# Collision of ultra-relativistic proton with strong magnetic field: Production of ultra-high energy photons and neutrinos 

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#### Abstract

Proton-proton interaction and photo-hadronic interaction in cosmic accelerators are the two main channels for the production of cosmic ultra-high energy photons and neutrinos (TeV-PeV). In this Letter, we use FWW approach to obtain the production of cosmic ultra-high energy photons and neutrinos from the collision between UHE proton with magnetic field which could be considered as the virtual photon in the rest frame of UHE proton. We name this as $p B$ process. The threshold for the occurrence of the $p B$ process is that the combination of the Lorentz factor of proton and the strength of the magnetic field is about $\gamma_{p} B \simeq 5 \times 10^{18}$ Gauss. Beyond this threshold, the rate of energy loss of proton due to the $p B$ process is about three orders higher than that due to the synchrotron radiation of proton in the same magnetic field. The $p B$ process might potentially happen in the atmosphere of white dwarfs, neutron stars or even that of stellar massive black holes.


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## 1. Introduction

Six years after its completion, IceCube has detected more than 80 neutrinos with energies between 100 TeV and 10 PeV [1]. ${ }^{1}$ For the production of ultra-high energy (UHE) neutrinos whose energy is above TeV , generally a cosmic accelerator for UHE protons and a target which could be either protons or radiation field are needed. In this Letter, we suggest that magnetic field could be another target. As seen by an UHE proton, the static magnetic field is equivalent to electromagnetic field, the collision between the UHE proton and the equivalent electromagnetic field (virtual photons) could produce UHE pions $\left(\pi^{0, \pm}\right)$, decay of pions produce UHE photons and neutrinos, which is very similar to the photo-hadronic interaction. We name the collision of the UHE proton with magnetic field as the " $p B$ " process.

The original idea of virtual quanta is from Fermi (1924), Weizsacker (1934), and Williams (1935), who proposed that the static electric field of nuclei as seen by a relativistic electron is equivalent to the electromagnetic field, the scattering of the electromagnetic field by the relativistic electron produces the so called bremsstrahlung radiation [2-5]. The Fermi-Weizsacker-Williams

[^0](FWW) method was also successfully applied to unify the synchrotron radiation and the inverse Compton scattering [6]. Applying the FWW method, Zhang \& Yuan (1998) obtained the quantum-limited synchro-curvation radiation formulae, which is very difficult to obtain in quantum electrodynamics [7]. Only recently, there appears an attempt to derive the quantum theory of synchro-curvation radiation from first principles $[8,9]$.

As protons strongly couple to meson field, proton synchrotron emission in a strong magnetic field could also produce pions and other mesons, similar to the production of photons. $\pi^{0}$ and other heavy meson productions of proton synchrotron emission have been studied in semi-classical approximations [10,11]. This process is also calculated in a fully relativistic and quantum-mechanical way [12] including the effect of proton anomalous magnetic moment in a strong magnetic field of $10^{18} \mathrm{G}$. Using scaling relations [13], the decay width can be extrapolated to the proton energy $\sim \mathrm{TeV}$ in magnetic fields of $10^{15} \mathrm{G}$, where the Landau numbers of the initial and final protons are $n_{i, f} \sim 10^{12}-10^{13}$.

However, in the above quantum-mechanical analysis of the pion $\left(\pi^{0}\right)$ production from ultra-relativistic proton in strong magnetic field, only one channel for the production of $\pi$ meson are considered: $p+B \rightarrow p+\pi^{0}$ (direct-pion production). Actually, there are many channels for the photo-hadronic interaction, such as, $\Delta$-resonance, $N$-resonance, direct-pion production and multi-pion production, and the dominate process is $\Delta$-resonance ([14,15]).

And the quantum-mechanical calculations is only valid for the case of the strong uniform straight magnetic field. If not, it is very difficult to solve the Dirac equation of proton in a curve magnetic field. Therefore, in this letter we discuss the $\pi^{0, \pm}$ production from the point view of FWW approach. There are several advantages in our method. First, the contributions from all channels for the production of $\pi^{0, \pm}$ are considered by using the total $p \gamma$ photon-meson cross section (for example, see Figure 1 in $[14,16]$ ). Second, the production rate for $\pi^{ \pm}$is also provided in our method, the decay of $\pi^{ \pm}$is one of the main production mechanisms of ultra high energy neutrinos in astrophysical sources. Third, according to our argument, our results are also valid in curved magnetic fields, even chaotic magnetic fields, provided that the Larmor radius of proton is less than the curvature radii of the local magnetic field [17]. Finally, our method is physical transparent, which is very helpful for understanding the results obtained in more theoretical calculations.

## 2. Semi-quantitative analysis

As a first step, a semi-quantitative analysis of the $p B$ process is presented to show the basic idea of " $p B$ " process via the FWW approach, the detailed analysis can be found in the following discussion.

Suppose there is a uniform straight magnetic field $B$ along $z-$ axis direction in the laboratory frame $\Sigma$. Now an UHE proton with Lorentz factor $\gamma_{p}$ is colliding with the magnetic field. Ignore the proton velocity in the $z$-axis, the electromagnetic field in the proton instantaneous rest frame $\Sigma^{\prime}$ can be estimated via the Lorentz transformation to be
$E^{\prime}=\gamma_{p} B\left(\frac{v_{y}}{c},-\frac{v_{x}}{c}, 0\right) ; B^{\prime}=\gamma_{p} B(0,0,1)$,
where $\vec{v}$ is the velocity of proton in the laboratory frame $\Sigma$. For ultra-relativistic particle, $\gamma_{p} \gg 1$, and $v \rightarrow c$, so the effect of passing segment of the field resembles an electromagnetic wave propagating in the $-\vec{v}$ direction, as seen by the observer in the instantaneous rest frame of proton. The apparent Poynting energy flux $S^{\prime}$ in the rest frame is $\sim c \gamma_{p}^{2} B^{2} /(4 \pi)$.

First, collision of the UHE proton with the magnetic field lead to the energy loss of the proton via the synchrotron radiation. Using the FWW approach, we can obtain the total energy loss rate of the proton synchrotron radiation, which is given by the product of the Poynting energy flux $S^{\prime}$ and the proton Thomson cross section $\sigma_{\mathrm{T}}^{p}=8 \pi e^{4} /\left(3 m_{p}^{2} c^{4}\right)$, where $m_{p}$ is the mass of proton, viz.,
$P_{\mathrm{syn}} \simeq \sigma_{\mathrm{T}}^{p} S^{\prime} \simeq \frac{2 \gamma^{2} e^{4} B^{2}}{3 m_{p}^{2} c^{3}} \simeq \sigma_{\mathrm{T}}^{p} c \gamma_{p}^{2} U_{\mathrm{B}}$,
here $U_{\mathrm{B}}=B^{2} /(8 \pi)$ is the energy density of magnetic fields in the laboratory frame. This is the standard result for classical synchrotron radiation [6].

If we replace the Thomson cross section $\sigma_{\mathrm{T}}^{p}$ with the cross section of photo-meson interaction, we can estimate the energy loss rate of the $p B$ process. It is well known that the cross section of the photon-meson interaction is energy-dependent. For simplicity, we first consider the resonant interaction, which dominates the process of photo-meson interaction. The cross section of the resonant interaction is $\sigma_{\mathrm{res}} \simeq 200 \mu$ barn for the photons with the energy between 0.2 GeV and 0.5 GeV in the rest frame of proton. The total rate of the energy loss of the UHE proton in the resonant region is
$P_{p B} \simeq \sigma_{\mathrm{res}} S^{\prime}=\sigma_{\mathrm{res}} c \gamma_{p}^{2} U_{\mathrm{B}}$.


Fig. 1. Schematic shows a segment of the trajectory of proton in the laboratory frame. When $t=0$, proton is at point $O^{\prime}$, the origin of $\Sigma^{\prime}$ frame at $t=0$, and when $t=\Delta t$, proton moves to point A along a circular orbit. During the period of the movement of proton, the inertial rest frame of proton $\Sigma^{\prime}$ moves to a new place along $x^{\prime}$-axis with a constant velocity $v=R \omega_{0}$.

In the FWW approach, the typical frequency of the equivalent electromagnetic field is the cyclotron frequency $\omega_{\mathrm{c}}^{\prime}$ of proton in its rest frame,
$\omega_{\mathrm{c}}^{\prime}=\frac{\gamma_{p} e B}{m_{p} c}$.
Therefore, for the occurrence of the $p B$ process, the energy of the equivalent photon should be in the range of 0.2 GeV and 0.5 GeV , i.e.
$0.2 \mathrm{GeV}<\hbar \omega_{c}^{\prime}=\frac{\gamma_{p} \hbar e B}{m_{p} c}<0.5 \mathrm{GeV}$,
which requires the product of proton Lorentz factor $\gamma_{p}$ and strength of magnetic field in the range of
$0.2 B_{\mathrm{c}}^{p}<\gamma_{p} B<0.5 B_{\mathrm{c}}^{p}$.
Here $B_{\mathrm{c}}^{p}=m_{p}^{2} c^{3} /(\hbar e)=1.5 \times 10^{20} \mathrm{G}$ is the critical magnetic field of proton.

The Thomson cross section of proton is $\sigma_{\mathrm{T}}^{p} \approx 0.20 \mu$ barn, compare with Eq. (2) and Eq. (3), it is evident that the $p B$ process dominates the energy loss of proton, when the $p B$ process switches on.

## 3. Proton magnetic field process

In the following, more detailed derivation of the rate of the energy loss of UHE proton, and relevant emissivity of UHE neutrinos are given. As well known, a relativistic proton moves in a circular orbit in the laboratory frame $\Sigma$,
$x=R \sin \omega_{0} t, \quad y=R \cos \omega_{0} t$,
where $R$ is the radius, $\omega_{0}=e B / \gamma_{p} m_{p} c$ is the cyclotron frequency of proton in the laboratory frame $\Sigma$. The proton velocity in the instantaneous rest frame $\Sigma^{\prime}$ is
$v_{x}^{\prime}=\frac{d x^{\prime}}{d t^{\prime}}=\frac{d t}{d t^{\prime}} \frac{d x^{\prime}}{d t}=-\frac{v\left(1-\cos \omega_{0} t\right)}{1-\frac{v^{2}}{c^{2}} \cos \omega_{0} t}$
$v_{y}^{\prime}=\frac{d y^{\prime}}{d t^{\prime}}=\frac{d t}{d t^{\prime}} \frac{d y^{\prime}}{d t}=-\frac{v \sin \omega_{0} t}{\gamma\left(1-\frac{v^{2}}{c^{2}} \cos \omega_{0} t\right)}$
As it is shown in Fig. 1, when $t=\Delta t$, the proton has a deviation from the origin point $O^{\prime}$ in its own rest frame $\Sigma^{\prime}$. In the regime of the classical radiation, which means that the motion of particle is classical, and the Landau quantization is unimportant, the acceleration of proton in its own rest frame $\Sigma^{\prime}$ is due to the force of
equivalent electric field $E^{\prime}\left(t^{\prime}\right)$. Moreover, using the Fourier analysis of the equivalent electric field $E^{\prime}\left(t^{\prime}\right)$, we can obtain the spectral Poynting flux $S^{\prime}\left(\omega^{\prime}\right)$ (energy per unit area per unit frequency) of the equivalent incident radiation, which is given by [6],

$$
\begin{align*}
S^{\prime}\left(\omega^{\prime}\right) & =c\left|\frac{1}{2 \pi} \int_{-\infty}^{\infty} E^{\prime} e^{i \omega^{\prime} t^{\prime}} d t^{\prime}\right|^{2} \\
& =c\left(\frac{\omega^{\prime} m_{p}}{2 \pi e}\right)^{2}\left|\int_{-\infty}^{\infty} v_{\perp}^{\prime}\left(t^{\prime}\right) e^{i \omega^{\prime} t^{\prime}} d t^{\prime}\right|^{2} . \tag{10}
\end{align*}
$$

This flux can be decomposed into two polarization modes ( $1,0,0$ ) and $(0,1,0)$,
$S^{\prime}\left(\omega^{\prime}\right)=c\left(\frac{\omega^{\prime} m_{p}}{2 \pi e}\right)^{2}\left(\left|I_{x}^{\prime}\right|^{2}+\left|I_{y}^{\prime}\right|^{2}\right)$,
where
$I_{x}^{\prime}=\int_{-\infty}^{\infty} v_{x}^{\prime}\left(t^{\prime}\right) e^{i \omega^{\prime} t^{\prime}} d t^{\prime}$,
$I_{y}^{\prime}=\int_{-\infty}^{\infty} v_{y}^{\prime}\left(t^{\prime}\right) e^{i \omega^{\prime} t^{\prime}} d t^{\prime}$.
Substitute Eq. (8)-(9) into Eqs. (12)-(13), and make the change of variable $t^{\prime} \rightarrow t$, and ignore terms of order $\lesssim 1 / \gamma_{p}^{2}$, the integral in Eqs. (12)-(13) can be obtained in terms of the modified Bessel functions (see also ref. [5,18]),

$$
\begin{align*}
I_{x}^{\prime} & \approx \frac{\sqrt{2} v}{\omega_{0} \gamma_{p}^{2}} \int_{-\infty}^{\infty} e^{i \frac{3}{2} \xi\left(a+\frac{1}{3} a^{3}\right)} d a \\
& =\frac{2 \sqrt{6}}{3}\left(\frac{c}{\omega_{0} \gamma_{p}^{2}}\right) K_{1 / 3}(\xi)  \tag{14}\\
I_{y}^{\prime} & \approx \frac{2 v}{\omega_{0} \gamma_{p}^{2}} \int_{-\infty}^{\infty} a e^{i \frac{3}{2} \xi\left(a+\frac{1}{3} a^{3}\right)} d a \\
& =\frac{4 \sqrt{3}}{3}\left(\frac{c}{\omega_{0} \gamma_{p}^{2}}\right) K_{2 / 3}(\xi), \tag{15}
\end{align*}
$$

where,
$a=\frac{\sqrt{2}}{2} \gamma_{p} \omega_{0} t, \quad \xi=\frac{2 \sqrt{2} \omega^{\prime}}{3 \gamma_{p}^{2} \omega_{0}}$.
From the properties of modified Bessel functions, it is evident that the flux peaks at $\xi \simeq 1$, that is,
$\omega_{\text {peak }}^{\prime} \simeq \gamma_{p}^{2} \omega_{0}=\frac{\gamma_{p} e B}{m_{p} c}=\omega_{c}^{\prime}$
Eventually, the Poynting flux in the instantaneous rest frame of proton reads
$S^{\prime}\left(\omega^{\prime}\right)=\frac{2 m_{p}^{2} c^{3}}{3 \pi^{2} e^{2}}\left(\frac{\omega^{\prime}}{\omega_{0} \gamma_{p}^{2}}\right)^{2}\left[K_{1 / 3}^{2}(\xi)+2 K_{2 / 3}^{2}(\xi)\right]$.
In order to get the power spectrum of the virtual photon $d^{2} P^{\prime} /$ $d \epsilon_{\mathrm{r}}^{\prime} d A^{\prime}$ (energy per unit time, per unit area and per unit photon

Table 1
Parameters for the $\Delta$-resonance (LR) and the higher resonance ( HR ). The range of energy of photons for the occurrence of resonance is determined by $\epsilon_{\min }$ and $\epsilon_{\max }$, $\sigma$ is the total cross section, and $K$ is the ratio of energy loss of proton [16].

| IT | $\epsilon_{\min }[\mathrm{GeV}]$ | $\epsilon_{\max }[\mathrm{GeV}]$ | $\sigma[\mu$ barn $]$ | $K$ |
| :--- | :--- | :--- | :--- | :--- |
| LR | 0.2 | 0.5 | 200 | 0.22 |
| HR | 0.5 | 1.2 | 90 | 0.39 |

energy), the Poynting flux should be divided by the cyclotron period in the instantaneous frame $2 \pi /\left(\gamma_{p}^{2} \omega_{0}\right)$,

$$
\begin{align*}
\frac{d^{2} P^{\prime}}{d \epsilon_{\mathrm{r}}^{\prime} d A^{\prime}}= & \frac{m_{p} c^{2} \gamma_{p} B}{3 \pi^{3} e \hbar}\left(\frac{\omega^{\prime}}{\omega_{0} \gamma_{p}^{2}}\right)^{2}\left[K_{1 / 3}^{2}(\xi)+2 K_{2 / 3}^{2}(\xi)\right] \\
\simeq & 4.7 \times 10^{51} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}\left(\frac{\gamma_{p} B}{B_{\mathrm{c}}^{p}}\right)^{-1}\left(\frac{\epsilon_{\mathrm{r}}^{\prime}}{m_{p} c^{2}}\right)^{2} \\
& {\left[K_{1 / 3}^{2}(\xi)+2 K_{2 / 3}^{2}(\xi)\right] } \tag{19}
\end{align*}
$$

where $\epsilon_{\mathrm{r}}^{\prime}$ is the energy of virtual photon in the particle instantaneous frame, and $\xi$ could be re-expressed as
$\xi=\frac{2 \sqrt{2}}{3}\left(\frac{\gamma_{p} B}{B_{\mathrm{c}}^{p}}\right)^{-1}\left(\frac{\epsilon_{\mathrm{r}}^{\prime}}{m_{p} c^{2}}\right)$
When $\omega^{\prime} \sim \omega_{\text {peak }}^{\prime}$, the power spectrum of virtual photons reaches its peak flux of

$$
\begin{equation*}
\frac{d^{2} P^{\prime}}{d \epsilon_{\mathrm{r}}^{\prime} d A^{\prime}} \sim \frac{\left(\gamma_{p} B\right)^{2} c}{4 \pi^{2} \hbar \omega_{\mathrm{peak}}^{\prime}}, \tag{21}
\end{equation*}
$$

which is in a good agreement with the result from the semiquantitative analysis.

Based on the spectrum of incident equivalent photons, the rate of the energy loss of proton can be estimated as follows,
$P_{\mathrm{p}, \text { loss }}\left(E_{p}\right)=\sum_{\mathrm{IT}}\left[K^{\mathrm{IT}} E_{p} \Gamma^{\mathrm{IT}}\left(E_{p}\right)\right]$,
where IT stands for the types of interaction, $K^{I T}$ is the averaged ratio of the energy loss of proton after the corresponding interaction, and $\Gamma^{I T}$ is the rate of interaction. The photo-meson interaction can be classified into three types: the resonant, direct, and multi-pion production. We use the simplified model B (Sim-B) in Hümmer (2010) [16], and concentrate on the resonances first. The total cross section of the resonances is dominated by the $\Delta(1232)$-resonance (LR) at low energy, and the higher resonances (HR) at high energy. The parameters for both resonances are shown in Table 1, and the parameters for the other types of interaction can be found in Hümmer (2010).

In this study, the interaction rate can be written as
$\Gamma^{\mathrm{IT}}\left(E_{p}, B\right)=\gamma_{p}^{-1} \int_{\epsilon_{\min }^{\mathrm{T}}}^{\epsilon_{\max }^{\mathrm{T}}} \epsilon_{\mathrm{r}}^{-1} \frac{d^{2} P^{\prime}}{d \epsilon_{\mathrm{r}}^{\prime} d A^{\prime}}\left(\epsilon_{\mathrm{r}}, \gamma_{p}, B\right) \sigma^{\mathrm{IT}}\left(\epsilon_{\mathrm{r}}\right) d \epsilon_{\mathrm{r}}$,
where $\epsilon_{\min }^{\mathrm{IT}}$ and $\epsilon_{\max }^{\mathrm{IT}}$ show the energy range of photons $\left(\epsilon_{\mathrm{r}}\right)$ for the occurrence of the resonance interactions, $\sigma^{\mathrm{IT}}\left(\epsilon_{\mathrm{r}}\right)$ is the total cross section of the corresponding resonant interaction in the proton rest frame.

Fig. 2 shows rates of energy loss of proton in the magnetic field with strength of $B=10^{11} \mathrm{G}$. It is clearly shown in the figure that as the increase of the energy of proton, the rate of energy loss due to the $p B$ process increases dramatically, and dominates the loss of energy. This happens at $\gamma_{p} \sim 10^{8}$, which is consistent with


Fig. 2. The rate of energy loss of ultra-relativistic proton as a function of its energy. The total rate due to the $p B$ process are shown with the black solid line, and that due to the synchrotron radiation with the blue solid line, while the red dashed line shows the contribution from the resonant interaction. The strength of the magnetic field is taken to be $B=10^{11} \mathrm{G}$.


Fig. 3. The rate of energy loss of ultra-relativistic proton as a function of its energy in different magnetic field: $B=10^{10} \mathrm{G}$ (red lines) and $B=10^{11} \mathrm{G}$ (black lines). Solid lines show the rates due to the $p B$ process, while the dashed lines show the rates due to the synchrotron radiation.

Eq. (6). To be more realistic, the contributions of the rate of energy loss due to the direct and multi-pion productions are also shown in Fig. 2. In the range of low energy, the resonances dominate the loss of energy, and the multi-pion production dominates the energy loss in the range of high energy.

The rates of energy loss of proton in the magnetic field with different strength are shown in Fig. 3. When $\gamma_{p} B \gtrsim 5 \times 10^{18} \mathrm{G}$, the rate due to the $p B$ process exceeds that due to the synchrotron radiation, and the $p B$ process dominates the energy loss of proton. In the resonances dominated region, the rate due to the $p B$ process is $\sim$ three orders greater than that due to the synchrotron radiation. Considering the broadened spectrum of incident equivalent photons, these results are consistent with our semi-quantitative analysis.

The $p B$ process produces the secondary gamma-ray photons and neutrinos, which resembles the conventional $p \gamma$ process. Nonthermal TeV-PeV photons can result from the decay of $\pi^{0}\left(\pi^{0} \rightarrow\right.$ $\gamma+\gamma)$. The Universe is not transparent to these photons from the cosmic distance, tens of TeV photons can annihilate with the cosmic background radiation into electron/positron pairs. Therefore, generally only softer and fainter photon spectrum would be observed.

However, the high-energy neutrinos can reach the Earth without annihilation. The total power of neutrino emission can be estimated as $P_{\nu}=\frac{3}{4} P_{p, \text { loss }}$, where the factor $\frac{3}{4}$ is the averaged energy fraction transferred from pion to neutrinos in the decay chains of $\pi^{+} \rightarrow \nu_{\mu} \mu^{+} \rightarrow v_{\mu} e^{+} \nu_{e} \bar{v}_{\mu}$ and $\pi^{-} \rightarrow \bar{v}_{\mu} \mu^{-} \rightarrow \bar{v}_{\mu} e^{-} \bar{v}_{e} v_{\mu}$, if the four final state leptons equally share the pion energy [19]. The total luminosity of neutrinos emission, which depends on the distribution of the number of injecting proton $N_{p}\left(E_{p}\right)$, can be given as
$L_{\nu, 0}=N_{p}\left(E_{p}\right) P_{\nu}$.
The primary pions and secondary muons would also undergo cooling or re-acceleration before they decay into neutrinos. The final neutrinos luminosity can be estimated as $L_{v}=f_{\mathrm{c}} L_{v, 0}$, where $f_{\mathrm{c}}$ is a correction factor for cooling or re-acceleration, which depends on the sources.

The initial neutrino flavor ratio at the source is $f_{e}: f_{\mu}: f_{\tau}=$ ( $\frac{1}{3}: \frac{2}{3}: 0$ ). For propagation over cosmological distances, due to the effect of neutrino oscillations, the three neutrino flavors approximately mix to the ratio of $(1: 1: 1)$ on the earth.

## 4. Conclusions and discussion

Pion production by proton synchrotron was also studied in semi-classical approximations [ 10,11 ] and in an exact quantum field theory treatment $[12,13]$. For $\pi^{0}$ meson, the result of semiclassical approximations and our result are basically on the same order of magnitude near the threshold energy. Our result has an additional $\pi^{0}$ hump via the multi-pion channel in the high energy region, which is not shown in the semi-classical approximation.

Note that the threshold energy of $\Delta(1232)$-resonance is $\sim 0.2$ GeV , which is less than the rest mass of proton. Therefore, near the resonance energy,
$\gamma_{p} B / B_{\mathrm{c}}^{p} \lesssim 1$,
the quantum effect has little influence on the rate of energy loss of proton [6]. Definitely, above the energy equivalent to the rest mass of proton, the effect of Landau energy levels of proton should be taken into account. The quantum result which studied in more realistic magnetic fields of $10^{15} \mathrm{G}$ and proton energies of $\sim \mathrm{TeV}$, argues that the anomalous magnetic moment of the proton can enhance the proton decay width significantly [13]. In our study, the Fourier analysis of virtual photon spectrum only considers the time evolution of electric field $\vec{E}(t)$. As the effect of anomalous magnetic moment is associated with magnetic field, this effect needs to use different cross section calculated by the full quantum field theory. This will be taken into consideration in our future study.

There are several potential astrophysical environments for the occurrence of the $p B$ process: first, UHE cosmic rays can be accelerated up to $10^{21} \mathrm{eV}$ in astrophysical sources, including active galactic nuclei, gamma-ray bursts, starburst galaxies and so on [20]. These cosmic rays can interact with the magnetic field of white dwarfs ( $B \sim 10^{6-9} \mathrm{G}$ ) in old stellar clusters [21] in their propagations to the Earth, which might make the $p B$ process take place. Second, neutron stars, such as magnetars, could have very strong magnetic field with strength up-to $B \sim 10^{14-15} \mathrm{G}$. During the merger of binary neutron stars, high magnetic fields might be produced [22-25], protons and other heavy elements can be accelerated and interact with magnetic field in the same system to produce UHE neutrinos and gamma-ray photons via the $p B$ process. Third, it is generally believed that it is difficult to produce UHE photons and neutrinos during the merger of binary black holes, because there is no conventional targets for the $p p / p \gamma$ interaction. However, it is proposed that a small amount of gases surrounding
a new-born black hole after the merger of binary black holes could support a strong magnetic field of $10^{11} \mathrm{G}$ in a limited time [26], which is helpful for the occurrence of the $p B$ process.

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