

# Study of the $a_1(1260)$ resonance in the $\gamma p \rightarrow \pi^+ \pi^+ \pi^- n$ reaction\*

Xu Zhang(张旭)<sup>1,2</sup> Ju-Jun Xie(谢聚军)<sup>1,2;1)</sup>

<sup>1</sup>Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

<sup>2</sup>University of Chinese Academy of Sciences, Beijing 101408, China

**Abstract:** Within an effective Lagrangian approach and resonance model, we study the  $\gamma p \rightarrow a_1(1260)^+ n$  and  $\gamma p \rightarrow \pi^+ \pi^+ \pi^- n$  reactions via the  $\pi$ -exchange mechanism. For the  $\gamma p \rightarrow \pi^+ \pi^+ \pi^- n$  reaction, we perform a calculation of the differential and total cross-sections by considering the contributions of the  $a_1(1260)$  intermediate resonance decaying into  $\rho\pi$  and then into  $\pi^+ \pi^+ \pi^-$ . Besides, the non-resonance process is also considered. With a lower mass of  $a_1(1260)$ , the experimental data for the invariant  $\pi^+ \pi^+ \pi^-$  mass distributions can be fairly well reproduced. For the  $\gamma p \rightarrow a_1(1260)^+ n$  reaction, with the model parameters, the total cross-section is of the order of  $10 \mu\text{b}$  at the photon beam energy  $E_\gamma \sim 2.5 \text{ GeV}$ . It is expected that the model calculations in this work could be tested by future experiments.

**Keywords:** photoproduction, mass distributions, total cross-section,  $a_1(1260)$

**PACS:** 14.40.Be **DOI:** 10.1088/1674-1137/43/6/064104

## 1 Introduction

The  $a_1(1260)$  resonance with quantum numbers  $J^{PC} = 1^{++}$  is a candidate for the chiral partner of  $\rho$  meson [1-3]. It is also described as a  $q\bar{q}$  state in the Nambu-Jona-Lasino model [4, 5]. Apart from the quark model, it is considered as a gauge boson of the local hidden symmetry [6, 7]. By using the chiral unitary approach,  $a_1(1260)$  is a state arising from the interactions of pairs of hadrons in coupled channels [8, 9]. In addition, the nature of  $a_1(1260)$  has also been investigated using the  $\tau$  decay spectrum into three pions [10-12], and multi-pion decays of light vector mesons [13, 14]. Recently, the  $a_1(1260)$  resonance was studied in Ref. [15] in the decay of  $\tau \rightarrow \nu_\tau \pi^- a_1(1260)$  through a triangle mechanism.

The dynamically generated nature of  $a_1(1260)$  has been tested in the radiative decay process. The decay of  $a_1(1260)$  into  $\pi\gamma$  in Ref. [16] was also studied in Refs. [17, 18] and found to be in agreement with the experimental data if  $a_1(1260)$  is associated with the dynamically generated picture. In Ref. [19], the lattice result for the coupling constant of  $a_1(1260)$  into the  $\rho\pi$  channel is

similar to the one obtained in Ref. [8]. Recently, the production of  $a_1(1260)$  in the  $\pi^- p \rightarrow a_1(1260)^- p$  reaction within the effective Lagrangian approach was studied in Ref. [20] based on the results of [8]. Besides, it was found that the elementary  $q\bar{q}$  component of  $a_1(1260)$  is comparable to the hadronic composite [21-23]. Using the chiral unitary approach, the large  $N_c$  behavior of the  $a_1(1260)$  state was investigated in Ref. [22], and it was found that  $q\bar{q}$  is not the main component of  $a_1(1260)$ .

Based on the values obtained by two different experimental groups [24, 25], it is estimated that the mass and Breit-Wigner width of  $a_1(1260)$  is  $M_{a_1(1260)} = 1230 \pm 40 \text{ MeV}$  and  $\Gamma_{a_1(1260)} = (250 - 600) \text{ MeV}$ , respectively [26]<sup>2)</sup>. The large uncertainties of the mass and width of  $a_1(1260)$  in the Particle Data Group (PDG) [26] show that the knowledge of  $a_1(1260)$  is very limited. Therefore, a study of  $a_1(1260)$  photoproduction could be helpful to determine the mass and width of this resonance.

Meson photoproduction off a proton provides one of the most direct platforms to extract information about the hadronic structure [27, 28]. We should point out that in the experiments, no signal representing  $a_1(1260)^+ n$  photo-

Received 8 January 2019, Published online 10 May 2019

\* Supported by the National Natural Science Foundation of China (11475227, 11735003), and the Youth Innovation Promotion Association CAS (2016367)

1) E-mail: xiejujun@impcas.ac.cn

2) The result reported by COMPASS provides the mass  $M_{a_1(1260)} = 1255 \pm 6_{-17}^{+7} \text{ MeV}$  and width  $\Gamma_{a_1(1260)} = 367 \pm 9_{-25}^{+28} \text{ MeV}$  [24], while the analysis results of Ref. [25] are  $M_{a_1(1260)} = 1225 \pm 9 \pm 20 \text{ MeV}$  and  $\Gamma_{a_1(1260)} = 430 \pm 24 \pm 31 \text{ MeV}$ .



Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI. Article funded by SCOAP<sup>3</sup> and published under licence by Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

production [29-33] could be isolated even though the  $\pi\gamma$  radiative width of  $a_1(1260)$  very likely exceeds that of  $a_2(1320)$  [16, 34-36]. The absence of the  $J^{PC} = 1^{++}$  state in charge exchange photoproduction is puzzling. In this paper, by investigating the  $\gamma p \rightarrow a_1(1260)^+ n$  process within the  $\pi$ -exchange mechanism, we calculate its total cross-section. The  $\pi^+\pi^+\pi^-$  mass distribution and the total cross-section of  $\gamma p \rightarrow \pi^+\pi^+\pi^- n$  are studied. In addition, we consider the non-resonance contributions to the  $\gamma p \rightarrow \pi^+\pi^+\pi^- n$  resonance, which involve nucleon pole terms. Other contributions, which involve  $\Delta(1232)$  and nucleon excited states, can be removed based on the  $\pi^+ n$  invariant mass spectrum from the experiments [33].

The article is organized as follows. After the introduction, we present the reaction mechanism of  $a_1(1260)$  photoproduction. The possible background relevant to the production of  $a_1(1260)$  is discussed and the  $\pi^+\pi^+\pi^-$  mass distribution is presented in Sec. 3. This work ends with a discussion and conclusion.

## 2 $\gamma p \rightarrow a_1(1260)^+ n$ reaction

In this section, we discuss the  $a_1(1260)$  production mechanism. Fig. 1 shows the basic tree-level Feynman diagram for the  $a_1(1260)$  production in the  $\gamma p \rightarrow a_1(1260)^+ n$  reaction via the  $\pi$ -exchange process.

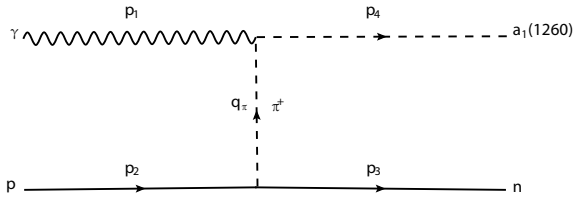


Fig. 1. Feynman diagram for the  $\gamma p \rightarrow a_1(1260)^+ n$  reaction via  $\pi$ -exchange.

For the  $\pi NN$  vertex, we adopt the commonly used effective Lagrangian

$$\mathcal{L} = -ig_{\pi NN} \bar{N} \gamma_5 (\vec{\tau} \cdot \vec{\pi}) N = -ig_{\pi NN} (\bar{p} \gamma_5 p \pi^0 + \sqrt{2} \bar{p} \gamma_5 n \pi^+ + \sqrt{2} \bar{n} \gamma_5 p \pi^- - \bar{n} \gamma_5 n \pi^0), \quad (1)$$

where the standard value,  $g_{\pi NN}^2/4\pi = 14.4$ , is adopted as in Refs. [37, 38]. In addition, the form factor is introduced for suppressing the vertex coupling when one or two interacting particles go off-shell. For the  $\pi NN$  vertex, the form factor satisfies the relation

$$F_{\pi NN}(q_\pi) = \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - q_\pi^2}, \quad (2)$$

where  $\Lambda_\pi$  is a cut-off parameter [39, 40], which will be discussed in the following.  $q_\pi$  is the momentum of the exchanged  $\pi$  meson.

The vertex depicting the interaction of  $a_1(1260)$  and  $\pi\gamma$  is [17, 18]

$$t_{a_1^+ \rightarrow \pi^+ \gamma} = g_{a_1 \pi \gamma} \left( g^{\mu\nu} - \frac{p_\gamma^\mu p_{a_1}^\nu}{p_\gamma \cdot p_{a_1}} \right) \varepsilon_\mu(p_{a_1}) \varepsilon_\nu(p_\gamma), \quad (3)$$

where  $\varepsilon_\mu(p_{a_1})$  and  $\varepsilon_\nu(p_\gamma)$  are the polarization vectors corresponding to  $a_1(1260)$  and photon, respectively.

With the vertex above, we can easily get the partial decay width of  $a_1 \rightarrow \pi\gamma$ ,

$$\Gamma_{a_1 \rightarrow \pi\gamma} = \frac{g_{a_1 \pi \gamma}^2}{24\pi M_{a_1}^3} (M_{a_1}^2 - m_\pi^2), \quad (4)$$

where  $M_{a_1} = 1230$  MeV is the nominal mass of  $a_1(1260)$ . Using the partial decay width  $\Gamma_{a_1 \rightarrow \pi\gamma} = 640 \pm 246$  keV of  $a_1(1260)$  as listed in PDG [26], we get  $g_{a_1 \pi \gamma} = 244 \pm 94$  MeV, where the error is from the uncertainties of  $\Gamma_{a_1 \rightarrow \pi\gamma}$  and the mass of  $a_1(1260)$ . In the following calculations, we take the average value  $g_{a_1 \pi \gamma} = 244$  MeV.

With the above integrands, one can get the scattering amplitude of the  $\gamma(p_1)p(p_2) \rightarrow a_1(1260)^+(p_4) + n(p_3)$  process as

$$\mathcal{M} = \frac{-\sqrt{2}ig_{\pi NN}g_{a_1\pi\gamma}}{q_\pi^2 - m_\pi^2} \bar{u}(p_3) \gamma_5 u(p_2) \times \left( g^{\mu\nu} - \frac{p_1^\mu p_4^\nu}{p_1 \cdot p_4} \right) \varepsilon_\mu(p_4) \varepsilon_\nu(p_1) F_{\pi NN}(q_\pi). \quad (5)$$

By defining  $s = (p_1 + p_2)^2$ , the corresponding unpolarized differential cross-section reads

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{32\pi s} \frac{|\vec{p}_4^{\text{c.m.}}|}{|\vec{p}_1^{\text{c.m.}}|} \left( \frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2 \right), \quad (6)$$

where  $\theta$  is the scattering angle of  $a_1^+$  meson relative to the beam direction in the c.m. frame, while  $\vec{p}_1^{\text{c.m.}}$  and  $\vec{p}_4^{\text{c.m.}}$  are the three-momenta of the initial photon and the final  $a_1^+$ , respectively.

In Fig. 2, the solid, dashed and dotted lines are obtained with  $\Lambda_\pi = 1.0, 1.3$  and  $1.6$  GeV, respectively. From Fig. 2 one can see that the total cross-section via  $\pi$  exchange increases very rapidly close to the threshold, and

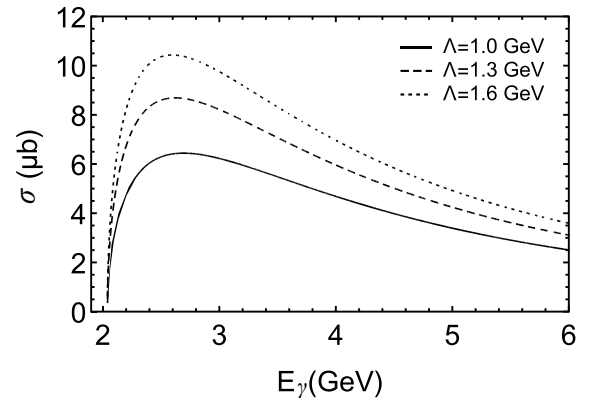


Fig. 2. Dependence of the total cross-section of  $\gamma p \rightarrow a_1(1260)^+ n$  as a function of  $E_\gamma$ .

the peak position of the total cross-section is  $E_\gamma \sim 2.6$  GeV. The total cross-section is proportional to  $g_{a_1, \pi\gamma}^2$ , which indicates that the cross-section is proportional to the partial decay width  $\Gamma_{a_1 \rightarrow \pi\gamma}$ . Since the exact value of  $\Gamma_{a_1 \rightarrow \pi\gamma}$  is not determined by theory or experiment, in this work we take  $\Gamma_{a_1 \rightarrow \pi\gamma} = 640$  keV. The result is comparable with the cross-section of  $a_2(1320)$  photoproduction [41].

### 3 $\gamma p \rightarrow \pi^+ \pi^+ \pi^- n$ reaction

Next, we consider the  $\gamma p \rightarrow a_1(1260)^+ n \rightarrow \rho^0 \pi^+ n \rightarrow \pi^+ \pi^+ \pi^- n$  and  $\gamma p \rightarrow \rho^0 p \rightarrow \pi^+ \pi^+ \pi^- n$  processes. Here  $\gamma p \rightarrow \rho^0 p \rightarrow \pi^+ \pi^+ \pi^- n$  can occur via the nucleon pole term [42].

#### 3.1 $\gamma p \rightarrow a_1(1260)^+ n \rightarrow \rho^0 \pi^+ n \rightarrow \pi^+ \pi^+ \pi^- n$ reaction

The  $\gamma p \rightarrow a_1(1260)^+ n \rightarrow \rho^0 \pi^+ n \rightarrow \pi^+ \pi^+ \pi^- n$  reaction with  $\pi$  exchange is shown in Fig. 3, where the relevant kinematic variables are shown. As discussed in the introduction, we take the coupling of  $a_1(1260)$  to the  $\rho\pi$  channel as obtained in Ref. [8].

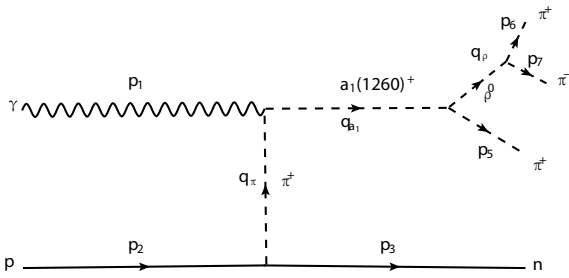


Fig. 3. Feynman diagram for the  $\gamma p \rightarrow a_1(1260)^+ n \rightarrow \rho^0 \pi^+ n \rightarrow \pi^+ \pi^+ \pi^- n$  reaction via  $\pi$  exchange.

The  $a_1^+ \rho^0 \pi^+$  vertex can be written as

$$-i t_1 = -i \frac{g_{a_1 \rho \pi}}{\sqrt{2}} \varepsilon_{a_1}^\mu \varepsilon_\mu, \quad (7)$$

where  $\varepsilon_{a_1}$  and  $\varepsilon$  are the polarization vectors of  $a_1(1260)$  and  $\rho$ , respectively.  $g_{a_1 \rho \pi}$  is the coupling of  $a_1(1260)$  to  $\rho\pi$ . We take  $g_{a_1 \rho \pi} = (-3795 + i2330)$  MeV as obtained in Ref. [8], where only the  $S$ -wave interaction was considered. Note that there is also a  $D$ -wave contribution to the  $a_1 \rho \pi$  vertex as investigated in Ref. [43], where the  $D$ -wave contribution was found to be small.

For the vertex of  $a_1(1260)^+$  interacting with  $\rho^0 \pi^+$ , we also introduce a form factor  $F_{a_1 \rho \pi}$ , which is

$$F_{a_1 \rho \pi}(q_{a_1}) = \frac{\Lambda_{a_1}^4}{\Lambda_{a_1}^4 + (q_{a_1}^2 - M_{a_1}^2)^2}, \quad (8)$$

with a typical value of  $\Lambda_{a_1} = 1.5$  GeV as in Refs. [20, 44].

The  $a_1(1260)$  propagator is

$$G_{a_1}^{\alpha\beta}(q_{a_1}) = i \frac{-g^{\alpha\beta} + q_{a_1}^\alpha q_{a_1}^\beta / M_{a_1}^2}{q_{a_1}^2 - M_{a_1}^2 + i M_{a_1} \Gamma_{a_1}}, \quad (9)$$

where the width  $\Gamma_{a_1}$  is dependent on its four-momentum squared, and we can take the form as in Refs. [45, 46],

$$\Gamma_{a_1} = \Gamma_0 + \Gamma_{3\pi}, \quad (10)$$

where  $\Gamma_{3\pi}$  is the decay width for the process  $a_1(1260) \rightarrow \rho\pi \rightarrow 3\pi$  [44], and  $\Gamma_0$  is the decay width for the other processes. Following the experimental result in Ref. [24] for the total decay width of  $a_1(1260)$ , we take  $\Gamma_0 = 201$  MeV for  $\Gamma_{a_1} = 367$  MeV at  $\sqrt{q_{a_1}^2} = 1230$  MeV.

For the structure of the  $\rho\pi\pi$  vertex, we use the general interaction as,

$$\mathcal{L}_{PPV} = -ig \langle V^\mu [P, \partial_\mu P] \rangle, \quad (11)$$

where  $\langle \rangle$  stands for the trace in  $SU(3)$ , and  $g = \frac{m_V}{2f}$ , with  $m_V = m_\rho$ , and  $f = 93$  MeV is the pion decay constant. The  $\rho\pi\pi$  vertex can then be written as

$$-it = -i \sqrt{2} g (p_7 - p_6)_\lambda \varepsilon^\lambda(p_4). \quad (12)$$

For the vertex of  $\rho$  interacting with  $\pi\pi$ , we also introduce a form factor  $F_{\rho\pi\pi}$ , which satisfies the relation

$$F_{\rho\pi\pi}(q_\rho) = \frac{\Lambda_\rho^4}{\Lambda_\rho^4 + (q_\rho^2 - m_\rho^2)^2}, \quad (13)$$

with a typical value of  $\Lambda_\rho = 1.5$  GeV as used in Ref. [44].

The  $\rho$  propagator is

$$G_\rho^{\sigma\lambda}(q_\rho) = i \frac{-g^{\sigma\lambda} + q_\rho^\sigma q_\rho^\lambda / m_\rho^2}{q_\rho^2 - m_\rho^2 + i m_\rho \Gamma_\rho}, \quad (14)$$

where  $\Gamma_\rho$  is energy dependent. Because the dominant decay channel of  $\rho$  is  $\pi\pi$ , we take

$$\Gamma_\rho(M_{\text{inv}}^2) = \Gamma_{\text{on}} \left( \frac{q_{\text{off}}}{q_{\text{on}}} \right)^3 \frac{m_\rho}{M_{\text{inv}}}, \quad (15)$$

with  $\Gamma_{\text{on}} = 149.1$  MeV, and

$$q_{\text{on}} = \frac{\sqrt{m_\rho^2 - 4m_\pi^2}}{2}, \quad (16)$$

$$q_{\text{off}} = \frac{\sqrt{M_{\text{inv}}^2 - 4m_\pi^2}}{2}, \quad (17)$$

where  $M_{\text{inv}}^2 = q_\rho^2 = (p_6 + p_7)^2$  or  $(p_5 + p_7)^2$  is the invariant mass squared of the  $\pi^+ \pi^-$  system. We take  $m_\rho = 775.26$  MeV in this work.

It is worth to mention that the parametrization of the width of  $\rho$  meson shown in Eq. (15) is meant to take into account the phase space of each decay mode as a function of the energy [40, 47, 48]. In the present work we take explicitly the phase space for the  $P$ -wave decay of the  $\rho$  into two pions.

We finally obtain the scattering amplitude for the diagram shown in Fig. 3,

$$\begin{aligned}
 \mathcal{M}_I = & \frac{\sqrt{2}i g_{\pi NN} g_{a_1 \pi \gamma} \bar{u}(p_3) \gamma_5 u(p_2)}{q_\pi^2 - m_\pi^2} \left( g^{\mu\nu} - \frac{p_1^\mu q_{a_1}^\nu}{p_1 \cdot q_{a_1}} \right) \\
 & \times \varepsilon_\nu(p_1) G_{\mu\sigma}^{a_1}(q_{a_1}) F_{\pi NN}(q_\pi) F_{a_1 \rho \pi}(q_{a_1}) (g_{\rho\pi} g) \\
 & \times \left( G_\rho^{\sigma\lambda}(p_6 + p_7) (p_7 - p_6)_\lambda F_{\rho\pi\pi}(p_6 + p_7) \right. \\
 & \left. + (G_\rho^{\sigma\lambda}(p_5 + p_7) (p_7 - p_5)_\lambda F_{\rho\pi\pi}(p_5 + p_7)) \right). \quad (18)
 \end{aligned}$$

### 3.2 $\gamma p \rightarrow \rho^0 p \rightarrow \pi^- \pi^+ p \rightarrow \pi^+ \pi^+ \pi^- n$ reaction

Besides the resonance contribution of the  $a_1(1260)$  resonance, we study another kind of reaction mechanism for the  $\gamma p \rightarrow \pi^+ \pi^+ \pi^- n$  reaction, which is depicted in Fig. 4, where we have considered the contribution from  $\gamma p \rightarrow \rho^0 p \rightarrow \pi^+ \pi^- \pi^+ n$ . In Fig. 4, the relevant kinematic variables are also shown.

To compute the contribution of Fig. 4, we take the interaction density for  $\rho\gamma\pi$  as [49, 50],

$$\mathcal{L}_{\rho\gamma\pi} = \frac{e g_{\rho\gamma\pi}}{m_\rho} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \rho_\nu \partial_\alpha A_\beta \pi, \quad (19)$$

where  $A_\beta, \pi$  and  $\rho_\nu$  denote the fields of the photon,  $\pi$  and  $\rho$ , respectively. The coupling constant  $g_{\rho\gamma\pi}$  can be obtained from the experimental decay width  $\Gamma_{\rho^0 \rightarrow \pi^0 \gamma}$  [26], which leads to  $g_{\rho\gamma\pi} = 0.76$ .

Other vertexes are the same as given above. With the above preparation, we get the transition amplitude for the diagram shown in Fig. 4,

$$\begin{aligned}
 \mathcal{M}_{II} = & \frac{-\sqrt{2} g_{\pi NN} g_{\pi NN} e g_{\rho\gamma\pi}}{q_\pi^2 - m_\pi^2} \frac{e g_{\rho\gamma\pi}}{m_\rho} g F_{\pi NN}(q_\pi) \bar{u}(p_3) \gamma_5 \\
 & \times \left( \frac{(\not{p}_3 + \not{p}_5) + m_p}{(p_3 + p_5)^2 - m_p^2} \gamma_5 u(p_2) F_N(p_3 + p_5) \epsilon^{\mu\nu\alpha\beta} \right. \\
 & \times (p_6 + p_7)_\alpha p_{1\beta} \epsilon_\nu G_\rho^{\mu\sigma} (p_6 + p_7) (p_7 - p_6)_\sigma F_{\rho\pi\pi}(p_6 + p_7) \\
 & + \frac{(\not{p}_3 + \not{p}_6) + m_p}{(p_3 + p_6)^2 - m_p^2} \gamma_5 u(p_2) F_N(p_3 + p_6) \epsilon^{\mu\nu\alpha\beta} (p_5 + p_7)_\alpha \\
 & \left. \times p_{1\beta} \epsilon_\nu G_\rho^{\mu\sigma} (p_5 + p_7) (p_7 - p_5)_\sigma F_{\rho\pi\pi}(p_5 + p_7) \right), \quad (20)
 \end{aligned}$$

with

$$F_N(q_p) = \frac{\Lambda_N^4}{\Lambda_N^4 + (q_p^2 - m_p^2)^2}, \quad (21)$$

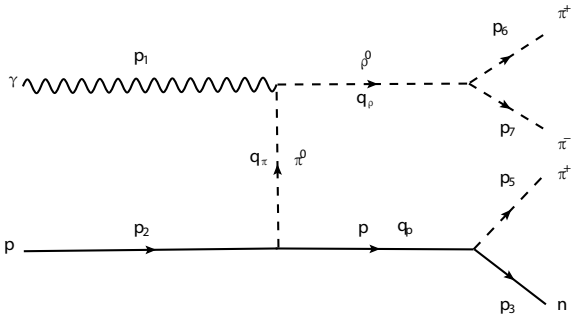


Fig. 4. Feynman diagram for the  $\gamma p \rightarrow \rho^0 \pi^+ n \rightarrow \pi^+ \pi^+ \pi^- n$  reaction via  $\pi$  exchange.

where  $\Lambda_\pi = 0.6$  GeV and  $\Lambda_N = 0.5$  GeV are taken from Refs. [49, 50, 51]. This choice of the cut-off leads to a satisfactory explanation of the  $\rho^0$  photoproduction at low energies. Note that the value of  $\Lambda_\pi$  is different from the one we used for the  $\gamma p \rightarrow na_1(1260)^+$  production. Other cut-off parameters are the same as given above.

### 3.3 Numerical results

The total cross-section of the  $\gamma p \rightarrow \pi^+ \pi^+ \pi^- n$  reaction can be obtained by integrating the invariant amplitude in the four-body phase space:

$$\begin{aligned}
 d\sigma(\gamma p \rightarrow \pi^+ \pi^+ \pi^- n) = & \frac{1}{2!} \frac{2m_p \cdot 2m_n}{4|p_1 \cdot p_2|} \left( \frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2 \right) \\
 & \times (2\pi)^4 d\phi_4(p_1 + p_2; p_3; p_5, p_6, p_7), \quad (22)
 \end{aligned}$$

with

$$\mathcal{M} = \mathcal{M}_I + \mathcal{M}_{II}, \quad (23)$$

where  $2!$  is a statistical factor for the final two  $\pi^+$  mesons, and the four-body phase space is defined as [26]

$$\begin{aligned}
 d\phi_4(p_1 + p_2; p_3; p_5, p_6, p_7) = & \frac{1}{16(2\pi)^8 \sqrt{s}} |\vec{p}_6^{*a}| |\vec{p}_5^{*b}| |\vec{p}_3| d\Omega_6^{*a} d\Omega_5^{*b} d\Omega_3 dM_{\pi^+ \pi^-} dM_{\pi^+ \pi^+ \pi^-}, \quad (24)
 \end{aligned}$$

where  $|\vec{p}_6^{*a}|$  and  $\Omega_6^{*a}$  are the three-momentum and solid angle of the out-going  $\pi^+$  in the c.m. frame of the final  $\pi^+ \pi^-$  system,  $|\vec{p}_5^{*b}|$  and  $\Omega_5^{*b}$  are the three-momentum and solid angle of the out-going  $\pi^+$  in the c.m. frame of the final  $\pi^+ \pi^+ \pi^-$  system, and  $|\vec{p}_3|$  and  $\Omega_3$  are the three-momentum and solid angle of the out-going  $n$  in the c.m. frame of the initial  $\gamma p$  system. In the above equation,  $M_{\pi^+ \pi^-}$  is the invariant mass of the  $\pi^+ \pi^-$  two body system, and  $M_{\pi^+ \pi^+ \pi^-}$  is the invariant mass of the  $\pi^+ \pi^+ \pi^-$  three body system, and  $s = (p_1 + p_2)^2$  is the invariant mass squared of the initial  $\gamma p$  system.

In Ref. [33], the  $\gamma p \rightarrow \pi^+ \pi^+ \pi^- n$  reaction was studied in the photon energy range 4.8–5.4 GeV. The  $3\pi$  mass distributions are measured from the  $1^{++}(\rho\pi)_S$  partial wave. In Fig. 5, we show the theoretical results,  $c_1 d\sigma/dM_{\pi^+ \pi^+ \pi^-}$ , for the  $\pi^+ \pi^+ \pi^-$  invariant mass distributions for the  $\gamma p \rightarrow \pi^+ \pi^+ \pi^- n$  reaction at  $E_\gamma = 5.1$  GeV, compared with the experimental measurements of Ref. [33]. The theoretical results are obtained with  $c_1 = 21.5$  and  $c_1 = 18$  for  $M_{a_1} = 1080$  and 1230 MeV, respectively, which have been adjusted to the experimental data reported by the CLAS collaboration [33]. From Fig. 5, it is seen that the bump structure around 1.4–1.6 GeV may account for the nuclear pole contribution. If we use  $M_{a_1} = 1080$  MeV, the  $\pi^+ \pi^+ \pi^-$  invariant mass distributions agree well with the experimental data. On the other hand, the theoretical results with  $M_{a_1} = 1230$  MeV can not describe the bump structure around 1.1 GeV.

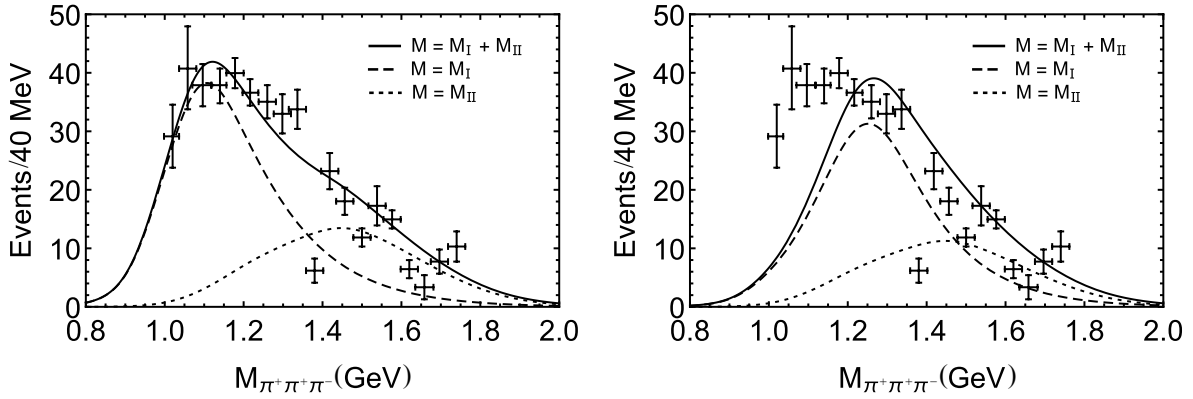


Fig. 5. The  $3\pi$  invariant mass spectrum for the  $\gamma p \rightarrow \pi^+\pi^+\pi^-n$  process compared with the data obtained by the CLAS collaboration from the  $1^{++}(\rho\pi)_S$  partial wave [33]. Left and right plots correspond to  $M_{a_1} = 1080$  and  $1230$  MeV, respectively.

In addition to the differential cross-section, we also calculated the total cross-section for the  $\gamma p \rightarrow \pi^+\pi^+\pi^-n$  process as a function of the photon beam energy  $E_\gamma$ . The results are shown in Fig. 6, where one can see that the total cross-section increases rapidly near the threshold,

and the peak of the total cross-section is at  $E_\gamma = 2.5$  and  $2.9$  GeV corresponding to  $M_{a_1} = 1080$  and  $1230$  MeV, respectively. The differential and total cross-sections could be checked in future experiments, such as those at CLAS.

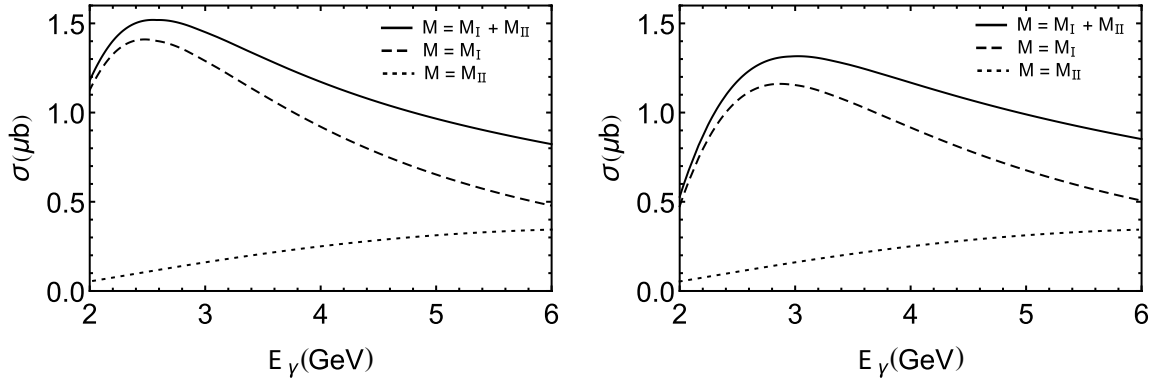


Fig. 6. Total cross-section for the  $\gamma p \rightarrow \pi^+\pi^+\pi^-n$  process. Left and right plots correspond to  $M_{a_1} = 1080$  and  $1230$  MeV, respectively.

#### 4 Conclusion and discussion

In recent years, it has been found that the  $a_1(1260)$  resonance, although long accepted as an ordinary  $q\bar{q}$  state, can be dynamically generated from the pseudoscalar-meson-vector-meson interaction, and therefore qualify as a pseudoscalar-vector molecule. In this work, we have proposed to study the  $a_1(1260)$  resonance in the photoproduction process. Since  $a_1(1260)$  was observed in the radiative decay of  $a_1(1260)^+ \rightarrow \pi^+\gamma$ , the  $\gamma p \rightarrow a_1(1260)^+n$  reaction by  $\pi$  meson exchange is the main process for producing  $a_1(1260)$ . Our numerical results show that the total cross-section of  $\gamma p \rightarrow a_1(1260)^+n$  is of the order of  $10 \mu\text{b}$ , which is comparable with the cross-section for photoproduction of  $a_2(1320)$ .

In addition, taking the coupling constant obtained

from the picture where the  $a_1(1260)$  resonance is a dynamically generated state from pseudoscalar-meson-vector-meson interaction, the  $\pi^+\pi^+\pi^-$  invariant mass distributions from the  $\gamma p \rightarrow \pi^+\pi^+\pi^-n$  reaction were studied. With  $M_{a_1} = 1080$  MeV, we can describe the experimental data for the  $\pi^+\pi^+\pi^-$  invariant mass distributions fairly well. The total cross-section of the  $\gamma p \rightarrow \pi^+\pi^+\pi^-n$  reaction was also studied using the model parameters determined from a comparison with the experimental data for the  $\pi^+\pi^+\pi^-$  invariant mass distributions. It is expected that our model calculations could be tested by future experiments with the  $\gamma p \rightarrow \pi^+\pi^+\pi^-n$  reaction at the photon beam energy  $E_\gamma$  around  $2.5\sim 2.9$  GeV.

*One of us (Xu Zhang) would like to thank Yin Huang for helpful discussions.*

## References

- 1 S. Weinberg, *Phys. Rev. Lett.*, **18**: 507 (1967)
- 2 C. W. Bernard, A. Duncan, J. LoSecco et al, *Phys. Rev. D*, **12**: 792 (1975)
- 3 G. Ecker, J. Gasser, A. Pich et al, *Nucl. Phys. B*, **321**: 311 (1989)
- 4 A. Dhar and S. R. Wadia, *Phys. Rev. Lett.*, **52**: 959 (1984)
- 5 A. Hosaka, *Phys. Lett. B*, **244**: 363 (1990)
- 6 M. Bando, T. Kugo, and K. Yamawaki, *Phys. Rept.*, **164**: 217 (1988)
- 7 N. Kaiser and U. G. Meissner, *Nucl. Phys. A*, **519**: 671 (1990)
- 8 L. Roca, E. Oset, and J. Singh, *Phys. Rev. D*, **72**: 014002 (2005), arXiv:hep-ph/0503273
- 9 M. F. M. Lutz and E. E. Kolomeitsev, *Nucl. Phys. A*, **730**: 392 (2004), arXiv:nucl-th/0307039
- 10 D. Gomez Dumm, A. Pich, and J. Portoles, *Phys. Rev. D*, **69**: 073002 (2004), arXiv:hep-ph/0312183
- 11 M. Wagner and S. Leupold, *Phys. Rev. D*, **78**: 053001 (2008), arXiv:0801.0814 [hep-ph]
- 12 D. G. Dumm, P. Roig, A. Pich et al, *Phys. Lett. B*, **685**: 158 (2010), arXiv:0911.4436 [hep-ph]
- 13 N. N. Achasov and A. A. Kozhevnikov, *Phys. Rev. D*, **71**: 034015 (2005), arXiv:hep-ph/0412077
- 14 P. Lichard and J. Juran, *Phys. Rev. D*, **76**: 094030 (2007), arXiv:hep-ph/0601234
- 15 L. R. Dai, L. Roca, and E. Oset, arXiv: 1811.06875 [hep-ph]
- 16 M. Zielinski et al, *Phys. Rev. Lett.*, **52**: 1195 (1984)
- 17 L. Roca, A. Hosaka, and E. Oset, *Phys. Lett. B*, **658**: 17 (2007), arXiv:hep-ph/0611075
- 18 H. Nagahiro, L. Roca, A. Hosaka et al, *Phys. Rev. D*, **79**: 014015 (2009), arXiv:0809.0943 [hep-ph]
- 19 C. B. Lang, L. Leskovec, D. Mohler et al, *JHEP*, **1404**: 162 (2014), arXiv:1401.2088 [hep-lat]
- 20 C. Cheng, J. J. Xie, and X. Cao, *Commun. Theor. Phys.*, **66**(6): 675 (2016), arXiv:1609.00442 [nucl-th]
- 21 H. Nagahiro, K. Nawa, S. Ozaki et al, *Phys. Rev. D*, **83**: 111504 (2011)
- 22 L. S. Geng, E. Oset, J. R. Pelaez et al, *Eur. Phys. J. A*, **39**: 81 (2009)
- 23 Z. H. Guo and J. A. Oller, *Phys. Rev. D*, **93**: 096001 (2016)
- 24 M. Alekseev et al, *Phys. Rev. Lett.*, **104**: 241803 (2010), arXiv:0910.5842 [hep-ex]
- 25 P. d'Argent et al, *JHEP*, **1705**: 143 (2017), arXiv:1703.08505 [hep-ex]
- 26 M. Tanabashi et al (Particle Data Group), *Phys. Rev. D*, **98**(3): 030001 (2018)
- 27 Y. Huang, J. J. Xie, J. He et al, *Chin. Phys. C*, **40**(12): 124104 (2016), arXiv:1604.05969 [nucl-th]
- 28 H. Xing, C. S. An, J. J. Xie et al, *Phys. Rev. D*, **98**(9): 094007 (2018), arXiv:1807.11151 [hep-ph]
- 29 Y. Eisenberg et al, *Phys. Rev. Lett.*, **23**: 1322 (1969)
- 30 G. T. Condo, T. Handler, W. M. Bugg et al, *Phys. Rev. D*, **48**: 3045 (1993)
- 31 W. Struczinski et al, *Nucl. Phys. B*, **108**: 45 (1976)
- 32 J. Ballam et al, *Phys. Rev. D*, **5**: 15 (1972)
- 33 M. Nozar et al, *Phys. Rev. Lett.*, **102**: 102002 (2009), arXiv:0805.4438 [hep-ex]
- 34 S. Cihangir et al, *Phys. Lett. B*, **117**: 119 (1982)
- 35 V. V. Molchanov et al, *Phys. Lett. B*, **521**: 171 (2001), arXiv:hep-ex/0109016]
- 36 E. N. May et al, *Phys. Rev. D*, **16**: 1983 (1977)
- 37 K. Tsushima, A. Sibirtsev, A. W. Thomas et al, *Phys. Rev. C*, **59**, 369 (1999); *Phys. Rev. C*, **61**, 029903 (2000), doi: 10.1103/Phys. Rev. C 61.029903, 10.1103/Phys. Rev. C 59.369 [nucl-th/9801063]
- 38 R. Machleidt, K. Holinde, and C. Elster, *Phys. Rept.*, **149**: 1 (1987)
- 39 J. J. Xie and B. S. Zou, *Phys. Lett. B*, **649**: 405 (2007), arXiv:nucl-th/0701021
- 40 J. J. Xie, B. S. Zou, and H. C. Chiang, *Phys. Rev. C*, **77**: 015206 (2008), arXiv:0705.3950 [nucl-th]
- 41 Y. Huang, J. J. Xie, X. R. Chen et al, *Int. J. Mod. Phys. E*, **23**: 1460002 (2014), arXiv:1308.3382 [hep-ph]
- 42 Y. Huang, J. He, X. R. Chen et al, *Phys. Rev. C*, **91**(6): 065202 (2015), arXiv:1412.7947 [nucl-th]
- 43 N. Isgur, C. Morningstar, and C. Reader, *Phys. Rev. D*, **39**: 1357 (1989)
- 44 X. Zhang and J. J. Xie, *Commun. Theor. Phys.*, **70**(1): 060 (2018), arXiv:1712.05572 [nucl-th]
- 45 J. J. Xie and E. Oset, *Eur. Phys. J. A*, **51**: 111 (2015), arXiv:1412.3234 [nuclth]
- 46 J. J. Xie, E. Oset, and L. S. Geng, *Phys. Rev. C*, **93**: 025202 (2016), arXiv:1509.06469 [nucl-th]
- 47 H. C. Chiang, E. Oset, and L. C. Liu, *Phys. Rev. C*, **44**: 738 (1991)
- 48 C. Hanhart, Y. S. Kalashnikova and A. V. Nefediev, *Phys. Rev. D*, **81**: 094028 (2010), arXiv:1002.4097 [hep-ph]
- 49 Y. s. Oh and T. S. H. Lee, *Phys. Rev. C*, **69**: 025201 (2004), arXiv:nucl-th/0306033
- 50 Y. s. Oh, A. I. Titov, and T. S. H. Lee, *Phys. Rev. C*, **63**: 025201 (2001), arXiv:nucl-th/0006057
- 51 A. I. Titov, T.-S. H. Lee, H. Toki et al, *Phys. Rev. C*, **60**: 035205 (1999)