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Heavy particle non-decoupling in flavor-changing gravitational interactions

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The flavor-changing gravitational process, $d \to s +$ graviton, is evaluated at the one-loop level in the standard electroweak theory with on-shell renormalization. The results that we present in the 't Hooft–Feynman gauge are valid for on- and off-shell quarks and for all external and internal quark masses. We show that there exist non-decoupling effects of the internal heavy top quark in interactions with gravity. A naive argument taking account of the quark Yukawa coupling suggests that the amplitude of the process $d \to s +$ graviton in the large top quark mass limit would possibly acquire an enhancement factor m_t^2/M_W^2, where m_t and M_W are the top quark and the W-boson masses, respectively. In practice this leading enhancement is absent in the renormalized amplitude due to cancellation. Thus the non-decoupling of the internal top quark takes place at the $\mathcal{O}(1)$ level. The flavor-changing two- and three-point functions are shown to satisfy the Ward–Takahashi identity, which is used as a consistency check for the aforementioned cancellation of the $\mathcal{O}(m_t^2/M_W^2)$ terms. Among the $\mathcal{O}(1)$ non-decoupling terms, we sort out those that can be regarded as due to the effective Lagrangian in which quark bilinear forms are coupled to the scalar curvature.

Subject Index B06, B32, B56, B57, E00

1. Introduction

The discoveries of gravitational waves at frequencies $f > 10$ Hz by the LIGO and Virgo Collaborations via a binary black hole merger and a binary neutron star inspiral have been hailed as a major milestone of gravitational wave astronomy [1–4]. The gravitational wave is now expected to be an exquisite tool not only to study astronomical objects such as black holes and neutron stars, but also to probe viable extensions of general relativity as well as what lies beyond the Standard Model (BSM) of elementary particles. It would be extremely interesting if we could look into the early universe before the time of last scattering by searching for gravitational waves.

The recent analyses of the 12.5-yr pulsar timing array data at frequencies $f \sim 1$/yr by the NANOGrav Collaboration [5] in search of a stochastic gravitational wave background [6–8] are also of particular importance and are encouraging enough for us to speculate on BSM: cosmic strings or supermassive black holes as possible sources of gravitational waves, first-order phase transitions in the dark sector, new scenarios of leptogenesis induced by gravitational backgrounds, and so on. In search for new avenues of BSM with the help of stochastic gravitational waves, it sometimes occurs that one has to deal with gravitational interactions of heavy unknown particles, in particular on the quantum level. In such a case we are necessarily forced to pay attention to heavy particle mass effects on physical observables.

Bearing these new directions in our mind, we would like to present in this paper an example in which heavy particles running along internal loops in the gravitational backgrounds induce potentially large and important new types of interactions. Recall that an important issue in particle physics incorporating possible heavy particles has been whether heavy particles have power-suppressed and therefore negligibly small effects in low-energy processes (decoupling), or that their effects may be observable in the form of new induced interactions in the limit of very large mass (non-decoupling). To keep our investigation within a reasonable size, we specifically study the loop-induced flavor-changing process

$$d \to s + \text{graviton} \tag{1}$$

in the standard electroweak interactions, instead of launching into BSM studies. In our case the top quark is supposed to be the heavy particle as opposed to all the other light quarks. The reason for computing Eq. (1) is that the process (1) is analogous to the $d \to s + \gamma$ and $d \to s +$ gluon (Penguin) processes and that the latter two processes are known to exhibit top quark non-decoupling effects in low-energy decay phenomena. It is quite natural to expect that similar non-decoupling phenomena would take place in Eq. (1) and we will argue in the present paper that this expectation is in fact the case. So far as we know, this is the first example of the non-decoupling of the internal heavy top quark in *gravitational interactions of light quarks.*

One of the sources of the non-decoupling may be searched for in the unphysical scalar field coupling to quarks with a strength proportional to the quark masses. This can be seen most apparently in the Feynman rules in the 't Hooft–Feynman gauge, which we will use throughout. We are particularly interested in whether or not the process (1) would be enhanced by the large factor m_t^2/M_W^2, where m_t and M_W are the top quark and W-boson masses, respectively. A quick glance over the Feynman rules, in fact, tells us that apparently this large factor comes into the Feynman amplitude as the coupling of an exchanged unphysical scalar field to an internal top quark. However, we will show in the present paper by explicit calculation that this enhancement factor disappears due to cancellation among the terms of $\mathcal{O}(m_t^2/M_W^2)$ in the renormalized transition amplitude[1]. The breaking of the top quark decoupling thus takes place mildly on the $\mathcal{O}(1)$ level. We will confirm that the cancellation of the $\mathcal{O}(m_t^2/M_W^2)$ terms is consistent with the Ward–Takahashi identity associated with the invariance under the general coordinate transformation.

Appelquist and Carazzone [9] pointed out in the mid-1970s that virtual effects of heavy unknown particles can be safely neglected in low-energy phenomena, provided that coupling constants are all independent of heavy particle masses. This fact is often referred to as the decoupling theorem, which provides us with an effective strategy to handle low-energy experimental data without worrying much about unknown new physics. In the course of the development of particle physics towards the end of the last century, however, the tables have been turned: we now believe that non-decoupling phenomena are much more interesting than decoupled cases

[1] When we say "terms of $\mathcal{O}(m_t^2/M_W^2)$", it is implicitly assumed that logarithmically corrected terms such as $(m_t^2/M_W^2)\log(m_t^2/M_W^2)$ are also included. Likewise, $\mathcal{O}(1)$ terms are assumed to include $\log(m_t^2/M_W^2)$ terms as well.

and that we might be able to have a glimpse of the high-energy contents of the future theory of elementary particles by investigating non-decoupling phenomena.

In the standard electroweak theory, the Higgs boson and unphysical scalar fields are coupled to quarks with a strength proportional to the quark masses. For large quark masses, as for the case of the top quark, the breaking of the decoupling theorem is naturally expected and in fact non-decoupling phenomena are ubiquitous in the Standard Model. They include Higgs boson production in the *pp*-collision via gluon fusion process through a top quark loop [10–12], the various decay processes of the Higgs boson involving heavy quarks [13–18], heavy quark effects on the K_0–$\overline{K}_0$ and B_0–$\overline{B}_0$ mixings [19–24], etc. It is very interesting to see whether a similar explanation for non-decoupling of heavy top quark effects would also work in gravitational interactions of light quarks. This is actually a strong motivating force for us to examine Eq. (1).

After submitting the present paper for publication, we learned that the process (1) had once been computed in the 't Hooft–Feynman gauge and in the unitary gauge by Degrassi et al. [25] and was investigated by Corianò et al. [26,27] for a different purpose from ours. Their elaborate calculations, however, are not quite suitable for our use since they put external quarks on the mass shell, while we would like to make the non-decoupling phenomena manifest by studying *off-shell* effective interactions in the large top quark mass limit. Our one-loop calculation is made to this end.

The present paper is organized as follows. First of all we explain in Sect. 2 the method of putting "weight" on the fermion fields in the curved background to render the Feynman rules to be discussed in Sect. 3 a little simpler. The self-energy type $d \to s$ transition in Minkowski space is evaluated in Sect. 4, the result of which is closely connected with the counter terms eliminating the divergencies associated with Eq. (1) as argued in Sect. 5. In Sect. 6 we compute all the one-loop Feynman diagrams associated with Eq. (1). The renormalization constants prepared in Sect. 5 are shown in Sect. 7 to be instrumental to eliminate all the ultraviolet divergences in Eq. (1). It is argued in Sect. 8 that unrenormalized and renormalized quantities associated with Eq. (1) satisfy the same Ward–Takahashi identity. The terms in the renormalized transition amplitude behaving asymptotically as $\mathcal{O}(m_t^2/M_W^2)$ in the large top quark mass limit are investigated in Sect. 9 and are shown to vanish via mutual cancellation. The $\mathcal{O}(1)$ terms for the large top quark mass are also discussed in Sect. 10, highlighting those that can be expressed by an operator of quark bilinear form coupled to the scalar curvature. Section 11 is devoted to summarizing the present paper. Various definitions of Feynman parameter integrations are collected in Appendix A and some combinations thereof are defined in Appendix B.

2. Dirac fermions in a gravitational field

Techniques of loop calculations involving Dirac fermions in the curved spacetime, our central concern in studying Eq. (1), were discussed a long time ago by Delbourgo and Salam [(R. Delbourgo and A. Salam, unpublished data, 1972), 28] in connection with anomalies [29,30]. They took due account of the "weight factors" of fermions [31], which we now recapitulate while setting up our notations. Hereafter in this section we will use Greek indices μ, ν, etc. for labeling general coordinates and indices a, b, etc. for labeling the coordinates in a locally inertial coordinate system. The latter indices are raised and lowered by the Minkowski metric η^{ab} and η_{ab}, respectively.

The Lagrangian of fermions in the curved spacetime is as usual given by

$$\mathcal{L}_{\rm Dirac} = \sqrt{-g}\left\{\frac{i}{2}\left(\overline{\psi}\,\gamma^\mu\,\nabla_\mu\psi - \nabla_\mu\overline{\psi}\,\gamma^\mu\,\psi\right) - \overline{\psi}\,m\,\psi\right\}, \tag{2}$$

where our notations are

$$\gamma^\mu = e^\mu{}_a\gamma^a, \tag{3}$$

$$\nabla_\mu\psi = \partial_\mu\psi - \frac{i}{4}\omega_{\mu ab}\sigma^{ab}\,\psi\ ,\ \nabla_\mu\overline{\psi} = \partial_\mu\overline{\psi} + \frac{i}{4}\overline{\psi}\,\omega_{\mu ab}\sigma^{ab}, \tag{4}$$

$$\sigma^{ab} = \frac{i}{2}(\gamma^a\gamma^b - \gamma^b\gamma^a), \tag{5}$$

and $g = \det(g_{\mu\nu})$. The relation between the spacetime metric $g_{\mu\nu}$ and the vierbein $e_\mu{}^a$ is given as usual by $g_{\mu\nu} = e_\mu{}^a\,e_\nu{}^b\,\eta_{ab}$. The spin connection $\omega_{\mu ab}$ is expressed in terms of the vierbein as

$$\omega_{\mu ab} = \frac{1}{2}\,e^\nu{}_a\left(\partial_\mu e_{\nu b} - \partial_\nu e_{\mu b}\right) - \frac{1}{2}\,e^\nu{}_b\left(\partial_\mu e_{\nu a} - \partial_\nu e_{\mu a}\right) - \frac{1}{2}\,e^\rho{}_a e^\sigma{}_b\left(\partial_\rho e_{\sigma c} - \partial_\sigma e_{\rho c}\right)e_\mu{}^c. \tag{6}$$

Noting the identity of gamma matrices

$$\gamma^\mu\sigma^{ab} + \sigma^{ab}\gamma^\mu = e^\mu{}_c\left(\gamma^c\sigma^{ab} + \sigma^{ab}\gamma^c\right) = -2e^\mu{}_c\,\varepsilon^{abcd}\,\gamma_d\gamma^5, \tag{7}$$

we are able to cast the Dirac Lagrangian (2) into

$$\mathcal{L}_{\rm Dirac} = \sqrt{-g}\left\{\frac{i}{2}\left(\overline{\psi}\,\gamma^\mu\partial_\mu\psi - \partial_\mu\overline{\psi}\gamma^\mu\psi\right) - \overline{\psi}m\psi\right\} - \frac{1}{4}\sqrt{-g}\left(\overline{\psi}\,e^\mu{}_a\omega_{\mu bc}\,\varepsilon^{abcd}\gamma_d\,\gamma^5\psi\right). \tag{8}$$

In order to facilitate perturbative calculations in Sect. 6 we would like to absorb $\sqrt{-g}$ on the right-hand side of Eq. (8) into dynamical fields as much as possible, putting a weight factor $(-e)^{1/4}$ on the Dirac fields

$$\Psi \equiv (-e)^{1/4}\,\psi, \qquad \overline{\Psi} \equiv (-e)^{1/4}\,\overline{\psi}, \tag{9}$$

where

$$(-e) = \det\left(e_\mu{}^a\right) = \sqrt{-g}. \tag{10}$$

In terms of the weighted Dirac fields (9), the Dirac Lagrangian (8) turns out to be

$$\mathcal{L}_{\rm Dirac} = \frac{i}{2}\,\tilde{e}^\mu{}_a\left(\overline{\Psi}\,\gamma^a\partial_\mu\Psi - \partial_\mu\overline{\Psi}\gamma^a\Psi\right) - \sqrt{-e}\,\overline{\Psi}m\Psi - \frac{1}{4}\overline{\Psi}\,\tilde{e}^\mu{}_a\,\tilde{\omega}_{\mu bc}\,\varepsilon^{abcd}\gamma_d\,\gamma^5\Psi. \tag{11}$$

Here we have introduced a weighted vierbein

$$\tilde{e}^\mu{}_a = \sqrt{-e}\,e^\mu{}_a \tag{12}$$

and $\tilde{\omega}_{\mu ab}$ is defined analogously to Eq. (6) by

$$\tilde{\omega}_{\mu ab} = \frac{1}{2}\,\tilde{e}^\nu{}_a\left(\partial_\mu\tilde{e}_{\nu b} - \partial_\nu\tilde{e}_{\mu b}\right) - \frac{1}{2}\,\tilde{e}^\nu{}_b\left(\partial_\mu\tilde{e}_{\nu a} - \partial_\nu\tilde{e}_{\mu a}\right) - \frac{1}{2}\,\tilde{e}^\rho{}_a\tilde{e}^\sigma{}_b\left(\partial_\rho\tilde{e}_{\sigma c} - \partial_\sigma\tilde{e}_{\rho c}\right)\tilde{e}_\mu{}^c. \tag{13}$$

To arrive at Eq. (11), use has been made of an identity

$$e^\mu{}_a\,\omega_{\mu bc}\,\varepsilon^{abcd} = \frac{1}{\sqrt{-e}}\,\tilde{e}^\mu{}_a\,\tilde{\omega}_{\mu bc}\,\varepsilon^{abcd}. \tag{14}$$

As we see in Eq. (11), the factor $\sqrt{-e}$ appears only in the mass term. Also note the relation

$$\sqrt{-e} = \left\{\det\left(\tilde{e}^\mu{}_a\right)\right\}^{1/(D-2)}, \tag{15}$$

where D is the number of spacetime dimensions. We will use the dimensional method for regularization and we do not set $D = 4$.

Putting the weight on the fields as in Eqs. (9) and (12) changes the choice of dynamical variables and will lead us to a different set of Feynman rules. It has been known, however, that the point transformation of dynamical variables does not alter the structure of the S-matrix [32–34] and therefore we need not worry much about the choice of variables. In the meantime, although the weighted field method renders loop calculation a little simpler, it hinders us from comparing our calculation directly with the preceding ones by Degrassi et al. [25] and by Corianò et al. [26,27] who did not put weight on the vierbein or fermion fields, either.

3. The electroweak theory in the curved background

We are going to work with the standard $SU(2)_L \times U(1)_Y$ electroweak theory embedded in the curved background field with the metric $g_{\mu\nu}$. Deviation from the Minkowski spacetime is described, in terms of the vierbein, as

$$\tilde{e}^{\mu}{}_{a} = \eta^{\mu}{}_{a} + \kappa\, h^{\mu}{}_{a}, \tag{16}$$

where $\kappa = \sqrt{8\pi G}$, G being Newton's constant. In terms of the metric, fluctuations are expressed as

$$\tilde{g}^{\mu\nu} = \tilde{e}^{\mu}{}_{a}\tilde{e}^{\nu}{}_{b}\,\eta^{ab} = \eta^{\mu\nu} + \kappa\ (h^{\mu\nu} + h^{\nu\mu}) + \kappa^2 h^{\mu\lambda}h^{\nu}{}_{\lambda}, \tag{17}$$

where Greek and Latin indices are no longer distinguished and indices of $h^{\mu\nu}$ are raised and lowered by the Minkowski metric. Also from here we assume that $h^{\mu\nu}$ is symmetric, i.e., $h^{\mu\nu} = h^{\nu\mu}$. Also note that Eq. (15) gives rise to the formula

$$\sqrt{-e} = 1 + \frac{\kappa}{D-2}\eta_{\mu}{}^{a}h^{\mu}{}_{a} + \cdots\cdots . \tag{18}$$

In the R_ξ gauge in the curved background, we add the following gauge-fixing terms to the action:

$$\begin{aligned}\mathcal{L}_{\text{g.f.}} = &-\frac{1}{\xi}\sqrt{-g}\left|g^{\lambda\rho}\nabla_\lambda W_\rho - i\,\xi\ M_W\ \chi\right|^2 - \frac{1}{2\xi'}\sqrt{-g}\left(g^{\mu\nu}\ \nabla_\mu Z_\nu + \xi'\ M_Z\ \chi_0\right)^2\\ &- \frac{1}{2\alpha}\sqrt{-g}\left(g^{\mu\nu}\ \nabla_\mu A_\nu\right)^2,\end{aligned} \tag{19}$$

where ξ, ξ', and α are gauge parameters. The masses of W- and Z-bosons are denoted by M_W and M_Z, respectively. The electromagnetic field is denoted by A_μ and χ and χ_0 are charged and neutral unphysical scalar fields, respectively. In our actual calculations we will use the $\xi = 1$ 't Hooft–Feynman gauge, in which the W-boson propagator is very much simplified and is most convenient to deal with. The second and third terms in Eq. (19) are not relevant to our later calculations but are included here just for completeness. The gravitational field is an external field and therefore the general covariance is not gauge-fixed.

The electroweak Lagrangian in the curved space is given in the power-series expansion in κ, namely,

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \kappa\ h^{\mu\nu}T_{\mu\nu} + \mathcal{O}(\kappa^2), \tag{20}$$

$$T_{\mu\nu} = T^{(W)}_{\mu\nu} + T^{(\chi)}_{\mu\nu} + \sum_q T^{(\text{q})}_{\mu\nu} + T^{(\text{qW})}_{\mu\nu} + T^{(\text{q}\chi)}_{\mu\nu}, \tag{21}$$

where $\mathcal{L}_{\text{SM}}$ is the standard electroweak Lagrangian in the flat Minkowski space and the second term in the expansion in κ in Eq. (20) corresponds to the one-graviton emission. Since we will not consider graviton loops, the Einstein–Hilbert gravitational action is not included in Eq. (20). The summation in the third term of Eq. (21) is taken over all quark flavors ($q = u$, d, s, …) and each term in Eq. (21) is respectively given by

$$T^{(W)}_{\mu\nu} = V_{\mu\nu}{}^{\sigma\tau\lambda\rho}\ (\partial_\sigma W^\dagger_\tau)(\partial_\lambda W_\rho) + 2\,M_W^2\ \eta^\tau{}_{(\mu}\eta_{\nu)}{}^\rho\ W^\dagger_\tau W_\rho$$
$$+\frac{2}{\xi}\,\eta^\sigma{}_{(\mu}\,\eta_{\nu)}{}^\tau\,\eta^{\lambda\rho}\ \left(W^\dagger_\tau\,\partial_\sigma\partial_\lambda W_\rho\right) + \frac{2}{\xi}\,\eta^\sigma{}_{(\mu}\,\eta_{\nu)}{}^\rho\,\eta^{\lambda\tau}\ \left(W_\rho\,\partial_\lambda\partial_\sigma W^\dagger_\tau\right), \tag{22}$$

$$T^{(\chi)}_{\mu\nu} = \partial_\mu\chi^\dagger\,\partial_\nu\chi + \partial_\nu\chi^\dagger\,\partial_\mu\chi - \eta_{\mu\nu}\,\frac{2}{D-2}\,\xi\,M_W^2\chi^\dagger\chi, \tag{23}$$

$$T^{(\mathrm{q})}_{\mu\nu} = \frac{i}{2}\,\overline{\Psi}_q\left(\gamma_\mu\overleftrightarrow{\partial_\nu} + \gamma_\nu\overleftrightarrow{\partial_\mu}\right)\,\Psi_q - \frac{1}{D-2}\,\eta_{\mu\nu}\overline{\Psi}_q\,m_q\Psi_q, \tag{24}$$

$$T^{(\mathrm{qW})}_{\mu\nu} = -\,\frac{g}{2\sqrt{2}}\left(\overline{U_L}\gamma_\mu V_{\mathrm{CKM}}D_L W^\dagger_\nu + \overline{U_L}\gamma_\nu V_{\mathrm{CKM}}D_L W^\dagger_\mu\right) + (\mathrm{h.c.}), \tag{25}$$

$$T^{(\mathrm{q}\chi)}_{\mu\nu} = \eta_{\mu\nu}\,\frac{1}{D-2}\frac{\sqrt{2}}{v}\,\chi^\dagger\ \left(\overline{U_R}\,\mathcal{M}_u\,V_{\mathrm{CKM}}\,D_L - \overline{U_L}\,V_{\mathrm{CKM}}\mathcal{M}_d\,D_R\right) + (\mathrm{h.c.}). \tag{26}$$

The quantity $V_{\mu\nu}{}^{\sigma\tau\lambda\rho}$ in Eq. (22) is defined by

$$V_{\mu\nu}{}^{\sigma\tau\lambda\rho} = -2\,\eta^\sigma{}_{(\mu}\,\eta_{\nu)}{}^\lambda\,\eta^{\tau\rho} - 2\,\eta^\tau{}_{(\mu}\,\eta_{\nu)}{}^\rho\,\eta^{\sigma\lambda} + 2\,\eta^\tau{}_{(\mu}\,\eta_{\nu)}{}^\lambda\,\eta^{\sigma\rho} + 2\,\eta^\sigma{}_{(\mu}\,\eta_{\nu)}{}^\rho\,\eta^{\tau\lambda}$$
$$+\frac{2}{D-2}\,\eta_{\mu\nu}\left(\eta^{\sigma\lambda}\,\eta^{\tau\rho} - \eta^{\tau\lambda}\,\eta^{\sigma\rho} + \frac{1}{\xi}\,\eta^{\sigma\tau}\,\eta^{\lambda\rho}\right), \tag{27}$$

and the symmetrization with respect to indices in the pair of parentheses in Eq. (27) is done in the following manner:

$$A_{(\sigma}B_{\tau)} \equiv \frac{1}{2}\left(A_\sigma B_\tau + A_\tau B_\sigma\right). \tag{28}$$

The symbol of the left–right derivative in Eq. (24) is defined by

$$\overleftrightarrow{\partial_\mu} = \frac{1}{2}\left(\overrightarrow{\partial_\mu} - \overleftarrow{\partial_\mu}\right). \tag{29}$$

The Cabibbo–Kobayashi–Maskawa (CKM) matrix is denoted by V_{CKM} in Eqs. (25) and (26) and the diagonal mass matrices of up- and down-type quarks are given, respectively, by

$$\mathcal{M}_u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix},\ \mathcal{M}_d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}. \tag{30}$$

The left (L)- and right (R)-handed quarks are projected as usual by

$$L = \frac{1-\gamma^5}{2}, \qquad R = \frac{1+\gamma^5}{2}, \tag{31}$$

and the projected up- and down-type quarks are expressed as

$$U_L = L\begin{pmatrix}\Psi_u \\ \Psi_c \\ \Psi_t\end{pmatrix},\ D_L = L\begin{pmatrix}\Psi_d \\ \Psi_s \\ \Psi_b\end{pmatrix}, \tag{32}$$

$$U_R = R\begin{pmatrix}\Psi_u \\ \Psi_c \\ \Psi_t\end{pmatrix}, \qquad D_R = R\begin{pmatrix}\Psi_d \\ \Psi_s \\ \Psi_b\end{pmatrix}, \tag{33}$$

in Eqs. (25) and (26). The $SU(2)_L$ gauge coupling is denoted by g in Eq. (25) and v in Eq. (26) is the vacuum expectation value.

Before closing this section, let us add a comment on the relation between the energy–momentum tensor and our $T_{\mu\nu}$. The conventional energy–momentum tensor is defined as the functional derivative of the action under the variation $\delta e^{\mu a}$, while $T_{\mu\nu}$ of Eq. (21) is the functional derivative of the action under $\delta\tilde{e}^{\mu a}$. The connection between the two types of functional derivatives is given by

$$\frac{\delta}{\delta e^{\mu a}} = \sqrt{-e}\frac{\delta}{\delta\tilde{e}^{\mu a}} - \frac{1}{2}\sqrt{-e}e_{\mu a}\, e^{\lambda b}\frac{\delta}{\delta\tilde{e}^{\lambda b}}, \tag{34}$$

and therefore the conserved energy–momentum tensor in the flat-space limit is a linear combination of $T_{\mu\nu}$ given by

$$T_{\mu\nu} - \frac{1}{2}\,\eta_{\mu\nu}\,\eta^{\lambda\rho}\,T_{\lambda\rho}. \tag{35}$$

The Ward–Takahashi identity, which will be discussed later in Sect. 8, is associated with Eq. (35).

4. Self-energy type $d \to s$ transition

In the present section, we would like to evaluate the self-energy type diagram depicted in Fig. 1 and to consider the corresponding counter terms in Fig. 2 in order to eliminate the ultraviolet divergences. The eventual goal of the present work is to uncover the non-decoupling nature of the internal heavy top quark in the low-energy process (1), which is induced at the loop levels. There are eight one-loop diagrams of two different types, which will be shown later in Sect. 6 (i.e., Figs. 3 and 4). There the gravitons ($h_{\mu\nu}$) are expressed by double wavy lines and are attached to vertices in Fig. 3 and to internal propagators in Fig. 4. The internal quark propagator consists of $j =$ top (t), charm (c), and up (u) quarks and we are interested in the large top quark mass behavior of the amplitudes of Figs. 3 and 4.

These one-loop contributions to Eq. (1) will be computed in Sect. 6 and they turn out to be ultraviolet divergent. The divergences should be subtracted by using the corresponding counter term Lagrangian $\widehat{\mathcal{L}}_{\text{c.t.}}$ in the curved spacetime, which should be diffeomorphism invariant and will be given in Sect. 5. Diagrammatically the counter term in $\widehat{\mathcal{L}}_{\text{c.t.}}$ with one external graviton is denoted by a cross in Fig. 2(b). As it turns out, the flat spacetime limit $\mathcal{L}_{\text{c.t.}}$ of $\widehat{\mathcal{L}}_{\text{c.t.}}$ should serve as the counter term Lagrangian that is supposed to eliminate the divergences associated with the self-energy type $d \to s$ transition $\Sigma(p)$ in the Minkowski space (without graviton emission). The vertex associated with $\mathcal{L}_{\text{c.t.}}$ is denoted by a cross in Fig. 2(a).

Now, by reversing this, we may proceed in the following way: after computing $\Sigma(p)$ in this section, we first work out in Sect. 5.1 the renormalization constants contained in $\mathcal{L}_{\text{c.t.}}$, and then we deduce in Sect. 5.2 an explicit form of $\widehat{\mathcal{L}}_{\text{c.t.}}$ by employing the diffeomorphism invariance argument. We will confirm in Sect. 7 that our $\widehat{\mathcal{L}}_{\text{c.t.}}$ thus obtained is necessary and sufficient to eliminate all the divergences that appear in the one-loop induced d–s–graviton vertex to be computed in Sect. 6. Keeping these procedures in mind, we would like to start with the calculation of $\Sigma(p)$ in the flat Minkowski space. The $d \to s$ transition takes place at the one-loop level via W- and charged unphysical scalar boson (χ) exchanges as depicted in Fig. 1. The Feynman rules in the 't Hooft–Feynman gauge ($\xi = 1$) lead us to

$$\Sigma(p) = \sum_{j=t,c,u} (V_{\text{CKM}})^*_{js}(V_{\text{CKM}})_{jd}\left\{\mathcal{S}^{(a)}(p) + \mathcal{S}^{(b)}(p)\right\}, \tag{36}$$

where we have defined the integrations of the following forms:

$$\mathcal{S}^{(a)}(p) \equiv -i\left(\frac{-ig}{\sqrt{2}}\right)^2 \mu^{4-D}\int \frac{d^Dq}{(2\pi)^D}\,\gamma_\alpha L\frac{i}{\gamma\cdot(p-q)-m_j}\,\gamma_\beta L\frac{-i\,\eta^{\alpha\beta}}{q^2-M_W^2}, \tag{37}$$

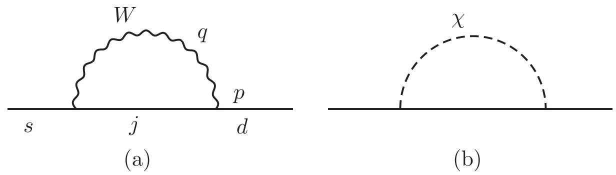

Fig. 1. The self-energy type $d \to s$ transition via (a) W- and (b) charged unphysical scalar (χ)-boson exchanges. The intermediate quark is denoted by j ($j = t, c, u$).

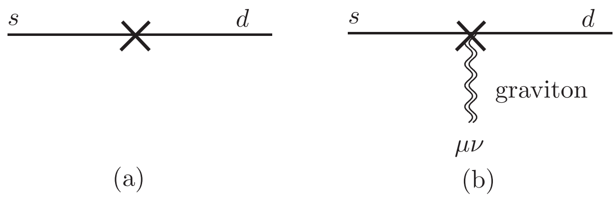

Fig. 2. The counter term diagram of the (a) $d \to s$ transition and (b) d–s–graviton vertex. The insertion of counter terms is indicated by a cross and the double wavy line in (b) denotes an emitted graviton ($h_{\mu\nu}$).

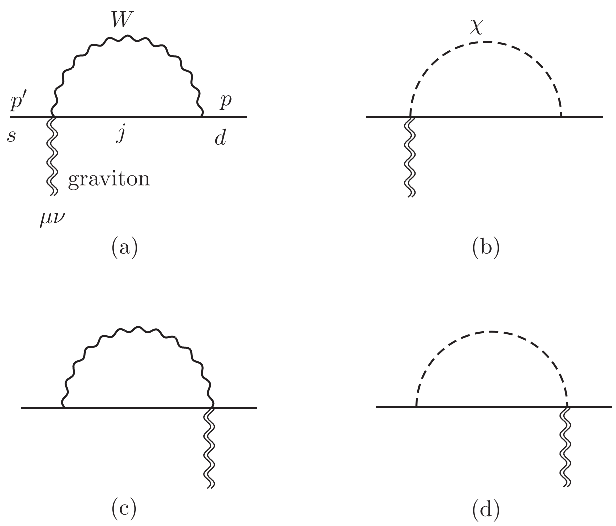

Fig. 3. Diagrams with a graviton attached to vertices.

$$\mathcal{S}^{(b)}(p) \equiv -i\left(\frac{-ig}{\sqrt{2}}\right)^2 \frac{\mu^{4-D}}{M_W^2}\int \frac{d^Dq}{(2\pi)^D}\left(m_jR - m_sL\right)\frac{i}{\gamma\cdot(p-q)-m_j}\left(m_jL - m_dR\right) \times \frac{i}{q^2 - M_W^2}, \tag{38}$$

corresponding to Figs. 1(a) and (b), respectively. Here μ is the mass scale of the D-dimensional regularization method.

The integrations in Eqs. (37) and (38) are rather standard and we find

$$\mathcal{S}^{(a)}(p) = \frac{-g^2}{(4\pi)^2}\left[\frac{1}{D-4} + \frac{1}{2} + f_1(p^2)\right]\gamma\cdot p\, L, \tag{39}$$

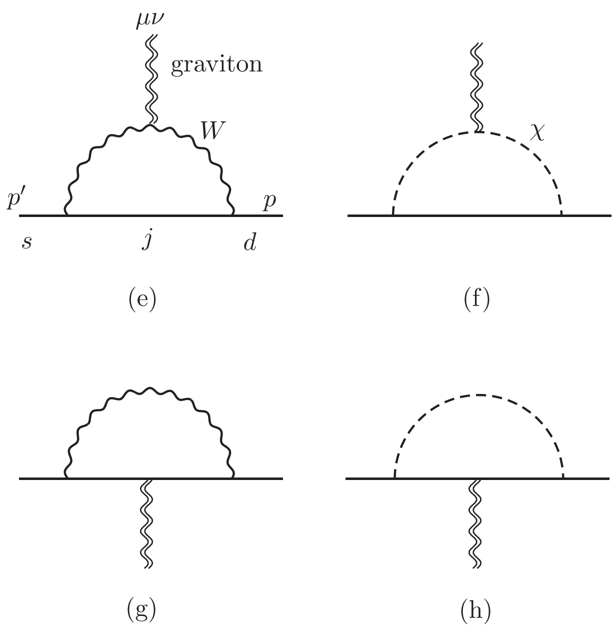

Fig. 4. Diagrams with a graviton attached to internal propagators.

$$\mathcal{S}^{(b)}(p) = \frac{-g^2}{(4\pi)^2}\left[\frac{1}{D-4}\left\{\frac{1}{2M_W^2}\,\gamma\cdot p\,(m_j^2\,L + m_s m_d R) - \frac{m_j^2}{M_W^2}(m_s L + m_d R)\right\}\right.$$
$$\left. + \frac{1}{2M_W^2}\left\{f_1(p^2)\,\gamma\cdot p\,(m_j^2\,L + m_s m_d\,R) - f_2(p^2)\,m_j^2(m_s L + m_d R)\right\}\right], \qquad (40)$$

where $f_1(p^2)$ and $f_2(p^2)$ are defined respectively by Eqs. (A1) and (A2) in Appendix A. Note that terms independent of m_j that are present in $\mathcal{S}^{(a)}$ and $\mathcal{S}^{(b)}$ will disappear after the j-summation in Eq. (36) because of the unitarity of the CKM matrix, V_{CKM}, i.e.,

$$\sum_{j=t,c,u}(V_{\mathrm{CKM}})^*_{js}(V_{\mathrm{CKM}})_{jd} = 0. \qquad (41)$$

Also remember that both $f_1(p^2)$ and $f_2(p^2)$ contain m_j^2 in their definitions (A1), (A2). Therefore putting Eqs. (39) and (40) together, we end up with the formula of the self-energy type $d \to s$ transition:

$$\Sigma(p) = \frac{-g^2}{(4\pi)^2}\sum_{j=t,c,u}(V_{\mathrm{CKM}})^*_{js}(V_{\mathrm{CKM}})_{jd}$$
$$\times\left[f_1(p^2)\,\gamma\cdot p\,L + \frac{1}{D-4}\left\{\frac{m_j^2}{2M_W^2}\,\gamma\cdot p\,L - \frac{m_j^2}{M_W^2}(m_s L + m_d R)\right\}\right.$$
$$\left. + \frac{1}{2M_W^2}\left\{f_1(p^2)\,\gamma\cdot p\left(m_j^2\,L + m_s m_d\,R\right) - f_2\left(p^2\right)\,m_j^2\,(m_s L + m_d R)\right\}\right]. \qquad (42)$$

5. Infinity subtraction procedure

5.1 *Counter terms in the flat spacetime*

Let us now move to the subtraction of infinities from $\Sigma(p)$, taking into account the counter terms, which are of the following form:

$$\begin{aligned}\mathcal{L}_{\text{c.t.}} &= Z_L\, \overline{\psi_{sL}}\, i\gamma \cdot \overleftrightarrow{\partial}\ \psi_{d\,L} + Z_R\, \overline{\psi_{sR}}\, i\gamma \cdot \overleftrightarrow{\partial}\ \psi_{d\,R} \\ &\quad + Z_{Y1}\, \overline{\psi_{sR}} m_s \psi_{d\,L} + Z_{Y2}\, \overline{\psi_{sL}}\, m_d \psi_{d\,R} \\ &\quad + (\text{h.c.}). \end{aligned} \tag{43}$$

Here the wave-function renormalization constants, Z_L, Z_R, Z_{Y1}, and Z_{Y2} take care of the mixing between d- and s-quarks under renormalization.

The contribution of Eq. (43) to the $d \to s$ transition is depicted in Fig. 2(a) and is written as

$$\Sigma_{\text{c.t.}}(p) = Z_L \gamma \cdot p\, L + Z_R \gamma \cdot p\, R + Z_{Y1} m_s L + Z_{Y2} m_d R\,. \tag{44}$$

The renormalization constants are arranged so that the renormalized $d \to s$ transition amplitude

$$\Sigma_{\text{ren}}(p) = \Sigma(p) + \Sigma_{\text{c.t.}}(p) \tag{45}$$

is finite. In other words the renormalization constants are given in the following form:

$$Z_L = \frac{g^2}{(4\pi)^2} \sum_{j=t,c,u} (V_{\text{CKM}})^*_{js} (V_{\text{CKM}})_{jd} \left\{ \frac{m_j^2}{2M_W^2} \cdot \frac{1}{D-4} - c_1(m_j) \right\}, \tag{46}$$

$$Z_R = \frac{g^2}{(4\pi)^2} \sum_{j=t,c,u} (V_{\text{CKM}})^*_{js} (V_{\text{CKM}})_{jd} \ \left\{ -c_2(m_j) \right\}, \tag{47}$$

$$Z_{Y1} = \frac{g^2}{(4\pi)^2} \sum_{j=t,c,u} (V_{\text{CKM}})^*_{js} (V_{\text{CKM}})_{jd} \left\{ -\frac{m_j^2}{M_W^2} \cdot \frac{1}{D-4} - c_3(m_j) \right\}, \tag{48}$$

$$Z_{Y2} = \frac{g^2}{(4\pi)^2} \sum_{j=t,c,u} (V_{\text{CKM}})^*_{js} (V_{\text{CKM}})_{jd} \left\{ -\frac{m_j^2}{M_W^2} \cdot \frac{1}{D-4} - c_4(m_j) \right\}, \tag{49}$$

in order to subtract the $D = 4$ pole terms in $\Sigma(p)$. Here $c_1(m_j)$, $c_2(m_j)$, $c_3(m_j)$, and $c_4(m_j)$ are all finite and should be determined by specifying the subtraction conditions.

Now we adopt the on-shell subtraction conditions [22] in such a way that the renormalized self-energy $\Sigma_{\text{ren}}(p)$ should satisfy the following conditions:

$$\begin{aligned} \Sigma_{\text{ren}}\, \Psi_d &= 0, \qquad \text{for } p^2 = m_d^2, \\ \overline{\Psi}_s\, \Sigma_{\text{ren}} &= 0, \qquad \text{for } p^2 = m_s^2. \end{aligned} \tag{50}$$

Each of the conditions in Eq. (50) gives rise to two constraints on Σ_{ren}: one for the left-handed part and the other for the right-handed part. We have therefore four constraints in total in Eq. (50), which in turn determine the four constants $c_1(m_j)$, $c_2(m_j)$, $c_3(m_j)$, and $c_4(m_j)$.

In order to determine these constants on the basis of Eq. (50), let us note that Eq. (45) is written explicitly as

$$\Sigma_{\rm ren}(p) = \frac{-g^2}{(4\pi)^2} \sum_{j=t,c,u} (V_{\rm CKM})^*_{js}(V_{\rm CKM})_{jd}$$
$$\times \left[\left\{ c_1(m_j) + f_1(p^2)\left(1 + \frac{m_j^2}{2M_W^2}\right)\right\} \gamma \cdot p\, L + \left\{ c_2(m_j) + \frac{m_s\, m_d}{2M_W^2} f_1(p^2)\right\} \gamma \cdot p\, R \right.$$
$$\left. + \left\{ c_3(m_j) - \frac{m_j^2}{2M_W^2} f_2(p^2) \right\} m_s L + \left\{ c_4(m_j) - \frac{m_j^2}{2M_W^2} f_2(p^2) \right\} m_d R \right]. \tag{51}$$

The subtraction conditions (50) then turn out to be

$$\left\{ c_1(m_j) + f_1\left(m_d^2\right)\left(1 + \frac{m_j^2}{2M_W^2}\right)\right\} m_d + \left\{ c_4(m_j) - \frac{m_j^2}{2M_W^2} f_2\left(m_d^2\right)\right\} m_d = 0, \tag{52}$$

$$\left\{ c_2(m_j) + \frac{m_s m_d}{2M_W^2} f_1\left(m_d^2\right)\right\} m_d + \left\{ c_3(m_j) - \frac{m_j^2}{2M_W^2} f_2\left(m_d^2\right)\right\} m_s = 0, \tag{53}$$

$$\left\{ c_1(m_j) + f_1\left(m_s^2\right)\left(1 + \frac{m_j^2}{2M_W^2}\right)\right\} m_s + \left\{ c_3(m_j) - \frac{m_j^2}{2M_W^2} f_2\left(m_s^2\right)\right\} m_s = 0, \tag{54}$$

$$\left\{ c_2(m_j) + \frac{m_s m_d}{2M_W^2} f_1\left(m_s^2\right)\right\} m_s + \left\{ c_4(m_j) - \frac{m_j^2}{2M_W^2} f_2\left(m_s^2\right)\right\} m_d = 0, \tag{55}$$

and we have worked out the following solutions to Eqs. (52)–(55):

$$c_1(m_j) = \frac{1}{m_d^2 - m_s^2}\left[-\left\{ m_d^2 f_1\left(m_d^2\right) - m_s^2 f_1\left(m_s^2\right)\right\}\left(1 + \frac{m_j^2}{2M_W^2}\right)\right.$$
$$\left. - \frac{m_d^2 m_s^2}{2M_W^2}\left\{ f_1\left(m_d^2\right) - f_1\left(m_s^2\right)\right\} + \frac{(m_d^2 + m_s^2)m_j^2}{2M_W^2}\left\{ f_2\left(m_d^2\right) - f_2\left(m_s^2\right)\right\}\right], \tag{56}$$

$$c_2(m_j) = \frac{m_d m_s}{m_d^2 - m_s^2}\left[-\left\{ f_1\left(m_d^2\right) - f_1\left(m_s^2\right)\right\}\left(1 + \frac{m_j^2}{2M_W^2}\right)\right.$$
$$\left. - \frac{1}{2M_W^2}\left\{ m_d^2 f_1\left(m_d^2\right) - m_s^2 f_1\left(m_s^2\right)\right\} + \frac{m_j^2}{M_W^2}\left\{ f_2\left(m_d^2\right) - f_2\left(m_s^2\right)\right\}\right], \tag{57}$$

$$c_3(m_j) = \frac{m_d^2}{m_d^2 - m_s^2}\left[\left\{ f_1\left(m_d^2\right) - f_1\left(m_s^2\right)\right\}\left(1 + \frac{m_j^2}{2M_W^2} + \frac{m_s^2}{2M_W^2}\right)\right.$$
$$\left. + \frac{m_j^2}{2M_W^2}\left\{ -\frac{m_d^2 + m_s^2}{m_d^2} f_2\left(m_d^2\right) + 2 f_2\left(m_s^2\right)\right\}\right], \tag{58}$$

$$c_4(m_j) = \frac{m_s^2}{m_d^2 - m_s^2}\left[\left\{ f_1\left(m_d^2\right) - f_1\left(m_s^2\right)\right\}\left(1 + \frac{m_j^2}{2M_W^2} + \frac{m_d^2}{2M_W^2}\right)\right.$$
$$\left. + \frac{m_j^2}{2M_W^2}\left\{ \frac{m_d^2 + m_s^2}{m_s^2} f_2\left(m_s^2\right) - 2 f_2\left(m_d^2\right)\right\}\right]. \tag{59}$$

5.2 *Counter terms in the curved spacetime*

So much for the counter terms in the flat Minkowski space; let us now think about the generalization to the curved background case. The counter terms in the curved background come out naturally by extending Eq. (43) to a diffeomorphism invariant form, i.e.,

$$\begin{aligned}\widehat{\mathcal{L}}_{\text{c.t.}} = & Z_L\, \overline{\Psi_{sL}}\, i\gamma^a \overleftrightarrow{\nabla}_\mu\, \Psi_{d\,L}\, \tilde{e}^\mu{}_a + Z_R\, \overline{\Psi_{sR}}\, i\gamma^a \overleftrightarrow{\nabla}_\mu\, \Psi_{d\,R}\, \tilde{e}^\mu{}_a \\ & + Z_{Y1}\sqrt{-e}\, \overline{\Psi_{sR}}\, m_s \Psi_{d\,L} + Z_{Y2}\sqrt{-e}\, \overline{\Psi_{sL}}\, m_d\, \Psi_{d\,R} \\ & + (\text{h.c.}),\end{aligned} \tag{60}$$

where the quark fields Ψ_d and Ψ_s are weighted by $(-e)^{1/4}$. The vierbein $\tilde{e}^\mu{}_a$ in Eq. (60) is expanded in κ and thereby we get

$$\widehat{\mathcal{L}}_{\text{c.t.}} = \widehat{\mathcal{L}}^{(0)}_{\text{c.t.}} + \kappa \widehat{\mathcal{L}}^{(1)}_{\text{c.t.}} + \cdots, \tag{61}$$

where $\widehat{\mathcal{L}}^{(0)}_{\text{c.t.}}$ coincides with the flat-space counter term (43). The next term $\widehat{\mathcal{L}}^{(1)}_{\text{c.t.}}$, on the other hand, is expressed as

$$\begin{aligned}\widehat{\mathcal{L}}^{(1)}_{\text{c.t.}} = h^{\mu\nu}\Bigg[& Z_L\, \overline{\Psi_{sL}}\, i\gamma_{(\mu} \overleftrightarrow{\nabla}_{\nu)}\, \Psi_{d\,L} + Z_R\, \overline{\Psi_{sR}}\, i\gamma_{(\mu} \overleftrightarrow{\nabla}_{\nu)}\, \Psi_{d\,R} \\ & + \frac{1}{D-2} Z_{Y1}\, \eta_{\mu\nu}\, \overline{\Psi_{sR}}\, m_s \Psi_{d\,L} + \frac{1}{D-2} Z_{Y2}\, \eta_{\mu\nu}\, \overline{\Psi_{sL}}\, m_d\, \Psi_{d\,R} \\ & + (\text{h.c.}) \Bigg],\end{aligned} \tag{62}$$

and gives rise to the contribution depicted in Fig. 2(b). As we will confirm later in Sect. 7 explicitly, Eq. (62) eliminates the divergences in the one-graviton emission vertex-type diagrams (Figs. 3 and 4). It is to be noted that the renormalization constants, Z_L, Z_R, Z_{Y1}, and Z_{Y2}, are playing two roles: one is to render the self-energy type diagrams (Fig. 1) finite, and the other is to make the one-graviton emission vertex finite. This is due to the fact that the two counter term Lagrangians (43), (62) should combine into a diffeomorphism invariant form (60).

It should be added herewith that Degrassi et al. [25] and Corianò et al. [26] also previously discussed renormalization of the vertex of Eq. (1). They took a sum of the vertex-type and self-energy type diagrams to find mutual cancellation of divergences. This cancellation is consistent with our procedure of eliminating divergences simultaneously in both self-energy type and vertex-type diagrams via Z_L, Z_R, Z_{Y1}, and Z_{Y2}.

Incidentally the coefficient $1/(D-2)$ in front of Z_{Y1} and Z_{Y2} in Eq. (62), which comes from the formula (18), gives rise to a finite deviation from $\frac{1}{2}Z_{Y1}$ and $\frac{1}{2}Z_{Y2}$, namely,

$$\begin{aligned}\frac{1}{D-2}Z_{Y1} &= \left\{\frac{1}{2} - \frac{D-4}{2(D-2)}\right\} Z_{Y1} \\ &= \frac{1}{2}Z_{Y1} + \frac{g^2}{(4\pi)^2} \sum_{j=t,c,u} (V_{\text{CKM}})^*_{js} (V_{\text{CKM}})_{jd} \frac{m_j^2}{M_W^2} \times \frac{1}{4},\end{aligned} \tag{63}$$

as we take the $D \to 4$ limit. The same formula also applies to $\frac{1}{D-2}Z_{Y2}$, i.e.,

$$\frac{1}{D-2}Z_{Y2} = \frac{1}{2}Z_{Y2} + \frac{g^2}{(4\pi)^2}\sum_{j=t,c,u}(V_{\rm CKM})^*_{js}(V_{\rm CKM})_{jd}\frac{m_j^2}{M_W^2} \times \frac{1}{4}. \tag{64}$$

6. Gravitational flavor-changing vertices

Now that we have the counter term Lagrangian (62) at our hand, we are well prepared to handle the divergences that appear in evaluating Eq. (1). The relevant Feynman diagrams for Eq. (1) may be classified into two types: those with two internal propagators (Fig. 3) and those with three internal propagators (Fig. 4). The latter diagrams are expressed necessarily by double integrals with respect to the Feynman parameters, the former by single ones. We will keep the external quarks off-shell, refraining from using the Dirac equation throughout. We will never use any approximation of the magnitude of the quark masses until Sect. 9, where the large top quark mass limit of the *d*–*s*–graviton vertex is investigated.

6.1 *A graviton attached to the charged current vertex*

Let us begin with the calculation of Fig. 3 in which graviton lines are attached to the charged current vertices. Applications of the Feynman rules give us the following sum:

$$\Gamma^{(\text{Fig. 3})}_{\mu\nu}(p,p') = \sum_{j=t,c,u}(V_{\rm CKM})^*_{js}(V_{\rm CKM})_{jd}\left\{\mathcal{G}^{(a)}_{\mu\nu} + \mathcal{G}^{(b)}_{\mu\nu} + \mathcal{G}^{(c)}_{\mu\nu} + \mathcal{G}^{(d)}_{\mu\nu}\right\}, \tag{65}$$

where for each diagram in Fig. 3 we define respectively the integrations

$$\mathcal{G}^{(a)}_{\mu\nu} \equiv \frac{i\kappa g^2}{4}\mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\left(\gamma_\mu\eta_{\nu\alpha} + \gamma_\nu\eta_{\mu\alpha}\right)L\frac{i}{\gamma\cdot(p-q) - m_j}\gamma_\beta L\frac{-i\,\eta^{\alpha\beta}}{q^2 - M_W^2}, \tag{66}$$

$$\mathcal{G}^{(b)}_{\mu\nu} \equiv \frac{i\kappa g^2}{2(D-2)}\cdot\frac{1}{M_W^2}\mu^{4-D}\,\eta_{\mu\nu}\int\frac{d^Dq}{(2\pi)^D}\frac{i}{q^2 - M_W^2} \times (m_jR - m_sL)\frac{i}{\gamma\cdot(p-q) - m_j}(m_jL - m_dR), \tag{67}$$

$$\mathcal{G}^{(c)}_{\mu\nu} \equiv \frac{i\kappa g^2}{4}\mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\gamma_\beta L\frac{i}{\gamma\cdot(p'-q) - m_j}\left(\gamma_\mu\eta_{\nu\alpha} + \gamma_\nu\eta_{\mu\alpha}\right)L\frac{-i\,\eta^{\alpha\beta}}{q^2 - M_W^2}, \tag{68}$$

$$\mathcal{G}^{(d)}_{\mu\nu} \equiv \frac{i\kappa g^2}{2(D-2)}\cdot\frac{1}{M_W^2}\mu^{4-D}\,\eta_{\mu\nu}\int\frac{d^Dq}{(2\pi)^D}\frac{i}{q^2 - M_W^2} \times (m_jR - m_sL)\frac{i}{\gamma\cdot(p'-q) - m_j}(m_jL - m_dR). \tag{69}$$

Note that the factor $1/(D-2)$ in front of Eqs. (67) and (69) is due to the second term of Eq. (18). On comparing Eqs. (67) and (69) with Eq. (38), one can immediately see a simple relation

$$\mathcal{G}^{(b)}_{\mu\nu} = \frac{\kappa}{D-2}\,\eta_{\mu\nu}\,\mathcal{S}^{(b)}(p), \qquad \mathcal{G}^{(d)}_{\mu\nu} = \frac{\kappa}{D-2}\,\eta_{\mu\nu}\,\mathcal{S}^{(b)}(p'). \tag{70}$$

The evaluation of the above Feynman integrations is rather standard and we simply list the results below:

$$\mathcal{G}^{(a)}_{\mu\nu} = \frac{\kappa g^2}{(4\pi)^2}\,G_1(p^2)\left\{\gamma_{(\mu}p_{\nu)} - \frac{1}{2}\eta_{\mu\nu}\gamma\cdot p\right\}L, \tag{71}$$

$$\mathcal{G}^{(c)}_{\mu\nu} = \frac{\kappa g^2}{(4\pi)^2}\, G_1(p'^2)\left\{\gamma_{(\mu}p'_{\nu)} - \frac{1}{2}\eta_{\mu\nu}\gamma\cdot p'\right\} L, \tag{72}$$

$$\mathcal{G}^{(b)}_{\mu\nu} = \frac{\kappa g^2}{(4\pi)^2}\cdot\frac{1}{4M_W^2}\left[\left\{-G_1(p^2)+\frac{1}{2}\right\}\eta_{\mu\nu}\gamma\cdot p\left(m_j^2 L + m_s m_d R\right)\right.$$
$$\left. + \left\{G_2(p^2) - 1\right\} m_j^2 \eta_{\mu\nu}\left(m_s L + m_d R\right)\right], \tag{73}$$

$$\mathcal{G}^{(d)}_{\mu\nu} = \frac{\kappa g^2}{(4\pi)^2}\cdot\frac{1}{4M_W^2}\left[\left\{-G_1(p'^2)+\frac{1}{2}\right\}\eta_{\mu\nu}\gamma\cdot p'\left(m_j^2 L + m_s m_d R\right)\right.$$
$$\left. + \left\{G_2(p'^2) - 1\right\} m_j^2 \eta_{\mu\nu}\left(m_s L + m_d R\right)\right]. \tag{74}$$

The functions $G_1(p^2)$ and $G_2(p^2)$ are defined respectively by Eqs. (B1) and (B2) in Appendix B. One can confirm that the formulae of $\mathcal{G}^{(b)}_{\mu\nu}$ and $\mathcal{G}^{(d)}_{\mu\nu}$ are nothing but those obtained from Eq. (40) by the relation (70).

6.2 *A graviton attached to the internal propagators*

Another set of Feynman diagrams depicted in Fig. 4 are those in which the graviton is attached to internal lines. Let us define

$$\Gamma^{(\text{Fig. 4})}_{\mu\nu}(p, p') = \sum_{j=t,c,u} (V_{\text{CKM}})^*_{js}(V_{\text{CKM}})_{jd}\left\{\mathcal{G}^{(e)}_{\mu\nu} + \mathcal{G}^{(f)}_{\mu\nu} + \mathcal{G}^{(g)}_{\mu\nu} + \mathcal{G}^{(h)}_{\mu\nu}\right\}, \tag{75}$$

where each term in the brackets on the right-hand side corresponds to each diagram in Fig. 4 and is given by

$$\mathcal{G}^{(e)}_{\mu\nu} \equiv -\frac{\kappa g^2}{2}\mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}\,\gamma^\tau L\frac{i}{\gamma\cdot q - m_j}\gamma^\rho L\cdot\frac{-i}{(p-q)^2 - M_W^2}\frac{-i}{(p'-q)^2 - M_W^2}$$
$$\times\left[V_{\mu\nu\sigma\tau\lambda\rho}\Big|_{\xi=1}(p'-q)^\sigma(p-q)^\lambda + 2M_W^2\,\eta_{\tau(\mu}\,\eta_{\nu)\rho}\right.$$
$$\left. - 2\,\eta_{\sigma(\mu}\eta_{\nu)\tau}\,\eta_{\lambda\rho}\,(p-q)^\sigma(p-q)^\lambda - 2\,\eta_{\sigma\tau}\eta_{\lambda(\mu}\eta_{\nu)\rho}(p'-q)^\lambda(p'-q)^\sigma\right], \tag{76}$$

$$\mathcal{G}^{(f)}_{\mu\nu} \equiv -\frac{\kappa g^2}{2}\frac{1}{M_W^2}\mu^{4-D}\int\frac{d^Dq}{(2\pi)^D}(m_jR - m_sL)\frac{i}{\gamma\cdot q - m_j}(m_jL - m_dR)$$
$$\times\frac{i}{(p'-q)^2 - M_W^2}\frac{i}{(p-q)^2 - M_W^2}$$
$$\times\left\{(p'-q)_\mu(p-q)_\nu + (p'-q)_\nu(p-q)_\mu - \frac{2}{D-2}\eta_{\mu\nu}M_W^2\right\}, \tag{77}$$

$$
\begin{aligned}
\mathcal{G}^{(g)}_{\mu\nu} \equiv & -\frac{\kappa g^2}{2}\mu^{4-D}\int \frac{d^D q}{(2\pi)^D}\frac{-i\eta_{\alpha\beta}}{q^2-M_W^2} \\
& \times \gamma^\alpha L \frac{i}{\gamma\cdot(p'-q)-m_j} \\
& \times \left\{\frac{1}{4}\gamma_\mu(p+p'-2q)_\nu + \frac{1}{4}\gamma_\nu(p+p'-2q)_\mu - \frac{1}{D-2}\eta_{\mu\nu}m_j\right\} \\
& \times \frac{i}{\gamma\cdot(p-q)-m_j}\gamma^\beta L, && (78)\\
\mathcal{G}^{(h)}_{\mu\nu} \equiv & -\frac{\kappa g^2}{2}\frac{1}{M_W^2}\mu^{4-D}\int \frac{d^D q}{(2\pi)^D}\frac{i}{q^2-M_W^2} \\
& \times (m_j R - m_s L)\frac{i}{\gamma\cdot(p'-q)-m_j} \\
& \times \left\{\frac{1}{4}\gamma_\mu(p+p'-2q)_\nu + \frac{1}{4}\gamma_\nu(p+p'-2q)_\mu - \frac{1}{D-2}\eta_{\mu\nu}m_j\right\} \\
& \times \frac{i}{\gamma\cdot(p-q)-m_j}(m_j L - m_d R). && (79)
\end{aligned}
$$

Now the calculations of the above integrals are again straightforward but tedious since there are many types of gamma-matrix combinations and tensor structures. We just list our final formulae:

$$
\begin{aligned}
\mathcal{G}^{(e)}_{\mu\nu} = & \frac{\kappa g^2}{(4\pi)^2}\Bigg[G_3(p,p')\eta_{\mu\nu}\gamma\cdot p + G_3(p',p)\eta_{\mu\nu}\gamma\cdot p' + G_4(p,p')\gamma_{(\mu}p_{\nu)} + G_4(p',p)\gamma_{(\mu}p'_{\nu)} \\
& + \left\{-2f_7(p,p')p_\mu p_\nu + 2f_8(p,p')p'_\mu p'_\nu + 2f_9(p,p')p_{(\mu}p'_{\nu)}\right\}\gamma\cdot p \\
& + \left\{-2f_7(p',p)p'_\mu p'_\nu + 2f_8(p',p)p_\mu p_\nu + 2f_9(p',p)p_{(\mu}p'_{\nu)}\right\}\gamma\cdot p' \\
& + f_{10}(p,p')\gamma\cdot p'\gamma_{(\mu}p_{\nu)}\gamma\cdot p + f_{10}(p',p)\gamma\cdot p'\gamma_{(\mu}p'_{\nu)}\gamma\cdot p\Bigg]L, && (80)\\
\mathcal{G}^{(f)}_{\mu\nu} = & \frac{\kappa g^2}{(4\pi)^2}\frac{1}{M_W^2}\Bigg[G_5(p,p')\eta_{\mu\nu}\gamma\cdot p + G_5(p',p)\eta_{\mu\nu}\gamma\cdot p' \\
& + G_6(p,p')\gamma_{(\mu}p_{\nu)} + G_6(p',p)\gamma_{(\mu}p'_{\nu)} \\
& + \left\{-f_{11}(p',p)p'_\mu p'_\nu - f_7(p,p')p_\mu p_\nu + f_{12}(p',p)p_{(\mu}p'_{\nu)}\right\}\gamma\cdot p \\
& + \left\{-f_{11}(p,p')p_\mu p_\nu - f_7(p',p)p'_\mu p'_\nu + f_{12}(p,p')p_{(\mu}p'_{\nu)}\right\}\gamma\cdot p'\Bigg](m_j^2 L + m_s m_d R) \\
& + \frac{\kappa g^2}{(4\pi)^2}\frac{1}{M_W^2}\Bigg[G_7(p,p')\eta_{\mu\nu} \\
& + \left\{f_{13}(p,p')p'_\mu p'_\nu + f_{13}(p',p)p_\mu p_\nu - f_{14}(p,p')p_{(\mu}p'_{\nu)}\right\}\Bigg]m_j^2(m_s L + m_d R), && (81)
\end{aligned}
$$

$$\mathcal{G}^{(g)}_{\mu\nu} = \frac{\kappa g^2}{(4\pi)^2}\Bigg[G_8(p,p')\eta_{\mu\nu}\gamma\cdot p + G_8(p',p)\eta_{\mu\nu}\gamma\cdot p' + G_9(p,p')\gamma_{(\mu}p_{\nu)} + G_9(p',p)\gamma_{(\mu}p'_{\nu)}$$
$$+\left\{2f_{17}(p,p')p_\mu p_\nu - 2f_{19}(p',p)p'_\mu p'_\nu - 2f_{21}(p,p')p_{(\mu}p'_{\nu)}\right\}\gamma\cdot p$$
$$+\left\{2f_{17}(p',p)p'_\mu p'_\nu - 2f_{19}(p,p')p_\mu p_\nu - 2f_{21}(p',p)p_{(\mu}p'_{\nu)}\right\}\gamma\cdot p'$$
$$+ f_{22}(p,p')\gamma\cdot p'\gamma_{(\mu}p_{\nu)}\gamma\cdot p + f_{22}(p',p)\gamma\cdot p'\gamma_{(\mu}p'_{\nu)}\gamma\cdot p\Bigg]L, \tag{82}$$

$$\mathcal{G}^{(h)}_{\mu\nu} = \frac{\kappa g^2}{(4\pi)^2}\frac{1}{M_W^2}\Bigg[G_{10}(p,p')\eta_{\mu\nu}\gamma\cdot p + G_{10}(p',p)\eta_{\mu\nu}\gamma\cdot p'$$
$$+ G_{11}(p,p')\gamma_{(\mu}p_{\nu)} + G_{11}(p',p)\gamma_{(\mu}p'_{\nu)}$$
$$+\left\{-\frac{1}{2}f_{17}(p,p')p_\mu p_\nu + \frac{1}{2}f_{28}(p',p)p'_\mu p'_\nu - \frac{1}{2}f_{29}(p,p')p_{(\mu}p'_{\nu)}\right\}\gamma\cdot p$$
$$+\left\{-\frac{1}{2}f_{17}(p',p)p'_\mu p'_\nu + \frac{1}{2}f_{28}(p,p')p_\mu p_\nu - \frac{1}{2}f_{29}(p',p)p_{(\mu}p'_{\nu)}\right\}\gamma\cdot p'$$
$$+\frac{1}{4}f_{22}(p,p')\gamma\cdot p'\gamma_{(\mu}p_{\nu)}\gamma\cdot p + \frac{1}{4}f_{22}(p',p)\gamma\cdot p'\gamma_{(\mu}p'_{\nu)}\gamma\cdot p\Bigg](m_j^2 L + m_s m_d R)$$
$$+\frac{\kappa g^2}{(4\pi)^2}\frac{1}{M_W^2}\Bigg[G_{12}(p,p')\eta_{\mu\nu}$$
$$+\left\{\frac{1}{2}f_{24}(p,p')p_\mu p_\nu + \frac{1}{2}f_{24}(p',p)p'_\mu p'_\nu + \frac{1}{2}f_{23}(p,p')p_{(\mu}p'_{\nu)}\right\}$$
$$+\left\{-\frac{1}{4}f_{20}(p,p')\gamma_{(\mu}p_{\nu)} - \frac{1}{4}f_{20}(p',p)\gamma_{(\mu}p'_{\nu)}\right\}\gamma\cdot p$$
$$+\gamma\cdot p'\left\{-\frac{1}{4}f_{20}(p,p')\gamma_{(\mu}p_{\nu)} - \frac{1}{4}f_{20}(p',p)\gamma_{(\mu}p'_{\nu)}\right\}$$
$$+\frac{1}{4}f_{25}(p,p')\eta_{\mu\nu}\gamma\cdot p'\gamma\cdot p\Bigg]m_j^2(m_s L + m_d R). \tag{83}$$

Here we have introduced various kinds of Feynman parameter integrations $f_i(p, p')$, all of which are collected in Appendix A. Some combinations $G_i(p, p')$ $(i = 3, \ldots, 12)$ of $f_i(p, p')$ are defined in Appendix B.

7. Cancellation of ultraviolet divergences

We are now ready to sum up the ultraviolet divergences that appear in the graviton emission vertex

$$\Gamma_{\mu\nu}(p,p') \equiv \Gamma^{(\text{Fig. 3})}_{\mu\nu}(p,p') + \Gamma^{(\text{Fig. 4})}_{\mu\nu}(p,p'). \tag{84}$$

As we see in the formulae of Appendix B, the quantities $G_1(p^2)$, $G_2(p^2)$, and $G_i(p, p')(i = 3, \ldots, 11)$ all have a pole term $1/(D-4)$. In Eq. (71), for instance, we notice that $G_1(p^2)$ is not accompanied by m_j^2 or any j-dependent factors and therefore the pole term in $G_1(p^2)$ in Eq. (71) does not survive the j-(=t, c and u) summation because of the unitarity relation (41). The same comment applies to many of the other pole terms. Namely, the pole terms survive the j-summation only when multiplied by j-dependent factors such as m_j. It is noteworthy that not only the divergences in Figs. 3(a) and (c) but also those of Figs. 4(e) and (g) disappear after summation over j. Putting the remaining ultraviolet divergent terms all together, we end up with the following expression for the divergences:

$$\begin{aligned}\Gamma_{\mu\nu}(p,p')\\&=\frac{\kappa g^2}{(4\pi)^2}\sum_j (V_{\rm CKM})^*_{js}(V_{\rm CKM})_{jd}\\&\times\left[\left(-\frac{1}{4}\cdot\frac{1}{D-4}\right)\gamma_{(\mu}(p+p')_{\nu)}\frac{m_j^2}{M_W^2}L+\eta_{\mu\nu}\left(\frac{1}{2}\cdot\frac{1}{D-4}\right)\frac{m_j^2}{M_W^2}(m_sL+m_dR)\right.\\&\left.+\text{(finite terms)}\right].\end{aligned}\tag{85}$$

Note that divergences proportional to $\eta_{\mu\nu}\,\gamma\cdot(p+p')$ disappear in Eq. (85) via mutual cancellation.

The divergences in Eq. (85) should be compared with the counter term contributions $\Gamma^{\rm c.t.}_{\mu\nu}(p,p')$ due to Eq. (62) (Fig. 2(b)), namely,

$$\begin{aligned}\Gamma^{\rm c.t.}_{\mu\nu}(p,p')\\&=\frac{\kappa}{2}Z_L\gamma_{(\mu}(p+p')_{\nu)}L+\frac{\kappa}{2}Z_R\gamma_{(\mu}(p+p')_{\nu)}R\\&+\frac{\kappa}{D-2}Z_{Y1}\eta_{\mu\nu}m_sL+\frac{\kappa}{D-2}Z_{Y2}\eta_{\mu\nu}m_dR\\&=\frac{\kappa g^2}{(4\pi)^2}\sum_j (V_{\rm CKM})^*_{js}(V_{\rm CKM})_{jd}\\&\times\left[\frac{1}{4}\cdot\frac{1}{D-4}\cdot\frac{m_j^2}{M_W^2}\gamma_{(\mu}(p+p')_{\nu)}L-\frac{1}{2}\cdot\frac{1}{D-4}\cdot\frac{m_j^2}{M_W^2}\eta_{\mu\nu}(m_sL+m_dR)\right]\\&+\text{(finite terms)}.\end{aligned}\tag{86}$$

Apparently, the $D=4$ pole terms in Eq. (85) are canceled out by the corresponding counter term contributions in Eq. (86). This type of cancellation is the same as that known for a long time in the d–s–γ vertex analyses [20,24].

We have thus confirmed the finiteness of the sum

$$\Gamma^{\rm ren}_{\mu\nu}(p,p')=\Gamma_{\mu\nu}(p,p')+\Gamma^{\rm c.t.}_{\mu\nu}(p,p'),\tag{87}$$

which we now call the renormalized d–s–graviton vertex. The S-matrix element for the process (1) is now given a finite value through Eq. (87). When we deal with S-matrix elements in general, renormalization of external lines usually has to be taken into account. In our case, however, the renormalized two-point function $\Sigma_{\rm ren}(p)$ vanishes due to the subtraction conditions (50) once we put external d- and s-quarks on the mass shell, and therefore it does not seem to affect the S-matrix element of Eq. (1). This, however, does not necessarily mean that external line renormalization does not play a role in the computation of the S-matrix. Recall that the renormalization constants Z_L, Z_R, Z_{Y1}, and Z_{Y2} contain finite terms $c_1(m_j)$, $c_2(m_j)$, $c_3(m_j)$, and $c_4(m_j)$, respectively, as we see in Eqs. (46)–(49). These finite terms are taken over in $\Gamma^{\rm ren}_{\mu\nu}(p,p')$

after the pole term cancellation in Eq. (87). Also remember that these terms are all shared by the two-point function $\Sigma_{\rm ren}(p)$ as we see in Eq. (51). The finite terms $c_i(m_j)\,(i = 1, \ldots, 4)$ in $\Gamma^{\rm ren}_{\mu\nu}(p, p')$ and those in $\Sigma_{\rm ren}(p)$ are two sides of the same coin and are closely linked. In this sense the two-point function $\Sigma_{\rm ren}(p)$ is an integral part in computing S-matrix elements.

8. Ward–Takahashi identity

In the present paper the gravitational field is always treated as an external field and the invariance properties associated with the general coordinate transformation are reflected in the Feynman integrals. Such invariance properties ought to be expressed in the form of Ward–Takahashi identities among the Green's functions, whose field theoretical derivation, however, would be rather involved due to the existence of unphysical modes. Here we would like to use a much more naive "bottom-up" method. Namely, we deal with the linear combinations

$$\mathcal{G}^{(X)}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta^{\lambda\rho}\mathcal{G}^{(X)}_{\lambda\rho}, \qquad (X = a, b, \ldots, h) \tag{88}$$

corresponding to Eq. (35), multiply the Feynman integrals (88) by $(p - p')^\mu$, shuffle the integrands in an algebraic way without performing the integrations, and eventually associate Eq. (88) with the integrals of $d \to s$ self-energy type diagrams, $\mathcal{S}^{(a)}$ of Eq. (37) and $\mathcal{S}^{(b)}$ of Eq. (38). The identity that we thus find is

$$\begin{aligned}(p - p')^\mu &\left\{\Gamma_{\mu\nu}(p, p') - \frac{1}{2}\eta_{\mu\nu}\eta^{\lambda\rho}\Gamma_{\lambda\rho}(p, p')\right\} \\ &= \kappa\left\{p'_\nu\ \Sigma(p) - p_\nu\ \Sigma(p') + \frac{1}{4}\Sigma(p')\gamma\cdot(p - p')\gamma_\nu + \frac{1}{4}\gamma_\nu\gamma\cdot(p - p')\Sigma(p)\right\}.\end{aligned} \tag{89}$$

Very curiously, the counter terms (44) and (86) also satisfy the identity of the same form, namely,

$$\begin{aligned}(p - p')^\mu &\left\{\Gamma^{\rm c.t.}_{\mu\nu}(p, p') - \frac{1}{2}\eta_{\mu\nu}\eta^{\lambda\rho}\Gamma^{\rm c.t.}_{\lambda\rho}(p, p')\right\} \\ &= \kappa\left\{p'_\nu\ \Sigma_{\rm c.t.}(p) - p_\nu\ \Sigma_{\rm c.t.}(p') + \frac{1}{4}\Sigma_{\rm c.t.}(p')\gamma\cdot(p - p')\gamma_\nu + \frac{1}{4}\gamma_\nu\gamma\cdot(p - p')\Sigma_{\rm c.t.}(p)\right\}.\end{aligned} \tag{90}$$

Combining Eqs. (89) and (90) we find that the renormalized quantities (45) and (87) also satisfy the same identity:

$$\begin{aligned}(p - p')^\mu &\left\{\Gamma^{\rm ren}_{\mu\nu}(p, p') - \frac{1}{2}\eta_{\mu\nu}\eta^{\lambda\rho}\Gamma^{\rm ren}_{\lambda\rho}(p, p')\right\} \\ &= \kappa\left\{p'_\nu\ \Sigma_{\rm ren}(p) - p_\nu\ \Sigma_{\rm ren}(p') + \frac{1}{4}\Sigma_{\rm ren}(p')\gamma\cdot(p - p')\gamma_\nu + \frac{1}{4}\gamma_\nu\gamma\cdot(p - p')\Sigma_{\rm ren}(p)\right\}.\end{aligned} \tag{91}$$

We have checked the consistency of our Feynman integrations by referring to these identities. Note that, if external quarks are on the mass shell, the identity (91) reduces to the transversality condition

$$(p - p')^\mu\left\{\Gamma^{\rm ren}_{\mu\nu}(p, p') - \frac{1}{2}\eta_{\mu\nu}\eta^{\lambda\rho}\Gamma^{\rm ren}_{\lambda\rho}(p, p')\right\} = 0, \qquad \text{(on shell)}, \tag{92}$$

due to the subtraction conditions (50).

In the present paper all of the Feynman integrations are performed in the 't Hooft–Feynman gauge. For the above-mentioned analyses of the Ward–Takahashi identity, however, we have confirmed explicitly that Eqs. (89)–(92) are all valid in the general R_ξ gauge. Incidentally the Ward–Takahashi identity associated with Eq. (1) was also worked out by Corianò et al. [26].

Our identity (91) is essentially the same as theirs except for the difference due to the weight factor $(-e)^{1/4}$ on the quark fields.

9. The large top quark mass limit

Looking at the results of the graph calculations in Sect. 6, we notice immediately that the squared masses of the intermediate quarks, i.e., m_j^2 $(j = u, c, t)$ appear explicitly in Eqs. (73), (74), (81), and (83) besides those in the Feynman integrations. The origin of this m_j-dependence is traced back to the coupling of the unphysical scalar field to the quarks. Furthermore we notice that the renormalization constants (46), (48), and (49), have the factor m_j^2/M_W^2 as a coefficient of the $D = 4$ pole terms. The finite terms $c_i(m_j)\,(i = 1, \ldots, 4)$ in the renormalization constants also contain m_j^2/M_W^2 explicitly, as we see in Eqs. (56), (57), (58), and (59). We are very much interested in whether or not such an explicit linear dependence on m_j^2/M_W^2 could survive the summation of all the diagrams, as the large factor m_t^2/M_W^2 (≈ 4.62) of the top quarks would have an enhancement effect on the process (1).

Up to Sect. 8, we did not use any approximation with respect to the magnitude of the quark masses. In the present section, however, since we are going to pay attention to the large top quark mass behavior of our loop calculations, we suppose that we can neglect all the other quark masses together with external momenta squared, p^2, p'^2, and $(p - p')^2$. We now have to perform the Feynman parameter integrations explicitly under this approximation, which can be done in a straightforward way. After such calculations, however, our formulae would be extremely cluttered and it is easy for us to lose sight of the essential points. Therefore, in order to have a clear insight into our calculation, we suppose an additional relation $m_t^2 \gg M_W^2$. This relation is used only to inspect the structure of power series expansion with respect to m_t^2/M_W^2.

As mentioned above, the most dominant terms in the large top quark mass limit come from the unphysical scalar exchange diagrams, i.e., Figs. 3(b), (d) and 4(f), (h). Therefore we collect all those terms that contain m_t^2 at the front, take the $m_t^2/M_W^2 \to \infty$ limit in the parameter integration, and then arrive at the following formula:

$$
\begin{aligned}
\Gamma_{\mu\nu}(p, p') = {} & \frac{\kappa g^2}{(4\pi)^2}\,\frac{m_t^2}{M_W^2}\,(V_{\mathrm{CKM}})^*_{ts}(V_{\mathrm{CKM}})_{td} \\
& \times \Bigg[\left\{ -\frac{1}{4}\cdot\frac{1}{D-4} - \frac{1}{8}\log\left(\frac{m_t^2}{4\pi\mu^2 e^{-\gamma_E}}\right) + \frac{3}{16} \right\} \gamma_{(\mu}(p+p')_{\nu)} L \\
& + \left\{ \frac{1}{2}\cdot\frac{1}{D-4} + \frac{1}{4}\log\left(\frac{m_t^2}{4\pi\mu^2 e^{-\gamma_E}}\right) - \frac{1}{2} \right\} \eta_{\mu\nu}(m_s L + m_d R) \\
& + \mathcal{O}\left(\frac{1}{m_t^2}\right) \Bigg].
\end{aligned} \tag{93}
$$

Note that terms proportional to $\eta_{\mu\nu}\gamma \cdot (p + p')$ have disappeared in Eq. (93) after mutual cancellation.

The pole terms at $D = 4$ in Eq. (93) are to be canceled by the corresponding ones in the counter terms that also contain m_t^2/M_W^2 at the beginning. The renormalization constants may be expressed in the following way:

$$
Z_L \approx \frac{g^2}{(4\pi)^2}(V_{\mathrm{CKM}})^*_{ts}(V_{\mathrm{CKM}})_{td}\,\frac{m_t^2}{M_W^2}\left(\frac{1}{2}\cdot\frac{1}{D-4} - \tilde{c}_1\right), \tag{94}
$$

$$Z_R \approx \frac{g^2}{(4\pi)^2}(V_{\rm CKM})^*_{ts}(V_{\rm CKM})_{td}\ \frac{m_t^2}{M_W^2} \times (-\tilde{c}_2), \tag{95}$$

$$Z_{Y1} \approx \frac{g^2}{(4\pi)^2}(V_{\rm CKM})^*_{ts}(V_{\rm CKM})_{td}\ \frac{m_t^2}{M_W^2}\left(-\frac{1}{D-4}\ - \tilde{c}_3\right), \tag{96}$$

$$Z_{Y2} \approx \frac{g^2}{(4\pi)^2}(V_{\rm CKM})^*_{ts}(V_{\rm CKM})_{td}\ \frac{m_t^2}{M_W^2}\left(-\frac{1}{D-4}\ - \tilde{c}_4\right). \tag{97}$$

Here four quantities $\tilde{c}_i$ $(i = 1, \ldots, 4)$ are extracted respectively from $c_i(m_t)$ $(i = 1, \ldots, 4)$ as coefficients of those proportional to m_t^2/M_W^2, namely,

$$\tilde{c}_1 = \frac{1}{m_d^2 - m_s^2}\left[-\frac{1}{2}\left\{m_d^2 f_1\left(m_d^2\right) - m_s^2 f_1\left(m_s^2\right)\right\} + \frac{(m_d^2 + m_s^2)}{2}\left\{f_2\left(m_d^2\right) - f_2\left(m_s^2\right)\right\}\right], \tag{98}$$

$$\tilde{c}_2 = \frac{m_d m_s}{m_d^2 - m_s^2}\left[-\frac{1}{2}\left\{f_1\left(m_d^2\right) - f_1\left(m_s^2\right)\right\} + \left\{f_2\left(m_d^2\right) - f_2\left(m_s^2\right)\right\}\right], \tag{99}$$

$$\tilde{c}_3 = \frac{m_d^2}{m_d^2 - m_s^2}\left[\frac{1}{2}\left\{f_1\left(m_d^2\right) - f_1\left(m_s^2\right)\right\} + \frac{1}{2}\left\{-\frac{m_d^2 + m_s^2}{m_d^2}\ f_2\left(m_d^2\right) + 2 f_2\left(m_s^2\right)\right\}\right], \tag{100}$$

$$\tilde{c}_4 = \frac{m_s^2}{m_d^2 - m_s^2}\left[\frac{1}{2}\left\{f_1\left(m_d^2\right) - f_1\left(m_s^2\right)\right\} + \frac{1}{2}\left\{\frac{m_d^2 + m_s^2}{m_s^2}\ f_2\left(m_s^2\right) - 2 f_2\left(m_d^2\right)\right\}\right]. \tag{101}$$

Recall that the original definitions of $f_1(p^2)$ and $f_2(p^2)$ contain m_j as we see in Eqs. (A1), and (A2). Here, however, we understand that all m_j in f_1 and f_2 in Eqs. (98), (99), (100) and (101) have been replaced by the top quark mass m_t, namely,

$$f_1(p^2) = \int_0^1 dx\,(1-x)\log\left\{\frac{-x(1-x)p^2 + x m_t^2 + (1-x)M_W^2}{4\pi\mu^2 e^{-\gamma_E}}\right\}, \tag{102}$$

$$f_2(p^2) = \int_0^1 dx\log\left\{\frac{-x(1-x)p^2 + x m_t^2 + (1-x)M_W^2}{4\pi\mu^2 e^{-\gamma_E}}\right\}. \tag{103}$$

The approximate equality "$\approx$" in Eqs. (94), (95), (96), and (97) means that we have simply collected terms containing m_t^2/M_W^2 as an overall factor without going into the details of the m_t-dependence of $\tilde{c}_i$ $(i = 1, \ldots, 4)$ through f_1 and f_2.

We now look at the four quantities $\tilde{c}_i$ $(i = 1, \ldots, 4)$ more closely, namely, their m_t-dependence entering through f_1 and f_2. Taking the limit $m_t \to \infty$ while neglecting M_W^2 and p^2 in Eqs. (102) and (103), we immediately find the following asymptotic behavior:

$$f_1(p^2) = -\frac{3}{4} + \frac{1}{2}\log\left(\frac{m_t^2}{4\pi\mu^2 e^{-\gamma_E}}\right) + \mathcal{O}\left(\frac{M_W^2}{m_t^2}, \frac{p^2}{m_t^2}\right), \tag{104}$$

$$f_2(p^2) = -1 + \log\left(\frac{m_t^2}{4\pi\mu^2 e^{-\gamma_E}}\right) + \mathcal{O}\left(\frac{M_W^2}{m_t^2}, \frac{p^2}{m_t^2}\right). \tag{105}$$

Inserting Eqs. (104) and (105) into Eqs. (98), (99), (100), and (101), we obtain the large-m_t behavior of the four quantities $\tilde{c}_i$ $(i = 1, \ldots, 4)$ as follows:

$$\tilde{c}_1 = \frac{3}{8} - \frac{1}{4}\log\left(\frac{m_t^2}{4\pi\mu^2 e^{-\gamma_E}}\right) + \mathcal{O}\left(\frac{1}{m_t^2}\right), \tag{106}$$

$$\tilde{c}_2 = \mathcal{O}\left(\frac{1}{m_t^2}\right), \tag{107}$$

$$\tilde{c}_3 = -\frac{1}{2} + \frac{1}{2}\log\left(\frac{m_t^2}{4\pi\mu^2 e^{-\gamma_E}}\right) + \mathcal{O}\left(\frac{1}{m_t^2}\right), \tag{108}$$

$$\tilde{c}_4 = -\frac{1}{2} + \frac{1}{2}\log\left(\frac{m_t^2}{4\pi\mu^2 e^{-\gamma_E}}\right) + \mathcal{O}\left(\frac{1}{m_t^2}\right). \tag{109}$$

By putting these formulae into Eqs. (94), (95), (96), and (97), the four renormalization constants turn out in the leading order in m_t^2/M_W^2 to be

$$Z_L = \frac{g^2}{(4\pi)^2}(V_{\rm CKM})^*_{ts}(V_{\rm CKM})_{td}\ \frac{m_t^2}{M_W^2} \times \left\{\frac{1}{2}\cdot\frac{1}{D-4} - \frac{3}{8} + \frac{1}{4}\log\left(\frac{m_t^2}{4\pi\mu^2 e^{-\gamma_E}}\right) + \mathcal{O}\left(\frac{1}{m_t^2}\right)\right\}, \tag{110}$$

$$Z_R = \frac{g^2}{(4\pi)^2}(V_{\rm CKM})^*_{ts}(V_{\rm CKM})_{td}\ \frac{m_t^2}{M_W^2} \times \mathcal{O}\left(\frac{1}{m_t^2}\right), \tag{111}$$

$$Z_{Y1} = \frac{g^2}{(4\pi)^2}(V_{\rm CKM})^*_{ts}(V_{\rm CKM})_{td}\ \frac{m_t^2}{M_W^2} \times \left\{-\frac{1}{D-4} + \frac{1}{2} - \frac{1}{2}\log\left(\frac{m_t^2}{4\pi\mu^2 e^{-\gamma_E}}\right) + \mathcal{O}\left(\frac{1}{m_t^2}\right)\right\}, \tag{112}$$

$$Z_{Y2} = \frac{g^2}{(4\pi)^2}(V_{\rm CKM})^*_{ts}(V_{\rm CKM})_{td}\ \frac{m_t^2}{M_W^2} \times \left\{-\frac{1}{D-4} + \frac{1}{2} - \frac{1}{2}\log\left(\frac{m_t^2}{4\pi\mu^2 e^{-\gamma_E}}\right) + \mathcal{O}\left(\frac{1}{m_t^2}\right)\right\}. \tag{113}$$

We thus find that the counter term contribution to the vertex (86) is given in the $m_t \to \infty$ limit by

$$\begin{aligned}\Gamma^{\rm c.t.}_{\mu\nu}(p,p') &= \frac{\kappa}{2}Z_L\gamma_{(\mu}(p+p')_{\nu)}\,L + \frac{\kappa}{2}Z_R\gamma_{(\mu}(p+p')_{\nu)}\,R \\ &\quad + \frac{\kappa}{D-2}Z_{Y1}\eta_{\mu\nu}m_s L + \frac{\kappa}{D-2}Z_{Y2}\eta_{\mu\nu}m_d R \\ &= \frac{\kappa g^2}{(4\pi)^2}\frac{m_t^2}{M_W^2}(V_{\rm CKM})^*_{ts}(V_{\rm CKM})_{td} \\ &\quad \times \left[\left\{\frac{1}{4}\cdot\frac{1}{D-4} - \frac{3}{16} + \frac{1}{8}\log\left(\frac{m_t^2}{4\pi\mu^2 e^{-\gamma_E}}\right)\right\}\gamma_{(\mu}(p+p')_{\nu)}L\right. \\ &\quad + \left\{-\frac{1}{2}\cdot\frac{1}{D-4} + \frac{1}{4} - \frac{1}{4}\log\left(\frac{m_t^2}{4\pi\mu^2 e^{-\gamma_E}}\right) + \frac{1}{4}\right\}\eta_{\mu\nu}\,(m_s L + m_d R) \\ &\quad \left. + \mathcal{O}\left(\frac{1}{m_t^2}\right)\right].\end{aligned} \tag{114}$$

The fourth term "$+\frac{1}{4}$" in the curly brackets in the third line of Eq. (114) comes from the top quark contribution in the second term in Eqs. (63) and (64). It is quite remarkable that there occurs a cancellation among the leading terms in Eq. (93) and those in Eq. (114) and the renormalized vertex is not of the order of m_t^2/M_W^2 but of $\mathcal{O}(1)$, i.e.,

$$\Gamma^{\rm ren}_{\mu\nu}(p,p') = \Gamma_{\mu\nu}(p,p') + \Gamma^{\rm c.t.}_{\mu\nu}(p,p') = \mathcal{O}(1). \tag{115}$$

There is thus no enhancement by the factor m_t^2/M_W^2 in the d–s–graviton vertex in the large top quark mass limit.

The cancellation between the leading terms in $\Gamma_{\mu\nu}$ and $\Gamma^{\text{c.t.}}_{\mu\nu}$, however, is not totally unexpected. In fact we have seen in Eqs. (93) and (114) that the tensor-index- and gamma-matrix-structures of $\Gamma_{\mu\nu}$ and $\Gamma^{\text{c.t.}}_{\mu\nu}$ consist of two types, i.e., $\gamma_{(\mu}(p+p^{'})_{\nu)}L$ and $\eta_{\mu\nu}(m_s L + m_d R)$. With these two types only, it is impossible for $\Gamma^{\text{ren}}_{\mu\nu}$ to satisfy the gravitational transverse condition (92) on the mass shell of external quarks. The sum of the leading terms in $\Gamma_{\mu\nu}$ and $\Gamma^{\text{c.t.}}_{\mu\nu}$ has necessarily to vanish. Note that in subleading orders, there appear several other types of tensor-index- and gamma-matrix-structures and the transverse condition would become non-trivial.

The absence of the $\mathcal{O}(m_t^2/M_W^2)$ terms in $\Gamma^{\text{ren}}_{\mu\nu}$ may be seen in terms of Σ_{ren} on the basis of the Ward–Takahashi identity. Let us now take the large top quark mass limit in Eq. (42), i.e.,

$$\Sigma(p) = \frac{g^2}{(4\pi)^2}(V_{\text{CKM}})^*_{ts}(V_{\text{CKM}})_{td}\frac{m_t^2}{M_W^2} \times \left[\left\{-\frac{1}{2}\cdot\frac{1}{D-4} - \frac{1}{4}\log\left(\frac{m_t^2}{4\pi\mu^2 e^{-\gamma_E}}\right) + \frac{3}{8}\right\}\gamma\cdot p\, L + \left\{\frac{1}{D-4} + \frac{1}{2}\log\left(\frac{m_t^2}{4\pi\mu^2 e^{-\gamma_E}}\right) - \frac{1}{2}\right\}(m_s L + m_d R) + \mathcal{O}\left(\frac{1}{m_t^2}\right)\right]. \tag{116}$$

Then we combine Eq. (116) with $\Sigma_{\text{c.t.}}(p)$ in Eq. (44) with the four renormalization constants approximated by Eqs. (110), (111), (112), and (113):

$$\begin{aligned}\Sigma_{\text{c.t.}}(p) &= Z_L\gamma\cdot p\, L + Z_R\gamma\cdot p\, R + Z_{Y1}m_s L + Z_{Y2}m_d R \\ &= \frac{g^2}{(4\pi)^2}(V_{\text{CKM}})^*_{ts}(V_{\text{CKM}})_{td}\frac{m_t^2}{M_W^2} \\ &\quad\times\left[\left\{\frac{1}{2}\cdot\frac{1}{D-4} - \frac{3}{8} + \frac{1}{4}\log\left(\frac{m_t^2}{4\pi\mu^2 e^{-\gamma_E}}\right)\right\}\gamma\cdot p\, L \right.\\ &\quad\left. + \left\{-\frac{1}{D-4} + \frac{1}{2} - \frac{1}{2}\log\left(\frac{m_t^2}{4\pi\mu^2 e^{-\gamma_E}}\right)\right\}(m_s L + m_d R) + \mathcal{O}\left(\frac{1}{m_t^2}\right)\right].\end{aligned} \tag{117}$$

Here we find the leading terms of $\mathcal{O}(m_t^2/M_W^2)$ in Eqs. (116) and (117) canceling each other, and we end up with

$$\Sigma_{\text{ren}}(p) = \Sigma(p) + \Sigma_{\text{c.t.}}(p) = \mathcal{O}(1)\,. \tag{118}$$

The absence of the $\mathcal{O}(m_t^2/M_W^2)$ terms in $\Sigma_{\text{ren}}(p)$ is consistent with the Ward–Takahashi identity (91), whose left- and right-hand sides are both of $\mathcal{O}(1)$.

10. The $\mathcal{O}(1)$ effective interactions

In the previous section we discussed the seemingly most dominant terms behaving as $\mathcal{O}(m_t^2/M_W^2)$ when the limit $m_t \to \infty$ is taken, and have shown that these leading terms cancel among themselves. Equations (115) and (118) were our net results in Sect. 9. In the present section we turn our attention to the $\mathcal{O}(1)$ terms that are supposed to come next in the said limit. There are a variety of contributions to this order and it is not straightforward to classify all of them. For now we simply highlight a few characteristic terms that are described effectively by the operator

$$\sqrt{-g}\,\overline{\psi}_s\,(m_s L + m_d R)\ \psi_d\ \mathcal{R} = \sqrt{-e}\,\overline{\Psi}_s\,(m_s L + m_d R)\ \Psi_d\ \mathcal{R}. \tag{119}$$

Here the scalar curvature $\mathcal{R}$ should not be confused with the chiral projection R. The strange and down quark fields on the right-hand side of Eq. (119) are given the weight $(-e)^{1/4}$ ($\Psi_s = (-e)^{1/4}\psi_s$, $\Psi_d = (-e)^{1/4}\psi_d$).

In the weak field approximation as given in Eq. (16) we have

$$\sqrt{-g}\,\mathcal{R} = -2\kappa\left(\partial^\mu\partial^\nu - \eta^{\mu\nu}\partial^2\right)\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^\lambda{}_\lambda\right) + \mathcal{O}(\kappa^2)$$

$$= -2\kappa\left(\partial^\mu\partial^\nu + \frac{1}{2}\partial^2\eta^{\mu\nu}\right)h_{\mu\nu} + \mathcal{O}(\kappa^2), \tag{120}$$

and in the momentum space Eq. (120) becomes

$$2\kappa\left(k^\mu k^\nu + \frac{1}{2}k^2\eta^{\mu\nu}\right)h_{\mu\nu} + \mathcal{O}(\kappa^2). \tag{121}$$

Here k^μ is the graviton momentum, i.e., $k^\mu = p^\mu - p'^\mu$. Thus if we find in $\Gamma_{\mu\nu}(p, p')$ terms of the following combination of tensor-index and gamma-matrix structures:

$$\left\{\left(p - p'\right)_\mu\left(p - p'\right)_\nu + \frac{1}{2}\left(p - p'\right)^2\eta_{\mu\nu}\right\}\left(m_s L + m_d R\right), \tag{122}$$

then we are allowed to say that these terms are described effectively by the operator (119).

Looking at the explicit results of $\mathcal{G}^{(X)}_{\mu\nu}(X = a, b, \ldots, h)$ in Sect. 6 closely, we notice that only $\mathcal{G}^{(f)}_{\mu\nu}$ and $\mathcal{G}^{(h)}_{\mu\nu}$ contain terms that could possibly be given the structure of Eq. (122):

$$\mathcal{G}^{(f)}_{\mu\nu} = \frac{\kappa g^2}{(4\pi)^2}\frac{1}{M_W^2}\Bigg[G_7(p, p')\eta_{\mu\nu}$$

$$+\left\{f_{13}(p, p')p'_\mu p'_\nu + f_{13}(p', p)p_\mu p_\nu - f_{14}(p, p')p_{(\mu}p'_{\nu)}\right\}\Bigg]m_j^2(m_s L + m_d R)$$

$$+ \cdots\cdots, \tag{123}$$

$$\mathcal{G}^{(h)}_{\mu\nu} = \frac{\kappa g^2}{(4\pi)^2}\frac{1}{M_W^2}\Bigg[G_{12}(p, p')\eta_{\mu\nu}$$

$$+\left\{\frac{1}{2}f_{24}(p, p')p_\mu p_\nu + \frac{1}{2}f_{24}(p', p)p'_\mu p'_\nu + \frac{1}{2}f_{23}(p, p')p_{(\mu}p'_{\nu)}\right\}\Bigg]m_j^2(m_s L + m_d R)$$

$$+ \cdots\cdots. \tag{124}$$

In order to confirm that terms in Eqs. (123) and (124) are actually combined together to be given the structure of Eq. (122), we restrict our analyses to the following low-energy case:

$$p^2,\ p'^2,\ (p - p')^2 \ll M_W^2,\ m_j^2. \tag{125}$$

Note that we do not assume any particular relation between M_W and m_j ($j = t, c, u$).

Applying the approximation (125) to the quantities $f_{13}(p, p')$ and $f_{14}(p, p')$ in Eq. (123), and to $f_{23}(p, p')$ and $f_{24}(p, p')$ in Eq. (124), we just set $p^2 = p'^2 = (p - p')^2 = 0$ in the integral representations (A14), (A15), (A25), and (A26) in Appendix A. After performing double integration we get the following formulae for the two combinations of these functions:

$$f_{13}(0, 0) + \frac{1}{2}f_{24}(0, 0) = \frac{1}{m_j^2}F_1\left(\frac{m_j^2}{M_W^2}\right), \tag{126}$$

$$f_{14}(0, 0) - \frac{1}{2}f_{23}(0, 0) = \frac{2}{m_j^2}F_1\left(\frac{m_j^2}{M_W^2}\right), \tag{127}$$

where we have introduced a function

$$F_1(x) = \frac{x(3-x)}{8(1-x)^2} - \frac{x(2x^2-4x-1)}{12(1-x)^3}\log x. \tag{128}$$

Note that the function $F_1(x)$ is finite at $x = 1$, i.e., $\lim_{x\to 1} F_1(x) = 1/12$. Also note the asymptotic behavior, $F_1(x) \sim -\frac{1}{8} + \frac{1}{6}\log x$ for large x. It is remarkable that a common quantity $F_1(m_j^2/M_W^2)$ has appeared on the right-hand side of Eqs. (126) and (127). Thanks to this common quantity, the sum of all the terms with $p_\mu p_\nu$, $p'_\mu p'_\nu$, and $p_{(\mu}p'_{\nu)}$ in Eqs. (123) and (124) turns out to be a very concise one, i.e.,

$$\begin{aligned}&\left\{f_{13}(0,0) + \frac{1}{2}f_{24}(0,0)\right\}(p_\mu p_\nu + p'_\mu p'_\nu) - \left\{f_{14}(0,0) - \frac{1}{2}f_{23}(0,0)\right\}p_{(\mu}p'_{\nu)}\\ &= \frac{1}{m_j^2}F_1\left(\frac{m_j^2}{M_W^2}\right)(p-p')_\mu(p-p')_\nu.\end{aligned} \tag{129}$$

Let us now move to the remaining terms, $G_7(p, p')\eta_{\mu\nu}$ in Eq. (123) and $G_{12}(p, p')\eta_{\mu\nu}$ in Eq. (124). Recall that $G_7(p, p')$ contains $f_4(p, p')$, $f_6(p, p')$, and $f_{15}(p, p')$ as defined in Eq. (B7) and that $G_{12}(p, p')$ contains $f_{20}(p, p')$, $f_{24}(p, p')$, $f_{26}(p, p')$, and $f_{27}(p, p')$ as defined in Eq. (B12). We expand these functions in a Taylor series with respect to p^2, p'^2, and $(p-p')^2$ through the second order to meet with Eq. (129). After straightforward calculations we have found a formula:

$$\begin{aligned}G_7(p,p') + G_{12}(p,p') = G_7(0,0) + G_{12}(0,0) + \frac{(p^2+p'^2)}{m_j^2}F_2\left(\frac{m_j^2}{M_W^2}\right)\\ + \frac{(p-p')^2}{m_j^2}\cdot\frac{1}{2}F_1\left(\frac{m_j^2}{M_W^2}\right) + \cdots\cdots,\end{aligned} \tag{130}$$

where the ellipses denote higher-order terms in the Taylor expansion and are neglected. Here we have defined another function:

$$F_2(x) = \frac{x+x^2}{8(1-x)^2} + \frac{x^2}{4(1-x)^3}\log x. \tag{131}$$

This function is also free from singularity at $x = 1$, i.e., $\lim_{x\to 1} F_2(x) = 1/24$. The third term in Eq. (130) that contains this function $F_2(m_j^2/M_W^2)$ would be described by an operator of a different type from Eq. (119), and we will not delve into it hereafter. It is noteworthy that the quantity $F_1(m_j^2/M_W^2)$ has again appeared as the coefficient of $(p-p')^2$ in Eq. (130).

Those related to the graviton momentum $(p-p')^\mu$ are thus summed up with the common coefficient $F_1(m_j^2/M_W^2)$ as

$$\begin{aligned}\mathcal{G}^{(f)}_{\mu\nu} + \mathcal{G}^{(h)}_{\mu\nu} = &\frac{\kappa g^2}{(4\pi)^2}\frac{1}{M_W^2}F_1\left(\frac{m_j^2}{M_W^2}\right)\left\{(p-p')_\mu(p-p')_\nu + \frac{1}{2}(p-p')^2\eta_{\mu\nu}\right\}\\ &\times(m_s L + m_d R)\\ &+\cdots\cdots.\end{aligned} \tag{132}$$

In terms of $\Gamma_{\mu\nu}(p, p')$, we have

$$\begin{aligned}\Gamma_{\mu\nu}(p,p') = &\frac{\kappa g^2}{(4\pi)^2}\frac{\mathcal{F}_1}{M_W^2}\left\{(p-p')_\mu(p-p')_\nu + \frac{1}{2}(p-p')^2\eta_{\mu\nu}\right\}(m_s L + m_d R)\\ &+\cdots\cdots,\end{aligned} \tag{133}$$

where the coefficient in front of the brackets

$$\mathcal{F}_1 = \sum_{j=t,c,u} (V_{\rm CKM})^*_{js}(V_{\rm CKM})_{jd}F_1\left(\frac{m_j^2}{M_W^2}\right) \tag{134}$$

depends on the top, charm, and up quark masses as well as the CKM matrix elements. Equation (133) is given the same tensor-index and gamma-matrix structure as Eq. (122). This is exactly what we expect to arise from the operator (119), and the effective Lagrangian becomes

$$\mathcal{L}^{\mathcal{R}}_{eff} = \frac{g^2}{(4\pi)^2}\frac{\mathcal{F}_1}{2M_W^2}\sqrt{-e}\,\overline{\Psi}_s\,(m_sL + m_dR)\ \Psi_d\ \mathcal{R}. \tag{135}$$

As we remarked before, the function F_1 has the asymptotic behavior

$$F_1\left(\frac{m_j^2}{M_W^2}\right) \sim -\frac{1}{8} + \frac{1}{6}\log\left(\frac{m_j^2}{M_W^2}\right) \quad \text{as} \quad \frac{m_j^2}{M_W^2} \to \infty, \tag{136}$$

and this formula shows clearly the $\mathcal{O}(1)$ non-decoupling effects of the heavy quark. Numerically, the top quark contribution to the coefficient $\mathcal{F}_1$ is dominant over the other two, as we find:

$$F_1\left(\frac{m_t^2}{M_W^2}\right) = 0.216\,86\ , \tag{137}$$

$$F_1\left(\frac{m_c^2}{M_W^2}\right) = -7.92\times 10^{-5}, \tag{138}$$

$$F_1\left(\frac{m_u^2}{M_W^2}\right) = -9.96\times 10^{-10}, \tag{139}$$

for $M_W = 80.379$ GeV, $m_t = 172.76$ GeV, $m_c = 1.27$ GeV, and $m_u = 2.16$ MeV [35]. This non-negligible effect of the heavy top quark is a manifestation of the $\mathcal{O}(1)$ non-decoupling effects.

Although our effective Lagrangian (135) is one of the most important results of the present paper, we do not attempt here to apply Eq. (135) to actual physical problems. Let us, however, bear in mind that Eq. (135) could be relevant to flavor-changing and CP-violating gravitational phenomena. In fact, the most dominant top quark contribution in Eq. (134) is accompanied by $(V_{\rm CKM})^*_{ts}(V_{\rm CKM})_{td}$, which is given by

$$(V_{\rm CKM})^*_{ts}(V_{\rm CKM})_{td} = \left(-c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta}\right)^*\left(s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta}\right), \tag{140}$$

according to the standard parametrization [35], and contains the CP-violating phase δ.

Finally we would like to add a comment on the comparison with the loop-induced $d \to s + \gamma$ transition, on which it has been pointed out [20,22] that the transition amplitude contains a term described effectively by the operator

$$\overline{\psi}_s\sigma^{\mu\nu}\,(m_sL + m_dR)\,\psi_d\ F_{\mu\nu}. \tag{141}$$

Here $F_{\mu\nu}$ is the electromagnetic field strength and $\sigma^{\mu\nu}$ is defined in Eq. (5). This operator reminds us of the Pauli term in quantum electrodynamics. It is also known [36–38] that in the loop-induced $d \to s +$ gluon transition, there also exist contributions described by the similar operator

$$\overline{\psi}_s\ T^a\,\sigma^{\mu\nu}\,(m_sL + m_dR)\ \psi_d\ F^a_{\mu\nu}, \tag{142}$$

where $F^a_{\mu\nu}$ is the field strength of the gluon field and T^a is the generator of the color gauge group.

A question naturally arises here: one may ask whether there exists a similar sort of contribution in the gravitational process (1). It is very tempting to postulate that the operator analogous to Eq. (142) would be

$$\sqrt{-g}\,\overline{\psi}_s \left\{\sigma^{ab},\ \sigma^{\mu\nu}\right\}(m_s L + m_d R)\ \psi_d\ R_{\mu\nu ab}. \tag{143}$$

Here the non-Abelian field strength $F^a_{\mu\nu}$ in Eq. (142) is replaced by the Riemann tensor defined in terms of the spin connection as

$$R_{\mu\nu ab} = \partial_\mu \omega_{\nu ab} - \partial_\nu \omega_{\mu ab} + \omega_{\mu a}{}^{c}\omega_{\nu cb} - \omega_{\nu a}{}^{c}\omega_{\mu cb} \quad \left(= e^{\lambda}{}_{a} e^{\rho}{}_{b} R_{\mu\nu\lambda\rho}\right). \tag{144}$$

The gauge group generator T^a in Eq. (142) has been replaced by the local Lorentz group generator σ^{ab}.

Now it is known that the Riemann tensor may be decomposed into three parts:

$$R_{\lambda\mu\nu\rho} = C_{\lambda\mu\nu\rho} - \frac{1}{2}\left(R_{\lambda\rho}g_{\mu\nu} - R_{\lambda\nu}g_{\mu\rho} + R_{\mu\nu}g_{\lambda\rho} - R_{\mu\rho}g_{\lambda\nu}\right) - \frac{1}{6}\mathcal{R}\ \left(g_{\lambda\nu}g_{\mu\rho} - g_{\lambda\rho}g_{\mu\nu}\right), \tag{145}$$

where the first term $C_{\lambda\mu\nu\rho}$ is the Weyl tensor and is traceless:

$$g^{\lambda\nu}C_{\lambda\mu\nu\rho} = 0, \quad g^{\mu\rho}C_{\lambda\mu\nu\rho} = 0. \tag{146}$$

Once we take the product of Eq. (145) with $\{\sigma^{\lambda\mu}, \sigma^{\nu\rho}\}$, we immediately find a relation:

$$\left\{\sigma^{\lambda\mu}, \sigma^{\nu\rho}\right\}\ R_{\lambda\mu\nu\rho} = 4\mathcal{R} + \left\{\sigma^{\lambda\mu}, \sigma^{\nu\rho}\right\}\ C_{\lambda\mu\nu\rho}. \tag{147}$$

The scalar curvature term $4\mathcal{R}$ on the right-hand side of Eq. (147), when plugged into Eq. (143), gives us the same operator as in Eq. (119), which has already been studied above. It is therefore crucial whether the contribution due to $\{\sigma^{\lambda\mu}, \sigma^{\nu\rho}\}C_{\lambda\mu\nu\rho}$ exists or not in the amplitudes in order for the operator (143) to be an effective one. Unfortunately in the weak field expansion (16), a straightforward calculation shows

$$\left\{\sigma^{\lambda\mu}, \sigma^{\nu\rho}\right\}\ R_{\lambda\mu\nu\rho} - 4\mathcal{R} = \mathcal{O}(\kappa^2). \tag{148}$$

This means that the Weyl tensor contribution $\{\sigma^{\lambda\mu}, \sigma^{\nu\rho}\}C_{\lambda\mu\nu\rho}$ is of the order of κ^2 and cannot be seen in our $\mathcal{O}(\kappa)$ calculation. To seek a gravitational analogue of Eqs. (141) and (142), we have to examine two graviton emission processes.

11. Summary

In the present paper we have investigated the loop-induced flavor-changing gravitational process (1) in the standard electroweak theory in order to see the non-decoupling effects of the heavy top quark running along an internal line. We have confirmed explicitly that the renormalization constants Z_L, Z_R, Z_{Y1}, and Z_{Y2} determined for the self-energy type $d \to s$ diagrams (Fig. 1) in flat space serve adequately to eliminate ultraviolet divergences in the one-graviton vertex diagrams (Figs. 3 and 4). It is pointed out that the unrenormalized and renormalized two- and three-point functions satisfy the same form of Ward–Takahashi identities, Eqs. (89) and (91), as quantum electrodynamics. We collected and examined the leading terms in the $m_t \to \infty$ limit in the renormalized transition amplitude that are proportional to m_t^2/M_W^2. We have found that these $\mathcal{O}(m_t^2/M_W^2)$ terms disappear by cancellation. The non-decoupling effects of the internal top quark thus take place at the $\mathcal{O}(1)$ level. Among the $\mathcal{O}(1)$ terms, we have noticed the contributions that are supposed to have come from the effective Lagrangian (135) that consists of a quark bilinear form coupled to the spacetime scalar curvature $\mathcal{R}$. The top quark effect is sizable in Eq. (135) and this is one of manifest forms of non-decoupling effects.

While the effective Lagrangian (135) looks concise, we did not find the Ricci tensor or the Weyl tensor counterpart within the present Standard Model calculation of Eq. (1) at the one-loop and one-graviton emission level. Perhaps in more sophisticated models such as supersymmetric gauge theories or grand unification models, in which several very heavy particles are supposed to exist, we could encounter various types of effective interactions as explored extensively by Ruhdorfer et al. [39]. Alternatively, such various interaction terms would arise in two-loop or higher level of calculations. Those non-trivial effective interactions with spacetime could cause intriguing effects if applied to the early universe. When the universe was expanding, the Riemann tensor, Ricci tensor, and scalar curvature in the ensuing effective Lagrangian have to be those of the Friedmann–Lemaître–Robertson–Walker metric, and the effective interactions among quarks would not respect the time-reversal invariance. The implications of such effective interactions would be extremely interesting and deserve further pursuit, but for now we have to leave these investigations for our future work.

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Appendix A. The Feynman parameter integrations

The following parameter integrations appear in evaluating the self-energy type $d \to s$ transition amplitudes:

$$f_1(p^2) = \int_0^1 dx\,(1-x)\log\left\{\frac{-x(1-x)p^2 + xm_j^2 + (1-x)M_W^2}{4\pi\mu^2 e^{-\gamma_E}}\right\}, \tag{A1}$$

$$f_2(p^2) = \int_0^1 dx\log\left\{\frac{-x(1-x)p^2 + xm_j^2 + (1-x)M_W^2}{4\pi\mu^2 e^{-\gamma_E}}\right\}, \tag{A2}$$

where γ_E is the Euler number. These functions also appear in the calculation of Fig. 3. Note that both Eqs. (A1) and (A2) are m_j-dependent, although the dependence is not made explicit on the left-hand side of Eqs. (A1) or (A2). The same comment applies to all the functions to be introduced hereafter in this appendix.

Combining propagators in Figs. 4(e) and (f) by using Feynman's parameters, the following combination commonly appears in the denominator:

$$\begin{aligned}\Delta_1 \equiv & -y(1-x-y)p^2 - x(1-x-y)p'^2 - xy(p-p')^2 \\ & + (x+y)M_W^2 + (1-x-y)m_j^2.\end{aligned} \tag{A3}$$

The parameter integrations involving Eq. (A3) that we used in Sect. 6 are as follows:

$$f_3(p,p') = M_W^2\int_0^1 dx\int_0^{1-x} dy\,\frac{y}{\Delta_1}, \tag{A4}$$

$$f_4(p,p') = \int_0^1 dx\int_0^{1-x} dy\,y\log\left\{\frac{\Delta_1}{4\pi\mu^2 e^{-\gamma_E}}\right\}, \tag{A5}$$

$$f_5(p,p') = M_W^2\int_0^1 dx\int_0^{1-x} dy\,\frac{y(x+y)}{\Delta_1}, \tag{A6}$$

$$f_6(p,p') = \int_0^1 dx \int_0^{1-x} dy\,(1-4y)\log\left\{\frac{\Delta_1}{4\pi\mu^2 e^{-\gamma_E}}\right\}, \tag{A7}$$

$$f_7(p,p') = \int_0^1 dx \int_0^{1-x} dy\,\frac{y^2(1-y)}{\Delta_1}, \tag{A8}$$

$$f_8(p,p') = \int_0^1 dx \int_0^{1-x} dy\,\frac{x^2(1+y)}{\Delta_1}, \tag{A9}$$

$$f_9(p,p') = \int_0^1 dx \int_0^{1-x} dy\,\frac{x(2y^2-1)}{\Delta_1}, \tag{A10}$$

$$f_{10}(p,p') = \int_0^1 dx \int_0^{1-x} dy\,\frac{(x+y)(1-2y)}{\Delta_1}, \tag{A11}$$

$$f_{11}(p,p') = \int_0^1 dx \int_0^{1-x} dy\,\frac{xy(1-y)}{\Delta_1}, \tag{A12}$$

$$f_{12}(p,p') = \int_0^1 dx \int_0^{1-x} dy\,\frac{x(1-x-y+2xy)}{\Delta_1}, \tag{A13}$$

$$f_{13}(p,p') = \int_0^1 dx \int_0^{1-x} dy\,\frac{x(1-x)}{\Delta_1}, \tag{A14}$$

$$f_{14}(p,p') = \int_0^1 dx \int_0^{1-x} dy\,\frac{(1-x-y+2xy)}{\Delta_1}, \tag{A15}$$

$$f_{15}(p,p') = M_W^2 \int_0^1 dx \int_0^{1-x} dy\,\frac{1}{\Delta_1}. \tag{A16}$$

Similarly when we combine propagators in Figs. 4(g) and (h) by using Feynman's parameters, the denominator turns out to be

$$\begin{aligned}\Delta_2 \equiv& -y(1-x-y)p^2 - x(1-x-y)p'^{\,2} - xy(p-p')^2 \\ &+ (x+y)m_j^2 + (1-x-y)M_W^2.\end{aligned} \tag{A17}$$

The parameter integrations containing Δ_2 are listed below:

$$f_{16}(p,p') = \int_0^1 dx \int_0^{1-x} dy\,(1-2y)\log\left\{\frac{\Delta_2}{4\pi\mu^2 e^{-\gamma_E}}\right\}, \tag{A18}$$

$$f_{17}(p,p') = \int_0^1 dx \int_0^{1-x} dy\,\frac{y(1-y)(1-2y)}{\Delta_2}, \tag{A19}$$

$$f_{18}(p,p') = \int_0^1 dx \int_0^{1-x} dy\,\frac{x(1-x)(1-2y)}{\Delta_2}, \tag{A20}$$

$$f_{19}(p,p') = \int_0^1 dx \int_0^{1-x} dy\,\frac{(1-x)(1-y)(1-2y)}{\Delta_2}, \tag{A21}$$

$$f_{20}(p,p') = \int_0^1 dx \int_0^{1-x} dy\,\frac{1-2y}{\Delta_2}, \tag{A22}$$

$$f_{21}(p,p') = \int_0^1 dx \int_0^{1-x} dy\,\frac{(1-y)(1-x-3y+4xy)}{\Delta_2}, \tag{A23}$$

$$f_{22}(p,p') = \int_0^1 dx \int_0^{1-x} dy\,\frac{(1-2y)(1-x-y)}{\Delta_2}, \tag{A24}$$

$$f_{23}(p, p') = \int_0^1 dx \int_0^{1-x} dy\, \frac{(x+y-4xy)}{\Delta_2}, \tag{A25}$$

$$f_{24}(p, p') = \int_0^1 dx \int_0^{1-x} dy\, \frac{y(1-2y)}{\Delta_2}, \tag{A26}$$

$$f_{25}(p, p') = \int_0^1 dx \int_0^{1-x} dy\, \frac{(1-x-y)}{\Delta_2}, \tag{A27}$$

$$f_{26}(p, p') = \int_0^1 dx \int_0^{1-x} dy\, \frac{y(1-y)}{\Delta_2}, \tag{A28}$$

$$f_{27}(p, p') = \int_0^1 dx \int_0^{1-x} dy\, \frac{xy}{\Delta_2}, \tag{A29}$$

$$f_{28}(p, p') = \int_0^1 dx \int_0^{1-x} dy\, \frac{xy(1-2y)}{\Delta_2}, \tag{A30}$$

$$f_{29}(p, p') = \int_0^1 dx \int_0^{1-x} dy\, \frac{y(1-3x-y+4xy)}{\Delta_2}. \tag{A31}$$

Appendix B. Functions G_i ($i = 1, \ldots, 12$)

Some combinations of Feynman parameter integrations are defined below:

$$G_1(p^2) = \frac{1}{D-4} + f_1(p^2), \tag{B1}$$

$$G_2(p^2) = \frac{2}{D-4} + f_2(p^2), \tag{B2}$$

$$G_3(p, p') = \frac{1}{3} \cdot \frac{1}{D-4} + \frac{1}{6} - f_3(p, p') + f_4(p, p'), \tag{B3}$$

$$\begin{aligned} G_4(p, p') = &-\frac{4}{3} \cdot \frac{1}{D-4} - \frac{5}{6} + \left(2 - \frac{p^2}{M_W^2} + \frac{2m_j^2}{M_W^2}\right) f_3(p, p') + \frac{p'^2}{M_W^2} f_3(p', p) \\ &- 4\, f_4(p, p') + 2\left(1 - \frac{m_j^2}{M_W^2}\right) f_5(p, p'), \end{aligned} \tag{B4}$$

$$G_5(p, p') = \frac{1}{6} \cdot \frac{1}{D-4} - \frac{1}{2} f_3(p, p') + \frac{1}{2} f_4(p, p'), \tag{B5}$$

$$G_6(p, p') = -\frac{1}{6} \cdot \frac{1}{D-4} - \frac{1}{2} f_6(p, p') - f_4(p, p'), \tag{B6}$$

$$G_7(p, p') = -\frac{1}{2} \cdot \frac{1}{D-4} - \frac{1}{2} f_6(p, p') - 2 f_4(p, p') + \frac{1}{2} f_{15}(p, p'), \tag{B7}$$

$$G_8(p, p') = \frac{1}{6} \cdot \frac{1}{D-4} + \frac{1}{12} + \frac{1}{2} f_{16}(p, p') + m_j^2 f_{20}(p, p'), \tag{B8}$$

$$\begin{aligned} G_9(p, p') = &-\frac{1}{6} \cdot \frac{1}{D-4} - \frac{1}{6} - \frac{1}{2} f_{16}(p, p') - p^2 f_{17}(p, p') - p'^2 f_{18}(p, p') \\ &+ 2\, p \cdot p'\, f_{19}(p, p') - m_j^2 f_{20}(p, p'), \end{aligned} \tag{B9}$$

$$G_{10}(p, p') = \frac{1}{12} \cdot \frac{1}{D-4} + \frac{1}{4} f_{16}(p, p') - \frac{1}{4} m_j^2 f_{20}(p, p'), \tag{B10}$$

$$G_{11}(p, p') = -\frac{1}{12} \cdot \frac{1}{D-4} - \frac{1}{24} - \frac{1}{4} f_{16}(p, p') + \frac{1}{4} p^2 f_{17}(p, p') + \frac{1}{4} p'^2 f_{18}(p, p') - \frac{1}{2} p \cdot p' f_{28}(p, p') + \frac{1}{4} m_j^2 f_{20}(p, p'), \tag{B11}$$

$$G_{12}(p, p') = -\frac{1}{8} - \frac{1}{4} p^2 f_{26}(p, p') - \frac{1}{4} p'^2 f_{26}(p', p) + \frac{1}{2} p \cdot p' f_{27}(p, p') + m_j^2 \left\{ \frac{1}{4} f_{20}(p, p') - \frac{1}{2} f_{24}(p, p') + f_{26}(p, p') \right\}. \tag{B12}$$

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