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# The dark dimension in a warped throat

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A B S T R A C T

By combining swampland conjectures with observational data, it was suggested that our universe should lie in an asymptotic region of the quantum gravity landscape. The generalized distance conjecture for dS, the smallness of the cosmological constant and astrophysical constraints led to a scenario with one mesoscopic large dimension of size $\ell \sim \Lambda^{-1/4} \sim 10^{-6}$ m. We point out that a strongly warped throat with its redshifted KK tower provides a natural string theoretic mechanism that realizes the scaling $m \sim \Lambda^{\alpha}$ with the factor $\alpha = 1/4$, the dark dimension being the one along the throat. We point out that in string theory it could be challenging to keep other KK towers heavy enough to avoid a conflict with astrophysical constraints on the number of extra large dimensions.

## 1. Introduction

Since the recent swampland program (see [1,2] for reviews) postulates that not all self-consistent quantum field theories admit a UV completion into a theory of quantum gravity, the notion of naturalness has to be changed. What seems natural from a purely low-energy point of view, could turn out to be in the swampland after all. This new notion has the potential to make quantum gravity and string theory much more predictive than initially thought. How such a logic can be made concrete was exemplified in the recent work of Montero-Vafa-Valenzuela [3]. By combining swampland conjectures with observational data, the authors suggest that our universe should lie in a specific corner of the quantum gravity landscape.

The starting point of [3] is the assumption that our universe is located in an asymptotic region of field space where the generalization of the AdS distance conjecture [4] to dS space applies. It states that for a cosmological constant $\Lambda$ approaching zero, a tower of states becomes light obeying the specific scaling behavior

$$m \sim |\Lambda|^{\alpha}. \tag{1.1}$$

Here a factor $\alpha \geq 1/2$ prevents scale separation between the internal dimensions and the radius of (A)dS space. It was conjectured

[4] that for AdS this is always the case. However, for dS the Higuchi bound [5] already requires that $\alpha \leq 1/2$. Taking one-loop corrections into account, it was argued in [3] that for four-dimensional dS the factor $\alpha$ should lie in the range $\frac{1}{4} \leq \alpha \leq \frac{1}{2}$.

Applying this to our universe with its observed tiny cosmological constant $\Lambda = 10^{-122} M_{\text{pl}}^4$ and taking bounds from deviations of Newton's gravitational force law into account, revealed that only the value $\alpha = 1/4$ can be consistent. Astrophysical constraints from the cooling/heating of neutron stars then led to a model of a single large extra dimension, dubbed the dark dimension, and a corresponding tower of KK modes

$$\Lambda^{\frac{1}{4}} = \lambda \, m_{\text{KK}} \tag{1.2}$$

with $10^{-4} < \lambda < 1$. It was further argued that the KK modes should include an excess of fermionic modes, leading to a kind of sterile neutrino species. Moreover, the species scale [6,7] related to this light tower of one-dimensional KK modes came out in the intermediate range $\Lambda_{\text{sp}} \sim 10^9 - 10^{10}$ GeV. This is tantalizingly close to the scale of $10^{11}$ GeV where due to the running of its self-coupling, the Higgs potential is believed to develop an instability. Interpreting the upper bound of $10^{10}$ GeV as the reason for the sharp cutoff of the flux of ultra-high-energy cosmic rays [8], led to the value $\lambda = 10^{-3}$. In [9], it was argued that such a single mesoscopic direction also changes the production rate of primordial black holes such that they can provide a large fraction of the dark matter of our universe. In [10] massive spin-2 KK modes along the dark dimension, dubbed dark gravitons, were proposed as dark matter

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candidates, leading to an explanation of the cosmological coincidence problem.

The nice aspect of this derivation is that it only involves a generic swampland conjecture and observational data. However, consistency with one swampland conjecture does not guarantee that this Dark Dimension Scenario is really consistent with quantum gravity. Therefore, it is an important question whether it can be realized in a fully fledged theory of quantum gravity, like string theory. In this note we take a first modest step in settling this question by pointing out that a commonly used aspect in string theory model building, a strongly warped throat, provides already a natural mechanism to get $\alpha = 1/4$ in the generalized distance conjecture for dS while providing a lightest one-dimensional KK tower.

On the downside, we point out that generically other KK modes and light states do not separate enough from the dark dimension to be compliant with astrophysical constraints. This is exemplified in the LVS as a typical class of models.

## 2. Realization in a warped throat

In string theory, realizations of AdS vacua are much better understood than dS vacua. AdS vacua can for instance be realized via tree-level flux compactifications and often give $\alpha = 1/2$ in (1.1). It is fair to say that the question of scale separation for AdS vacua is not yet completely settled, as for instance the DGKT vacua [11] would be a candidate for an AdS vacuum featuring scale separation.[1] Consistent with the dS swampland conjecture [13] (see also [14,15] for alternative arguments), it is fair to say that so far there does not exist any generically accepted construction of a controlled dS vacuum.

However, the KKLT construction [16] and the large volume scenario (LVS) [17] are well studied candidates. These start with an AdS minimum and then invoke an uplift mechanism to dS, that is usually the contribution of an anti-D3-brane. To allow a controlled balance of the AdS vacuum energy and the uplift energy one puts the anti D3-brane in a strongly warped throat. In this way, all energy contributions localized in the throat get redshifted by the small warp factor. By balancing the negative and positive contributions, the former AdS minimum can be uplifted to a dS one.

Following this prescription, let us now assume that in a type IIB orientifold set-up, by turning on 3-form fluxes on a Calabi-Yau manifold and taking non-perturbative effects into account, one has indeed arrived at a not necessarily supersymmetric AdS minimum with negative vacuum energy $V_{AdS}$. In order to be able to uplift, we also assume that the complex structure moduli stabilization involves one modulus $Z$ that controls the size of a 3-cycle, which for $Z \to 0$ approaches a conifold singularity. This situation has been analyzed in detail in e.g. [18,19] and here we simply state and use some of their results.

Denoting the total volume modulus of the Calabi-Yau as $\mathcal{V}$, for $\mathcal{V}|Z|^2 \ll 1$ one is in the regime of strong warping and the total geometry can be thought of as a long, strongly warped Klebanov-Strassler (KS) throat [20] of length $y_{UV}$ glued to a bulk Calabi-Yau manifold. Here $y$ denotes the radial direction in the KS metric. Following [21], the $N = 1$ supersymmetric low energy effective action for the two moduli $Z$ and $\mathcal{V}$ is described by a Kähler potential[2]

$$K = -2\log(\mathcal{V}) + \frac{2c' g_s M^2 |Z|^{\frac{2}{3}}}{\mathcal{V}^{\frac{2}{3}}} + \dots \qquad (2.1)$$

where the dots denote terms involving all the other complex structure and Kähler moduli of the CY threefold. Moreover the string coupling constant is related to the vacuum expectation values of the dilaton $g_s = e^{\langle \phi \rangle}$, which is the real part of the complex axio-dilaton $S = e^{-\phi} + iC_0$ field. Computing the periods close to a conifold point $|Z| \ll 1$ and turning on R-R three-form flux $M$ along the A-cycle and an NS-NS three-form flux $K$ along the B-cycle of the conifold results in a superpotential of the form

$$W = -\frac{M}{2\pi i} Z \log Z + iKSZ + \dots, \qquad (2.2)$$

where the dots again contain more flux induced tree-level contributions involving the complex structure moduli and non-perturbative terms involving the Kähler moduli. Then, the conifold modulus is stabilized at

$$Z \sim \exp\left(-\frac{2\pi K}{g_s M}\right), \qquad (2.3)$$

which self-consistently can be made exponentially small by choosing appropriate fluxes. For its mass one finds [19]

$$m_Z \simeq \frac{1}{(g_s M^2)^{\frac{1}{2}}} \left(\frac{|Z|}{\mathcal{V}}\right)^{\frac{1}{3}}. \qquad (2.4)$$

For an isotropic Calabi-Yau threefold one often considers the bulk KK tower of mass scale

$$M_{KK} \sim \frac{1}{\tau_b} \sim \frac{1}{\mathcal{V}^{\frac{2}{3}}}. \qquad (2.5)$$

However, a compactification with a strongly warped throat is a highly non-isotropic situation, so that this does not necessarily reflect the lowest KK scale. In [19] (see also [23]) it was shown to first approximation analytically, and confirmed numerically, that there exists a one-dimensional tower of redshifted KK modes that are mainly supported close to the tip of the KS throat. Their masses scale in the same way as the mass of the conifold modulus $Z$, i.e.[3]

$$m_{KK} \sim \frac{1}{(g_s M^2)^{\frac{1}{2}} y_{UV}} \left(\frac{|Z|}{\mathcal{V}}\right)^{\frac{1}{3}}. \qquad (2.6)$$

Note that while the warped modes contain a lower suppression by the volume, the exponentially small value of $Z$ will easily make their mass scale much smaller than the bulk KK scale. Control over the warped effective action requires $g_s M^2 \gg 1$ which additionally suppresses the warped KK scale. The strongly warped throat can be thought of as containing one very long direction along the radial $y$-direction of the throat and thus being effectively 5-dimensional at intermediate energy scales between $m_{KK}$ and $m_{KK}^{(1)}$. Here $m_{KK}^{(1)}$ denotes the mass scale of the second lightest tower of KK modes. This is not necessarily the bulk mass scale, as for instance we expect the KK modes localized on the $S^3$ in the KS throat to also become redshifted and thus be lighter than the bulk modes. We will come back to this point at the beginning of section 3.

The uplift contribution to the scalar potential for an anti D3-brane placed at the tip of the KS throat is given by [18,19]

$$V_{up} \sim \frac{1}{(g_s M^2)} \left(\frac{|Z|}{\mathcal{V}}\right)^{\frac{4}{3}}. \qquad (2.7)$$

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[1] In an explicit example [12], it leads to a value of $\alpha = \frac{7}{18}$.

[2] It was shown in [22] that this Kähler potential receives corrections when going off-shell. Note that here we are only interested in the behavior close to the minimum.

[3] More precisely, the numerical analysis [19] indicated that the localization of the KK modes close to the tip of the throat makes the KK masses (2.6) insensitive to the length of the throat $y_{UV}$ beyond a critical length $y_{UV} > y_{UV}^* = O(10)$. A further increase in throat length is not detected by the localized modes and the scaling with $y_{UV}$ stops.

We notice that parametrically (in the exponentially small quantity $|Z|/\mathcal{V}$) we have the relation

$$m_{\text{KK}} \sim \frac{1}{(g_s M^2)^{\frac{1}{4}} y_{\text{UV}}} |V_{\text{up}}|^{\frac{1}{4}} \tag{2.8}$$

between the warped KK mass and the uplift potential. Choosing fluxes in (2.3) such that $V_{\text{up}} \sim |V_{\text{AdS}}|$, we get a relation between the value of $Z$ and the cosmological constant of the AdS vacuum before the uplift

$$\frac{|Z|}{\mathcal{V}} \sim (g_s M^2)^{\frac{3}{4}} |V_{\text{AdS}}|^{\frac{3}{4}} . \tag{2.9}$$

If the uplift really works and there is a meta-stable dS minimum, the cosmological constant in the dS minimum is then given by $\Lambda = V_{\text{up}} + V_{\text{AdS}} \gtrsim 0$. Note that with respect to the exponentially small and therefore most relevant parameter $|Z|/\mathcal{V}$, the final cosmological constant $\Lambda$ scales in the same way as $V_{\text{up}}$ and $|V_{\text{AdS}}|$.

The usual landscape philosophy says that playing with the warp factor allows the cosmological constant to be tuned to hierarchically smaller values.[4] However, one has to keep in mind that according to (2.3) the VEV of $Z$ is determined by quantized fluxes. The number of fluxes one can use is expected to be bound from above by tadpole cancellation conditions,[5] the genuine quantum gravity constraints in string theory. Moreover, as shown [22] the minimum should not move too far away from its initial position. This suggests that in quantum gravity the actual tuning one is allowed to do in a controllable manner is limited. We include this tuning in our analysis by writing

$$\Lambda = \lambda' |V_{\text{up}}| \tag{2.10}$$

with $\lambda' < 1$. Finally we use eq. (2.8) to arrive at the relation

$$\Lambda^{\frac{1}{4}} = \left[ (g_s M^2)^{\frac{1}{4}} y_{\text{UV}} \lambda' \right] m_{\text{KK}} . \tag{2.11}$$

We observe that this is precisely the relation (1.2) for the dark dimension scenario with $\lambda = (g_s M^2)^{\frac{1}{4}} y_{\text{UV}} \lambda'$. Imposing the bound $\lambda > 10^{-4}$ from [3], guaranteeing that a small value of $\lambda$ does not change the scaling too much, leads to

$$\lambda' > \frac{10^{-4}}{(g_s M^2)^{\frac{1}{4}} y_{\text{UV}}} . \tag{2.12}$$

Consistent with what we just discussed, this restricts the amount of "fine tuning" one can perform to get a small cosmological constant.

Note that if the former AdS vacuum admits an uplift of this type, i.e. that fluxes can be found so that the uplift condition $V_{\text{up}} \sim |V_{\text{AdS}}|$ holds and a meta-stable vacuum appears, then it satisfies the scaling of the AdS distance conjecture with $\alpha = 1/4$, where the tower of states is given by KK modes in the long throat. Thus, not only the uplifted dS but also the AdS vacuum is scale separated.

We conclude that an uplift by an anti D3-brane in a strongly warped throat generically leads to a tower of one-dimensional KK-modes parametrically satisfying the (A)dS distance conjecture with

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[4] Additional tuning could also arise from a landscape of initial AdS minima.

[5] To quantify the generic relative tuning available for $V_{\text{up}}$ by choosing discrete fluxes in (2.3) we define the relative minimal distance $\lambda' \sim \frac{|Z_1|^{4/3} - |Z_2|^{4/3}}{|Z_1|^{4/3}}$ for two values $|Z_1| > |Z_2|$. Assuming that the tadpoles restrict the fluxes $M$ and $K$ to be smaller than $N \gg 1$ one can derive the lower bound

$$\lambda' \sim \left( 1 - e^{-\frac{8\pi}{3g_s} \frac{|K_2 M_1 - K_1 M_2|}{M_1 M_2}} \right) > \left( 1 - e^{-\frac{8\pi}{3g_s N^2}} \right) \sim \frac{8\pi}{3 g_s N^2} .$$

For realistic values $g_s = 1/10$ and $N = 200$ one gets $\lambda' > 2 \cdot 10^{-3}$.

$\alpha = 1/4$. In view of the unsettled control issues raised recently for both the KKLT scenario [24,25,23] and the LVS [26,27], one might wonder whether one can draw any lesson from our analysis in case this idea of initial AdS with subsequent uplifting does not really work. Clearly, our result persists as long as there exists a strongly warped KS throat and the final (quasi) dS vacuum or even quintessence potential is dominated by the energy scale in the strongly warped throat.

## 3. Intermediate mass scales

While this is quite a robust result, it is of course just a single aspect of a whole string model. Putting aside the not yet settled question whether dS vacua exist at all in string theory, there are more mass scales in the game, like the other moduli masses or other heavier KK towers that might still be in conflict with astrophysical bounds. Their presence will also change the estimate of the species scale, where gravity becomes strongly coupled $\Lambda_{\text{sp}}^{\text{grav}} = M_{\text{pl}}/\sqrt{N} = m_{\text{KK}}^{1/3} M_{\text{pl}}^{2/3} \sim 10^{9-10}$ GeV. We have already indicated that beyond the lightest one-dimensional tower of KK modes in the throat there might be more towers that are also red-shifted and hence lighter than $\Lambda_{\text{sp}}^{\text{grav}}$. Moreover, employing the emergence proposal [28–30], it was argued in [19] that the effective theory in the throat determined by the Kähler potential (2.1) and the superpotential (2.2) comes with its own cutoff $\Lambda_{\text{sp}}^{\text{throat}} \sim (g_s M^2 y_{\text{UV}})^{\frac{2}{3}} m_{\text{KK}}$ which is much lighter than $\Lambda_{\text{sp}}^{\text{grav}}$. Above this scale other non-perturbative states appear that need to be included in the effective action.

Clearly, this is a very generic aspect of realizing the dark dimension scenario via a warped throat. Finally, let us also discuss the appearance of other light moduli and KK towers for the case of an uplifted large volume scenario.

*Example: uplifted LVS*

For the definition of the LVS and its moduli stabilization scheme and the resulting mass scales we refer the reader to the existing literature [17,31]. Here we only need to know that there are two Kähler moduli, the volume modulus $\tau_b \simeq \mathcal{V}^{\frac{2}{3}}$ and $\tau_s$, where the second is stabilized at small radius by a non-perturbative effect, whereas the first is stabilized perturbatively by an intricate balancing of three terms at

$$\mathcal{V} \sim \sqrt{\tau_s} \, e^{a \tau_s} . \tag{3.1}$$

The value of the cosmological constant in the non-supersymmetric AdS minimum scales like

$$V_{\text{AdS}} \sim -\frac{1}{\tau_s} e^{-3a\tau_s} \sim \frac{1}{\mathcal{V}^3} . \tag{3.2}$$

The masses of the small and the large Kähler moduli scale as

$$m_{\tau_b} \sim \frac{1}{\mathcal{V}^{\frac{3}{2}}} , \qquad m_{\tau_s} \sim \frac{1}{\mathcal{V}} . \tag{3.3}$$

Recalling that the fluxes are chosen such that $V_{\text{up}} \sim |V_{\text{AdS}}|$, we express the value of the conifold modulus in terms of the volume

$$|Z| \sim (g_s M^2)^{\frac{3}{4}} \mathcal{V}^{-\frac{5}{4}} . \tag{3.4}$$

Then taking the scaling of the warped KK scale $m_{\text{KK}} \sim V_{\text{up}}^{\frac{1}{4}} \sim 1/\mathcal{V}^{\frac{3}{4}}$ and the (naive) bulk KK mass scale $M_{\text{KK}} \sim 1/\mathcal{V}^{\frac{2}{3}}$ into account we get the following hierarchy of mass scales

$$m_{\mathcal{V}} < m_{\tau_s} < m_{\text{KK}} < M_{\text{KK}} . \tag{3.5}$$

Thus, we see that all Kähler moduli (and also almost all the complex structure moduli) are lighter than the warped KK scale. In

particular this means that in LVS the conifold modulus is actually the heaviest complex structure modulus. Note that since $\mathcal{V}|Z|^2 \sim \mathcal{V}^{-3/2}$ we are indeed in the regime of strong warping. As expected, the bulk KK modes are indeed heavier than the ones arising in the throat. However, for their ratio one finds

$$\frac{M_{\text{KK}}}{m_{\text{KK}}} \sim \mathcal{V}^{\frac{1}{12}} \sim \Lambda^{-\frac{1}{36}} \sim 2 \cdot 10^3 \qquad (3.6)$$

so that the corresponding length scale in the bulk is only by a factor of $10^{-3}$ smaller than the length scale of the throat.[6] This puts the uplifted LVS in tension with the astrophysical bounds on KK modes in more than one large extra dimension. Moreover, this means that new physics appear below the species scale computed (naively) via the throat KK modes.

## 4. Conclusions

In this note we pointed out that a common aspect of string theoretic dS constructions naturally gives rise to the requirements of realizing the dark dimension scenario. We argued that under fairly generic assumptions, the non-isotropy caused by the presence of a strongly warped throat leads precisely to the required exponent $\alpha = 1/4$ in the (A)dS distance conjecture. The reason for this is simply that the energy density of an anti D3-brane at the tip of the throat and the mass scale of the one-dimensional tower of redshifted KK modes localized deep in the throat satisfy $m_{\text{KK}} \sim V_{\text{up}}^{\frac{1}{4}}$. Using the anti D3-brane as an uplift from an AdS minimum to dS parametrically correlates the uplift energy scale with the dS cosmological constant. The important numerical factor of $\lambda \sim 10^{-1} - 10^{-3}$ in the dark dimension scenario is then related to the flux dependent and therefore restricted "tuning" of $\Lambda$ relative to $V_{\text{up}}$. Our result is expected to persist even under the milder assumptions that there exists a strongly KS throat in the geometry and that the final quasi dS energy is dominated by the energy scale in the strongly warped throat.

On the downside, we pointed out that generically the warped throat will support more (towers of) light states below the gravity cutoff scale. In this respect, we identified more redshifted KK towers, non-perturbative states beyond the cutoff of the effective theory in the warped throat and, as shown concretely for the LVS, other light bulk moduli fields and KK towers. The appearance of such modes will lower the gravity cutoff and will be in conflict with the astrophysical constraints on the size of extra dimensions.

### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

### Data availability

No data was used for the research described in the article.

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[6] We can be a bit more precise by taking into account that the background is highly non-isotropic. In this case we better approximate $\text{Vol} = r_b^5 r_t$ and define the bulk KK scale as $M_{\text{KK}} \sim \frac{1}{r_b}$. In this way we find $\frac{M_{\text{KK}}}{m_{\text{KK}}} \sim \mathcal{V}^{1/10} \sim \Lambda^{-1/30} \sim 10^4$.