Physics Letters B 844 (2023) 138084

Contents lists available at ScienceDirect

Physics Letters B

journal homepage: www.elsevier.com/locate/physletb

A nearly Dirichlet Higgs for lower-scale warped extra dimensions

Mariana Frank [a], Nima Pourtolami [b], Manuel Toharia [c,*]

[a] Department of Physics, Concordia University, 7141 Sherbrooke St. West, Montreal, Quebec H4B 1R6, Canada
[b] National Bank of Canada, 1155 Metcalfe St., Montreal, Quebec H3B 4S9, Canada
[c] Department of Physics, Dawson College, 3040 Sherbrooke St. W, Montreal, Qc, H3Z 1A4, Canada

ARTICLE INFO

Article history:
Received 25 May 2023
Accepted 13 July 2023
Available online 20 July 2023
Editor: G.F. Giudice

ABSTRACT

We consider a minimal extension of the Standard Model in warped extra dimensions, with fields propagating in the bulk including a bulk SM-like Higgs doublet. We show that the Higgs can acquire a non-trivial oscillatory VEV, strongly localized towards the TeV brane, but such that its value at that brane could be highly suppressed due to its oscillatory behaviour. Within the minimal Randall-Sundrum metric background, this oscillatory VEV can alleviate the bounds coming from oblique precision electroweak parameters, such that the KK gluon mass can be around 3 TeV (instead of ~ 8 TeV for the usual non-oscillatory bulk Higgs). We also discuss the stability of the configuration as well as the naturalness of the model parameters.

1. Introduction

The original motivation for warped extra dimensional models was to solve the weak-Planck scale hierarchy by allowing gravity to propagate in the bulk of the extra dimension [1,2], stabilized by a Goldberger-Wise type mechanism [3–8]. Furthermore, by allowing the SM fermion fields to propagate into the bulk, it was found that the localization of fields along the extra dimension could provide an explanation for the observed masses and flavor mixing among quarks and leptons [9–14]. In these scenarios, while the electroweak symmetry breaking can still proceed through the standard Higgs mechanism, the AdS/CFT correspondence can also be used so that the Higgs appears as a composite pseudo-Goldstone boson of the strongly coupled theory [15,16]. In these models the Higgs boson field must be localized near the TeV boundary of the extra dimension in order to solve the hierarchy problem. However, it is also possible for the Higgs to leak into the bulk (bulk Higgs scenario). The benefit of this scenario is to alleviate some of the flavour bounds and precision electroweak tests plaguing the brane Higgs models [17–21]. Generally, in these models, the bounds from the precision flavour and electroweak processes push the mass scale of new particles to relatively large scales, no lower than ~ 8 TeV, which is not accessible to experiments yet. Two different solutions have been proposed to satisfy these bounds, while allowing for light enough new physics within the reach of the LHC: one is to extend the gauge group to include custodial symmetry [22,20], the other is to modify the warping of the space-time metric away from the pure AdS spacetime [23–27].

In this letter, we point out that the parameter space of the usual bulk Higgs, with no gauge group extensions or modified metric backgrounds, has not been fully explored. We study here a remaining region of parameter space and show that the bounds from precision electroweak tests are much less constraining in that region. The metric background is unchanged and the gauge groups are the usual SM groups, however the nontrivial Higgs VEV that we consider can have a substantial suppression on the TeV boundary.

2. The odd Higgs

We consider a scenario with one extra space dimension and assume a (properly stabilized) static spacetime background as

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2\,, \qquad (1)$$

where $\sigma(y)$ is a warp factor responsible for the exponential suppression of the mass scales from the UV brane, down to the IR brane, located at the two boundaries of the extra coordinate, $y = 0$ and $y = y_1$, respectively [1,2]. We assume that the dynamics responsible for stabilizing the setup (*i.e.* giving mass to the radion) do not back-react strongly on the background metric so that we can use the simple Randall-Sundrum (RS) metric with the warp factor given by $\sigma(y) = ky$. The parameter k is assumed to be of the order of the fundamental scale M_{Planck}, so that $ky_1 \simeq 34$ and the hierarchy between the Planck and the electro-weak scale is achieved naturally, and we will refer to the warped-down scale $\Lambda_{KK} = ke^{-ky_1}$ as the Kaluza Klein (KK) scale.

* Corresponding author.
E-mail addresses: mariana.frank@concordia.ca (M. Frank), nima.pourtolami@gmail.com (N. Pourtolami), mtoharia@dawsoncollege.qc.ca (M. Toharia).

https://doi.org/10.1016/j.physletb.2023.138084

The matter content of this scenario corresponds to a minimal 5D extension of the SM gauge group and with all fields propagating in the bulk [9,11,10], such that fermion field localizations along the extra dimension can address the flavor puzzle of the SM [19].

Electroweak symmetry breaking (EWSB) is induced by a single 5D bulk Higgs doublet (H) appearing in the Lagrangian density as

$$\mathcal{L}_5 \supset \sqrt{g}\left(-\frac{1}{4g_5^2}W_{MN}^2 - \frac{1}{g_5'^2}B_{MN}^2 - |D^M H|^2 - V(H)\right) - \sqrt{g}\left(\delta(y)\lambda_0(H) + \delta(y-y_1)\lambda_1(H)\right), \tag{2}$$

where the capital indices run through the 5 spacetime directions, $M=(\mu,5)$, while the Greek indices, $\mu, \nu, \ldots$, denote the usual 4D dimensions [28]. We first consider the simple case of a quadratic bulk Higgs potential with the addition of two brane localized potentials, such that the one located at $y=y_1$ triggers EWSB. We define the parameters in Eq. (2) as

$$V(H) = \frac{1}{2}\mu_B^2 H^2\,, \tag{3}$$

$$\lambda_0(H) = \frac{1}{2}M_0 H^2\,, \tag{4}$$

$$\lambda_1(H) = \frac{1}{2}M_1 H^2 + \frac{1}{2}\gamma_1 H^4\,. \tag{5}$$

The 5D Higgs doublet can be expanded around a nontrivial VEV profile $v_{odd}(y)$ in a similar way as in the SM

$$H = \frac{1}{\sqrt{2}}e^{ig_5\Pi}\begin{pmatrix}0\\ v(y)+h(x,y)\end{pmatrix}. \tag{6}$$

Within the simple RS metric, the static non-trivial Higgs VEV profile, $v_{odd}(y)$, has to satisfy the bulk equation

$$v'' - 4kv' - \mu_B^2 v = 0, \tag{7}$$

and to obey the boundary conditions

$$v'(0) = M_0\, v(0)\,, \tag{8}$$

$$v'(y_1) = -M_1 v(y_1) - 2\gamma_1 v(y_1)^3\,. \tag{9}$$

There are two different types of nontrivial solutions[1] to Eq. (7), depending on the size of the bulk mass parameter.[2] We distinguish two cases:

- If ($\mu_B^2 \geq -4k^2$) then the nontrivial solution is

$$v_{usual}(y) = v_0\left(e^{aky} - \frac{M_0/k - a}{M_0/k - 4 + a}e^{(4-a)ky}\right) \tag{10}$$

  where we have introduced the real parameter a as $ak = 2k + \sqrt{\mu_B^2 + 4k^2}$ and we have imposed the boundary condition at $y=0$. The boundary condition at $y=y_1$ yields and equation for the amplitude v_0 in terms of the parameters k, M_0, M_1, γ_1 and μ_B^2. This solution is the usual nontrivial VEV solution used in the literature when considering a bulk Higgs mechanism.
- If ($\mu_B^2 \leq -4k^2$) then the nontrivial solution is

$$v_{odd}(y) = v_0 e^{2ky}\left(\sin(bky) + \frac{b}{(M_0/k-2)}\cos(bky)\right) \tag{11}$$

  where the real parameter b is defined as $bk = \sqrt{-\mu_B^2 - 4k^2}$ and we have imposed the boundary condition at $y=0$. The second boundary condition at $y=y_1$ will constrain the amplitude v_0 in terms of the parameters k, M_0, M_1, γ_1 and μ_B^2. This solution can yield a Dirichlet-like and more delocalized (but still solving the hierarchy problem) bulk Higgs, which we refer to subsequently as the "odd" Higgs.

Even though the last solution does not necessarily have *odd* parity under orbifold reflection (*i.e.* exact Dirichlet boundary conditions), it contains that possibility. The most interesting region in the parameter space of this novel VEV profile is where the first term dominates and hence, the solution is almost odd (or Dirichlet-like). As we will see shortly, stability related bounds on the free parameter b (fixed by the bulk Higgs mass, μ_B^2), are such that $b \leq \pi/(ky_1)$. It is therefore generally expected that $b \ll (M_0/k - 2)$, and hence, Eq. (11) reduces to $v_{odd}(y) \simeq v_{odd}^{(--)}(y)$, where we refer to the pure *sine* solution as the $(--)$ odd Higgs profile $v_{odd}^{(--)}(y) = v_0 e^{2ky}\sin(bky)$. In the case where $b \gg (M_0/k-2)$ (possible whenever M_0/k is really close to 2), the profile becomes $v_{odd}(y) \simeq v_{odd}^{(+-)}(y)$, where now we refer to the pure *cosine* solution as the $(+-)$ odd Higgs solution, $v_{odd}^{(+-)}(y) = \tilde{v}_0 e^{2ky}\cos(bky)$, where $\tilde{v}_0$ is the proportionality constant of this solution.

We must mention here that $\mu_B^2 < -4k^2$ violates the Breitenlohner-Freedman bound for the full AdS space [29]. However, for a *slice* of AdS, the boundary effects slightly modify the stability threshold [30], and therefore, it is possible for μ_B^2 to be slightly below $-4k^2$. Stability considerations still impose strict constraints on μ_B^2 and in particular, on the VEV which should not have any nodes within the interval [31–34]. This implies that in order to obtain stable, oscillatory VEV solutions we must have[3]: $-4k^2 - \pi^2/y_1^2 \leq \mu_B^2 \leq -4k^2$. This condition is the origin of the bound, $b \leq \pi/(ky_1)$ mentioned earlier, and since $(ky_1) \simeq 34$, we require: $0 \leq b \lesssim 0.1$. We will return to stability issues in Section 5.

Next, we consider the physical Higgs perturbations $h(x,y) = \sum_n h_x^n(x)h_n(y)$ around the nontrivial VEV. The profiles $h_n(y)$ of the Higgs (KK) modes satisfy the equation

$$h_n'' - 4kh_n' - \mu_B^2 h_n + m_n^2 e^{2ky}h_n = 0, \tag{12}$$

with boundary conditions

$$h_n'(0) = M_0\, h_n(0)\,, \tag{13}$$

$$h_n'(y_1) = -(M_1 - 6\gamma_1 v(y_1)^2)h_n(y_1). \tag{14}$$

It is apparent that with $m_n = 0$, the bulk Higgs profile and the background VEV satisfy identical equations of motion but with different boundary conditions. (Although for our choice of brane potentials, the boundary conditions at $y=0$ are the same.)

Assume that the mass m_0^2 of the lowest mode (which will be identified as the SM Higgs) is small compared to the KK scale, so that the problem can be treated perturbatively. We have

$$h_0(y) = \alpha\; v(y)\; (1 + \mathcal{O}(m_0^2/\Lambda_{KK}^2))\,, \tag{15}$$

where the KK scale is as before $\Lambda_{KK} = ke^{-ky_1} \sim \mathcal{O}$ (TeV), and with α a proportionality factor to be fixed by the canonical normalization of the Higgs mode. Then, the Higgs mass can be written as (see for example [25])

$$m_0^2 = -\frac{2v^2(y_1)e^{-4ky_1}\left(M_1 + \frac{v'(y_1)}{v(y_1)}\right)}{\int_0^{y_1} dy e^{-2ky}v^2(y)}\left(1 + \mathcal{O}\left(\frac{m_0^2}{\Lambda_{KK}^2}\right)\right), \tag{16}$$

[1] The trivial solution $v=0$ is always possible. This corresponds to the electroweak symmetric background and, if it is unstable, it will trigger EWSB.

[2] Note that the sign of the real parameter μ_B^2 can in principle be either positive or negative.

[3] Strictly speaking, this is the condition for the existence of stable, nontrivial solutions with Dirichlet boundary conditions on both branes.

which is valid for perturbations around any nontrivial VEV. In particular, perturbing around $v_{usual}(y)$ one finds

$$(m_0^2)_{usual} \simeq -8(a-1)\left(\frac{M_1}{2k}+\frac{a}{2}\right)\Lambda_{KK}^2, \tag{17}$$

with $a \geq 2$. This result shows the amount of tuning needed in the usual bulk Higgs setup, since, in order to satisfy Eq. (17) we must first require M_1 to be negative (for the Higgs mass to be positive) and second, an $\mathcal{O}(1\%–0.1\%)$ cancellation between $M_1/(2k)$ and $a/2$, which are, in principle, independent parameters, originating from the brane and bulk Higgs potentials respectively.

Moving on to the case of the "odd" Higgs, we take[4] $v_{odd}^{(--)}$ and obtain

$$(m_0^2)_{odd} \simeq -8(1+b^2)F(b)\,\Lambda_{KK}^2, \tag{18}$$

where we have defined

$$F(b) = \frac{\sin^2(bky_1)(2+\frac{M_1}{k}+b\cot(bky_1))}{1+b^2-\cos(2bky_1)-b\sin(2bky_1)}. \tag{19}$$

This expression is only valid for small m_0^2, and in general it is possible to fix M_1 such that $F(b)$ is suppressed.

We can also see that suppression of $\sin(bky_1)$ at the boundary (*i.e.*, $bky_1 \sim \pi$), will lead to a suppression of the Higgs mass and we find

$$(m_0^2)_{odd} \simeq 8\frac{\sin(bky_1)}{b}\Lambda_{KK}^2. \tag{20}$$

Moreover, the sign of $\sin(bky_1)$ indicates whether the Higgs mass is tachyonic or not. In particular, for $bky_1 = (\pi+\epsilon)$, with ϵ small, the Higgs mass becomes tachyonic. When $bky_1 = \pi$, the Higgs will be exactly massless. This result agrees with the stability criterion introduced earlier, since $bky_1 = \pi + \epsilon$ means that the corresponding VEV would have a node within the interval.

Next, we analyse how this odd VEV improves the precision electroweak bounds on the model parameters.

3. Precision electroweak tests

We now consider the effects of integrating out the gauge KK modes on the precision electroweak parameters by calculating the corrections to the T and S parameters [17]. These can be easily computed with a set of surprisingly compact and simple integrals (see [25] for details), given by

$$\alpha T = s_W^2 m_Z^2 y_1 \int_0^{y_1} dy e^{2ky}(1-\Omega_h(y))^2 \tag{21}$$

$$\alpha S = 8 s_W^2 c_W^2 m_Z^2 y_1 \int_0^{y_1} dy e^{2ky}\left(1-\frac{y}{y_1}\right)(1-\Omega_h(y)) \tag{22}$$

with $s(c)_W$ being $\sin\theta_W(\cos\theta_W)$, and the function $\Omega_h(y)$ defined in terms of the light SM-like Higgs profile, $h_0(y)$, as:

$$\Omega_h(y) = \frac{\int_0^y dy' e^{-2ky'} h_0^2(y')}{\int_0^{y_1} dy' e^{-2ky'} h_0^2(y')}. \tag{23}$$

Note that because the mass of h_0 is small compared to the KK scale, its wave function is proportional to the nontrivial VEV $v_{odd}(y)$, *i.e.* $h_0(y) \propto v_{odd}(y)$. We consider the two different VEVs discussed in the previous section, namely the usual bulk Higgs VEV and the odd Higgs VEV,

$$v_{usual}(y) \simeq v_0 e^{aky} \tag{24}$$

$$v_{odd}(y) \simeq v_0 e^{2ky}\sin(bky) \tag{25}$$

where, for simplicity, we have only kept the leading term in each case. Full formulations are easily obtained, and used only for later numerical computations, since they do not affect our discussion here.

One can find expressions for the T and S parameters in each case, by evaluating the integrals analytically. We ignore warped down terms, and also assume that $v_{odd}(y)$ is highly suppressed near the TeV brane. We find

$$\alpha T_{usual} = s_W^2 \frac{m_Z^2}{\Lambda_{KK}^2}(ky_1)\frac{(a-1)^2}{a(2a-1)}, \tag{26}$$

$$\alpha S_{usual} = 8 s_W^2 c_W^2 \frac{m_Z^2}{\Lambda_{KK}^2}\frac{a^2-1}{4a^2}, \tag{27}$$

and

$$\alpha T_{odd} = s_W^2 \frac{m_Z^2}{\Lambda_{KK}^2}(ky_1)\frac{17}{648}\left(1+\mathcal{O}(b^2,\epsilon_b)\right), \tag{28}$$

$$\alpha S_{odd} = 8 s_W^2 c_W^2 \frac{m_Z^2}{\Lambda_{KK}^2}\frac{5}{64}\left(1+\mathcal{O}(b^2,\epsilon_b)\right), \tag{29}$$

where ϵ_b is defined as $b = \pi/(ky_1) - \epsilon_b$, and assumed to be small, so that the Higgs profile is almost Dirichlet on the $y = y_1$ boundary. Because $\mu_B^2/k^2 = -4 - b^2$, a very small value for ϵ_b means that the bulk Higgs mass parameter μ_B^2 is very close to the stability limit of $-4k^2 - \pi^2/(y_1)^2$.

We focus first on the T parameter as, due to the volume factor enhancement of $(ky_1) \sim 34$, it is the most constraining of the oblique parameters. The usual bulk Higgs result depends on the parameter a, and is such that the least constrained result is obtained for $a = 2$ (*i.e.* a relatively delocalized Higgs VEV), where the numerical factor from the integral becomes $1/6$. When a is very large (corresponding to a brane localized Higgs VEV), the factor becomes $1/2$. Therefore the bound on the KK scale coming from the T parameter is $\sqrt{3}$ times smaller for a delocalized bulk Higgs as compared to a brane Higgs. The same integral factor in the odd Higgs case is $17/648$, meaning that the bound on the KK scale is $\sqrt{648/17 \times 6} \sim 2.5$ times better (*i.e.* weaker) for the odd Higgs case compared to the most delocalized usual Higgs case ($a = 2$).

For the less constraining S parameter, again, the odd Higgs corrections are more suppressed. However, here the S parameter correction is larger relative to the T parameter correction. Indeed comparing the two cases again

$$\frac{T_{usual}}{S_{usual}} = \frac{ky_1}{8c_W^2}\frac{4a^2(a-1)^2}{(a^2-1)a(2a-1)} \sim 0.89\frac{ky_1}{8c_W^2}\ (a=2), \tag{30}$$

and

$$\frac{T_{odd}}{S_{odd}} = \frac{ky_1}{8c_W^2}\frac{136}{405} \sim 0.34\frac{ky_1}{8c_W^2}. \tag{31}$$

Thus for the odd Higgs case the S parameter is relatively more important. The effect is to push the overall electroweak corrections towards a more favourable direction in the $S-T$ plane, allowing for lower KK scales than those allowed by only considering the T

[4] Here, for simplicity, we assumed $b \ll (M_0/k - 2)$. This will not change any qualitative features as we will see later.

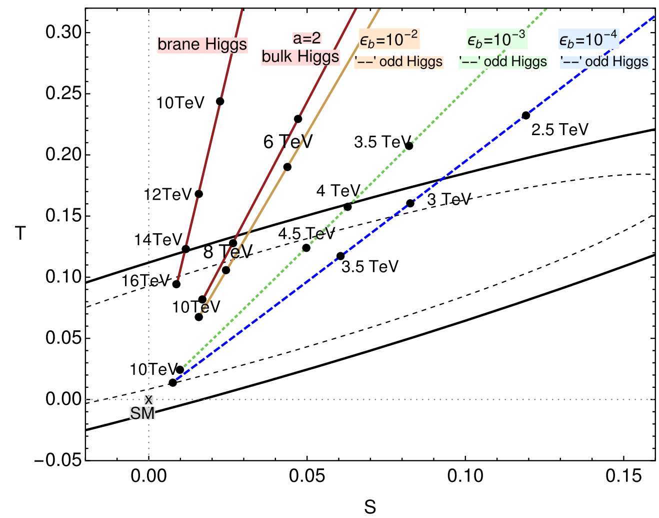

Fig. 1. Allowed region in the $S-T$ parameter plane at the 95% and 68% CL [35], as well as the S and T predictions (from tree-level KK gauge boson exchange) for a bulk Higgs scenario in the three regimes of brane Higgs, bulk Higgs and odd Higgs. Each curve is obtained by varying the KK scale which we parametrize using the physical mass of the lightest KK gauge boson $(M_{KK})_1$.

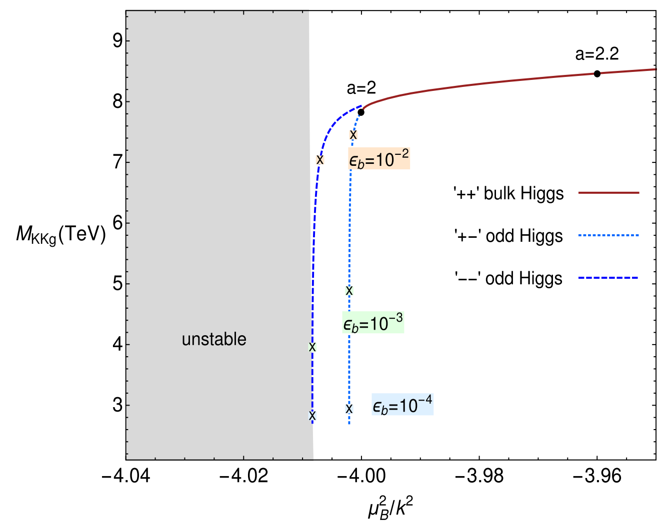

Fig. 2. 95% CL lower bound from S and T parameters constraints on the mass of the first KK gauge boson as a function of the (negative) bulk Higgs mass coefficient, μ_B^2. The grey region at the left side represents the region (in μ_B^2/k^2) where the bulk Higgs mass parameter is smaller than the lower bound set by stability requirements and is thus ruled out. The parameter ϵ_b denotes the mass difference (in units of k^2) between μ_B^2 and the stability limit. When $\mu_B^2 \geq -4k^2$, the scenario enters the usual bulk Higgs regime and is commonly parametrized by the Higgs wave function coefficient $a \geq 2$.

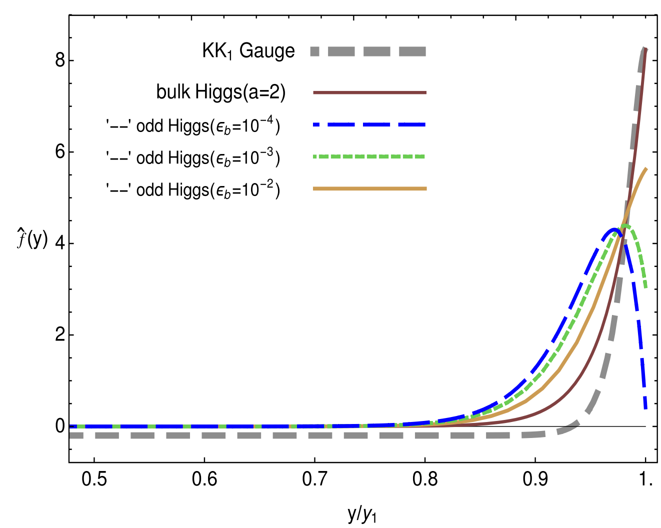

Fig. 3. Profiles of the "physical" wave-functions, $\hat{f}(y) = e^{-ky} v_{odd}(y)$, of the lightest Higgs mode in the bulk Higgs regime with $a = 2$ (solid, dark brown) and in the odd Higgs regime with $\epsilon_b = 10^{-4}$ (dashed, blue), $\epsilon_b = 10^{-3}$ (dotted, light green) and $\epsilon_b = 10^{-2}$ (light orange). Also shown is the wave-function of the first heavy KK gauge mode (grey, thick dashed). The overlap between Higgs and KK gauge bosons is clearly suppressed as the Higgs wave function becomes diminished near the brane. This leads to a suppressed contribution to the T parameter.

parameter.[5] This is shown in Fig. 1, where the predicted values for S and T arising from integrating out the KK electroweak bosons are traced for different values of the bulk Higgs parameter, μ_B^2, by varying the KK scale of the model, parametrized as the lightest KK gauge boson mass. As shown earlier, it is useful to parametrize this scale using the a and b parameters. When a is large (μ_B^2 large and positive) the Higgs sector corresponds to the brane Higgs limit and the bounds from the oblique parameters are the strongest. In this case the lightest KK gluon cannot be lighter than ~ 14 TeV at 95% CL. As the Higgs leaks out of the brane, by reducing a (or μ_B^2), the bounds improve. At $a = 2$ ($\mu_B^2 = -4k^2$) the usual limit of bulk Higgs delocalization is obtained and the bounds are such that the KK gluon mass should not be lower than ~ 8 TeV. As discussed earlier, the bulk Higgs mass can be lowered beyond $-4k^2$ as long it respects the stability bound $-4k^2 - \pi^2/y_1^2 \leq \mu_B^2$. The parameters S and T are evaluated numerically and show a striking improvement in their bounds, as the parameter ϵ_b is reduced, (*i.e.*, as the Higgs is more and more Dirichlet-like). There are, however, limiting values for each (*cf.* Eqs. (28) and (29), so that once ϵ_b is small enough, no further improvement can be obtained. Within the odd Higgs regime, the lowest mass of the KK gluon, consistent with the $S-T$ bounds is about 2.8 TeV. This represents the least constraining (calculable) limit on RS models without custodial symmetry or large metric back-reaction [25,36–38,26,39].

In Fig. 2, we consider again the tree level corrections to S and T from KK gauge bosons, but in this case we show the lowest possible KK gluon mass as a function of the bulk Higgs mass, μ_B^2, near the $\mu_B^2 = -4k^2$ region. As expected, the bounds improve slightly as the Higgs mass approaches $-4k^2$ from above. Once the mass decreases beyond that point, the bounds improve dramatically until reaching the stability limit. Note that, as mentioned earlier, the stability limit differs depending on the UV boundary conditions on the odd Higgs VEV. When the UV brane mass parameter, M_0, is large enough, the Higgs acquires the $(--)$ nontrivial VEV, while if there is a strong cancelation $(M_0/k - 2) \ll 1$ the VEV becomes the $(+-)$ VEV. In this case, the condition for the VEV to have no nodes within the interval is $b \leq \pi/(2ky_1)$, which yields a slightly different bound for μ_B^2. This appears in the figure as a vertical line for the $(+-)$ case, appearing at a different value of μ_B^2 than the vertical asymptote for the $(--)$ case. Note that the best bound on the KK scale is the same in both situations, and when the odd Higgs VEV has an expression containing both $\sin(bky)$ and $\cos(bky)$, the stability limit of μ_B^2 will lie between the limits of the $(--)$ and the $(+-)$ case. In the figure we show the region set by the $(--)$ solution as the grey region marked as unstable.

Finally, in Fig. 3, we show the Higgs profiles (and therefore VEVs) for different values of the bulk Higgs mass, μ_B^2. The profile

[5] Note that this effect is already present for the usual bulk Higgs, since the ratio T/S is about two times larger for a brane Higgs (a large) compared to a delocalized Higgs ($a = 2$). Therefore the bounds for $a = 0$ shift towards a more favourable direction in the $S-T$ plane and are thus improved further compared to the most constraining brane Higgs.

of the lightest KK gauge mode (same in all regimes) is also plotted for comparison. We observe that, as the Higgs mode becomes more and more Dirichlet-like, it leaks out of the boundary (*cf.* [38,26]), which will lead to weaker couplings with the KK gauge bosons and thus suppressed contributions to the S and T parameters.

4. Fermion masses and Yukawa couplings

Consider the following 5D up-quark sector Yukawa Lagrangian density

$$\mathcal{L}_Y = \sqrt{g}\left(\frac{Y_u^{bulk}}{\sqrt{k}} H Q U\right) + h.c., \tag{32}$$

where the 3×3 Yukawa coupling matrix Y_u^{bulk} is composed of $\mathcal{O}(1)$ dimensionless coefficients. From these 5D interactions one can extract the 4D Yukawa couplings (and the 4D mass terms) of the up-type quark zero modes (similarly for the down-type quark and the lepton sector of the theory). The 4D effective Yukawa couplings between the zero mode Higgs and quarks are obtained from the overlap integrals of the quarks and Higgs wave functions along the extra dimension. The normalized wave functions for the left-handed doublet and the right-handed singlet quarks are $q(y) = \sqrt{k}N_q e^{(2-c_q)ky}$ and $u(y) = \sqrt{k}N_u e^{(2+c_u)ky}$, respectively, where $N_q = \sqrt{\frac{(2c_q-1)}{1-e^{(1-2c_q)ky_1}}}$ and $N_u = \sqrt{\frac{(2c_u+1)}{-1+e^{(1+2c_u)ky_1}}}$ are canonical normalization factors.[6] The Yukawa couplings then are

$$\mathcal{L}_Y^{4D} = Y_u^{bulk}\sqrt{k}N_q N_u \int_0^{y_1} dy h_0(y) e^{-\Delta cky} + h.c., \tag{33}$$

where $\Delta c = c_q - c_u$. The canonically normalized Higgs profiles are given by:

$$h_{0usual}(y) = \sqrt{\frac{2(a-1)k}{(e^{2(a-1)ky_1}-1)}}e^{aky}, \tag{34}$$

and

$$h_{0odd}(y) = \sqrt{k}\frac{2ky_1}{\pi}e^{-ky_1}e^{2ky}\sin(bky)\,(1+\mathcal{O}(b^2)), \tag{35}$$

where in the case of the odd Higgs we have neglected terms of order $b^2 \lesssim 10^{-2}$ (but we keep all terms for numerical calculations). We can now calculate the 4D Yukawa couplings in each Higgs regime:

$$y_u^{usual} = \frac{\sqrt{2}}{(2-\Delta c)}\, Y_u^{bulk} N_q N_u e^{(1-\Delta c)ky_1} \tag{36}$$

and

$$y_u^{odd} = \frac{2}{(2-\Delta c)^2}\, Y_u^{bulk} N_q N_u e^{(1-\Delta c)ky_1}. \tag{37}$$

The two results are surprisingly similar, that is, both have the same exponential dependence on $\Delta c = c_q - c_u$. The ratio of these expressions is given by $\frac{\sqrt{2}}{2-\Delta c}$, which is essentially $\mathcal{O}(1)$, within the usual range of the c-values,[7] and thus the odd Higgs regime addresses the flavour puzzle of the SM in the same way as the usual bulk Higgs does, *i.e.*, through small masses and hierarchical mixing angles which are a reflection of the geographical location of the fermion wave functions along the fifth dimension.

[6] Note that, to simplify the notation, we omitted the flavour indices.

[7] This usual range is such that if the quark flavour structure is explained by the exponential factors in Δc, typical values are roughly found between $\Delta c \sim 1.2$ (lightest quarks) and $\Delta c \sim 0$ (top quark).

5. Existence, stability and naturalness

We first consider the stability conditions for the trivial Higgs VEV, *i.e.* $v = 0$. In this case there is no EWSB, but we can still study the spectrum of Higgs perturbations around the $v = 0$ vacuum. The Higgs KK modes in the unbroken phase still satisfy the same equation as Eq. (12) but now the boundary condition, Eq. (14), is modified:

$$h_n'(y_1) = -M_1 h_n(y_1). \tag{38}$$

We would like to know what is the most negative value for μ_B^2, before the lightest Higgs mode, h_0, becomes tachyonic. The threshold condition will be reached when the lowest mode h_0 is massless, *i.e.* $m_0^2 = 0$. Moreover, it is known that the largest eigenvalue of the Sturm-Liouville boundary value problem is the eigenvalue associated to the Dirichlet problem, which in this case is obtained in the limit of very large M_0 and M_1. Therefore, the absolute stability bound will be obtained for the value of μ_B^2 that generates a massless zero-mode ($m_0^2 = 0$) in the Dirichlet boundary value problem associated with Eq. (12). The massless mode profile solution with Dirichlet boundary conditions is $h_{0trivial}(y) = N_h e^{2ky}\sin bky$, with the parameter b fixed by the Dirichlet boundary condition as $b = \pi/(ky_1)$. But the parameter b depends on the bulk Higgs mass, $b^2k^2 = -\mu_B^2 - 4k^2$, and therefore the threshold value of the bulk mass parameter is $(\mu_B^2)_{min} = -4k^2(1+\pi^2/(2ky_1)^2)$. Below this value, the Dirichlet Higgs perturbations around the trivial background are unstable. This value represents the generalized Breitenlohner-Freedman bound [29] for a scalar field theory defined on a slice of AdS_5 [30] and is exactly the same as the stability bound for Higgs perturbations around the non-trivial odd Higgs VEV, mentioned earlier. Before returning to the EWSB phase, we discuss the more general case in which the boundary conditions are not Dirichlet. The (perturbative) expression for the mass of the lowest Higgs KK perturbation around the trivial vacuum, $v = 0$, has the same form as the mass of the lightest Higgs mode with the non-trivial VEV. The only difference is that it has opposite sign and is divided by a factor of 2, *i.e.*

$$m_{0trivial}^2 = -\frac{1}{2}m_{0non-trivial}^2, \tag{39}$$

where the expressions for $m_{0non-trivial}^2$ were given in Eqs. (17) and (18), depending on the regime of the nontrivial VEV. This means that whenever the unbroken phase is unstable, the broken phase will be stable and vice versa.

We now return to the case where the Higgs acquires a non-trivial VEV. In [34] it was shown that for any bulk scalar potential defined on a slice of AdS_5, with the scalar field respecting Dirichlet boundary conditions, any non-trivial scalar VEV solution with nodes in the interval would be unstable. In the case of the simple potential used in this paper, this result can be obtained easily, since the equation for the perturbations around a nontrivial VEV is the same as the one for the trivial vacuum. Indeed, one finds that the threshold at which a massless mode appears is precisely the point where the VEV has a node at the boundaries (it is Dirichlet-like). When the boundary node moves slightly into the bulk, the eigenvalue becomes slightly negative. For the case of a more general bulk Higgs potential, this no-node criterion continues to be useful in the determination of the absolute stability bound associated with the Higgs sector.

A related topic is the size of the Higgs potential parameters, namely the bulk mass coefficient, μ_B^2, the brane quadratic coefficients, M_0 and M_1, and the brane quartic coefficient, γ_1. As previously discussed, the parameter M_0 can remain to be $\mathcal{O}(1)$ (in units of k) without changing any properties of the odd Higgs VEV solution, Eq. (25). In the case of the bulk mass parameter, it is bound to be $\mu_B^2 > -4k^2 - \pi^2/(ky_1)^2 k^2$ and so it can easily remain $\mathcal{O}(1)$ in units of k^2.

Moreover, as we have seen earlier, in order to have a light Higgs the bulk mass parameter must be very close to the stability limit, *i.e.* close to $-4k^2$. This implies some degree of parameter tuning because one needs to require the function $F(b)$ to be small enough to generate the light Higgs mass (see Eq. (18)). If we define

$$\epsilon_b = \mu_B^2 + 4k^2 + \pi^2/y_1^2, \tag{40}$$

assuming small ϵ_b, the approximate expression for $F(b)$ (including the dependence on M_1) becomes

$$F(b) \simeq \frac{\epsilon_b k y_1}{b}\left((4 + M_1/k)\frac{\epsilon_b k y_1}{b} - 1\right), \tag{41}$$

from which we can easily identify two contributions to the overall coefficient. Since $(ky_1) \sim 34$ and $b \sim 10^{-1}$, one needs at least $\epsilon_b \sim 10^{-3}$ so that: $\left(\frac{\epsilon_b k y_1}{b}\right) \sim 10^{-1}$. This requirement on ϵ_b represents roughly a 0.1% tuning on the parameter μ_B^2 and we still need a further 10^{-1} factor from the term containing M_1 in order to obtain the overall 10^{-2} suppression for $F(b)$. We see that, while in the normal bulk Higgs case (Eq. (17)), M_1 has to be negative, in this scenario it can be positive or negative. In fact, with our previous choice, we see that a positive value of M_1/k lying between 4 and 5 can achieve the overall suppression of $F(b)$ without really much tuning of M_1. We conclude that in this regime, the required tuning to obtain a light Higgs is around 0.1% and the tuned parameter is (μ_B^2).[8]

The brane quartic coefficient γ_1 is fixed by the requirement of obtaining a light enough electroweak scale, *i.e.* generating the appropriate Z and W boson masses. The W mass can be approximated as

$$m_W^2 = \frac{g_5^2}{4y_1}\int_0^{y_1} dy\, v^2(y) e^{-2ky} \tag{42}$$

where $v(y)$ is the nontrivial Higgs VEV, which depending on the sign of $(4k^2 + \mu^2)$, can take either the usual form of Eq. (10) or the new oscillatory solution $v_{odd}(y)$ of Eq. (11).

The constant coefficient v_0 can be removed in favour of m_W^2, and using the IR boundary condition for the VEV, Eq. (9), we can solve for the quartic coefficient, γ_1, as

$$\gamma_1 = \frac{g_4^2}{16}\frac{m_h^2}{m_W^2}\left(\int_0^{y_1} dy \frac{e^{-2ky} v(y)^2}{e^{-2ky_1} v(y_1)^2}\right)^2. \tag{43}$$

When $v(y) = v_{usual}(y)$, one obtains $\gamma_1 = \frac{g_4^2}{16k^2}\frac{m_h^2}{m_W^2}\frac{1}{(2a-2)^2}$ which is naturally of $\mathcal{O}(1/k^2)$. However when $v(y) = v_{odd}(y)$ the coefficient γ_1 diverges if v_{odd} vanishes at the IR brane (*i.e.* when it becomes exactly Dirichlet-like). This is not surprising since Dirichlet boundary conditions require an infinite brane potential. However, $v_{odd}(y)$ does not exactly vanish at the IR brane as the Higgs mass should not be zero. The parameter ϵ_b as defined in Eq. (40) represents the deviation from the stability threshold (*i.e.*, a massless Higgs) and γ_1 can be easily obtained for different values of ϵ_b. For example, for a fixed volume factor $ky_1 = 34$ and $\epsilon_b = 10^{-3}$ we obtain $\gamma_1 \simeq \left(\frac{0.86}{k^2}\right)$, while for $\epsilon_b = 5\times 10^{-4}$ we get $\gamma_1 \simeq \left(\frac{7.1}{k^2}\right)$ and for $\epsilon_b = 10^{-4}$, $\gamma_1 \simeq \left(\frac{2.5\times 10^3}{k^2}\right)$. This behaviour reflects again that in order to suppress scalar field values at the boundary, one requires a large scalar brane potential. Moreover, we also see that the value of the quartic potential acquires very large values close to the region where the oblique parameters are most strongly suppressed. However if one insists in keeping γ_1 at most $\mathcal{O}(1/k^2)$ one can still have a suppressed T parameter for $\epsilon_b \sim 10^{-3}$ (see Figs. 1 and 2).

Finally we turn to the interplay between the Higgs background and the gravitational background of this scenario. So far we have assumed a static RS background, but it is known that since the background contains a massless graviscalar mode, the radion, it must be stabilized. The natural question is whether the back-reaction of the nontrivial Higgs VEV (which we neglected, assuming it to be small) will stabilize the gravitational background, namely if the radion will acquire a positive mass squared. Unfortunately, it was shown in [40] that when one considers static solutions for both the warp factor and a single scalar field (here, the Higgs), the radion will be tachyonic whenever the derivative of the scalar VEV vanishes inside the interval. Since this is precisely what happens with the odd Higgs VEV, the setup as is will not be gravitationally stable. However, RS type scenarios always require a mechanism to lift the massless radion. The simplest procedure is to add a new scalar singlet which would acquire a VEV and will back-react on the metric background generating a stabilizing potential for the radion [3–8]. This is the solution for our scenario as well, except that in our case the stabilizing scalar should lift the radion tachyonic mass generated by the Higgs nontrivial VEV. Since the Higgs VEV determines the electroweak scale (which is much smaller than the KK scale), one expects the scale of the radion tachyonic mass to also be small, and thus raising the radion mass should proceed in a fashion similar to that of the usual warped scenarios.

6. Conclusions

We considered the simplest RS warped scenario in which matter particles and the Higgs field propagate in the bulk and where the flavour structure of the SM is generated by the localization of the fermion fields along the extra space dimension. We found that a small region of parameter space within this setup, where the bulk Higgs mass is pushed beyond the naive $-4k^2$ threshold, has not been explored yet. Moreover, within that region the bounds from precision electroweak constraints are much milder than in the usual bulk Higgs regimes. In this region, we carefully addressed stability concerns associated to the extension of the parameter space.

Even though the regime considered here requires slightly more tuning than the usual bulk Higgs regime, and also can require a somewhat larger brane scalar potential, this scenario is very interesting thanks to its simplicity and its success in reducing the T parameter. Even if a small hierarchy problem is present with the setup, the oscillatory Higgs VEV can be exploited within different scenarios, either with custodial symmetry or with heavily modified metric backgrounds. Thus, while still addressing the hierarchy and the flavour problem of the Standard Model, the bounds and mass limits of the model are improved, making it possible to allow for lighter KK modes, which would be accessible at the LHC.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

[8] Note that, as we mentioned before, (*cf.* Eq. (17) and the discussion below it), in the usual bulk Higgs scenario there is also a tuning, $(M_1 + 2k + \sqrt{\mu_B^2 + 4k^2}) \sim \mathcal{O}(1\%)$. This tuning is initially slightly less 'fine' than in the odd Higgs case for a given KK scale. However, since the odd Higgs allows for a lower KK scale, the tuning ends up becoming of similar order if one considers a bulk Higgs scenario with heavier KK modes with masses closer to 10 TeV.

Data availability

No data was used for the research described in the article.

Acknowledgements

The work of M. F. is partly supported by NSERC through the grant number SAP105354. M. T. would like to thank FRQNT for financial support under grant number PRC-290000. This paper reflects solely the authors' personal opinions and does not represent the opinions of the authors' employers, present and past, in any way.

References

[1] L. Randall, R. Sundrum, A large mass hierarchy from a small extra dimension, Phys. Rev. Lett. 83 (1999) 3370–3373, https://doi.org/10.1103/PhysRevLett.83.3370, arXiv:hep-ph/9905221.
[2] L. Randall, R. Sundrum, An alternative to compactification, Phys. Rev. Lett. 83 (1999) 4690–4693, https://doi.org/10.1103/PhysRevLett.83.4690, arXiv:hep-th/9906064.
[3] W.D. Goldberger, M.B. Wise, Modulus stabilization with bulk fields, Phys. Rev. Lett. 83 (1999) 4922–4925, https://doi.org/10.1103/PhysRevLett.83.4922, arXiv:hep-ph/9907447.
[4] W.D. Goldberger, M.B. Wise, Phenomenology of a stabilized modulus, Phys. Lett. B 475 (2000) 275–279, https://doi.org/10.1016/S0370-2693(00)00099-X, arXiv:hep-ph/9911457.
[5] C. Csaki, J. Erlich, T.J. Hollowood, Y. Shirman, Universal aspects of gravity localized on thick branes, Nucl. Phys. B 581 (2000) 309–338, https://doi.org/10.1016/S0550-3213(00)00271-6, arXiv:hep-th/0001033.
[6] C. Csaki, M. Graesser, L. Randall, J. Terning, Cosmology of brane models with radion stabilization, Phys. Rev. D 62 (2000) 045015, https://doi.org/10.1103/PhysRevD.62.045015, arXiv:hep-ph/9911406.
[7] O. DeWolfe, D.Z. Freedman, S.S. Gubser, A. Karch, Modeling the fifth-dimension with scalars and gravity, Phys. Rev. D 62 (2000) 046008, https://doi.org/10.1103/PhysRevD.62.046008, arXiv:hep-th/9909134.
[8] N. Arkani-Hamed, S. Dimopoulos, J. March-Russell, Stabilization of submillimeter dimensions: the new guise of the hierarchy problem, Phys. Rev. D 63 (2001) 064020, https://doi.org/10.1103/PhysRevD.63.064020, arXiv:hep-th/9809124.
[9] H. Davoudiasl, J.L. Hewett, T.G. Rizzo, Bulk gauge fields in the Randall-Sundrum model, Phys. Lett. B 473 (2000) 43–49, https://doi.org/10.1016/S0370-2693(99)01430-6, arXiv:hep-ph/9911262.
[10] A. Pomarol, Gauge bosons in a five-dimensional theory with localized gravity, Phys. Lett. B 486 (2000) 153–157, https://doi.org/10.1016/S0370-2693(00)00737-1, arXiv:hep-ph/9911294.
[11] Y. Grossman, M. Neubert, Neutrino masses and mixings in nonfactorizable geometry, Phys. Lett. B 474 (2000) 361–371, https://doi.org/10.1016/S0370-2693(00)00054-X, arXiv:hep-ph/9912408.
[12] S. Chang, J. Hisano, H. Nakano, N. Okada, M. Yamaguchi, Bulk standard model in the Randall-Sundrum background, Phys. Rev. D 62 (2000) 084025, https://doi.org/10.1103/PhysRevD.62.084025, arXiv:hep-ph/9912498.
[13] T. Gherghetta, A. Pomarol, Bulk fields and supersymmetry in a slice of AdS, Nucl. Phys. B 586 (2000) 141–162, https://doi.org/10.1016/S0550-3213(00)00392-8, arXiv:hep-ph/0003129.
[14] H. Davoudiasl, J.L. Hewett, T.G. Rizzo, Experimental probes of localized gravity: on and off the wall, Phys. Rev. D 63 (2001) 075004, https://doi.org/10.1103/PhysRevD.63.075004, arXiv:hep-ph/0006041.
[15] R. Contino, Y. Nomura, A. Pomarol, Higgs as a holographic pseudo-Goldstone boson, Nucl. Phys. B 671 (2003) 148–174, https://doi.org/10.1016/j.nuclphysb.2003.08.027, arXiv:hep-ph/0306259.
[16] C. Csaki, A. Falkowski, A. Weiler, The flavor of the composite pseudo-Goldstone Higgs, J. High Energy Phys. 09 (2008) 008, https://doi.org/10.1088/1126-6708/2008/09/008, arXiv:0804.1954.
[17] M. Carena, A. Delgado, E. Ponton, T.M.P. Tait, C.E.M. Wagner, Warped fermions and precision tests, Phys. Rev. D 71 (2005) 015010, https://doi.org/10.1103/PhysRevD.71.015010, arXiv:hep-ph/0410344.
[18] S.J. Huber, Flavor violation and warped geometry, Nucl. Phys. B 666 (2003) 269–288, https://doi.org/10.1016/S0550-3213(03)00502-9, arXiv:hep-ph/0303183.
[19] K. Agashe, G. Perez, A. Soni, Flavor structure of warped extra dimension models, Phys. Rev. D 71 (2005) 016002, https://doi.org/10.1103/PhysRevD.71.016002, arXiv:hep-ph/0408134.
[20] K. Agashe, R. Contino, L. Da Rold, A. Pomarol, A custodial symmetry for $Zb\bar{b}$, Phys. Lett. B 641 (2006) 62–66, https://doi.org/10.1016/j.physletb.2006.08.005, arXiv:hep-ph/0605341.
[21] C. Csaki, C. Delaunay, C. Grojean, Y. Grossman, A model of lepton masses from a warped extra dimension, J. High Energy Phys. 10 (2008) 055, https://doi.org/10.1088/1126-6708/2008/10/055, arXiv:0806.0356.
[22] K. Agashe, A. Delgado, M.J. May, R. Sundrum, RS1, custodial isospin and precision tests, J. High Energy Phys. 08 (2003) 050, https://doi.org/10.1088/1126-6708/2003/08/050, arXiv:hep-ph/0308036.
[23] A. Falkowski, M. Perez-Victoria, Electroweak breaking on a soft wall, J. High Energy Phys. 12 (2008) 107, https://doi.org/10.1088/1126-6708/2008/12/107, arXiv:0806.1737.
[24] B. Batell, T. Gherghetta, D. Sword, The soft-wall standard model, Phys. Rev. D 78 (2008) 116011, https://doi.org/10.1103/PhysRevD.78.116011, arXiv:0808.3977.
[25] J.A. Cabrer, G. von Gersdorff, M. Quiros, Suppressing electroweak precision observables in 5D warped models, J. High Energy Phys. 05 (2011) 083, https://doi.org/10.1007/JHEP05(2011)083, arXiv:1103.1388.
[26] A. Carmona, E. Ponton, J. Santiago, Phenomenology of non-custodial warped models, J. High Energy Phys. 10 (2011) 137, https://doi.org/10.1007/JHEP10(2011)137, arXiv:1107.1500.
[27] S. Mert Aybat, J. Santiago, Bulk fermions in warped models with a soft wall, Phys. Rev. D 80 (2009) 035005, https://doi.org/10.1103/PhysRevD.80.035005, arXiv:0905.3032.
[28] F. del Aguila, M. Perez-Victoria, J. Santiago, Effective description of brane terms in extra dimensions, J. High Energy Phys. 10 (2006) 056, https://doi.org/10.1088/1126-6708/2006/10/056, arXiv:hep-ph/0601222.
[29] P. Breitenlohner, D.Z. Freedman, Stability in gauged extended supergravity, Ann. Phys. 144 (1982) 249, https://doi.org/10.1016/0003-4916(82)90116-6.
[30] M. Toharia, Odd tachyons in compact extra dimensions, arXiv:0803.2503, 2008.
[31] B. Grzadkowski, M. Toharia, Low-energy effective theory from a nontrivial scalar background in extra dimensions, Nucl. Phys. B 686 (2004) 165–187, https://doi.org/10.1016/j.nuclphysb.2004.03.007, arXiv:hep-ph/0401108.
[32] M. Toharia, M. Trodden, Metastable kinks in the orbifold, Phys. Rev. Lett. 100 (2008) 041602, https://doi.org/10.1103/PhysRevLett.100.041602, arXiv:0708.4005.
[33] M. Toharia, M. Trodden, Existence and stability of non-trivial scalar field configurations in orbifolded extra dimensions, Phys. Rev. D 77 (2008) 025029, https://doi.org/10.1103/PhysRevD.77.025029, arXiv:0708.4008.
[34] M. Toharia, M. Trodden, E.J. West, Scalar kinks in warped extra dimensions, Phys. Rev. D 82 (2010) 025009, https://doi.org/10.1103/PhysRevD.82.025009, arXiv:1002.0011.
[35] J. Haller, A. Hoecker, R. Kogler, K. Mönig, J. Stelzer, Status of the global electroweak fit with Gfitter in the light of new precision measurements, PoS ICHEP2022 (2022) 897, https://doi.org/10.22323/1.414.0897, arXiv:2211.07665.
[36] J.A. Cabrer, G. von Gersdorff, M. Quiros, Soft-wall stabilization, New J. Phys. 12 (2010) 075012, https://doi.org/10.1088/1367-2630/12/7/075012, arXiv:0907.5361.
[37] J.A. Cabrer, G. von Gersdorff, M. Quiros, Warped electroweak breaking without custodial symmetry, Phys. Lett. B 697 (2011) 208–214, https://doi.org/10.1016/j.physletb.2011.01.058, arXiv:1011.2205.
[38] M. Quiros, Higgs bosons in extra dimensions, Mod. Phys. Lett. A 30 (2015) 1540012, https://doi.org/10.1142/S021773231540012X, arXiv:1311.2824.
[39] B.M. Dillon, S.J. Huber, Non-custodial warped extra dimensions at the LHC?, J. High Energy Phys. 06 (2015) 066, https://doi.org/10.1007/JHEP06(2015)066, arXiv:1410.7345.
[40] J. Lesgourgues, L. Sorbo, Goldberger-wise variations: stabilizing brane models with a bulk scalar, Phys. Rev. D 69 (2004) 084010, https://doi.org/10.1103/PhysRevD.69.084010, arXiv:hep-th/0310007.