\mu}{e} $ [cm]	$< 1.8 \times 10^{-19}$ (95% C.L.) [21]	$\sim 6 \times 10^{-23}$ (PSI [22])
$ \frac{d_e}{e} $ [cm] (ACME)	$< 1.1 \times 10^{-29}$ (90% C.L.) [23]	(Advanced ACME [24])

Table 1. Experimental results for a_μ and upper limits for $\mathcal{B}(\mu \rightarrow e\gamma)$, $|d_\mu/e|$ and $|d_e/e|$.

Hence the discrepancy between the experimental and theoretical values amounts to [18]

$$\Delta a_\mu \equiv a_\mu(\text{BNL} + \text{FNAL}) - a_\mu^{\text{SM}} = (25.1 \pm 5.9) \times 10^{-10}, \quad (1.7)$$

which implies a significance at the 4.2σ level, slightly under the standard criterion of 5σ to claim a discovery. Nevertheless, this discrepancy is as large as the SM electroweak contribution to the muon $g - 2$ [19],

$$a_\mu^{\text{EW}} = (15.4 \pm 0.1) \times 10^{-10}, \quad (1.8)$$

which provides strong hints of bSM physics around the electroweak scale be responsible for it. Future goal of the ongoing efforts at FNAL [18] is to further reduce the existing uncertainty in the muon anomaly measurement by a factor of $1/4$.

For the electric dipole moment of the SM charged leptons, we show that they vanish identically at one-loop in minimal G2HDM due to the lack of CP violating phases in the products of related complex couplings as well as vanishing combinations of loop integrals. Similar conclusions can be obtained for the SM quarks. Higher loop contributions are needed to anticipate to achieve a nonzero electric dipole moments for the SM fermions in minimal G2HDM. We will not address this issue in this work.

Current experimental status of a_μ , $\mathcal{B}(\mu^+ \rightarrow e^+\gamma)$ and $|d_{e,\mu}/e|$ are summarized in table 1.

We lay out the paper as follows. In the next section, we will review the minimal particle content in G2HDM and write down the relevant interactions required for the one-loop computation for the form factors of the radiative decays $l_i \rightarrow l_j\gamma$. In section 3, we compute the magnetic and electric dipole form factors for the radiative decays. In the case of $i = j$ we also obtain the anomalous magnetic dipole moment and electric dipole moment for the lepton l_i . We will show that the electric dipole moment of the lepton vanishes identically at one-loop in G2HDM. Numerical analysis for $\mu \rightarrow e\gamma$ and Δa_μ is presented in section 4. We also present the impact of the viable parameter space on the spin-independent cross section for the sub-GeV dark matter direct search experiments. We conclude in section 5.

Analytical formulas for the form factors and the associated loop integrals are given in appendix A. In appendix B, we show that the form factors of the SM W boson loop obtained in the unitary and 't Hooft-Feynman gauges are equivalent. Discrepancies between our results and existing ones in the literature are clarified in the appendices. Some relevant Feynman rules in G2HDM are shown in appendix C. In appendix D, we demonstrate the

Scalar	SU(2) _L	SU(2) _H	U(1) _Y	U(1) _X	<i>h</i> -parity
$H = (H_1 \ H_2)^T$	2	2	$\frac{1}{2}$	$\frac{1}{2}$	(+, -)
$\Phi_H = (\Phi_1 \ \Phi_2)^T$	1	2	0	$\frac{1}{2}$	(-, +)
S	1	1	0	0	+

Table 2. Higgs scalars in the minimal G2HDM and their quantum number assignments.

Fermion	SU(3) _C	SU(2) _L	SU(2) _H	U(1) _Y	U(1) _X	<i>h</i> -parity
$Q_L = (u_L \ d_L)^T$	3	2	1	$\frac{1}{6}$	0	(+, +)
$U_R = (u_R \ u_R^H)^T$	3	1	2	$\frac{2}{3}$	$\frac{1}{2}$	(+, -)
$D_R = (d_R^H \ d_R)^T$	3	1	2	$-\frac{1}{3}$	$-\frac{1}{2}$	(-, +)
u_L^H	3	1	1	$\frac{2}{3}$	0	-
d_L^H	3	1	1	$-\frac{1}{3}$	0	-
$L_L = (\nu_L \ e_L)^T$	1	2	1	$-\frac{1}{2}$	0	(+, +)
$N_R = (\nu_R \ \nu_R^H)^T$	1	1	2	0	$\frac{1}{2}$	(+, -)
$E_R = (e_R^H \ e_R)^T$	1	1	2	-1	$-\frac{1}{2}$	(-, +)
ν_L^H	1	1	1	0	0	-
e_L^H	1	1	1	-1	0	-

Table 3. Fermions in the minimal G2HDM and their quantum number assignments.

well-known fact that only the magnetic and electric dipole moment form factors are relevant for the computations of the on-shell amplitude of $l_i \rightarrow l_j \gamma$.

2 Minimal G2HDM

In this section, we will briefly review the minimal G2HDM studied recently in [12–14]. The original model based on augmenting the SM electroweak gauge group $SU(2)_L \times U(1)_Y$ by a hidden gauge sector $SU(2)_H \times U(1)_X$ was introduced in ref. [25]. The main idea of G2HDM is to group the two Higgs doublets H_1 and H_2 in inert 2HDM (I2HDM) together to form a bifundamental irreducible representation of $SU(2)_L \times SU(2)_H$. Various refinements [26–28] and collider phenomenology [29–31] were pursued subsequently with the same particle content as the original model where the DM candidate is a complex scalar \mathcal{D} . In this work, as in [12–14], we will drop the triplet field Δ_H of the extra $SU(2)_H$ in the original model and propose the complex gauge boson field $\mathcal{W}^{(p,m)}$ as DM candidate rather than the complex scalar \mathcal{D} . For convenience, the scalar and fermion contents and their quantum numbers as well as *h*-parity in the model are tabulated in table 2 and 3 respectively. Our convention for the electric charge Q (in unit of e) is $Q = T_L^3 + Y$ where T_L^3 is the third component of the $SU(2)_L$ generators and Y is the hypercharge. S is the scalar field introduced to implement the Stueckelberg mechanism to provide a mass for the $U(1)_X$ gauge boson [32–35].

2.1 Higgs potential and spontaneous symmetry breaking

The most general Higgs potential which is invariant under both $SU(2)_L \times U(1)_Y \times SU(2)_H \times U(1)_X$ can be written down as follows

$$\begin{aligned}
 V = & -\mu_\Phi^2 \Phi_H^\dagger \Phi_H + \lambda_\Phi \left(\Phi_H^\dagger \Phi_H \right)^2 - \mu_H^2 \left(H^{\alpha i} H_{\alpha i} \right) + \lambda_H \left(H^{\alpha i} H_{\alpha i} \right)^2 \\
 & + \frac{1}{2} \lambda'_H \epsilon_{\alpha\beta} \epsilon^{\gamma\delta} \left(H^{\alpha i} H_{\gamma i} \right) \left(H^{\beta j} H_{\delta j} \right) + \lambda_{H\Phi} \left(H^\dagger H \right) \left(\Phi_H^\dagger \Phi_H \right) \\
 & + \lambda'_{H\Phi} \left(H^\dagger \Phi_H \right) \left(\Phi_H^\dagger H \right),
 \end{aligned} \tag{2.1}$$

where $(\alpha, \beta, \gamma, \delta)$ and (i, j) refer to the $SU(2)_H$ and $SU(2)_L$ indices respectively, all of which run from one to two, and $H^{\alpha i} = H_{\alpha i}^*$.

To study spontaneous symmetry breaking (SSB) in the model, we parameterize the Higgs fields according to standard practice

$$H_1 = \begin{pmatrix} G^+ \\ \frac{v+h_{\text{SM}}}{\sqrt{2}} + i\frac{G^0}{\sqrt{2}} \end{pmatrix}, \quad H_2 = \begin{pmatrix} \mathcal{H}^+ \\ \mathcal{H}_2^0 \end{pmatrix}, \quad \Phi_H = \begin{pmatrix} G_H^p \\ \frac{v_\Phi + \phi_H}{\sqrt{2}} + i\frac{G_H^0}{\sqrt{2}} \end{pmatrix} \tag{2.2}$$

where v and v_Φ are the only non-vanishing vacuum expectation values (VEVs) in H_1 and Φ_H fields respectively. H_2 does not develop VEV as in the case of I2HDM.

Theoretical constraints like bounded from below and perturbative unitarity of the above scalar potential can be found in our previous works [12, 13].

2.2 Interaction Lagrangian

Besides the unitary Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix

$$V_{\text{PMNS}} \equiv \left(U_\nu^L \right)^\dagger U_l^L, \tag{2.3}$$

defined in the left-handed lepton sector, we also need to introduce the following unitary mixing matrices in the right-handed lepton sector in G2HDM,

$$\begin{aligned}
 V_l^H & \equiv \left(U_l^R \right)^\dagger U_{l^H}^R, \\
 V_\nu^H & \equiv \left(U_\nu^R \right)^\dagger U_{\nu^H}^R.
 \end{aligned} \tag{2.4}$$

There are altogether 6 one-loop contributions to the $l_i - l_j - \gamma$ vertex in the minimal G2HDM. The Feynman diagrams are shown in figures 1 and 2. Here the self-energy diagrams are not explicitly shown. However they contribute to the γ^μ and $\gamma^\mu \gamma_5$ form factors in the amplitude which are important for the cancellation of ultraviolet divergences and the maintenance of gauge invariance. Figure 1 is the SM-like contributions with all h -parity even particles circulating inside the loop, while figure 2 is the new contributions from G2HDM with all h -parity odd particles circulating inside the loop. The QED vertex for a photon couples with W^\pm , l_i , l_i^H and \mathcal{H}^\pm are standard, they can be found in many textbooks and will be omitted in what follows.

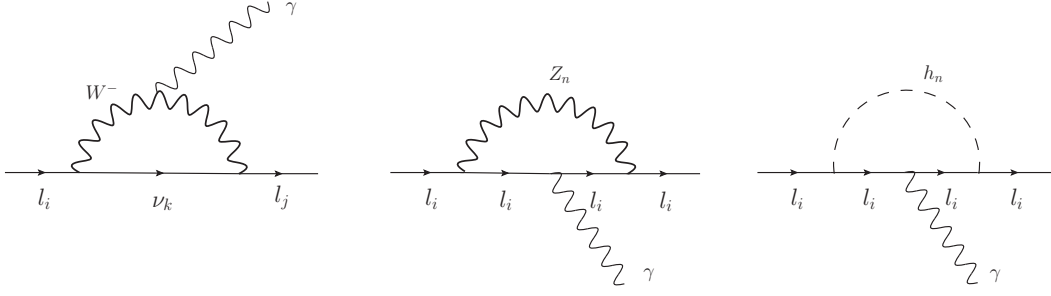


Figure 1. The one-loop SM-like contribution to $l_i - l_j - \gamma$ vertex from the SM W boson diagram (left panel), and contributions to $l_i - l_i - \gamma$ vertex from $\{Z_n\}$ diagram (center panel) and $\{h_n\}$ diagram (right panel) in G2HDM.

The first diagram in figure 1 is the contribution from the SM charged W^\pm boson. The relevant interaction Lagrangian is

$$\mathcal{L}^W \supset \frac{g}{2\sqrt{2}} \sum_{i,k} (V_{\text{PMNS}})_{ki} \bar{\nu}_k \gamma^\mu (1 - \gamma_5) l_i W_\mu^+ + \text{H.c.} \quad (2.5)$$

The second diagram in figure 1 is the contribution from the neutral gauge bosons $\{Z_n\}$. The relevant interaction Lagrangian is

$$\mathcal{L}^{\{Z_n\}} \supset \sum_n \sum_i \bar{l}_i \gamma_\mu (C_{Vn} + C_{An} \gamma_5) l_i Z_n^\mu, \quad (2.6)$$

where C_{Vn} and C_{An} are the vector and axial-vector coupling constants. Based on lepton universality, these couplings are independent of the charged lepton flavor i . Their expressions are given by $C_{Vn} = (C_{Ln} + C_{Rn})/2$ and $C_{An} = (-C_{Ln} + C_{Rn})/2$ with

$$C_{Ln} = \frac{g}{\cos \theta_W} \left(-\frac{1}{2} + \sin^2 \theta_W \right) O_{1n}^N, \quad (2.7)$$

$$C_{Rn} = \frac{g}{\cos \theta_W} \sin^2 \theta_W O_{1n}^N - \frac{1}{2} g_H O_{2n}^N - \frac{1}{2} g_X O_{3n}^N, \quad (2.8)$$

where θ_W is the weak mixing angle, g_X and g_H are gauge couplings of the $U(1)_X$ and $SU(2)_H$, respectively. O^N is a 3×3 orthogonal matrix that diagonalizes the following mass matrix in the basis of $(Z^{\text{SM}}, \mathcal{W}'_3, X)$

$$\mathcal{M}_Z^2 = \begin{pmatrix} m_Z^2 & -\frac{1}{2} g_H v m_Z & -\frac{1}{2} g_X v m_Z \\ -\frac{1}{2} g_H v m_Z & m_{\mathcal{W}'}^2 & \frac{1}{4} g_H g_X v_-^2 \\ -\frac{1}{2} g_X v m_Z & \frac{1}{4} g_H g_X v_-^2 & \frac{1}{4} g_X^2 v_+^2 + M_X^2 \end{pmatrix}, \quad (2.9)$$

where

$$m_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}, \quad (2.10)$$

$$m_{\mathcal{W}'} = \frac{1}{2} g_H \sqrt{v^2 + v_\Phi^2}, \quad (2.11)$$

$$v_\pm^2 = \left(v^2 \pm v_\Phi^2 \right), \quad (2.12)$$

and M_X is the Stueckelberg mass for the $U(1)_X$. We denote the physical mass eigenstates as Z_n ($n = 1, 2, 3$) with the mass ordering $M_{Z_1} \geq M_{Z_2} \geq M_{Z_3}$. In the parameter space choice in our numerical work, Z_1 will be identified as the Z boson of 91.1876 GeV [36] observed at LEP, Z_2 is the dark Z' and Z_3 is the dark photon γ' (or A' in some literature). They all have even h -parity.

The third diagram in figure 1 is the contribution from the neutral Higgs bosons $\{h_n\}$. The relevant interaction Lagrangian is

$$\mathcal{L}^{\{h_n\}} \supset - \sum_n \sum_i (O^H)_{1n} \frac{m_i}{v} \bar{l}_i l_i h_n, \quad (2.13)$$

where O^H is the mixing matrix between h_{SM} and ϕ_H ,

$$\begin{pmatrix} h_{\text{SM}} \\ \phi_H \end{pmatrix} = O^H \cdot \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_1 & \sin \theta_1 \\ -\sin \theta_1 & \cos \theta_1 \end{pmatrix} \cdot \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}. \quad (2.14)$$

The mixing angle θ_1 is given by

$$\tan 2\theta_1 = \frac{\lambda_{H\Phi} v v_\Phi}{\lambda_\Phi v_\Phi^2 - \lambda_H v^2}. \quad (2.15)$$

The masses of h_1 and h_2 are given by

$$m_{h_1, h_2}^2 = \lambda_H v^2 + \lambda_\Phi v_\Phi^2 \mp \sqrt{\lambda_H^2 v^4 + \lambda_\Phi^2 v_\Phi^4 + (\lambda_{H\Phi}^2 - 2\lambda_H \lambda_\Phi) v^2 v_\Phi^2}. \quad (2.16)$$

Depending on its mass, either h_1 or h_2 is identified as the observed Higgs boson h at the Large Hadron Collider (LHC). Currently the most precise measurement of the Higgs boson mass is $m_h = 125.38 \pm 0.14$ GeV [37]. In this work, we will identify the lighter state h_1 as h .

Since the gauge and Yukawa couplings in (2.6) and (2.13) respectively are all real and flavor diagonal, there are no contributions to $l_i \rightarrow l_j \gamma$ ($i \neq j$) and electric dipole moment of l_i from the interactions $\mathcal{L}^{\{Z_n\}}$ and $\mathcal{L}^{\{h_n\}}$. The only non-vanishing contribution to $l_i \rightarrow l_j \gamma$ ($i \neq j$) in SM at one-loop is the charged W^\pm from \mathcal{L}^W in (2.5). However it is well known that its amplitude is suppressed by the squared of neutrino masses due to GIM-like mechanism in the lepton sector. Furthermore, due to the unitarity of V_{PMNS} , $d_{l_i}(W^\pm)$ also vanishes at one-loop. See appendix A.

Next we turn to the new contributions in G2HDM.

The first diagram in figure 2 is the contribution from the dark Higgs \mathcal{D} which is a linear combination of two odd h -parity components \mathcal{H}_2^0 and G_H^m ,

$$\mathcal{D} = \cos \theta_2 \mathcal{H}_2^0 + \sin \theta_2 G_H^m, \quad (2.17)$$

where θ_2 is a mixing angle giving by

$$\tan 2\theta_2 = \frac{2v v_\Phi}{v_\Phi^2 - v^2}. \quad (2.18)$$

The mass of \mathcal{D} is

$$m_{\mathcal{D}}^2 = \frac{1}{2} \lambda'_{H\Phi} v_+^2, \quad (2.19)$$

where v_+^2 is defined in (2.12).

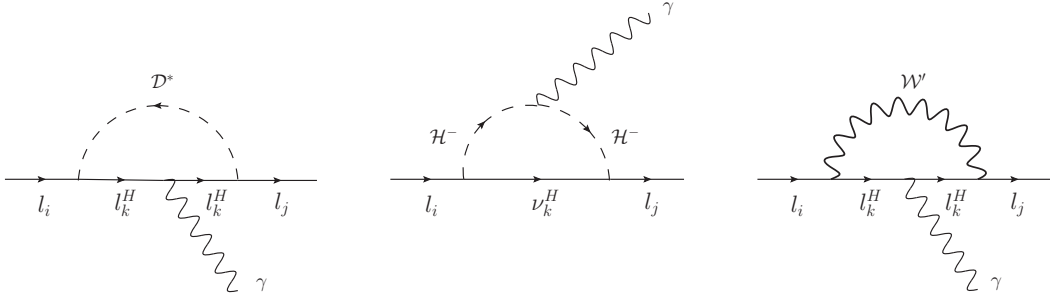


Figure 2. Three new contributions of \mathcal{D} , \mathcal{H}^+ and \mathcal{W}' to $l_i - l_j - \gamma$ vertex in G2HDM.

The relevant interaction Lagrangian is given by

$$\mathcal{L}^{\mathcal{D}} \supset \sum_{i,j} \bar{l}_i^H \left(y_{Sij}^{\mathcal{D}} + y_{Pij}^{\mathcal{D}} \gamma_5 \right) l_j \mathcal{D}^* + \text{H.c.}, \quad (2.20)$$

where the scalar and pseudoscalar Yukawa couplings $y_{Sij}^{\mathcal{D}}$ and $y_{Pij}^{\mathcal{D}}$ are given by

$$y_{S/Pij}^{\mathcal{D}} = \pm \frac{\sqrt{2}}{2v} \cos \theta_2 \left(V_l^{H\dagger} M_l \right)_{ij} + \frac{\sqrt{2}}{2v_\Phi} \sin \theta_2 \left(M_{lH} V_l^{H\dagger} \right)_{ij}, \quad (2.21)$$

with $M_l = \text{diag}(m_e, m_\mu, m_\tau)$ and $M_{lH} = \text{diag}(m_{l_1^H}, m_{l_2^H}, m_{l_3^H})$. Note that the ordering of the mass matrices are important in the Yukawa couplings (2.21). From (2.21), one obtains

$$y_{Pki}^{\mathcal{D}*} y_{Ski}^{\mathcal{D}} = \frac{1}{2} \left| (V_l^H)_{ik} \right|^2 \left(\frac{m_{l_k^H}^2}{v_\Phi^2} \sin^2 \theta_2 - \frac{m_{l_i}^2}{v^2} \cos^2 \theta_2 \right). \quad (2.22)$$

Thus $\text{Im} \left(y_{Pki}^{\mathcal{D}*} y_{Ski}^{\mathcal{D}} \right) = 0$. We don't expect the complex Yukawa couplings in $\mathcal{L}^{\mathcal{D}}$ to give rise a non-vanishing electric dipole moment d_{l_i} at one-loop, as shown in appendix A.

The second diagram in figure 2 is the contribution from the dark charged Higgs \mathcal{H}^\pm which has odd h -parity and a mass given by

$$m_{\mathcal{H}^\pm}^2 = \frac{1}{2} \left(\lambda'_{H\Phi} v_\Phi^2 - \lambda'_H v^2 \right). \quad (2.23)$$

The relevant interaction Lagrangian is given by

$$\mathcal{L}^{\mathcal{H}} \supset \sum_{i,j} \bar{\nu}_i^H \left(y_{Sij}^{\mathcal{H}} + y_{Pij}^{\mathcal{H}} \gamma_5 \right) l_j \mathcal{H}^+ + \text{H.c.}, \quad (2.24)$$

where the scalar and pseudoscalar Yukawa couplings $y_{Sij}^{\mathcal{H}}$ and $y_{Pij}^{\mathcal{H}}$ are given by

$$y_{S/Pij}^{\mathcal{H}} = \pm \frac{\sqrt{2}}{2v} \left(V_\nu^{H\dagger} M_\nu V_{\text{PMNS}} \right)_{ij}, \quad (2.25)$$

with $M_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$. Since the Yukawa couplings $y_{Sij}^{\mathcal{H}}$ and $y_{Pij}^{\mathcal{H}}$ are related, we expect $d_{l_i}(\mathcal{H}^\pm) = 0$ at one-loop. See appendix A for detail.

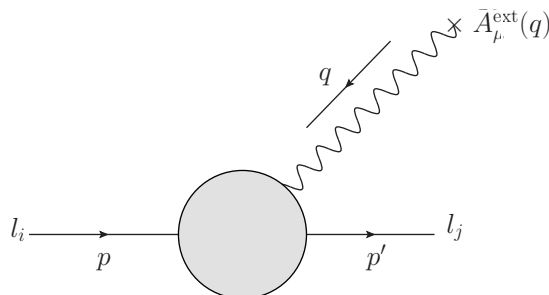


Figure 3. Feynman Diagram for the $l_i - l_j - \gamma$ vertex.

The third diagram in figure 2 is the contribution from the vector dark matter $\mathcal{W}'^{(p,m)}$ ($\equiv (\mathcal{W}'_1 \mp i\mathcal{W}'_2)/\sqrt{2}$) which is assumed to be the lightest h -parity odd particle in G2HDM. Its mass is given by (2.11). The relevant interaction Lagrangian is given by

$$\mathcal{L}^{\mathcal{W}'} \supset \sum_{i,j} \bar{l}_i^H \gamma^\mu \left(g_{V'ij}^{\mathcal{W}'} + g_{A'ij}^{\mathcal{W}'} \gamma_5 \right) l_j \mathcal{W}'_\mu + \text{H.c.}, \quad (2.26)$$

where the vector and axial gauge couplings $g_{V'ij}^{\mathcal{W}'}$ and $g_{A'ij}^{\mathcal{W}'}$ are given by

$$g_{V'ij}^{\mathcal{W}'} = g_{A'ij}^{\mathcal{W}'} = \frac{g_H}{2\sqrt{2}} \left(V_l^H \right)_{ij}^\dagger. \quad (2.27)$$

Since the vector and axial vector couplings $g_{V'ij}^{\mathcal{W}'}$ and $g_{A'ij}^{\mathcal{W}'}$ are the same, we expect $d_{l_i}(\mathcal{W}') = 0$ at one-loop. (See appendix A.) This is analogous to the SM charged W^\pm case where the vector and axial vector couplings are opposite sign to each other, there as is well-known we have $d_{l_i}(W^\pm) = 0$ at one-loop too.

In summary, we expect all the new flavor non-diagonal complex couplings from $\mathcal{L}^{\mathcal{D}}$, $\mathcal{L}^{\mathcal{H}}$ and $\mathcal{L}^{\mathcal{W}'}$ in G2HDM can give rise to contributions to $l_i \rightarrow l_j \gamma$ ($i \neq j$). Certainly they will all give non-vanishing contributions to a_{l_i} but not d_{l_i} at one-loop. The relevant Feynman rules are given in appendix C.

3 Magnetic and electric dipole form factors

The Lorentz invariant amplitude for a charged lepton l_i of flavor i scatters with an electromagnetic background field $\tilde{\mathcal{A}}_\mu^{\text{ext}}(q)$ to become another charged lepton l_j of flavor j as depicted in figure 3 is given by¹

$$i\mathcal{M}_{ji} = \bar{u}_j(p') \left(-ie\Gamma_{ji}^\mu \right) u_i(p) \tilde{\mathcal{A}}_\mu^{\text{ext}}(q), \quad (3.1)$$

with $-e$ ($e > 0$) and m_i are the electric charge and mass of l_i respectively, $q = (p' - p)$ is the momentum transfer, and the vertex function Γ_{ji}^μ can be decomposed as

$$\Gamma_{ji}^\mu = i\sigma^{\mu\nu} q_\nu \frac{m_i}{2} \left(A_{ji}^M + iA_{ji}^E \gamma_5 \right), \quad (3.2)$$

¹In general, the amplitude has six Lorentz decomposition form factors, however, due to the gauge invariance, only $\sigma^{\mu\nu} q_\nu$ and $\sigma^{\mu\nu} q_\nu \gamma_5$ terms are retained for on-shell photon where $q^2 = 0$ (see appendix D for a detailed discussion).

with A_{ji}^M and A_{ji}^E related to the transition magnetic and electric dipole form factors respectively.²

The effective Lagrangian that can reproduce the matrix element (3.1) with the associated vertex (3.2) is

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} e m_i \bar{l}_j \sigma^{\mu\nu} \left(A_{ji}^M + i\gamma_5 A_{ji}^E \right) l_i F_{\mu\nu}^{\text{ext}}, \quad (3.3)$$

where $F_{\mu\nu}^{\text{ext}}$ is the electromagnetic background field strength.

The above form factors A_{ji}^M and A_{ji}^E enable us to compute the decay rate for the process $l_i \rightarrow l_j \gamma$ ($i \neq j$) with the following spin-averaged matrix element squared

$$\overline{\sum} |\mathcal{M}_{ji}|^2 = \frac{e^2}{2} m_i^6 \left(1 - \frac{m_j^2}{m_i^2} \right)^2 \left(|A_{ji}^M|^2 + |A_{ji}^E|^2 \right). \quad (3.4)$$

We thus obtain the decay rate and branching ratio for $l_i \rightarrow l_j \gamma$

$$\Gamma(l_i \rightarrow l_j \gamma) = \frac{1}{32\pi} m_i^5 \left(1 - \frac{m_j^2}{m_i^2} \right)^3 e^2 \left(|A_{ji}^M|^2 + |A_{ji}^E|^2 \right), \quad (3.5)$$

$$\mathcal{B}(l_i \rightarrow l_j \gamma) = \frac{\Gamma(l_i \rightarrow l_j \gamma)}{\Gamma(l_i \rightarrow l_j \nu_i \bar{\nu}_j)} \cdot \frac{\Gamma(l_i \rightarrow l_j \nu_i \bar{\nu}_j)}{\Gamma_{l_i}}, \quad (3.6)$$

where³

$$\Gamma(l_i \rightarrow l_j \nu_i \bar{\nu}_j) = \frac{G_F^2 m_i^5}{192\pi^3} f\left(\frac{m_j}{m_i}\right), \quad (3.7)$$

with G_F is the Fermi constant and

$$f(x) = \left(1 - 8x^2 + 8x^6 - x^8 - 24x^4 \log x \right). \quad (3.8)$$

Therefore

$$\mathcal{B}(l_i \rightarrow l_j \gamma) = \frac{3(4\pi)^3 \alpha_{\text{EM}}}{2G_F^2} \cdot \frac{\left(1 - \frac{m_j^2}{m_i^2} \right)^3}{f\left(\frac{m_j}{m_i}\right)} \cdot \left(|A_{ji}^M|^2 + |A_{ji}^E|^2 \right) \cdot \mathcal{B}(l_i \rightarrow l_j \nu_i \bar{\nu}_j), \quad (3.9)$$

where $\alpha_{\text{EM}} = e^2/(4\pi)$. For $\mu \rightarrow e \bar{\nu}_e \nu_\mu$,

$$\mathcal{B}(\mu \rightarrow e \bar{\nu}_e \nu_\mu) \approx 100\%. \quad (3.10)$$

For $l_i \rightarrow l_j \gamma$ ($i \neq j$) in G2HDM there are 4 distinct non-vanishing contributions to each A_{ji}^M and A_{ji}^E ,

$$A_{ji}^{M/E} = A_{ji}^{M/E}(W) + A_{ji}^{M/E}(\mathcal{D}) + A_{ji}^{M/E}(\mathcal{H}) + A_{ji}^{M/E}(W'). \quad (3.11)$$

As is well known the SM contribution $A_{ji}^{M/E}(W)$ for $i \neq j$ from the W boson loop is vanishingly small and many orders below the current experimental sensitivities.

²For ease of comparisons of their analytical expressions presented in the appendices, we use the same notations A_{ji}^M and A_{ji}^E as in [8]. And they are understood to be evaluated at $q^2 = 0$.

³See for example the appendix in the textbook *Collider Physics*, Updated Edition, CRC Press 1996, by Barger and Phillips.

The anomalous magnetic dipole moment a_{l_i} of the charged lepton l_i can be identified as the coefficient of $(e/2m_i)i\sigma^{\mu\nu}q_\nu$ in the vertex $e\Gamma_{ii}^\mu$ of (3.2), i.e.

$$a_{l_i} = m_i^2 A_{ii}^M, \quad (\text{no sum on } i). \quad (3.12)$$

The electric dipole moment d_{l_i} of the charged lepton l_i is given by

$$\frac{d_{l_i}}{e} = \frac{m_i}{2} A_{ii}^E, \quad (\text{no sum on } i). \quad (3.13)$$

For the anomalous magnetic dipole moment of l_i in G2HDM, besides the well-known QED contribution $a_{l_i}(\gamma) = \alpha_{\text{EM}}/2\pi$, there are in general 6 distinct electroweak contributions to the a_{l_i} ,

$$a_{l_i} = \frac{\alpha_{\text{EM}}}{2\pi} + m_i^2 A_{ii}^M, \quad (\text{no sum on } i),$$

$$A_{ii}^M = A_{ii}^M(W) + A_{ii}^M(\{Z_n\}) + A_{ii}^M(\{h_n\}) + A_{ii}^M(\mathcal{D}) + A_{ii}^M(\mathcal{H}) + A_{ii}^M(\mathcal{W}'). \quad (3.14)$$

Analytical one-loop expressions for $A_{ji}^{M/E}$ are given in appendix A. There one will see all the A_{ii}^E s vanish at one-loop in G2HDM, hence the electric dipole moment d_{l_i} of l_i vanish too according to (3.13). These form factors $A_{ji}^{M/E}$ were also computed for general couplings in [8]. Aside from an overall factor of 2 in the form factors, we will discuss some minor discrepancies in the loop integrals between our results and [8] in appendix A.

4 Numerical results

In this section, we show numerical results for the cLFV process $\mu \rightarrow e\gamma$ and muon anomalous magnetic dipole moment with the parameter space in the model chosen to satisfy the current constraints for a sub-GeV non-abelian vector DM \mathcal{W}' . In particular, the scan data are adapted from ref. [14] in which the theoretical constraints on the scalar potential [12, 13], signal strength measurements from the LHC [38–40], dark photon physics [41, 42], electroweak precision measurements [36] including the recent W boson mass measurement at the CDF II [43] and constraints from DM searches including the DM relic density measured from Planck collaboration [44], DM direct detections [45–47] and Higgs invisible decays constraint from the LHC [48]. For the data points that satisfy the above mentioned constraints, the total DM annihilation cross section is of order $10^{-32} \text{ cm}^3 \cdot s^{-1}$ or below that is much lower than the current DM indirect detection constraints.

We set the mixing matrices in the right-handed lepton sector in the model to be

$$V_l^H = V_\nu^H = V_{\text{PMNS}}, \quad (4.1)$$

where V_{PMNS} is parameterized as

$$V_{\text{PMNS}} \equiv \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\text{CP}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\text{CP}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\text{CP}}} & c_{23}c_{13} \end{pmatrix}, \quad (4.2)$$

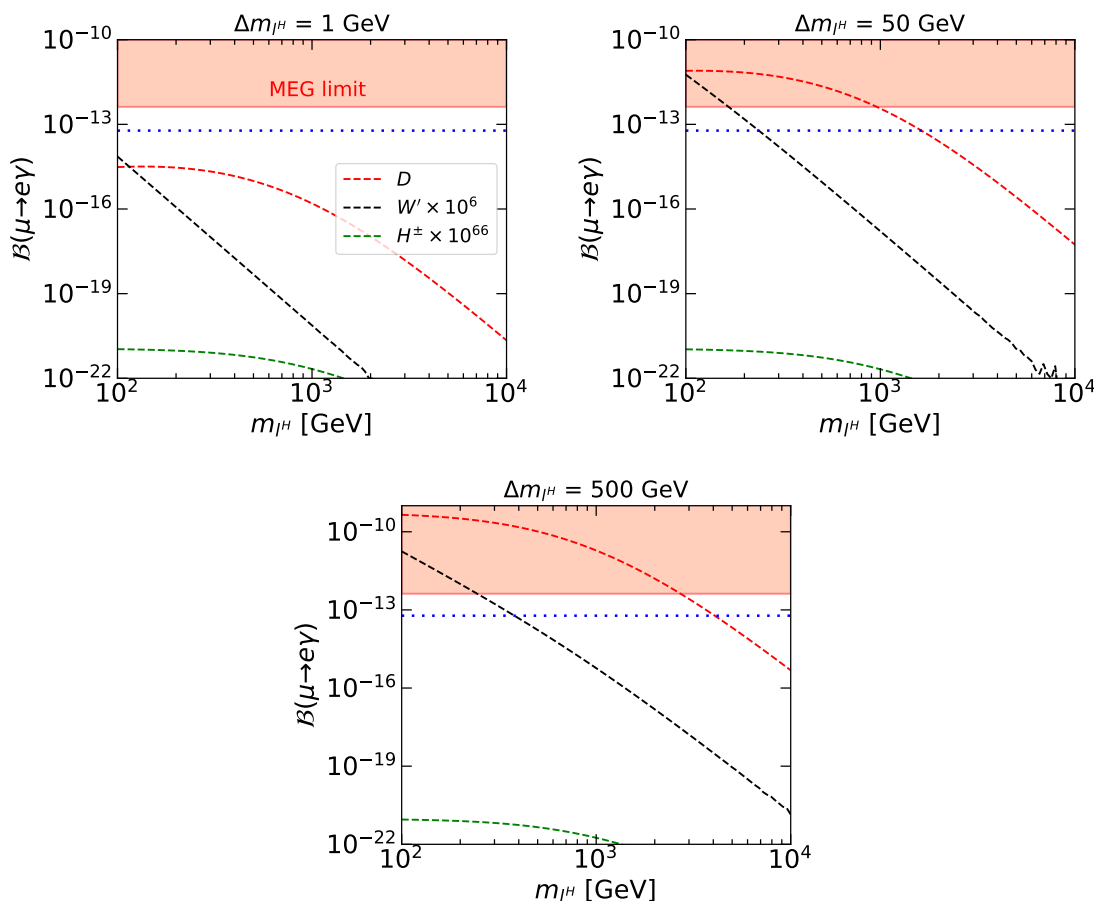


Figure 4. Branching ratio of $\mu \rightarrow e\gamma$ as a function of the heavy hidden lepton mass m_{lH} . Other parameters in the model are set to be $m_{h_2} = 292.50$ GeV, $m_{\mathcal{D}} = 766.07$ GeV, $m_{\mathcal{H}^\pm} = 848.13$ GeV, $m_{\mathcal{W}'}$ = 1.0 GeV, $\theta_1 = 0.030$ rad, $\theta_2 = 0.056$ rad, $M_X = 1.96$ GeV and $g_X = 2.5 \times 10^{-4}$. From the left to right and top to bottom panels, the mass splitting between the heavy hidden lepton generations is set to be $\Delta m_{lH} = 1$ GeV, 50 GeV and 500 GeV, respectively. The dashed red, black and green lines represent the contributions from \mathcal{D} boson, \mathcal{W}' boson multiplied by 10^6 and \mathcal{H}^\pm boson multiplied by 10^{66} respectively. The orange region is the excluded region at 90% C.L. from MEG collaboration [4] and the dotted blue line indicates the future sensitivity from MEG II [5].

where s_{ij} and c_{ij} stand for $\sin \theta_{ij}$ and $\cos \theta_{ij}$ respectively, and δ_{CP} is a Dirac CP violating phase. The current best-fit values using a normal ordering are given by [49]: $\theta_{12} = 33.44^{+0.77^\circ}_{-0.74^\circ}$, $\theta_{23} = 49.2^{+1.0^\circ}_{-1.3^\circ}$, $\theta_{13} = 8.57^{+0.13^\circ}_{-0.12^\circ}$, and $\delta_{CP} = 194^{+52^\circ}_{-25^\circ}$. We also set the heavy hidden lepton masses to be

$$M_{\nu H} = M_{lH} = \text{diag}(m_{lH}, m_{lH} + \Delta m_{lH}, m_{lH} + \Delta m_{lH}) \quad (4.3)$$

where the second and third generations are assumed degenerate, and Δm_{lH} is a mass splitting between the first and second (third) generations.

Figure 4 shows the branching ratio of $\mu \rightarrow e\gamma$ as a function of the heavy hidden lepton mass m_{lH} . Here we fixed other parameters in the model to be $m_{h_2} = 292.50$ GeV, $m_{\mathcal{D}} = 766.07$ GeV, $m_{\mathcal{H}^\pm} = 848.13$ GeV, $m_{\mathcal{W}'}$ = 1.0 GeV, $\theta_1 = 0.030$ rad, $\theta_2 = 0.056$ rad,

$M_X = 1.96 \text{ GeV}$ and $g_X = 2.5 \times 10^{-4}$. We note that this benchmark point satisfies all current constraints mentioned above. The mass splitting Δm_{lH} is fixed to be 1 GeV, 50 GeV and 500 GeV as respectively shown from the left to right and top to bottom panels in figure 4. From figure 4 one can see that the contribution from the \mathcal{D} boson diagram to the branching ratio of $\mu \rightarrow e\gamma$ is dominant. The SM W boson contribution is suppressed by the sums over of $(\Delta m_{i1}/m_W)^4$ with $i = 2, 3$ and Δm_{i1}^2 is the mass difference between the neutrino generations. Using global fit values for Δm_{i1}^2 from [49], one can obtain $\mathcal{B}(\mu \rightarrow e\gamma)_W \simeq 4.4 \times 10^{-55}$. The contribution from \mathcal{H}^\pm is similarly suppressed by the mass of neutrinos, whereas the contribution from \mathcal{W}' is negligible due to the smallness of the gauge coupling g_H that is $g_H \simeq 4.58 \times 10^{-4}$ for this benchmark point. For a fixed value of the mass splitting Δm_{lH} , the total branching ratio of $\mu \rightarrow e\gamma$ decreases when m_{lH} increases. When Δm_{lH} increases, the branching ratio of $\mu \rightarrow e\gamma$ from \mathcal{D} and \mathcal{W}' bosons increase, while the contribution from charged Higgs is almost unchanged. For large values of Δm_{lH} , the current limit from the MEG experiment can put a lower bound on m_{lH} . A larger mass splitting Δm_{lH} requires a larger m_{lH} . In particular, as shown on the top-right and bottom panel in figure 4, the heavy hidden lepton mass m_{lH} is required $\gtrsim 1$ and 3 TeV for fixing Δm_{lH} at 50 and 500 GeV, respectively. For small values of Δm_{lH} , the branching ratio of $\mu \rightarrow e\gamma$ is suppressed and thus escaping the MEG constraint (see the top-left panel in figure 4). We note that for the degenerate mass case, i.e. $\Delta m_{lH} = 0$, the contributions from new particles to the branching ratio of $\mu \rightarrow e\gamma$ vanishes. This is because, in this case, the form factors from \mathcal{D} , \mathcal{W}' and \mathcal{H}^\pm are proportional to the following factors⁴

$$A_{e\mu}^{M/E}(\mathcal{D} \text{ or } \mathcal{W}') \Big|_{\text{degenerate } l^H} \sim \sum_{k=1}^3 (V_{l^H})_{2k}^* (V_{l^H})_{1k} = 0, \quad (4.4)$$

$$A_{e\mu}^{M/E}(\mathcal{H}^\pm) \Big|_{\text{degenerate } \nu^H} \sim \sum_{k=1}^3 (V_{\nu^H})_{2k}^* (V_{\nu^H})_{1k} = 0. \quad (4.5)$$

Figure 5 shows 2σ favored parameter space for \mathcal{W}' as a sub-GeV M candidate in the model. The data points are projected on $(m_{\mathcal{W}'}, g_H)$ plane (left panel) and $(m_{\mathcal{D}}, |\sin \theta_2|)$ plane (right panel). The colors of circle points in the left and right panels of figure 5 indicate the values of $\mathcal{B}(\mu \rightarrow e\gamma)$ calculated from \mathcal{W}' diagram and \mathcal{D} diagram, respectively. Here we fixed $m_{lH} = 1 \text{ TeV}$ and $\Delta m_{lH} = 50 \text{ GeV}$. The contribution from the \mathcal{W}' diagram to $\mathcal{B}(\mu \rightarrow e\gamma)$ is linearly proportional to the gauge coupling g_H^2 . However due to the constraints from the dark matter direct detection and dark photon physics that required $g_H \lesssim 10^{-3}$ [12–14], the $\mathcal{B}(\mu \rightarrow e\gamma)$ from the \mathcal{W}' diagram is suppressed. In particular, $\mathcal{B}(\mu \rightarrow e\gamma)_{\mathcal{W}'} \lesssim 10^{-21}$ as shown in the left panel of figure 5. On the other hand, the contribution to $\mathcal{B}(\mu \rightarrow e\gamma)$ from the \mathcal{D} diagram is significant. The branching ratio is enhanced in the region of large mixing angle θ_2 and heavy mass region of \mathcal{D} boson. The current experimental data requires $0.018 \lesssim |\sin \theta_2| \lesssim 0.21$ for the 2σ favored region [14], which results $10^{-17} \lesssim \mathcal{B}(\mu \rightarrow e\gamma) \lesssim 10^{-10}$. We note that the $\mathcal{B}(\mu \rightarrow e\gamma)$ from the \mathcal{D} boson diagram peaks at a certain value of $m_{\mathcal{D}}$ depending on the mass of heavy hidden leptons.

⁴In general, we have $A_{ji}^{M/E}(\mathcal{D} \text{ or } \mathcal{W}' \text{ or } \mathcal{H}^\pm) \Big|_{\text{degenerate } f^H} = 0$ for $i \neq j$, which is just the manifestation of the well-known GIM-mechanism in SM.

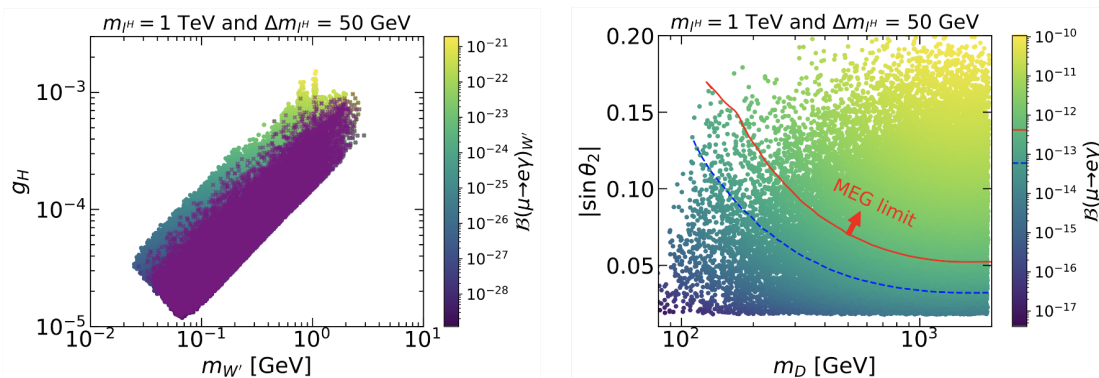


Figure 5. Viable DM parameter points spanned in $(m_{W'}, g_H)$ plane (left) and $(m_{\mathcal{D}}, |\sin \theta_2|)$ plane (right). The color of circle points indicates the contribution to $\mathcal{B}(\mu \rightarrow e\gamma)$ from the \mathcal{W}' diagram (left panel) and \mathcal{D} diagram (right panel). Here we fixed $m_{l_H} = 1$ TeV and $\Delta m_{l_H} = 50$ GeV. The solid red and dashed blue lines on the right panel are the MEG limit and future sensitivity from MEG II [5], respectively. The crossed purple points on the left panel are allowed by the current limit from MEG [4] which are obtained from the bound on $|\sin \theta_2|$ (solid red on the right panel) due to the relation given in eq. (4.6).

For $m_{l_H} = 1$ TeV and $\Delta m_{l_H} = 50$ GeV, the peak is at $m_{\mathcal{D}} \sim 1.8$ TeV. The current limit from the MEG experiment [4] can exclude a large portion of parameter space ($\sim 50\%$ of data points) in this enhanced region as shown by the red line on the right panel of figure 5. The future sensitivity from MEG II [5], as shown by the dashed blue line on the right panel of figure 5, can probe lower values of the mixing angle θ_2 and a smaller region of \mathcal{D} boson mass in the model. We note that the upper bound on $|\sin \theta_2|$ from the MEG experiment can be translated into a bound on the DM mass $m_{W'}$ and the gauge coupling g_H due to the following relation

$$g_H = \frac{2m_{W'}}{v} \times \begin{cases} |\sin \theta_2|, & \text{for } \theta_2 > 0, \\ |\cos \theta_2|, & \text{for } \theta_2 \leq 0, \end{cases} \quad (4.6)$$

which can be derived from eqs. (2.11) and (2.18). The allowed points after taking into account the MEG constraint projected on $(m_{W'}, g_H)$ plane is shown as the crossed purple points in the left panel of figure 5.

In figure 6, we show the 2σ favored parameter space on the plane of the total branching ratio of $\mu \rightarrow e\gamma$ and muon anomalous magnetic dipole moment Δa_μ . As mentioned above, the main contribution to the $\mathcal{B}(\mu \rightarrow e\gamma)$ is from the \mathcal{D} boson diagram and a large portion of parameter space can be excluded by the current MEG experiment. On the other hand, the main contributions to Δa_μ in the model are from \mathcal{D} and $Z_{2,3}$ diagrams. The contribution from the \mathcal{W}' diagram gives a negative value for Δa_μ , whereas the neutral Higgs and charged Higgs contributions are both suppressed for the current viable parameter space in the model. One can see in figure 6, the total contribution to Δa_μ is not reaching the 2σ region (shaded light blue) for the muon anomalous magnetic dipole moment measured at BNL [15–17] and FNAL [18]. We expect the Δa_μ can be enhanced in higher loop diagrams such as the two-loop Barr-Zee mechanism [50]. Calculation of these two-loop Barr-Zee contributions is thus highly desirable but nevertheless beyond the scope of this study. We hope to return to this issue in the future.

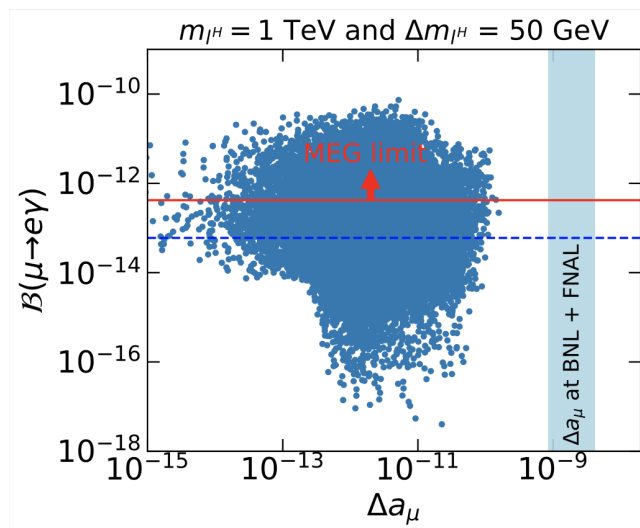


Figure 6. Viable DM parameter points spanned in the plane of the total branching ratio of $\mu \rightarrow e\gamma$ and muon anomalous magnetic dipole moment Δa_μ . Here we fixed $m_{lH} = 1$ TeV and $\Delta m_{lH} = 50$ GeV. The solid red and dashed blue lines are the current limit from MEG [4] and future sensitivity from MEG II [5], respectively. The shaded light blue band represents the 2σ region of Δa_μ measured at BNL [15–17] and FNAL [18].

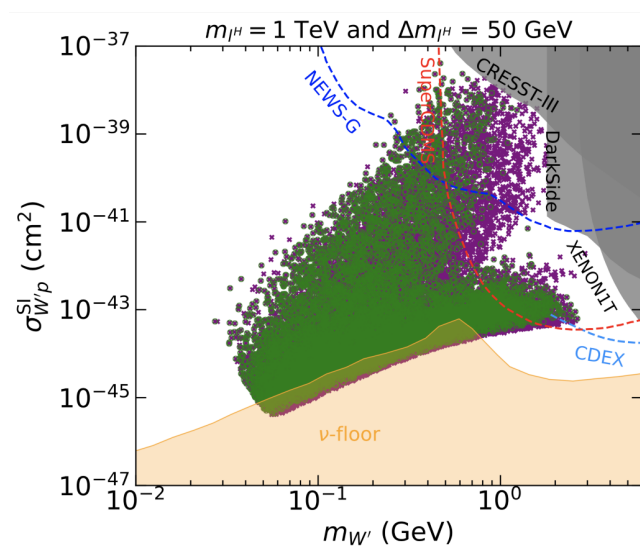


Figure 7. Favored data projected on the plane of the DM mass and spin independent DM-proton scattering cross section. Here we fixed $m_{lH} = 1$ TeV and $\Delta m_{lH} = 50$ GeV. The crossed purple points indicate the data satisfied the MEG constraint [4], while the circle green points indicate the data that can be probed by future experiment from MEG II [5]. The gray regions are the exclusion from CRESST-III [45], DarkSide-50 [46] and XENON1T [47] experiments. The dashed blue, red and light blue lines represent the future sensitivities from DM direct detection experiments at NEWS-G [51], SuperCDMS [52] and CDEX [53], respectively. Orange region is the neutrino floor background.

Figure 7 shows the allowed data points by the current MEG constraints (crossed purple) and the points that can be probed by future experiment from MEG II (circle green) on the DM direct detection plane. The predicted DM mass range in 2σ favored region is $\sim (0.02 - 3)$ GeV. The future sensitivity from MEG II can probe almost the entire viable range of DM mass. Interestingly, some data points with the DM mass at around 1 GeV can be probed complementarily by various future DM direct detection experiments including NEWS-G [51], SuperCDMS [52] and CDEX [53].

5 Conclusion

We computed the one-loop radiative decay rates for the charged lepton flavor violation processes $l_i \rightarrow l_j \gamma$, the anomalous magnetic dipole moment Δa_{l_i} and the electric dipole moment d_{l_i} of the charged lepton l_i at one-loop level in a minimal G2HDM. Besides the contribution from the SM W boson, the charged lepton flavor violation processes occurred at one-loop in G2HDM due to the new interactions of SM charged leptons with two h -parity odd particles — heavy hidden leptons (l^H or ν^H) and hidden dark scalars or gauge bosons (\mathcal{D} , $\mathcal{H}^\pm, \mathcal{W}'$). The contributions from these new interactions vanish when the heavy hidden lepton masses among generations degenerate.

We analyzed the $\mu \rightarrow e \gamma$ process and Δa_μ using a parameter space that favors a sub-GeV non-abelian vector dark matter $\mathcal{W}'^{(p,m)}$ in the model. The scan data is adapted from ref. [14] and they satisfy various constraints including the theoretical constraints on the scalar potential, the Higgs signal strength measurements from the LHC, the dark photon physics, the electroweak precision measurements including the recent W boson mass measurement at the CDF II, DM relic density measured from Planck collaboration, and from DM searches including the DM direct detections and the Higgs invisible width from the LHC.

We found that the branching ratio of $\mu \rightarrow e \gamma$ is significantly dependent on the heavy hidden lepton mass and the mass splitting between its generations. In particular, a heavier hidden lepton mass results in a smaller branching ratio of $\mu \rightarrow e \gamma$ while a larger mass splitting gives a larger branching ratio as shown in figure 4.

Among the new contributions to $\mu \rightarrow e \gamma$ in the model, the contribution from \mathcal{D} boson diagram is dominant. The branching ratio can be enhanced in the heavy \mathcal{D} boson mass region and the region of large mixing angle θ_2 between two odd h -parity bosons, \mathcal{H}_2^0 and G_H^m , which compose \mathcal{D} . The current constraint on the cLFV process from MEG can exclude a significant portion of the parameter space in the favored region obtained in previous studies. Although the contribution from the DM candidate \mathcal{W}' to the branching ratio of $\mu \rightarrow e \gamma$ is suppressed due to the smallness of new gauge coupling g_H , the DM parameters can be affected indirectly by the cLFV processes due to the relation between parameters in the model, especially the relation (4.6) between the mixing angle θ_2 , new gauge coupling g_H and DM mass $m_{\mathcal{W}'}$. We found that the future measurement at MEG II can probe almost the entire viable range of the DM mass which is $\sim (0.02 - 3)$ GeV and interestingly, the region at the DM mass around 1 GeV can be also probed by future DM direct detection experiments such as NEWS-G, SuperCDMS and CDEX.

In the viable DM parameter space, the total one-loop level contribution to Δa_μ in the model is not big enough to explain the 4.2σ level discrepancy between the theoretical value and the experimental results measured at the BNL and FNAL. We expect an enhanced contribution to Δa_μ at higher loop corrections, such as the Barr-Zee two-loop mechanism [50] for the neutrino magnetic dipole moments, can be anticipated to address the muon anomaly in the model.

In appendix A, we also showed that the electric dipole moment of charged lepton vanishes at one-loop in G2HDM. This is due to (1) the lack of CP violating phases (or in general imaginary parts) in products of generic but related complex vector and axial vector gauge couplings or scalar and pseudoscalar Yukawa couplings, and (2) vanishing combinations of Feynman loop integrals. Same conclusion can be drawn for the SM quarks in the model. Thus it is also interesting to investigate if the two-loop mechanisms like the Weinberg three-gluon operator [54–56] for the gluon chromo-electric dipole moment [57] and the Barr-Zee diagrams for the charged leptons [58] can generate a non-vanishing result for the electric dipole moments for the neutron and SM leptons respectively in G2HDM. For these two-loop calculations, we will reserve them for future tasks.

Acknowledgments

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A One-loop analytical formulae of A_{ji}^M and A_{ji}^E

In this appendix, we present the one-loop expressions for the transition magnetic and electric dipole form factors A_{ji}^M and A_{ji}^E from the six different contributions in G2HDM. For the gauge particle loops of W^\pm , $\{Z_n\}$ and $\mathcal{W}^{(p,m)}$, we use unitary gauge in the computation. For the computation of W^\pm loop in the 't Hooft-Feynman gauge, see appendix B.⁵ For convenience, we define $z = 1 - x - y$ in what follows.

A.1 W contribution — left diagram in figure 1

The transition magnetic and electric dipole form factors are given by

$$A_{ji}^M(W) = +\frac{1}{8\pi^2} \left(\frac{g}{2\sqrt{2}}\right)^2 \sum_k (V_{PMNS})_{kj}^* (V_{PMNS})_{ki} \times [\mathcal{I}(m_i, m_j, m_{\nu_k}, m_W) + \mathcal{I}(m_i, m_j, -m_{\nu_k}, m_W)] , \quad (\text{A.1})$$

$$A_{ji}^E(W) = -\frac{i}{8\pi^2} \left(\frac{g}{2\sqrt{2}}\right)^2 \sum_k (V_{PMNS})_{kj}^* (V_{PMNS})_{ki} \times [\mathcal{I}(m_i, -m_j, m_{\nu_k}, m_W) + \mathcal{I}(m_i, -m_j, -m_{\nu_k}, m_W)] , \quad (\text{A.2})$$

⁵The issue of gauge fixings in the model has been studied as given in an appendix in ref. [13].

respectively where the loop integral $\mathcal{I}(m_i, m_j, m_k, m_X)$ is

$$\begin{aligned}
& \mathcal{I}(m_i, m_j, m_k, m_X) \\
&= \int_0^1 dx \int_0^{1-x} dy \left\{ \frac{1}{-xz m_i^2 - xym_j^2 + xm_k^2 + (1-x)m_X^2} \right. \\
&\quad \times \left[\left((y + 2z(1-x)) + (z + 2y(1-x)) \frac{m_j}{m_i} - 3(1-x) \frac{m_k}{m_i} \right) \right. \\
&\quad \quad \left. \left. + \frac{m_i^2}{m_X^2} x^2 \left(z^2 + y^2 \frac{m_j^3}{m_i^3} + yz \frac{m_j}{m_i} \left(1 + \frac{m_j}{m_i} \right) - \frac{m_j m_k}{m_i^2} \right) \right] \right. \\
&\quad + \frac{1}{m_X^2} \left(x(1-z) + y + (x(1-y) + z) \frac{m_j}{m_i} - \frac{m_k}{m_i} \right) \\
&\quad + \frac{1}{m_X^2} \left(2 - x(3-4z) - 3y - z + (2-x(3-4y) - y - 3z) \frac{m_j}{m_i} \right) \\
&\quad \left. \times \log \left(\frac{m_X^2}{-xz m_i^2 - xym_j^2 + xm_k^2 + (1-x)m_X^2} \right) \right\}. \tag{A.3}
\end{aligned}$$

We note that this integral \mathcal{I} is for the diagram with two internal charged vector bosons X coupled to the external photon computed using the unitary gauge. The third line of eq. (A.3) comes from the product of the transverse pieces of the two vector boson propagators, while all the remaining terms are due to the product of the transverse and longitudinal pieces of these two propagators. The product of longitudinal pieces do not give rise to the contributions for the transition magnetic and electric dipole form factors. Our integral \mathcal{I} is denoted as $I_{k,3}^{++}$ in eq. (A.5) of [8]. Except for the fourth line of eq. (A.3), our formula agrees.⁶ The difference between our result of eq. (A.3) and eq. (A.5) of [8] is

$$\text{Diff} = \frac{(m_i - m_j)^2}{m_X^2} \int_0^1 dx \int_0^{1-x} dy \frac{xyz \left(x + y + (1-y) \frac{m_j}{m_i} - \frac{m_k}{m_i} \right)}{-xz m_i^2 - xym_j^2 + xm_k^2 + (1-x)m_X^2}. \tag{A.4}$$

Since the difference disappears in the case of $m_i = m_j$, one can't use the known result of charged lepton anomaly [59] to discriminate the two answers. However, see appendix B.

For $i = j$, there is no CP violating phase arise from the product $|(V_{PMNS})_{ki}|^2$ in (A.2), which implies the electric dipole moment d_i should be vanishing from the one-loop W diagram in SM. Indeed the sum of the two integrals in (A.2) vanishes when $m_i = m_j$!

A.2 $\{Z_n\}$ contribution — middle diagram in figure 1

The transition magnetic dipole form factor is given by

$$\begin{aligned}
A_{ji}^M(\{Z_n\}) &= + \frac{\delta_{ij}}{8\pi^2} \sum_n \left[(C_{Vn})^2 \mathcal{J}(m_i, m_i, m_i, m_{Z_n}) \right. \\
&\quad \left. + (C_{An})^2 \mathcal{J}(m_i, m_i, -m_i, m_{Z_n}) \right], \tag{A.5}
\end{aligned}$$

⁶Note that there are also a couple of trivial typos in the coefficients of the log term of $I_{k,3}^{++}$ in eq. (A.5) of [8].

where

$$\begin{aligned}
& \mathcal{J}(m_i, m_j, m_k, m_X) \\
&= - \int_0^1 dx \int_0^{1-x} dy \left\{ \frac{1}{-xzm_i^2 - xym_j^2 + (1-x)m_k^2 + xm_X^2} \right. \\
&\quad \times \left[2x \left((1-z) + (1-y) \frac{m_j}{m_i} - 2 \frac{m_k}{m_i} \right) \right. \\
&\quad \quad + \frac{m_i^2}{m_X^2} \left((1-x) \left(\frac{m_j}{m_i} - \frac{m_k}{m_i} \right) \left(z + y \frac{m_j}{m_i} \right) \left(1 - \frac{m_k}{m_i} \right) \right. \\
&\quad \quad \quad - z \left(\frac{m_j}{m_i} - \frac{m_k}{m_i} \right) \left((1-x(1-z)) + xy \frac{m_j^2}{m_i^2} \right) \\
&\quad \quad \quad \left. \left. - y \left(1 - \frac{m_k}{m_i} \right) \left(xz + (1-x(1-y)) \frac{m_j^2}{m_i^2} \right) \right) \right] \\
&\quad + \frac{1}{m_X^2} \left(y + z \frac{m_j}{m_i} - (1-x) \frac{m_k}{m_i} \right) \\
&\quad + \frac{1}{m_X^2} \left((1-3y) + (1-3z) \frac{m_j}{m_i} + (1-3x) \frac{m_k}{m_i} \right) \\
&\quad \left. \times \log \left(\frac{m_X^2}{-xzm_i^2 - xym_j^2 + (1-x)m_k^2 + xm_X^2} \right) \right\}. \tag{A.6}
\end{aligned}$$

We note that this integral of \mathcal{J} is for the diagram with one internal neutral gauge boson X exchange computed using the unitary gauge. The third line of eq. (A.6) comes from the transverse piece of the vector boson propagator, while the remaining terms come entirely from the longitudinal piece of the propagator. Our loop integral $(-\mathcal{J})$ corresponds to $I_{k,4}^{++}$ in eq. (A.6) of [8].⁷ Using our expression of \mathcal{J} in eq. (A.6) for the equal mass case of $m_i = m_j = m_\mu$ and setting $m_k = m_F$, one can easily reproduce the well-known expression of muon anomaly for a neutral gauge boson X with a general gauge coupling of a muon and another fermion F first obtained in [59].

For the transition electric dipole form factor, one finds

$$A_{ji}^E(\{Z_n\}) = 0, \tag{A.7}$$

which implies $d_{l_i}(\{Z_n\}) = 0$.

A.3 $\{h_n\}$ contribution — right diagram in figure 1

The transition magnetic dipole form factor is

$$A_{ji}^M(\{h_n\}) = \frac{\delta_{ij} m_i^2}{\pi^2 v^2} \sum_n (O^H)_{1n}^2 \mathcal{K}(m_i, m_i, m_i, m_{h_n}), \tag{A.8}$$

⁷We note that in the fourth line of our eq. (A.6), instead of the factor of $(1-x)$, eq. (A.6) of [8] got $(x-1)$.

with

$$\mathcal{K}(m_i, m_j, m_k, m_X) = \int_0^1 dx \int_0^{1-x} dy \times \left[\frac{x \left(y + z \frac{m_j}{m_i} \right) + (1-x) \frac{m_k}{m_i}}{-xym_i^2 - xzm_j^2 + (1-x)m_k^2 + xm_X^2} \right]. \quad (\text{A.9})$$

This loop integral \mathcal{K} is the same as $I_{k,1}^{++}$ in eq. (A.1) of [8].

As in the $\{Z_n\}$ case, one finds that the transition electric dipole form factor vanishes

$$A_{ji}^E(\{h_n\}) = 0, \quad (\text{A.10})$$

which implies $d_{l_i}(\{h_n\}) = 0$ as well.

A.4 \mathcal{D} contribution — left diagram in figure 2

The transition magnetic and electric dipole form factors are given by

$$A_{ji}^M(\mathcal{D}) = \frac{1}{8\pi^2} \left[\sum_k y_{S kj}^{\mathcal{D}*} y_{S ki}^{\mathcal{D}} \mathcal{K}(m_i, m_j, m_{l_k^H}, m_{\mathcal{D}}) + \sum_k y_{P kj}^{\mathcal{D}*} y_{P ki}^{\mathcal{D}} \mathcal{K}(m_i, m_j, -m_{l_k^H}, m_{\mathcal{D}}) \right], \quad (\text{A.11})$$

$$A_{ji}^E(\mathcal{D}) = \frac{i}{8\pi^2} \left[\sum_k y_{P kj}^{\mathcal{D}*} y_{S ki}^{\mathcal{D}} \mathcal{K}(m_i, -m_j, m_{l_k^H}, m_{\mathcal{D}}) + \sum_k y_{S kj}^{\mathcal{D}*} y_{P ki}^{\mathcal{D}} \mathcal{K}(m_i, -m_j, -m_{l_k^H}, m_{\mathcal{D}}) \right], \quad (\text{A.12})$$

where the summation is over all the heavy hidden charged leptons l_k^H running inside the loop. \mathcal{K} is defined already in (A.9).

For $i = j$, (A.11) reduces to

$$A_{ii}^M(\mathcal{D}) = \frac{1}{8\pi^2} \left[\sum_k |y_{S ki}^{\mathcal{D}}|^2 \mathcal{K}(m_i, m_i, m_{l_k^H}, m_{\mathcal{D}}) + \sum_k |y_{P ki}^{\mathcal{D}}|^2 \mathcal{K}(m_i, m_i, -m_{l_k^H}, m_{\mathcal{D}}) \right], \quad (\text{A.13})$$

with

$$|y_{(S,P) ki}^{\mathcal{D}}|^2 = \frac{1}{2} |(V_l^H)_{ik}|^2 \left(\frac{m_{l_i}^2}{v^2} \cos^2 \theta_2 + \frac{m_{l_k^H}^2}{v_\Phi^2} \sin^2 \theta_2 \pm \frac{m_{l_i} m_{l_k^H}}{v v_\Phi} \sin 2\theta_2 \right). \quad (\text{A.14})$$

We note that the possible new CP violating phase in V_l^H is cancelled out in $|y_{(S,P) ki}^{\mathcal{D}}|^2$.

On the other hand, for $i = j$, (A.12) reduces to

$$\begin{aligned} A_{ii}^E(\mathcal{D}) &= \frac{i}{8\pi^2} \sum_k \text{Re} \left(y_{P ki}^{\mathcal{D}*} y_{S ki}^{\mathcal{D}} \right) \left[\mathcal{K}(m_i, -m_i, m_{l_k^H}, m_{\mathcal{D}}) + \mathcal{K}(m_i, -m_i, -m_{l_k^H}, m_{\mathcal{D}}) \right] \\ &\quad - \frac{1}{8\pi^2} \sum_k \text{Im} \left(y_{P ki}^{\mathcal{D}*} y_{S ki}^{\mathcal{D}} \right) \left[\mathcal{K}(m_i, -m_i, m_{l_k^H}, m_{\mathcal{D}}) - \mathcal{K}(m_i, -m_i, -m_{l_k^H}, m_{\mathcal{D}}) \right] \\ &= 0, \end{aligned} \quad (\text{A.15})$$

due to the fact that $\mathcal{K}(m_i, -m_i, m_{l_k^H}, m_{\mathcal{D}}) + \mathcal{K}(m_i, -m_i, -m_{l_k^H}, m_{\mathcal{D}}) = 0$ and from (2.22) we have $\text{Im}(y_{P_{ki}}^{\mathcal{D}*} y_{S_{ki}}^{\mathcal{D}}) = 0$.

A.5 \mathcal{H}^\pm contribution — middle diagram in figure 2

The transition magnetic and electric dipole form factors are

$$A_{ji}^M(\mathcal{H}) = +\frac{1}{8\pi^2} \sum_k y_{S_{kj}}^{\mathcal{H}*} y_{S_{ki}}^{\mathcal{H}} \left[\mathcal{L}(m_i, m_j, m_{\nu_k^H}, m_{\mathcal{H}}) + \mathcal{L}(m_i, m_j, -m_{\nu_k^H}, m_{\mathcal{H}}) \right], \quad (\text{A.16})$$

$$A_{ji}^E(\mathcal{H}) = -\frac{i}{8\pi^2} \sum_k y_{S_{kj}}^{\mathcal{H}*} y_{S_{ki}}^{\mathcal{H}} \left[\mathcal{L}(m_i, -m_j, m_{\nu_k^H}, m_{\mathcal{H}}) + \mathcal{L}(m_i, -m_j, -m_{\nu_k^H}, m_{\mathcal{H}}) \right], \quad (\text{A.17})$$

where we have used $y_{P_{kj}}^{\mathcal{H}} = -y_{S_{kj}}^{\mathcal{H}}$ from (2.25) and the summation is over all the heavy hidden neutrinos ν_k^H running inside the loop. The loop integral \mathcal{L} is given by

$$\mathcal{L}(m_i, m_j, m_k, m_X) = -\int_0^1 dx \int_0^{1-x} dy \times \left[\frac{x \left(y + z \frac{m_j}{m_i} + \frac{m_k}{m_i} \right)}{-xym_i^2 - xzm_j^2 + xm_k^2 + (1-x)m_X^2} \right]. \quad (\text{A.18})$$

Our loop integral ($-\mathcal{L}$) is the same as $I_{k,2}^{++}$ in eq. (A.2) of [8].

For $i = j$, each term in both $A_{ii}^M(\mathcal{H})$ and $A_{ii}^E(\mathcal{H})$ is proportional to

$$|y_{S_{ki}}^{\mathcal{H}}|^2 = \frac{|\left(V_{\nu}^{H\dagger} M_{\nu} V_{\text{PMNS}} \right)_{ki}|^2}{2v^2}, \quad (\text{A.19})$$

which is real but may contain CP-violating phases from V_{PMNS} and V_{ν}^H . The effects from these CP-violating phases in $\Delta a_{li}(\mathcal{H}^\pm)$ are small due to the suppression from the neutrino masses. The important role of CP violating phases in the muon anomaly in MSSM coming from the charginos and neutrinos sectors has been emphasized previously in [60–62]. The electric dipole moment $d_{li}(\mathcal{H}^\pm)$ should be vanishing since the sum of the two integrals in (A.17) vanishes when $m_i = m_j$!

A.6 $\mathcal{W}'^{(p,m)}$ contribution — right diagram in figure 2

The transition magnetic and electric dipole form factors are

$$A_{ji}^M(\mathcal{W}') = +\frac{1}{8\pi^2} \sum_k g_{V_{kj}}^{\mathcal{W}'*} g_{V_{ki}}^{\mathcal{W}'} \left[\mathcal{J}(m_i, m_j, m_{l_k^H}, m_{\mathcal{W}'}) + \mathcal{J}(m_i, m_j, -m_{l_k^H}, m_{\mathcal{W}'}) \right], \quad (\text{A.20})$$

$$A_{ji}^E(\mathcal{W}') = +\frac{i}{8\pi^2} \sum_k g_{V_{kj}}^{\mathcal{W}'*} g_{V_{ki}}^{\mathcal{W}'} \left[\mathcal{J}(m_i, -m_j, m_{l_k^H}, m_{\mathcal{W}'}) + \mathcal{J}(m_i, -m_j, -m_{l_k^H}, m_{\mathcal{W}'}) \right], \quad (\text{A.21})$$

where we have used $g_{A_{kj}}^{\mathcal{W}'} = g_{V_{kj}}^{\mathcal{W}'}$ from (2.27) and the summation is over all the heavy hidden charged leptons l_k^H running inside the loop. The loop integral \mathcal{J} is given in (A.6). In the case of $i = j$, each term in both $A_{ii}^M(\mathcal{W}')$ and $A_{ii}^E(\mathcal{W}')$ is proportional to

$$|g_{V_{ki}}^{\mathcal{W}'}|^2 = \frac{g_H^2}{8} |(V_l^H)_{ik}|^2, \quad (\text{A.22})$$

which is real and contains no CP-violating phase. The electric dipole moment $d_{l_i}(\mathcal{W}')$ should be vanishing as one can check that the sum of the two integrals in (A.21) vanishes when $m_i = m_j$!

We note that all our results for the charged lepton anomalous magnetic dipole moments (where i and j are the same charged lepton) are consistent with eqs. (3), (4), (10) and (11) in [59] if we choose $q_F = q_X = q_H = -1$ in these formulas.

B Expression for $\mathcal{I}(m_i, m_j, m_k, m_X)$ in 't Hooft-Feynman gauge

In the SM, the W loop contribution can be evaluated in the 't Hooft-Feynman gauge. The longitudinal contributions from the two W propagators will be 'simulated' by three extra diagrams involving the couplings $\gamma G^+ G^-$ and $\gamma G^\pm W^\mp$ where G^\pm are the charged Goldstone bosons of W^\pm . The expression of $\mathcal{I}(m_i, m_j, m_k, m_X)$ in the 't Hooft-Feynman gauge is

$$\begin{aligned} & \mathcal{I}_{\text{'t Hooft-Feynman}}(m_i, m_j, m_k, m_X) \\ &= \int_0^1 dx \int_0^{1-x} dy \left(\frac{1}{-xz m_i^2 - x y m_j^2 + x m_k^2 + (1-x) m_X^2} \right) \\ & \times \left\{ \left[(y + 2z(1-x)) + (z + 2y(1-x)) \frac{m_j}{m_i} - 3(1-x) \frac{m_k}{m_i} \right] \right. \\ & \quad - \frac{m_i^2}{m_X^2} \left[x \left(1 - \frac{m_k}{m_i} \right) \left(\frac{m_j}{m_i} - \frac{m_k}{m_i} \right) \left(z + y \frac{m_j}{m_i} + \frac{m_k}{m_i} \right) \right] \\ & \quad \left. + y \left(1 - \frac{m_k}{m_i} \right) + z \left(\frac{m_j}{m_i} - \frac{m_k}{m_i} \right) \right\}, \quad (\text{B.1}) \end{aligned}$$

where we have defined $z = 1 - x - y$ as before.

One can integrate over the y variable in (B.1) and obtain a 1-dimension integral representation

$$\begin{aligned} & \mathcal{I}_{\text{'t Hooft-Feynman}}(m_i, m_j, m_k, m_X) \\ &= \int_0^1 dx \left\{ A(1-x) \right. \\ & \quad \left. + \frac{B}{x} \left(C_0 + C_1 x + C_2 x^2 \right) \log \left[\frac{m_X^2(1-x) - x(m_i^2(1-x) - m_k^2)}{m_X^2(1-x) - x(m_j^2(1-x) - m_k^2)} \right] \right\}, \quad (\text{B.2}) \end{aligned}$$

with

$$A = \frac{(m_i - m_k)(m_j - m_k) + 2m_X^2}{m_i(m_i + m_j)m_X^2}, \quad (\text{B.3})$$

$$B = \frac{1}{m_i(m_i - m_j)(m_i + m_j)^2 m_X^2}, \tag{B.4}$$

$$C_0 = \left(-2m_i^2 - 3m_i m_j - 2m_j^2 + 3(m_i + m_j)m_k + m_k^2 + 2m_X^2\right) m_X^2, \tag{B.5}$$

$$C_1 = (m_i^2 - m_k^2)(m_j^2 - m_k^2) + (2m_i + m_j - m_k)(m_i + 2m_j - m_k)m_X^2 - 2m_X^4, \tag{B.6}$$

$$C_2 = -m_i m_j \left((m_i - m_k)(m_j - m_k) + 2m_X^2\right). \tag{B.7}$$

Taking the limit of $m_j \rightarrow m_i$, (B.2) reduces to

$$\begin{aligned} & \mathcal{I}'_{\text{'t Hooft-Feynman}}(m_i, m_i, m_k, m_X) \\ &= \int_0^1 dx (1-x) \left[\frac{2(1-x) \left((2-x) - \frac{2m_k}{m_i} \right) - \frac{m_i^2}{m_X^2} x \left(1 - \frac{m_k}{m_i} \right)^2 \left((1-x) + \frac{m_k}{m_i} \right)}{m_X^2(1-x) - x(m_i^2(1-x) - m_k^2)} \right]. \end{aligned} \tag{B.8}$$

Multiplying the above result by $m_i^2 C_V^2 / 8\pi^2$ and let $m_i \rightarrow m_\mu$ reproduces the first vector coupling piece in eq. (4) (with $x \rightarrow 1-x$, $q_X = -1$ and $m_F = m_k$) of ref. [59], who first computed the anomaly a_μ for a charged X -loop with general gauge couplings of a muon and another neutral fermion F in the unitary gauge. The contribution from the axial coupling can be obtained by setting $C_V \rightarrow C_A$ and flipping the sign of the mass m_k in the above loop integral [59]. One can also reproduce the result of ref. [59] by starting directly from our result (A.3) in the unitary gauge, as mentioned earlier.

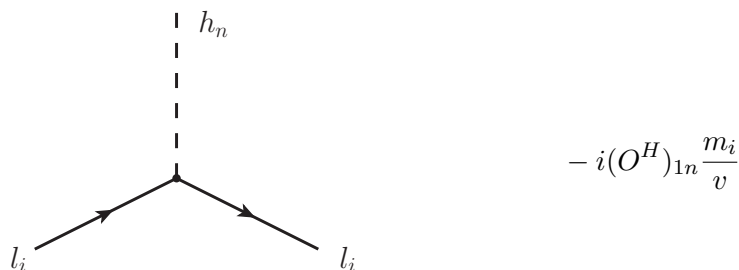
An 1-dimension integral representation for $\mathcal{I}(m_i, m_j, m_k, m_X)$ in the 't Hooft-Feynman gauge had been obtained previously in ref. [63]. Our result disagrees with this earlier result. One can show analytically that both of our expressions of $\mathcal{I}(m_i, m_j, m_k, m_X)$ in (A.3) and (B.1) (or equivalently (B.2)) from the unitary and 't Hooft-Feynman gauges respectively agree with each other. This can be done by integrating the integrand in (A.3) over y first and then subtract it with (B.2). The difference can then be shown to be zero by applying the following identity

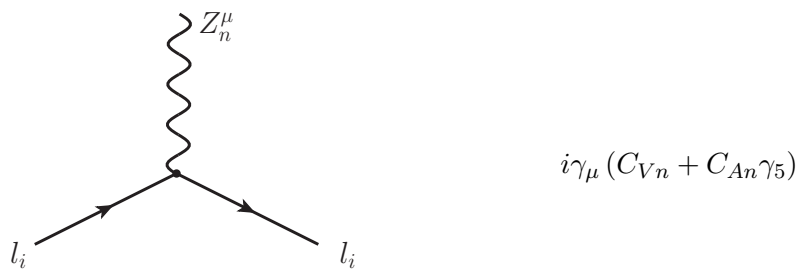
$$\int_0^1 dx g'(x) \log f(x) = g(x) \log f(x) \Big|_0^1 - \int_0^1 dx \frac{f'(x)g(x)}{f(x)} \tag{B.9}$$

to the log terms. The intermediate steps are tedious and not illuminative, we will omit them here.

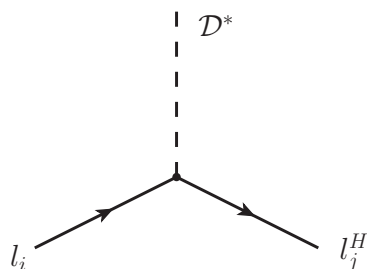
C Feynman rules

Some relevant Feynman rules for this work are listed as follows.

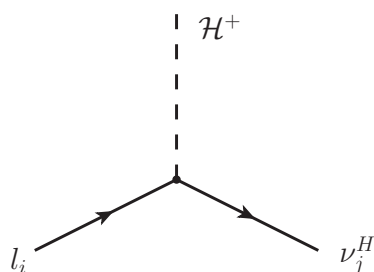




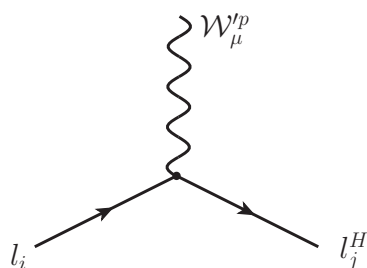
$$i\gamma_\mu (C_{Vn} + C_{An}\gamma_5)$$



$$i \left(y_{Sji}^D + y_{Pji}^D \gamma_5 \right)$$



$$i \left(y_{Sji}^H + y_{Pji}^H \gamma_5 \right)$$



$$i\gamma_\mu \left(g_{Vji}^{W'} + g_{Aji}^{W'} \gamma_5 \right)$$

D Amplitude of the on-shell $l_i \rightarrow l_j \gamma$ process

In general, the Lorentz invariant amplitude for $l_i \rightarrow l_j \gamma (i \neq j)$ as depicted in figure 3 is given by [64]

$$i\mathcal{M}_{ji} = \langle l_j | J_{\text{em}}^\mu | l_i \rangle \tilde{\mathcal{A}}_\mu^{\text{ext}}(q), \tag{D.1}$$

where $\tilde{\mathcal{A}}_\mu^{\text{ext}}(q)$ is the electromagnetic background field and

$$\langle l_j | J_{\text{em}}^\mu | l_i \rangle = \bar{u}_j(p') [i\sigma^{\mu\nu} q_\nu (A + B\gamma_5) + \gamma^\mu (C + D\gamma_5) + q^\mu (E + F\gamma_5)] u_i(p) \tag{D.2}$$

with $q = p' - p$ and A, B, C, D, E and F are the form factors. Using the electromagnetic gauge invariance, one has

$$\partial_\mu J_{\text{em}}^\mu = 0 \tag{D.3}$$

which yields the condition

$$-m_i(C - D\gamma_5) + m_j(C + D\gamma_5) + q^2(E + F\gamma_5) = 0 \quad (\text{D.4})$$

or $C = D = 0$ for the case of $m_i \neq m_j$ and on-shell photon ($q^2 = 0$). Furthermore, since $q^\mu \tilde{\mathcal{A}}_\mu^{\text{ext}} = 0$, the amplitude for the on-shell $l_i \rightarrow l_j \gamma$ process is then given as

$$i\mathcal{M}_{ji} = \bar{u}_j(p') [i\sigma^{\mu\nu} q_\nu (A + B\gamma_5)] u_i(p) \tilde{\mathcal{A}}_\mu^{\text{ext}}(q). \quad (\text{D.5})$$

To compare with the conventions established in eq. (3.1), we can identify

$$A = -ie \frac{m_i}{2} A_{ji}^M, \quad (\text{D.6})$$

$$B = e \frac{m_i}{2} A_{ji}^E. \quad (\text{D.7})$$

It is important to note that the amplitude in eq. (D.5) corresponds to a dimension-five operator and as such, it can only be induced from loop diagrams. Furthermore, as there can be no counterterm to absorb infinities, it is imperative that the amplitude in eq. (D.5) must be finite [64]. Since the self-energy diagrams contribute only to the C and D form factors, they are not relevant to the on-shell amplitude of $l_i \rightarrow l_j \gamma$. Of course these self-energy diagrams are necessarily included along with the 1PI diagrams to maintain the gauge invariance of QED!

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References

- [1] S.T. Petcov, *The Processes $\mu \rightarrow e + \gamma, \mu \rightarrow e + \bar{e}, \nu' \rightarrow \nu + \gamma$ in the Weinberg-Salam Model with Neutrino Mixing*, *Sov. J. Nucl. Phys.* **25** (1977) 340 [*Erratum ibid.* **25** (1977) 698] [*Erratum Yad. Fiz.* **25** (1977) 1336] [[INSPIRE](#)].
- [2] T.P. Cheng and L.-F. Li, *Nonconservation of Separate μ -Lepton and e -Lepton Numbers in Gauge Theories with $V + A$ Currents*, *Phys. Rev. Lett.* **38** (1977) 381 [[INSPIRE](#)].
- [3] F. Wilczek and A. Zee, *Rare Muon Decays, Natural Lepton Models, and Doubly Charged Leptons*, *Phys. Rev. Lett.* **38** (1977) 531 [[INSPIRE](#)].
- [4] MEG collaboration, *Search for the lepton flavour violating decay $\mu^+ \rightarrow e^+ \gamma$ with the full dataset of the MEG experiment*, *Eur. Phys. J. C* **76** (2016) 434 [[arXiv:1605.05081](#)] [[INSPIRE](#)].
- [5] MEG II collaboration, *MEG II experiment status and prospect*, *PoS NuFact2021* (2022) 120 [[arXiv:2201.08200](#)] [[INSPIRE](#)].
- [6] Y. Kuno and Y. Okada, *Muon decay and physics beyond the standard model*, *Rev. Mod. Phys.* **73** (2001) 151 [[hep-ph/9909265](#)] [[INSPIRE](#)].

- [7] A. de Gouvea and P. Vogel, *Lepton Flavor and Number Conservation, and Physics Beyond the Standard Model*, *Prog. Part. Nucl. Phys.* **71** (2013) 75 [[arXiv:1303.4097](#)] [[INSPIRE](#)].
- [8] M. Lindner, M. Platscher and F.S. Queiroz, *A Call for New Physics: The Muon Anomalous Magnetic Moment and Lepton Flavor Violation*, *Phys. Rept.* **731** (2018) 1 [[arXiv:1610.06587](#)] [[INSPIRE](#)].
- [9] R.H. Bernstein and P.S. Cooper, *Charged Lepton Flavor Violation: An Experimenter's Guide*, *Phys. Rept.* **532** (2013) 27 [[arXiv:1307.5787](#)] [[INSPIRE](#)].
- [10] P.Q. Hung, T. Le, V.Q. Tran and T.-C. Yuan, *Lepton Flavor Violating Radiative Decays in EW-Scale ν_R Model: An Update*, *JHEP* **12** (2015) 169 [[arXiv:1508.07016](#)] [[INSPIRE](#)].
- [11] P.Q. Hung, T. Le, V.Q. Tran and T.-C. Yuan, *Muon-to-Electron Conversion in Mirror Fermion Model with Electroweak Scale Non-Sterile Right-handed Neutrinos*, *Nucl. Phys. B* **932** (2018) 471 [[arXiv:1701.01761](#)] [[INSPIRE](#)].
- [12] R. Ramos, V.Q. Tran and T.-C. Yuan, *Complementary searches of low mass non-Abelian vector dark matter, dark photon, and dark Z'* , *Phys. Rev. D* **103** (2021) 075021 [[arXiv:2101.07115](#)] [[INSPIRE](#)].
- [13] R. Ramos, V.Q. Tran and T.-C. Yuan, *A Sub-GeV Low Mass Hidden Dark Sector of $SU(2)_H \times U(1)_X$* , *JHEP* **11** (2021) 112 [[arXiv:2109.03185](#)] [[INSPIRE](#)].
- [14] V.Q. Tran, T.T.Q. Nguyen and T.-C. Yuan, *Obliquely Scrutinizing a Hidden SM-like Gauge Model*, [arXiv:2208.10971](#) [[INSPIRE](#)].
- [15] MUON G-2 collaboration, *Measurement of the positive muon anomalous magnetic moment to 0.7 ppm*, *Phys. Rev. Lett.* **89** (2002) 101804 [[hep-ex/0208001](#)] [[INSPIRE](#)].
- [16] MUON G-2 collaboration, *Measurement of the negative muon anomalous magnetic moment to 0.7 ppm*, *Phys. Rev. Lett.* **92** (2004) 161802 [[hep-ex/0401008](#)] [[INSPIRE](#)].
- [17] MUON G-2 collaboration, *Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL*, *Phys. Rev. D* **73** (2006) 072003 [[hep-ex/0602035](#)] [[INSPIRE](#)].
- [18] MUON G-2 collaboration, *Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm*, *Phys. Rev. Lett.* **126** (2021) 141801 [[arXiv:2104.03281](#)] [[INSPIRE](#)].
- [19] T. Aoyama et al., *The anomalous magnetic moment of the muon in the Standard Model*, *Phys. Rept.* **887** (2020) 1 [[arXiv:2006.04822](#)] [[INSPIRE](#)].
- [20] A. Keshavarzi, K.S. Khaw and T. Yoshioka, *Muon $g - 2$: A review*, *Nucl. Phys. B* **975** (2022) 115675 [[arXiv:2106.06723](#)] [[INSPIRE](#)].
- [21] MUON (G-2) collaboration, *An Improved Limit on the Muon Electric Dipole Moment*, *Phys. Rev. D* **80** (2009) 052008 [[arXiv:0811.1207](#)] [[INSPIRE](#)].
- [22] A. Adelmann et al., *Search for a muon EDM using the frozen-spin technique*, [arXiv:2102.08838](#) [[INSPIRE](#)].
- [23] ACME collaboration, *Improved limit on the electric dipole moment of the electron*, *Nature* **562** (2018) 355 [[INSPIRE](#)].
- [24] *Advanced ACME*, <http://doylegroup.harvard.edu/edm/index.html>.
- [25] W.-C. Huang, Y.-L.S. Tsai and T.-C. Yuan, *G2HDM : Gauged Two Higgs Doublet Model*, *JHEP* **04** (2016) 019 [[arXiv:1512.00229](#)] [[INSPIRE](#)].

- [26] A. Arhrib, W.-C. Huang, R. Ramos, Y.-L.S. Tsai and T.-C. Yuan, *Consistency of a gauged two-Higgs-doublet model: Scalar sector*, *Phys. Rev. D* **98** (2018) 095006 [[arXiv:1806.05632](#)] [[INSPIRE](#)].
- [27] C.-T. Huang, R. Ramos, V.Q. Tran, Y.-L.S. Tsai and T.-C. Yuan, *Consistency of Gauged Two Higgs Doublet Model: Gauge Sector*, *JHEP* **09** (2019) 048 [[arXiv:1905.02396](#)] [[INSPIRE](#)].
- [28] C.-R. Chen, Y.-X. Lin, C.S. Nugroho, R. Ramos, Y.-L.S. Tsai and T.-C. Yuan, *Complex scalar dark matter in the gauged two-Higgs-doublet model*, *Phys. Rev. D* **101** (2020) 035037 [[arXiv:1910.13138](#)] [[INSPIRE](#)].
- [29] C.-R. Chen, Y.-X. Lin, V.Q. Tran and T.-C. Yuan, *Pair production of Higgs bosons at the LHC in gauged 2HDM*, *Phys. Rev. D* **99** (2019) 075027 [[arXiv:1810.04837](#)] [[INSPIRE](#)].
- [30] W.-C. Huang, H. Ishida, C.-T. Lu, Y.-L.S. Tsai and T.-C. Yuan, *Signals of New Gauge Bosons in Gauged Two Higgs Doublet Model*, *Eur. Phys. J. C* **78** (2018) 613 [[arXiv:1708.02355](#)] [[INSPIRE](#)].
- [31] W.-C. Huang, Y.-L.S. Tsai and T.-C. Yuan, *Gauged Two Higgs Doublet Model confronts the LHC 750 GeV diphoton anomaly*, *Nucl. Phys. B* **909** (2016) 122 [[arXiv:1512.07268](#)] [[INSPIRE](#)].
- [32] B. Kors and P. Nath, *A Stueckelberg extension of the standard model*, *Phys. Lett. B* **586** (2004) 366 [[hep-ph/0402047](#)] [[INSPIRE](#)].
- [33] B. Kors and P. Nath, *Aspects of the Stueckelberg extension*, *JHEP* **07** (2005) 069 [[hep-ph/0503208](#)] [[INSPIRE](#)].
- [34] D. Feldman, Z. Liu and P. Nath, *The Stueckelberg Z Prime at the LHC: Discovery Potential, Signature Spaces and Model Discrimination*, *JHEP* **11** (2006) 007 [[hep-ph/0606294](#)] [[INSPIRE](#)].
- [35] D. Feldman, Z. Liu and P. Nath, *The Stueckelberg Z' Extension with Kinetic Mixing and Milli-Charged Dark Matter From the Hidden Sector*, *Phys. Rev. D* **75** (2007) 115001 [[hep-ph/0702123](#)] [[INSPIRE](#)].
- [36] PARTICLE DATA GROUP collaboration, *Review of Particle Physics*, *PTEP* **2020** (2020) 083C01 [[INSPIRE](#)].
- [37] CMS collaboration, *A measurement of the Higgs boson mass in the diphoton decay channel*, *Phys. Lett. B* **805** (2020) 135425 [[arXiv:2002.06398](#)] [[INSPIRE](#)].
- [38] CMS collaboration, *Combined measurements of Higgs boson couplings in proton-proton collisions at $\sqrt{s} = 13$ TeV*, *Eur. Phys. J. C* **79** (2019) 421 [[arXiv:1809.10733](#)] [[INSPIRE](#)].
- [39] ATLAS collaboration, *Combined measurements of Higgs boson production and decay using up to 80 fb^{-1} of proton-proton collision data at $\sqrt{s} = 13$ TeV collected with the ATLAS experiment*, *Phys. Rev. D* **101** (2020) 012002 [[arXiv:1909.02845](#)] [[INSPIRE](#)].
- [40] ATLAS collaboration, *Combined measurements of Higgs boson production and decay using up to 139 fb^{-1} of proton-proton collision data at $\sqrt{s} = 13$ TeV collected with the ATLAS experiment*, *ATLAS-CONF-2021-053* (2021) [[INSPIRE](#)].
- [41] ATLAS collaboration, *Search for high-mass dilepton resonances using 139 fb^{-1} of pp collision data collected at $\sqrt{s} = 13$ TeV with the ATLAS detector*, *Phys. Lett. B* **796** (2019) 68 [[arXiv:1903.06248](#)] [[INSPIRE](#)].

- [42] M. Fabbrichesi, E. Gabrielli and G. Lanfranchi, *The Dark Photon*, [arXiv:2005.01515](#) [[INSPIRE](#)].
- [43] CDF collaboration, *High-precision measurement of the W boson mass with the CDF II detector*, *Science* **376** (2022) 170 [[INSPIRE](#)].
- [44] PLANCK collaboration, *Planck 2018 results. VI. Cosmological parameters*, *Astron. Astrophys.* **641** (2020) A6 [[arXiv:1807.06209](#)] [[INSPIRE](#)].
- [45] CRESST collaboration, *Results on MeV-scale dark matter from a gram-scale cryogenic calorimeter operated above ground*, *Eur. Phys. J. C* **77** (2017) 637 [[arXiv:1707.06749](#)] [[INSPIRE](#)].
- [46] DARKSIDE collaboration, *Low-Mass Dark Matter Search with the DarkSide-50 Experiment*, *Phys. Rev. Lett.* **121** (2018) 081307 [[arXiv:1802.06994](#)] [[INSPIRE](#)].
- [47] XENON collaboration, *Light Dark Matter Search with Ionization Signals in XENON1T*, *Phys. Rev. Lett.* **123** (2019) 251801 [[arXiv:1907.11485](#)] [[INSPIRE](#)].
- [48] ATLAS collaboration, *Search for invisible Higgs-boson decays in events with vector-boson fusion signatures using 139 fb^{-1} of proton-proton data recorded by the ATLAS experiment*, *JHEP* **08** (2022) 104 [[arXiv:2202.07953](#)] [[INSPIRE](#)].
- [49] I. Esteban, M.C. Gonzalez-Garcia, M. Maltoni, T. Schwetz and A. Zhou, *The fate of hints: updated global analysis of three-flavor neutrino oscillations*, *JHEP* **09** (2020) 178 [[arXiv:2007.14792](#)] [[INSPIRE](#)].
- [50] S.M. Barr, E.M. Freire and A. Zee, *A Mechanism for large neutrino magnetic moments*, *Phys. Rev. Lett.* **65** (1990) 2626 [[INSPIRE](#)].
- [51] M. Battaglieri et al., *US Cosmic Visions: New Ideas in Dark Matter 2017: Community Report*, in *U.S. Cosmic Visions: New Ideas in Dark Matter* College Park United States, March 23–25 2017 [[arXiv:1707.04591](#)] [[INSPIRE](#)].
- [52] SUPERCDMS collaboration, *Projected Sensitivity of the SuperCDMS SNOLAB experiment*, *Phys. Rev. D* **95** (2017) 082002 [[arXiv:1610.00006](#)] [[INSPIRE](#)].
- [53] CDEX collaboration, *CDEX Dark Matter Experiment: Status and Prospects*, *J. Phys. Conf. Ser.* **1342** (2020) 012067 [[arXiv:1712.06046](#)] [[INSPIRE](#)].
- [54] S. Weinberg, *Larger Higgs Exchange Terms in the Neutron Electric Dipole Moment*, *Phys. Rev. Lett.* **63** (1989) 2333 [[INSPIRE](#)].
- [55] E. Braaten, C.-S. Li and T.-C. Yuan, *The Evolution of Weinberg's Gluonic CP Violation Operator*, *Phys. Rev. Lett.* **64** (1990) 1709 [[INSPIRE](#)].
- [56] S. Weinberg, *Unitarity Constraints on CP Nonconservation in Higgs Exchange*, *Phys. Rev. D* **42** (1990) 860 [[INSPIRE](#)].
- [57] E. Braaten, C.S. Li and T.C. Yuan, *The Gluon Color - Electric Dipole Moment and Its Anomalous Dimension*, *Phys. Rev. D* **42** (1990) 276 [[INSPIRE](#)].
- [58] S.M. Barr and A. Zee, *Electric Dipole Moment of the Electron and of the Neutron*, *Phys. Rev. Lett.* **65** (1990) 21 [[INSPIRE](#)].
- [59] J.P. Leveille, *The Second Order Weak Correction to $(g - 2)$ of the Muon in Arbitrary Gauge Models*, *Nucl. Phys. B* **137** (1978) 63 [[INSPIRE](#)].
- [60] T. Ibrahim and P. Nath, *CP violation and the muon anomaly in $N = 1$ supergravity*, *Phys. Rev. D* **61** (2000) 095008 [[hep-ph/9907555](#)] [[INSPIRE](#)].

- [61] T. Ibrahim and P. Nath, *Effects of large CP violating phases on $g_\mu - 2$ in MSSM*, *Phys. Rev. D* **62** (2000) 015004 [[hep-ph/9908443](#)] [[INSPIRE](#)].
- [62] T. Ibrahim, U. Chattopadhyay and P. Nath, *Constraints on explicit CP violation from the Brookhaven muon $g - 2$ experiment*, *Phys. Rev. D* **64** (2001) 016010 [[hep-ph/0102324](#)] [[INSPIRE](#)].
- [63] E. Ma and A. Pramudita, *Exact Formula for $(\mu \rightarrow e\gamma)$ Type Processes in the Standard Model*, *Phys. Rev. D* **24** (1981) 1410 [[INSPIRE](#)].
- [64] T.P. Cheng and L.F. Li, *Gauge theory of elementary particle physics*, Oxford University Press (1984) [ISBN: 9780198519614].