Physics Letters B 744 (2015) 67–73

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Counting ghosts in the “ghost-free” non-local gravity

Ilya L. Shapiro [a,b,c,*]

[a] Département de Physique Théorique and Center for Astroparticle Physics, Université de Genève, 24 quai Ansermet, CH-1211 Genève 4, Switzerland
[b] Departamento de Física, ICE, Universidade Federal de Juiz de Fora, Campus Universitário, Juiz de Fora, 36036-330, MG, Brazil
[c] Tomsk State Pedagogical University and Tomsk State University, Tomsk, Russia

ARTICLE INFO

Article history:
Received 9 February 2015
Received in revised form 10 March 2015
Accepted 17 March 2015
Available online 21 March 2015
Editor: M. Cvetič

Keywords:
Quantum gravity
Massive ghosts
Higher derivatives
Renormalization
Non-local theories

ABSTRACT

In the recently proposed non-local theory of quantum gravity one can avoid massive tensor ghosts at the tree level by introducing an exponential form factor between the two Ricci tensors. We show that at the quantum level this theory has an infinite amount of massive unphysical states, mostly corresponding to complex poles.

1. Introduction

The general relativity (GR) is a very successful theory of gravity, but it is perhaps not an ultimate theory. One of the reasons is that the fourth derivative terms in the action of gravity become necessary as the UV completion of the theory at the semiclassical level [1] (see also [2,3] for the introduction and [4] for a recent pedagogical review). The same fourth-derivative terms make the theory of quantum gravity (QG) renormalizable [5]. On the other side, fourth derivatives lead to the massive ghosts in the physical spectrum of the theory, leading to the violation of unitarity.

The consistency of the fourth derivative quantum gravity (QG) can be, in principle, achieved by dealing with the dressed propagator instead of the classical one [6–8]. The main expectation is that the massive ghost poles become unstable and decay in the far future, such that the asymptotic *out*-state becomes free of ghosts. Unfortunately, the final conclusion concerning this approach requires a complete non-perturbative knowledge of the dressed propagator [9], which is unavailable.

Some years ago a completely different approach was proposed by Tomboulis [10]. The action of this new theory of QG has an infinite amount of derivatives. It was discovered a few years earlier by Tseytlin [11] that for some specially tuned form of the non-local action such a theory is free of ghosts at the tree level while the exponential form factors remove Newtonian singularity, similar to the much simpler fourth derivative gravity [5]. The approach of [11] (see also [12]) was to use this action in the framework of string theory, as an alternative of the Zwiebach ghost-killing transformation of the background fields [13–15]. In string theory the ghost-free non-local action is a kind of a “final product”, which is not supposed to gain further quantum corrections.[1] On the contrary, if one takes the same model as a basis of quantum gravity [10], the following three important questions should be answered:

- First, how to quantize the non-local theory?
- Second, what is the power counting in a theory with infinite amount of derivatives?
- The third and most difficult question is what happens with the ghost-free structure of the theory after the quantum corrections are taken into account?

Concerning the first point, the quantization of non-local theories has been discussed in the literature [17] and is relatively well-understood. The second issue has been explored in [10] and

* Correspondence to: Departamento de Física, ICE, Universidade Federal de Juiz de Fora, Campus Universitário, Juiz de Fora, 36036-330, MG, Brazil.
E-mail address: shapiro@fisica.ufjf.br.

[1] However, this does not make it free of ambiguity related to the third and higher powers of curvature, similar to the one discussed in [16].

http://dx.doi.org/10.1016/j.physletb.2015.03.037

in the more recent publications [18–20]. The main conclusion is that the power counting in the non-local theory of [10] is the same as in the local higher derivative superrenormalizable QG suggested earlier in [21]. Moreover, in both cases there is a chance to make such a QG theory finite. This can be certainly achieved in the local case [21] and very likely in the non-local one.[2]

In the present work we will mainly address the third question. There are strong arguments that at least the most simple example of the non-local theory suggested in [10] does not remain free of ghost-like states at the quantum level. The last means that the quantum corrections lead to an infinite amount of the ghost-like states in the dressed propagator. The relation between ghosts in the classical (naked) and dressed propagators is almost opposite to what was expected in the fourth-derivative renormalized theory of QG [5–8].

The paper is organized as follows. In Section 2 we present a brief review of the non-local gravity which is ghost-free at the tree-level. In Section 3 we explain the power counting in the non-local model, here our consideration mainly follows previous publications [10,18] and [20], but we try to make it more transparent, especially by comparing to the local superrenormalizable QG case [21]. Some relevant details concerning Lagrangian quantization of the non-local theory are settled in Appendix A. In Section 4 it is shown how the ghost-free structure is violated by quantum corrections to the propagator. In Section 5 we discuss the modified Newtonian limit in the non-local theory and the possible role played by the "hidden" ghosts. Finally, in the last section we draw our conclusions.

2. Non-local ghost-free models

The simplest way to count degrees of freedom in QG is based on the analysis of the tree-level propagator on the flat background. In most of the theories this procedure gives the same result as canonical quantization [22,3]. In order to explore the flat-space propagator, the relevant part of the classical action is at most bilinear in the curvature tensor,

$$S = \int d^4x\sqrt{-g}\,\Big\{-\frac{1}{\kappa^2}\,R + R\,F_1(\Box)\,R + R_{\mu\nu}\,F_2(\Box)\,R^{\mu\nu} + R_{\mu\nu\alpha\beta}\,F_3(\Box)\,R^{\mu\nu\alpha\beta}\Big\}. \tag{1}$$

Here $\kappa^2 = 16\pi G$ and $F_{1,2,3}$ are functions of d'Alembertian operator. The cosmological constant term is set to zero, following the standard treatment [5]. In order to simplify the action, let us note that the difference between the term $R_{\mu\nu\alpha\beta}F_3(\Box)R^{\mu\nu\alpha\beta}$ and the combination $4R_{\mu\nu}F_3(\Box)R^{\mu\nu} - RF_3(\Box)R$ is proportional to the term of the third power in curvature, $\mathcal{O}(R^3_{...})$ (see, e.g., [21,23]). Therefore one can cast the relevant part of the action (1) in the form

$$S = \int d^4x\sqrt{-g}\,\Big\{-\frac{1}{\kappa^2}\,R + \frac{1}{2}\,C_{\mu\nu\alpha\beta}\,\Phi(\Box)\,C_{\mu\nu\alpha\beta} + \frac{1}{2}\,R\,\Psi(\Box)\,R\Big\}, \tag{2}$$

where $C_{\mu\nu\alpha\beta}$ is the Weyl tensor. The function Ψ is responsible for the spin-0 part of the propagator and the function Φ for the spin-2 part. For the sake of simplicity, we can mainly concentrate on the spin-2 sector. The consideration for the Ψ-part would be very similar. After the Fourier transformation, the relevant equation for defining the poles of the propagator is [10]

$$p^2\left[1 + \kappa^2 p^2 \Phi(-p^2)\right] = 0\,. \tag{3}$$

One can see that there is always a massless pole corresponding to gravitons. For a constant Φ there is also a massive pole corresponding to a spin-2 ghost, which may be also a tachyon. For a non-constant polynomial function Φ there are always ghost-like poles, real or complex [21]. However, one can choose the function Φ in such a way that there will not be any other spin-2 pole, except the graviton $p^2 = 0$. The simplest example of this sort is [11]

$$1 + \kappa^2 p^2 \Phi(-p^2) = e^{\alpha p^2}\,, \tag{4}$$

where α is some constant of the dimension $mass^{-2}$. One can find other entire functions which have the same features [10,18], but for the sake of simplicity we consider only (4).

Let us remember that the exponential function has two remarkable properties. The equation $\exp z = 0$ has no real solutions and only one very peculiar solution

$$z = -\infty + i\times 0 \tag{5}$$

on the extended complex plane. At the same time, already the equation $\exp z = A \neq 0$ has infinitely many complex solutions, the same is true for

$$e^z = Az^2\log z\,, \tag{6}$$

which is the typical case for the exponential theory with logarithmic quantum corrections. These well-known features of exponential function mean, in our case, that the absence of massive ghosts in the spin-2 part of the propagator of the theory (4) is the result of an absolutely precise tuning of the function $\Phi(-p^2)$. If this tuning is violated by the loop corrections, then the ghosts-like states will emerge in an infinite number. For instance, any polynomial addition to the exponential function produce infinitely many complex solutions.

One important note is in order. The expression "ghosts-like states" means that these states are not exactly the "classical" massive ghosts, that means states with positive square of mass and negative kinetic energy. In the present case there are mostly complex poles, that means a complex "square of mass" and complex "kinetic energy". This situation makes the particle interpretation of these states rather complicated. We postpone the discussion of this issue until another publication and will call these states simply ghosts in what follows.

If the theory with more ghosts should be qualified worst, then the exponential gravity (4) with violated absolute tuning is worse than the polynomial version of superrenormalizable QG [21] (see also the next section), because the last has only finite amount of ghosts. So, the main question concerning the theory of exponential gravity (4) is whether one can preserve an absolute tuning of (4) at the quantum level. In the next sections we consider this issue starting from the strongest effect related to the UV divergences and related logarithmic running. For comparison, we also present considerations for the mentioned polynomial model of QG.

3. Power-counting in local and non-local QG

Before discussing the dressed propagator and possible violation of the absolute tuning in (4), let us shortly review the renormalization properties of the theory (2) and some its natural extensions. A brief survey of the Lagrangian quantization of the theories such as (2) or (7) with some details related to non-local versions of the theory can be found in Appendix A.

[2] In the odd space–time dimensions this can be easily proved in [18].

3.1. Polynomial higher-derivative gravity

The action of the general superrenormalizable polynomial model can be written as

$$S = S_{EH} + \int d^4x\sqrt{-g}\,\Big\{d_1 R^2_{\mu\nu\alpha\beta} + d_2 R^2_{\mu\nu} + d_3 R^2 + \dots + c_1 R_{\mu\nu\alpha\beta}\Box^k R^{\mu\nu\alpha\beta} + c_2 R_{\mu\nu}\Box^k R^{\mu\nu} + c_3 R\Box^k R + \dots + b_{1,2,..} R^{k+2}_{...}\Big\}, \qquad (7)$$

where the omitted terms and $b_{1,2,..}R^{k+2}_{...}$ denote the set of all covariant local terms with the derivatives up to the order $2k+4$. The action includes not only quadratic in curvature terms, but also generic $\mathcal{O}(R^3_{...})$ terms, and so on. $d_{1,2,3}$, $c_{1,2,3}$, $\dots$, $b_{1,2,\dots}$ are arbitrary coefficients.

In order to explore the superficial degree of divergence of the theory one needs two relations, namely

$$D + d = \sum_{l_{int}} (4 - r_l) - 4n + 4 + \sum_{\nu} K_\nu \qquad (8)$$

for the power counting, and the topological relation

$$l_{int} = p + n - 1\,. \qquad (9)$$

In these formulas l_{int} is the number of internal lines with the inverse power of momenta r_l in the propagator, n is the number of vertices with K_ν derivatives and p is the number of loops. On the *l.h.s.*, d is the number of derivatives acting on the external lines of a given diagram and D is its superficial degree of divergence.

In the theory (7) the most divergent diagrams correspond to the vertices with maximal number of derivatives, $K_\nu = 2k+4$. One can always formulate the theory (see [5,24,21] and Appendix A of the present work) in such a way that $r_l \equiv 2k+4$ for all fields. Then it is an easy exercise to combine (8) and (9), and the result is [21]

$$d = 4 + k(1 - p) \qquad (10)$$

for the logarithmically divergent diagrams with $D = 0$. The last relation shows that the versions of QG with $k \geq 3$ have only one-loop divergences. This means, the higher order contributions may be also divergent, but they become finite after we renormalize the one-loop sub-diagrams. Furthermore, the possible counterterms may have only four, two and zero mass dimensions. In other words, only the terms in the first line of (7) needs to be renormalized. All terms with derivatives higher than four are not running. At the same time, the coefficients of these higher derivative terms define the running of the cosmological and Newton constants and of the coefficients d_1, d_2 and d_3.

The last two observations which will be used in the rest of the paper and (as we shall see in what follows) can be applied also to the exponential gravity, are as follows:

- The running of the parameters G, ρ_Λ and $d_{1,2,3}$ is gauge-fixing independent, because the classical equations of motion have more derivatives than the counterterms. In order to understand this statement, let us remember that the gauge-fixing dependence disappears on-shell (see, e.g., [26] for further references on the subject). The practical application of this feature to QG was discussed in [25].
- The β-functions for the Newton constant and the ones of $d_{1,2,3}$ are given by bi-linear and linear combinations of the coefficients of the $\mathcal{O}(R^3_{...})$ and $\mathcal{O}(R^4_{...})$ terms in the action (7). Therefore, one can provide to these β-functions any desirable values by changing the corresponding coefficients.[3] The remarkable exception is the β-function for the cosmological constant derived in [21]. This unique β-function is completely defined by the coefficients $c_{1,2,3}$ in (7).

3.2. Exponential gravity

In the exponential gravity theory the power counting formula (8) has no much sense, because it leads to an indefinite output of the $\infty-\infty$ type. At the same time the topological relation (9) is working well and shows that the theory is superrenormalizable [10,18,20]. Let us consider this point.

Each propagator gives contribution of infinite negative powers of momenta, let us call it $\mathcal{I}$. With the vertices the situation is more complex, because there are vertices with different powers of momenta. Without loss of generality one can consider only the diagrams with maximal divergence, when each vertex gives contribution $-\mathcal{I}$. It is important that the two symbols $\mathcal{I}$ and $-\mathcal{I}$ correspond to the same power of infinity, for otherwise the relation should become more complicated.[4] Then it is clear that the diagram with more internal lines than vertices will be automatically convergent and the diagram with more vertices than internal lines will be strongly divergent. The relation (9) tells us that the difference is $l_{int} - n = p - 1$. This means that only the one-loop diagrams with $p = 1$ can be divergent. At the same time, the presence of the exponential form factor does not change the degree of divergence of the one-loop diagrams.

The power counting in the exponential gravity is performed by the topological relation (9), without the formula (8). Nevertheless, the result is exactly the same as in the polynomial theory (7) for $k \geq 3$. Namely, the divergences show up only at the one-loop level, and the counterterms have zero, two and four derivatives of the metric only. In other words, the possible counterterms have the form

$$\Delta S = \int d^4x\sqrt{-g}\,\{a_1 R^2_{\mu\nu\alpha\beta} + a_2 R^2_{\mu\nu} + a_3 R^2 + a_4\Box R + a_5 R + a_6\}\,. \qquad (11)$$

The divergent coefficients $a_{1,2,\dots,6}$ are at most $\mathcal{O}(1/(n-4))$ in dimensional regularization.

Similar to the polynomial case, there is a chance to specially tune the $\mathcal{O}(R^2_{...})$, $\mathcal{O}(R^3_{...})$ and $\mathcal{O}(R^4_{...})$-terms in the action (7) such that the divergent coefficients $a_{1,2,\dots,6}$ in (11) can be adjusted to have desirable values. In the case of exponential QG one should try to provide these divergent coefficients to become zero, because the possible running would violate an absolute tuning requested by the ghost-free structure of the exponential gravity. In case of the logarithmic divergences of the form (11), the equation for the poles of the propagator has the form (6) with $z = p^2$ and $A = 4a_1 + a_2$ for the spin-2 sector of the propagator. As we already know, if such corrections take place, then the dressed propagator has infinitely many ghost-like states. So, if we intend to keep the ghost-free structure at the quantum level, the first thing to do is to require that the theory should be finite. We shall discuss this subject further in the next section.

---

[3] Of course, this does not mean that the explicit derivation of these β-functions would not be interesting. Since the potential result is a possibility to obtain exact β-functions in some model of quantum gravity, this calculation would worth the requested hard work anyway. From the physical side, different choices of these coefficients may correspond to different physical properties of the theory, so such a calculation would be quite relevant.

[4] This means, in particular, that the value of α in Eq. (4) must be identical for both functions $\Phi(\Box)$ and $\Psi(\Box)$ in (2), for otherwise the theory would be badly non-renormalizable.

4. Quantum corrections and dressed propagator

As we already know, the power counting in the theory with exponential form factors (4) is exactly the same as in the polynomial theory (7) with $k \geq 3$. One can see this similarity in the following way. Imagine we replace the exponential function in (4) by the partial sum of its Taylor expansion,

$$P_N(\alpha p^2) = \sum_{l=0}^{N} \frac{(\alpha p^2)^l}{l!}\,. \qquad (12)$$

For a sufficiently large N the theory will be superrenormalizable, exactly as in the exponential case. At the same time, there will be N roots of the polynomial $P_N(\alpha p^2)$ at the complex plane. The number of these roots is growing with larger N. When $N \to \infty$ the power counting remains the same. At the same time, the number of poles becomes infinite, so the theory gains an infinite amount of ghost-like poles, most of them complex. However, it happens that all these poles converge to the very special infinite point (5). Then the theory is ghost-free at the tree-level, but there is a danger that the absolute tuning may be broken by the one-loop corrections, which require very special attention.

The one-loop effective action in the theory (2) is given by the expression [24] (see Appendix A for details)

$$\Gamma^{(1)}_{div} = \frac{i}{2}\,\mathrm{Tr\,Log}\,\hat{H} - i\,\mathrm{Tr\,Log}\,\hat{H}_{ghost} + \frac{i}{2}\,\mathrm{Tr\,Log}\,\hat{Y}\,. \qquad (13)$$

The last two terms are contributions of ghost and weight operators. Both of them have standard form plus some part related to the term $\mathrm{Tr}\,\Box$. This term is discussed in Appendix A, where we argue that its contribution has the form (11) with fixed coefficients. Therefore, the main question is whether the finiteness can be provided by changing the action S in such a way that the operator

$$\hat{H} = \frac{1}{2\sqrt{-g}}\,\frac{\delta^2 S}{\delta g_{\alpha\beta}\delta g_{\rho\sigma}} \qquad (14)$$

provides a cancellation of the first term in the formula (13) with the divergences coming from the last two terms of the same expression.

It is obvious that one cannot achieve this goal by using the original action (2), because both functions Ψ and Φ are proportional to the same expression

$$\Psi = c_1\,e^{-\alpha\Box}\,, \qquad \Phi = c_2\,e^{-\alpha\Box}\,. \qquad (15)$$

It is easy to see that by changing the coefficients c_1 and c_2 one modifies only the cosmological constant-type counterterm, and not the fourth-derivative ones, which are relevant for Eq. (6). Therefore, in order to provide finiteness one has to generalize the action (2). As it was discussed for the polynomial QG, this can be done by adding $\mathcal{O}(R^3_{\dots})$- and $\mathcal{O}(R^4_{\dots})$-type terms. The explicit calculation in this theory would be quite difficult and also there is no real need to make it. Let us instead present a general evaluation of the possible effect of the $\mathcal{O}(R^3_{\dots})$-type terms. The general form of the terms with a minimal possible non-local insertion is

$$\frac{1}{M^2}\int \sqrt{-g}\,R_{\dots}R_{\dots}\,e^{-\alpha\Box}\,R_{\dots}\,, \qquad (16)$$

where M is a new massive parameter. Then the operator (14) will have a general non-minimal structure (after an appropriate gauge-fixing)

$$\hat{H} \propto \hat{\Box} + M^{-2}\,\hat{\mathcal{D}}^{\mu\nu}\nabla_\mu\nabla_\nu + \hat{\Pi}\,, \qquad (17)$$

where all the operators act in the space of quantum metrics, and $\hat{\mathcal{D}}^{\mu\nu}$ and $\hat{\Pi}$ are proportional to the curvature tensor. The contribution of $\mathrm{Tr\,Log}\,\hat{H}$ for the operators of the form (17) is known, in particular it was elaborated recently in [28] by means of the generalized Schwinger–DeWitt technique [34]. One meets the one-loop divergences which are given by an infinite series in curvatures $\hat{\mathcal{D}}^{\mu\nu}$ and $\hat{\Pi}$, and the (super)renormalizability of the theory is completely broken, so we have to look for some generalization.

Another possibility is to modify the expression (16) by introducing further non-localities. The possible solution is to consider the non-local terms of the general form

$$\int \sqrt{-g}\,R_{\dots}\frac{1}{\Box}R_{\dots}\,e^{-\alpha\Box}\,R_{\dots}\,. \qquad (18)$$

The one-loop divergences in the theories of similar type were already considered in the literature [35]. Let us note that the expression (18) still leaves us a lot of freedom in the choice of the action, because of the numerous possible tensor structures and corresponding coefficients. Since the number of the possible tensor structures in the operator $\hat{H}$ is restricted, there is a good chance to meet such a combination of terms in (18) which would lead to the operator

$$\hat{H} \propto \hat{\Box}^2 + \hat{V}^{\mu\nu}\nabla_\mu\nabla_\nu + \hat{U}\,, \qquad (19)$$

plus some contribution of the operator $\hat{\Box}^{-1}$ which can be factorized out in a standard way (see, e.g., Chapter 9 of [3]). In the expression (19) one still has the freedom to choose the operator $\hat{V}^{\mu\nu}$, which is proportional to the curvature tensor. As a result, it is possible to manipulate the divergent part of effective action (11) and to provide the desirable pre-fixed values for the coefficients $a_{1,2\dots6}$. In particular, there is a chance to obtain a finite QG in this way.

The situation may be even more simple if we include the $\mathcal{O}(R^4_{\dots})$-type terms with an additional $\Box^{-2}$ insertion. In this case the relevant operator will have the form

$$\hat{H} \propto \hat{\Box}^3 + \hat{V}^{\mu\nu\alpha\beta}\nabla_\mu\nabla_\nu\nabla_\alpha\nabla_\beta + \hat{U}^{\mu\nu}\nabla_\mu\nabla_\nu + \hat{W}\,, \qquad (20)$$

similar to the one we dealt with in [21]. The operator $\hat{U}^{\mu\nu}$ will be linearly proportional to the coefficients of the $\mathcal{O}(R^4_{\dots})$-type terms. On the other hand, linear dependence will also take place between $\hat{U}^{\mu\nu}$ and the fourth-derivative terms in (11). Therefore, there are pretty good chances to provide finiteness in the exponential QG theory by means of a special choice of the coefficients of the $\mathcal{O}(R^4_{\dots})$-type terms with an appropriate non-local insertion.

Indeed, the possibility to have a finite theory in the non-local case is not so certain as in the polynomial QG (7). In case of the non-finite theory the ghost-free structure will be certainly violated. So, let us be generous to the exponential QG and simply assume that the non-local theory can be made finite in the way we described above. As we shall see right now, this is still not sufficient to prevent the theory from the ghost-like states. The consistent theory of QG should include quantization of matter fields, not only the metric. The matters fields of the spin-0, spin-1/2 and spin-1 contribute to the divergences in the form of Eq. (11) [2,3]. For the illustration purpose, let us reproduce the complete form factors of the one-loop quantum corrections to the Φ-function in Eq. (2), derived in [27,29] for massive scalar and fermion fields,

$$\bar{\Gamma}^{(1)}_{scal} = \frac{1}{32\pi^2}\int d^4x\sqrt{-g}\,C_{\rho\sigma\alpha\beta}\Big[\frac{1}{60\,(4-n)} + \frac{1}{120}\,\ln\Big(\frac{4\pi\mu^2}{m^2}\Big) + \frac{1}{2}\,k^s_W(a)\Big]C^{\rho\sigma\alpha\beta} \qquad (21)$$

and

$$\bar{\Gamma}^{(1)}_{ferm} = \frac{1}{32\pi^2}\int d^4x\sqrt{-g}\,C_{\rho\sigma\alpha\beta}\Big[\frac{1}{10\,(4-n)} + \frac{1}{20}\ln\Big(\frac{4\pi\mu^2}{m_f^2}\Big) + \frac{1}{2}k_W^f(a)\Big]C^{\rho\sigma\alpha\beta}\,, \tag{22}$$

where

$$k_W(a) = \frac{240A + 20a^2 + 3a^4}{450\,a^4}\,, \tag{23}$$

$$k_W^f(a) = \frac{300Aa^2 - 480\,A - 40a^2 + 19\,a^4}{225\,a^4} \tag{24}$$

and we used notations

$$A = 1 - \frac{1}{a}\ln\frac{1+a/2}{1-a/2} \quad \text{and} \quad a^2 = \frac{4\Box^2}{\Box^2 - 4m^2}\,. \tag{25}$$

The contributions to the Ψ-function are qualitatively similar [29], but we do not reproduce them here for the sake of brevity.

The first observation is that the divergences in the Weyl-squared sector have the same sign independent on whether we take scalar, fermion, massless or massive vector. This well-known feature of the divergences [2] means that no cancellation of the overall contribution to the Φ-function due to supersymmetry is possible. Therefore, if there is no cancellation with the divergences coming from the QG sector, the expression (4) gains $p^4\log p^2$ contribution due to the matter fields loops and this is certainly sufficient to have infinitely many ghost-like excitations at the quantum level.

The second important point is that, even if the cancellation of the divergences really takes place, it is not sufficient to preserve the ghost-free structure of the theory even at the one-loop level. The reason is that both expressions (23) and (24) have an infinite set of sub-logarithmic contributions, and those cannot be canceled by the QG part. The situation may be, in principle, different in a strictly massless theory of matter, when the cancellation in the leading-log part may be sufficient.

Furthermore, even if one can adjust the QG contribution to make the theory completely free of divergences in the presence of matter fields, this would be true only at the one-loop level. Let us remember that the exponential QG has only one-loop divergences, but for the matter loops this is not so. Starting from the second loop, matter fields produce the form factors with higher powers of $\log(\Box/\mu^2)$ in the UV. Then the compensation seems to be completely impossible. Let us note that the β-functions in the matter sector are not affected by QG in all superrenormalizable models, as it was explained in Section 5 of [21].

One can naturally expect that the same breaking will take place in the case of pure QG without matter. Indeed, any quantum theory, including (2) may produce sub-logarithmic contributions in the dressed propagator, and then an absolute fine-tuning leading to the ghost-free structure will be violated. Unfortunately, the explicit results concerning sub-logarithmic contributions in QG are not available, but there cannot be much doubts about their existence. And the last is sufficient for violating an absolute tuning leading to the ghost-free structure of exponential gravity (4). So we have to conclude that the exponential QG has an infinite set of massive ghost-like states at the quantum level.

5. Note concerning Newtonian singularity

There is a very interesting and simple relation between the renormalizability of the QG theory and the absence of Newtonian singularity at the classical level. This relation was first noted by Stelle in [5], the main formula for the modified Newtonian potential is

$$\varphi(r) = -GM\Big[\frac{1}{r} - \frac{4}{3}\frac{e^{-m_{(2)}r}}{r} + \frac{1}{3}\frac{e^{-m_{(0)}r}}{r}\Big]\,. \tag{26}$$

Here $m_{(2)}$ and $m_{(0)}$ are masses of the spin-2 ghost and spin-0 massive particle which are present in the spectrum of the fourth-derivative gravity. It is easy to see that near the origin $r=0$ the contribution of these two massive degrees of freedom exactly cancels the one of the graviton, such that the limit of the modified Newtonian potential $\varphi(r)$ at $r\to 0$ is free of singularity. In our recent work [23] it was shown that the same cancellation takes place in the more general theory of gravity with the action (7), if additional degrees of freedom in this theory correspond to the non-degenerate real massive poles.[5] For a while, there is no proof that the same effect takes place in the case of complex massive poles.

It is remarkable that the non-singular modified Newtonian limit takes place also in the exponential gravity theory (4). This was originally found by Tseytlin in [11]. Recently, exactly the same non-singular solution has been rediscovered in the papers [18,30,20]. An unfortunate detail apparently related to Ref. [30] is the evaluation of our preprint [23] which was done by Biswas et al. in [20]. On page 3 of their manuscript authors say "*It was not until recently though, that concrete criteria for any covariant gravitational theory (including infinite-derivative theories) to be free from ghosts and tachyons around the Minkowski vacuum was obtained by Biswas, Gerwick, Koivisto and Mazumdar (BGKM) [23,24], see also [25] for a recent re-derivation of the same results using auxiliary field methods.*".[6]

I believe that the evaluation of our work which was done in [20] is not correct for at least two reasons. First, in our Ref. [23] there is no use of auxiliary fields. Second, we did not explore or discuss "concrete criteria for any covariant gravitational theory (including infinite-derivative theories) to be free from ghosts and tachyons around the Minkowski vacuum" and moreover we even did not deal with the "infinite-derivative theories" in [23]. As it was already mentioned above, our work [23] is about Newtonian singularity in the polynomial theory (7), so it is not easy to understand what the observation of [20] actually means.

Coming back to the relation between (super)renormalizability of QG and the absence of Newtonian singularity, perhaps the most intriguing aspect is that the theory without real poles can be free of singularities. One can note that the polynomial theory with a form factor given by the partial sums of the exponential function (12) has no real poles in the propagator. From the other side, the limit at $N\to\infty$ is free of Newtonian singularity, according to [11] and to the consequent publications on the subject [12,30]. Let us remember that the polynomials with growing N have growing amount of massive poles. One most natural physical interpretation is as follows. These poles are organized in such a way that they lead to the cancellation of singularity in a way similar to (26) and to the more complicated relations discussed in [23]. Then the cancellation of singularity in the exponential case of [11] is nothing else but the same effect coming from an infinite amount of the "hidden" ghosts. This conjecture is something interesting to verify, in our opinion it would give better understanding of the relation between local and non-local models of QG.

[5] The spin-0 contribution was elaborated much earlier in [32].

[6] Here *[23,24]* correspond to [30] and [31] and *[25]* to the citation [23] of the present work. Let me stress that our paper [23] is devoted to the Newtonian singularity in the polynomial theory (7) and hence we did not repeat the result of [11] and consequent works such as [12] and [30], which calculated the modified Newtonian limit in the exponential theory case.

6. Conclusions and discussions

The main conflict of QG is between renormalizability and unitarity. In order to have renormalizable or superrenormalizable QG, one has to include higher derivatives into the starting action. Higher derivatives lead to ghosts and/or tachyons and excluding these unphysical states from the spectrum produces violation of unitarity. Since higher derivatives emerge already at the semiclassical level, apparently there is no way to avoid them, so the question is how to deal with the massive ghosts.

Some qualitatively new approach to the problem of QG was suggested in [10] and recently elaborated further in [18,19] and [20]. This new approach assumes that the starting theory is chosen in such a way that the higher derivative theory is free of ghosts from the very beginning. Such a choice implies that the theory must be non-local, in a way explored earlier by Efimov et al. [17].

We have shown that in the case of exponential QG the usual power counting evaluation must be modified and the main role is played by the topological relation between the number of vertices and internal lines. After all, the renormalization properties of the non-local theory of [10] are very similar to the ones of the superrenormalizable QG, introduced earlier in [21]. In particular, this means that the β-functions of the matter fields are not affected by QG and that the β-functions in the gravitational sector can vanish, rending the theory finite.

While the classical theory of exponential gravity is ghost-free, the quantum corrections may easily lead to the dressed propagator which has infinitely many complex ghost-like poles. We have shown that this scenario is unavoidable if the theory is not finite. The finiteness in such a theory is possible by tuning the $\mathcal{O}(R^3_{...})$ and $\mathcal{O}(R^4_{...})$-type non-local terms in the action. This may guarantee the absence of the strongest logarithmic corrections and the result can be extended even to the one-loop theory with quantum matter included. However, at higher loops this tuning breaks down. On the other hand, even the one-loop contributions of massive matter fields have well-established sub-logarithmic contributions, which cannot be canceled in the exponential QG model. The final conclusion is that an infinite amount of unphysical complex poles emerge in the theory at the quantum level.

In our opinion, an improved understanding of the role of ghosts is one of the most relevant issues for QG. In particular, the main lesson which one should learn from the comparison of polynomial and exponential models of QG is the importance of the models with complex poles, which were not covered by the consideration of [21]. It would be interesting to treat this case in detail in both quantum field theory framework and in the more phenomenological way proposed recently in [33].

Acknowledgements

Author is grateful to CNPq, FAPEMIG and ICTP for partial support of his work and to the Département de Physique Théorique and Center for Astroparticle Physics at the Université de Genève for partial support and kind hospitality during his sabbatical period. I am also grateful to Theoretisch-Physikalisches-Institut of the Friedrich-Schiller-Universität in Jena for the warm hospitality during the time when the first version of this paper was completed.

Appendix A. Brief review of Lagrangian quantization

Let us consider the Faddeev–Popov procedure for the theory of QG based on the action (2) with possible additional terms of the $\mathcal{O}(R^3_{...})$- and $\mathcal{O}(R^4_{...})$-type. Higher order terms have no much importance, because they do not affect the divergences, in case of the "right" distribution of non-local exponential factors. Our treatment will cover both polynomial and exponential choices of the form factors $\Phi = \Phi(\Box)$ and $\Psi = \Psi(\Box)$ in Eq. (2). As we have noted in the main text, in the exponential case the theory can be superrenormalizable only if the functions Φ and Ψ have the same exponential factor, let's call it $\exp(-\alpha\Box)$. In the polynomial QG of Ref. [21] the requirement is less rigid, namely Φ and Ψ should be polynomials of the same order. For the sake of simplicity, let us assume that $\Phi = \Psi$. Our consideration will be partially repeating the one of [5,3] and [21] and we include it mainly to provide consistent presentation and to discuss special features of the non-local case.

We assume that the quantum metric $h_{\mu\nu}$ is defined as $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$. Let us introduce the gauge fixing condition in the form

$$S_{\rm gf} = \int d^4x \, \chi_\mu \, Y^{\mu\nu} \, \chi_\nu \,, \tag{27}$$

with the following form of the gauge-fixing and weight functions:

$$\chi_\mu = \partial_\lambda h^\lambda_\mu - \beta \, \partial_\mu h$$
$$Y^{\mu\nu} = -\frac{1}{\tau} \, \Omega(\Box) \, \big(g^{\mu\nu}\Box + \gamma \, \nabla^\mu \nabla^\nu\big) \,. \tag{28}$$

Here $h = h_{\mu\nu}\eta_{\mu\nu}$ and β, τ, γ are gauge-fixing parameters. Furthermore, $\Omega(\Box)$ is a function, which can be chosen by the convenience criteria. Our first purpose is to have a non-degenerate bilinear form of the action, therefore it is useful to choose $\Omega(\Box) = \Phi(\Box) = \Psi(\Box)$. As it was discussed in the main text, the divergences do not depend on the choice of the gauge fixing. For this reason we will not discuss the most general form of the weight function, which may be dependent on the curvature tensor. Also, the gauge-fixing parameters can be chosen to make the consideration simpler.

The most general bilinear form of the action on the flat background is

$$\begin{aligned} S^{(2)} = \frac{1}{2} \int d^4x \, h^{\kappa\omega} \Big\{ & k_1 \delta_{\kappa\omega,\rho\sigma} \Box^2 + k_2 g_{\kappa\omega} g_{\rho\sigma} \Box^2 \\ & + k_3 \big(g_{\kappa\omega}\partial_\rho\partial_\sigma + g_{\rho\sigma}\partial_\kappa\partial_\omega\big) \\ & + k_4 \big(g_{\kappa\rho}\partial_\omega\nabla_\sigma + g_{\kappa\sigma}\partial_\omega\nabla_\rho + g_{\omega\rho}\partial_\rho\partial_\sigma + g_{\omega\sigma}\partial_\kappa\partial_\rho\big) \\ & + k_5 \partial_\omega\partial_\kappa\partial_\rho\partial_\sigma \Big\} h^{\rho\sigma} \,. \end{aligned} \tag{29}$$

Here k_i, $i = 1, \dots, 5$ are some functions of $\Box$, which depend on the choice of the theory. In case of the polynomial theory (7) they are polynomials, while in the case of the exponential theory they are all proportional to $\Box \exp\big(-\alpha\Box\big)$. The explicit form of these functions can be found in [30] and [18], but we do not need them here.

Since the gauge-fixing parameters β, τ and γ do not affect divergences, one can choose them in such a way that the bilinear form of the overall action $S^{(2)} + S_{\rm gf}$ becomes minimal. This means that the tensor structures proportional to k_3, k_4 and k_5 cancel. Then, since the remaining k_1 and k_2 will be proportional to $\Phi(\Box)$, the propagator of the quantum metric has $r_l = 4 + 2N$ for the $\Phi(\Box)$ being polynomial of order N, and $r_l = \mathcal{I}$ for the exponential.

In order to apply the power counting relations (8) and (9), one has to provide that the Faddeev–Popov ghosts have the same power of momenta r_l in the bilinear form of their action. This can be achieved by using the method suggested by Fradkin and Tseytlin [24]. The presence of extra derivatives in the form factors of the initial action does not affect the scheme [21], in both polynomial and exponential models. Let us introduce the modified form of the ghost action,

$$S_{gh} = \int d^4x\sqrt{-g}\,\bar{C}_\alpha\, Y^\alpha_{\ \beta}\, M^\beta_{\ \gamma}\, C^\gamma\,,$$
$$\text{where}\quad M^\beta_{\ \gamma} = \frac{\delta\chi^\beta}{\delta h_{\rho\sigma}}\, R_{\rho\sigma.\gamma}\,, \tag{30}$$

where $R_{\rho\sigma.\gamma}$ is the generator of gauge transformations of $h_{\mu\nu}$ in the background field method. An extra insertion of the weight function in the definition of the ghost action in (30) provides that for the quantum metric and for the Faddeev–Popov ghosts there will be the same r_l in the formula (8).

The effective action Γ is defined as [24]

$$e^{i\Gamma[g_{\mu\nu}]} = \left(\text{Det}\, Y^\beta_\alpha\right)^{-1/2} \int dh_{\mu\nu} d\bar{C}_\alpha dC^\beta\, e^{iS+iS_{gf}+iS_{gh}}\,. \tag{31}$$

The remaining problem is how to evaluate the functional determinant $\text{Det}\, Y^\beta_\alpha$ in the last expression. For the polynomial QG theory (7), this operator is of the standard sort,

$$Y^\beta_\alpha = \Box^{2k}\left(\Box^2\,\delta^\alpha_\beta + \lambda\nabla^\alpha\nabla_\beta\right) + \mathcal{O}(R_{...}) \times \nabla^{2k}_{...} + \dots\,, \tag{32}$$

considered in [21]. This type of operator can be elaborated by the generalized Schwinger–DeWitt technique of Barvinsky and Vilkovisky [34]. The divergent contribution of this expression is $\mathcal{O}(R^2_{...})$, confirming the power counting-based analysis.

In the exponential case the contribution of the weight operator in (32) is more complicated. It is easy to see that $\text{Det}\, Y^\beta_\alpha$ is factorized into the product of determinants of the two operators. One of these operators is trivial and for the second one has to evaluate

$$\log\,\text{Det}\,\Omega(\Box) = \log\,\text{Det}\,\exp\left(-\alpha\Box\right) = \text{Tr}\,\log\exp\left(-\alpha\Box\right)$$
$$= \text{Tr}\left(-\alpha\Box\right). \tag{33}$$

The last expression has quadratic divergences, but it does not mean there are no logarithmic ones too, as usual. The evaluation of it can be performed by local momentum representation or by the Schwinger–DeWitt technique. Unfortunately, the operator (33) seems to be unappropriate for the technique of [34]. However, there are strong reasons to suppose that the result will be qualitatively the same as for the polynomial case. In order to see this, one has to regard the $\exp\left(-\alpha\Box\right)$ as a limit of the expression (12). In order to complete the story, one has to note that a qualitatively different result for the divergent part of (33) would mean one more addition to Eq. (6). This would further enforce the main conclusion of the present paper, concerning the presence of the infinite set of ghosts in the dressed propagator of the exponential theory.

References

[1] R. Utiyama, B.S. DeWitt, J. Math. Phys. 3 (1962) 608.
[2] N.D. Birrell, P.C.W. Davies, Quantum Fields in Curved Space, Cambridge Univ. Press, Cambridge, 1982.
[3] I.L. Buchbinder, S.D. Odintsov, I.L. Shapiro, Effective Action in Quantum Gravity, IOP Publishing, Bristol, 1992.
[4] I.L. Shapiro, Class. Quantum Gravity 25 (2008) 103001.
[5] K.S. Stelle, Phys. Rev. D 16 (1977) 953.
[6] E. Tomboulis, Phys. Lett. B 70 (1977) 361;
E. Tomboulis, Phys. Lett. B 97 (1980) 77;
E. Tomboulis, Phys. Rev. Lett. 52 (1984) 1173.
[7] A. Salam, J. Strathdee, Phys. Rev. D 18 (1978) 4480.
[8] I. Antoniadis, E.T. Tomboulis, Phys. Rev. D 33 (1986) 2756.
[9] D.A. Johnston, Nucl. Phys. B 297 (1988) 721.
[10] E.T. Tomboulis, Superrenormalizable gauge and gravitational theories, arXiv:hep-th/9702146.
[11] A.A. Tseytlin, Phys. Lett. B 363 (1995) 223, arXiv:hep-th/9509050.
[12] W. Siegel, Stringy gravity at short distances, arXiv:hep-th/0309093.
[13] B. Zwiebach, Phys. Lett. B 156 (1985) 315.
[14] S. Deser, A.N. Redlich, Phys. Lett. B 176 (1986) 350.
[15] A.A. Tseytlin, Phys. Lett. B 176 (1986) 92.
[16] A.L. Maroto, I.L. Shapiro, Phys. Lett. B 414 (1997) 34.
[17] G.V. Efimov, Non-Local Interactions of Quantum Fields, Nauka, Moscow, 1977 (in Russian).
[18] L. Modesto, Phys. Rev. D 86 (2012) 044005, arXiv:1107.2403;
L. Modesto, Astron. Rev. 8 (2) (2013) 4, arXiv:1202.3151;
L. Modesto, L. Rachwal, Nucl. Phys. B 889 (2014) 228, arXiv:1407.8036.
[19] L. Modesto, S. Tsujikawa, Phys. Lett. B 727 (2013) 48–56, arXiv:1307.6968.
[20] S. Talaganis, T. Biswas, A. Mazumdar, Towards understanding the ultraviolet behavior of quantum loops in infinite-derivative theories of gravity, arXiv:1412.3467.
[21] M. Asorey, J.L. López, I.L. Shapiro, Int. J. Mod. Phys. A 12 (1997) 5711.
[22] D.M. Gitman, I.V. Tyutin, Quantization of Fields with Constraints, Springer-Verlag, Berlin, Heidelberg, 1990.
[23] L. Modesto, T. de Paula Netto, I.L. Shapiro, On Newtonian singularities in higher derivative gravity models, arXiv:1412.0740.
[24] E.S. Fradkin, A.A. Tseytlin, Nucl. Phys. B 201 (1982) 469.
[25] I.L. Shapiro, A.G. Jacksenaev, Phys. Lett. B 324 (1994) 284.
[26] P.M. Lavrov, I.L. Shapiro, J. High Energy Phys. 1306 (2013) 086, arXiv:1212.2577.
[27] E.V. Gorbar, I.L. Shapiro, J. High Energy Phys. 02 (2003) 021, arXiv:hep-ph/0210388.
[28] T. de Paula Netto, I.L. Shapiro, Phys. Rev. D 89 (2014) 104037, arXiv:1402.3152.
[29] E.V. Gorbar, I.L. Shapiro, J. High Energy Phys. 06 (2003) 004, arXiv:hep-ph/0303124.
[30] T. Biswas, E. Gerwick, T. Koivisto, A. Mazumdar, Phys. Rev. Lett. 108 (2012) 031101.
[31] T. Biswas, T. Koivisto, A. Mazumdar, Nonlocal theories of gravity: the flat space propagator, arXiv:1302.0532.
[32] I. Quandt, H.-J. Schmidt, Astron. Nachr. 312 (1991) 97, arXiv:gr-qc/0109005.
[33] F. de O. Salles, I.L. Shapiro, Phys. Rev. D 89 (2014) 084054, arXiv:1401.4583;
F. de O. Salles, I.L. Shapiro, Phys. Rev. D 90 (2014) 129903(E) (Erratum).
[34] A.O. Barvinsky, G.A. Vilkovisky, Phys. Rep. 119 (1985) 1.
[35] S. Ichinose, Phys. Lett. B 251 (1990) 39.