Generalized geometrical coupling for vector field localization on thick brane in asymptotic anti–de Sitter spacetime

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It is known that a five-dimensional free vector field A_M cannot be localized on Randall-Sundrum (RS)-like thick branes—namely, the thick branes embedded in asymptotic anti–de Sitter spacetime. To localize a vector field on the RS-like thick brane, an extra coupling term should be introduced. We generalize the geometrical coupling mechanism by adding two mass terms $(\alpha R g^{MN} A_M A_N + \beta R^{MN} A_M A_N)$ to the action. We decompose the fundamental vector field A_M into three parts: transverse vector part $\hat{A}_\mu$ and scalar parts ϕ and A_5. Then we find that the transverse vector part $\hat{A}_\mu$ decouples from the scalar parts. To eliminate the tachyonic modes of $\hat{A}_\mu$, the two coupling parameters α and β should satisfy a relation. Combining the restricted condition, we can get a combination parameter as $\gamma = \frac{3}{2} \pm \sqrt{1+12\alpha}$. Only if $\gamma > 1/2$ can the zero mode of $\hat{A}_\mu$ be localized on the RS-like thick brane. We also investigate the resonant character of the vector part $\hat{A}_\mu$ for a general RS-like thick brane with a warp factor $A(z) = -\ln(1+k^2z^2)/2$ by choosing the relative probability method. The results show that the massive resonant Kaluza-Klein modes can exist only for $\gamma > 3$. The number of resonant Kaluza-Klein states increases with the combination parameter γ, and the lifetime of the first resonant state can be as long as our Universe's. This indicates that the vector resonances might be considered one of the candidates of dark matter. Combining the conditions of experimental observations, the constraint shows that the parameter k has a lower limit with $k \gtrsim 10^{-17}$ eV, the combination parameter γ should be greater than 57, and, accordingly, the mass of the first resonant state should satisfy $m_1 \gtrsim 10^{-15}$ eV.

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I. INTRODUCTION

In recent decades, brane world theories have received a lot of attention for success in solving the gauge hierarchy and cosmological constant problems [1,2]. In the brane world scenario, our Universe is a 3-brane embedded in a higher-dimensional bulk. The well-known Randall-Sundrum (RS) models [1] (including the RS-1 and RS-2 models) involve one extra dimension with a nontrivial warp factor due to the underlying anti–de Sitter (AdS) geometry.

In the RS thin brane models and their generalizations, the branes have no thickness, and there are no dynamical mechanisms responsible for their formation. To investigate the dynamical generation of branes and their internal structure, domain wall (or thick brane) models were presented; for more details on thick brane models, see Refs. [3,4]. One of the features of a thick brane is that it is usually generated by one or more background scalar fields coupled with gravity.

In thick brane models, various fundamental matter fields are living in the higher-dimensional bulk. Therefore, to construct a more realistic brane world, on which the four-dimensional gravity and matter fields in the standard model should be localized, it is very necessary and significant to provide an effective localization mechanism for the bulk gravity and matter fields. The results of Refs. [5–11] show

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that four-dimensional gravity can be localized on the thick branes generated by the background scalar field(s) in a five-dimensional asymptotic AdS spacetime. As shown in Refs. [12–17], a free massless scalar field can also be localized on the thick branes. For a Dirac fermion field, without introducing the scalar-fermion coupling (also called the Yukawa coupling) [18–24] or fermion-gravity coupling [25], it has no normalizable zero mode in five-dimensional RS-like brane models. Unfortunately, Ref. [26] gave the essence of the "no-go theorem" in the thin brane limit (the RS-2 model with an infinite extra dimension): that the localization for a vector field seems to require a richer brane structure, for example, the de Sitter brane [27–30], the brane world with a finite extra dimension [31], or a six-dimensional stringlike model [32].

A lot of works have been devoted to find a mechanism for vector field localization, and the literature shows a wide variety of ideas. Kehagias and Tamvakis proposed a dilaton coupling between the vector field and the background scalar field [33]. This mechanism has been widely applied in different thick brane models [34–40]. Chumbes *et al.* proposed a coupling function between the vector field and the background scalar field [41]. Vaquera-Araujo and Corradini introduced a Yukawa-like coupling—namely, a Stueckelberg-like action—to realize the localization of the vector field [42].

Recently, Zhao *et al.* [43] presented another localization mechanism of the vector field A_M; i.e., they introduced a mass term $\alpha R g^{MN} A_M A_N$, with R and g_{MN} being the five-dimensional scalar curvature and metric, respectively. They found that only for a special coupling parameter $\alpha = -1/16$ can the vector part $\hat{A}_\mu$ be localized on the thick brane, and there are no tachyonic modes. Then, Alencar *et al.* introduced other forms of the mass term: $\beta R^{MN} A_M A_N$ and $\beta G^{MN} A_M A_N$ with R^{MN} and G^{MN} the Ricci tensor and Einstein tensor [44,45]. While in all these mechanisms, to eliminate tachyonic vector modes, the massive parameter α or β should be fixed since there are no more degrees of freedom for the coupling parameter. As a result, the effective potential of the vector Kaluza-Klein (KK) modes is fixed and usually there are no resonant vector KK modes quasilocalized on the brane.

Inspired by the above works, we generalize the mass term to the following one,

$$-\frac{1}{2}(\alpha R g^{MN} A_M A_N + \beta R^{MN} A_M A_N), \tag{1}$$

since both terms are possible couplings. Then we study the localization and quasilocalization of the vector field on the thick brane. Quasilocalized massive KK modes might be a candidate for dark matter. Note that the consistency conditions for this kind of localization mechanism were just investigated in Ref. [46].

We decompose the vector field A_M into three parts: the transverse component $\hat{A}_\mu$ (the transverse vector part), the longitudinal component $\partial_\mu \phi$ (the scalar part), and the fifth component A_5 (the scalar part). Here the latin indices (M, $N = 0, 1, 2, 3, 5$) stand for the five-dimensional coordinate indices, and the greek indices ($\mu, \nu = 0, 1, 2, 3$) correspond to the brane coordinate indices. We find that the transverse vector part $\hat{A}_\mu$ decouples with the scalar parts ϕ and A_5. Besides, to eliminate the tachyonic modes of $\hat{A}_\mu$, the two parameters in the coupling term (1), α and β, should satisfy the relation $\beta = -1 - 8\alpha \pm \sqrt{1 + 12\alpha}$. With this constraint, we can get a combination parameter $\gamma = \frac{3}{2} \pm \sqrt{1 + 12\alpha}$, and the localized condition for the transverse vector part $\hat{A}_\mu$ is $\gamma > 1/2$. More importantly, we can find the resonant states under this restrictive condition. We investigate the resonant character of $\hat{A}_\mu$ with the general RS-like thick brane warp factor $A(z) = -\ln(1 + k^2 z^2)/2$. These resonant states can be considered for dark matter.

The remaining parts of the paper are organized as follows. In Sec. II, we introduce the generalized model of the vector field. Then we calculate the localization of the transverse part of a five-dimensional vector on the thick brane in Sec. III. After that, we study the resonant character of the transverse vector part in Sec. IV. Finally, we conclude with our results in Sec. V.

II. THE GENERALIZED GEOMETRICAL COUPLING MECHANISM OF THE VECTOR FIELD

The vector field can be localized on the thick brane by considering the geometrical coupling term, e.g., the coupling between the vector field and the Ricci scalar (or the Ricci tensor). In this paper, we consider the generalized geometrical coupling (1). Then the full five-dimensional action for the vector field A_M is given by

$$S = -\frac{1}{4}\int d^5x\sqrt{-g}(F^{MN}F_{MN} + 2(\alpha R g^{MN} + \beta R^{MN})A_M A_N), \tag{2}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ is the field strength. Note that the simplest mass term ($M^2 g^{MN} A_M A_N$) is not considered here since we cannot make sure that the localization of the vector zero mode and the exclusion of the tachyonic vector modes for any RS-like brane. The form of the geometrical coupling mechanism can be considered a massive term of the vector field which is related to the extra dimension. For such a massive vector, the U(1) gauge symmetry is violated. If we want to recover the five-dimensional gauge invariant, we should introduce a scalar field using the Stueckelberg mechanism. The five-dimensional gauge invariant massive vector field action can be expressed as

$$S = \int d^5x\sqrt{-g}\left(-\frac{1}{4}F^{MN}F_{MN} - \frac{1}{2}Q^{MN}(R)(A_M - \partial_M \mathcal{B})(A_N - \partial_N \mathcal{B})\right), \tag{3}$$

where $\mathcal{B}$ is a dynamical scalar field coming from the Stueckelberg mechanism and $Q^{MN} = \alpha g^{MN} R + \beta R^{MN}$. With the gauge transformation $A_M \to A_M + \partial_M \xi$, $\mathcal{B} \to \mathcal{B} + \xi$, the action (3) remains gauge invariant. In this paper, we consider the localization of the zero mode of the four-dimensional component A_μ, whose effective action will keep the gauge invariant in four-dimensional spacetime, which is consistent with the known four-dimensional theory. If we want to keep the gauge invariant in five-dimensional spacetime, we should add the Stueckelberg field $\mathcal{B}$, which introduces a new degree of freedom. By choosing a gauge, the redundant degree of freedom will be eliminated.

We decompose the vector A_M in the following way:

$$A_M = (\hat{A}_\mu + \partial_\mu \phi, A_5), \tag{4}$$

where $\hat{A}_\mu$ is the transverse component with the transverse condition $\partial_\mu \hat{A}^\mu = 0$ and $\partial_\mu \phi$ is the longitudinal component.

We adopt the following metric ansatz to describe the five-dimensional spacetime:

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \tag{5}$$

where the warp factor $A(y)$ is a function of the extradimensional coordinate y. So, the Ricci scalar and the nonvanishing components of the Ricci tensor can be expressed as

$$R = -4(5A'^2 + 2A''), \tag{6}$$

$$R^{\mu\nu} = -(4A'^2 + A'') g^{\mu\nu}, \tag{7}$$

$$R^{55} = -4(A'^2 + A''), \tag{8}$$

where the prime denotes the derivative with respect to y. The components of the mass terms in the action (3) can the written as

$$\alpha R g^{\mu\nu} + \beta R^{\mu\nu} = \mathcal{W} g^{\mu\nu}, \tag{9}$$

$$\alpha R g^{55} + \beta R^{55} = \mathcal{G} g^{55}, \tag{10}$$

where

$$\mathcal{W} = -4(5\alpha + \beta) A'^2 - (8\alpha + \beta) A'', \tag{11}$$

$$\mathcal{G} = -4(5\alpha + \beta) A'^2 - 4(2\alpha + \beta) A''. \tag{12}$$

By substituting the decomposition (4) into the action (3), we can split it into two parts,

$$S = S_V(\hat{A}_\mu) + S_S(\phi, A_5), \tag{13}$$

where

$$S_V = -\frac{1}{4} \int d^5 x (\hat{F}_{\lambda\mu} \hat{F}_{\nu\rho} \eta^{\lambda\nu} \eta^{\mu\rho} + 2\partial_5 \hat{A}_\mu \partial^5 \hat{A}_\nu \eta^{\mu\nu} e^{2A} + 2\mathcal{W} \hat{A}_\mu \hat{A}_\nu \eta^{\mu\nu} e^{2A}), \tag{14}$$

$$S_S = -\frac{1}{2} \int d^5 x e^{2A} (\eta^{\mu\nu} g^{55} (\partial_5 \partial_\mu \phi)(\partial_5 \partial_\nu \phi) + \mathcal{W} \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \eta^{\mu\nu} g^{55} \partial_\mu A_5 \partial_\nu A_5 + \mathcal{G} e^{2A} g^{55} A_5 A_5 - 2\eta^{\mu\nu} g^{55} \partial_\mu A_5 (\partial_5 \partial_\nu \phi)), \tag{15}$$

where $\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu$. The above result shows that the transverse vector part $\hat{A}_\mu$ decouples from the scalar parts. So, we consider separately only the localization condition and resonant character of the transverse vector part $\hat{A}_\mu$.

III. LOCALIZATION OF THE TRANSVERSE VECTOR PART OF THE VECTOR FIELD ON THICK BRANE

In this section, we consider the localization of the vector part $\hat{A}_\mu$ independently. We make the following KK decomposition:

$$\hat{A}_\mu(x, y) = \sum_n a_\mu^{(n)}(x^\nu) \tilde{\rho}_n(y), \tag{16}$$

where $a_\mu^{(n)}(x^\nu)$ is the four-dimensional vector KK mode and $\tilde{\rho}_n(y)$ is the corresponding extradimensional profile (also called the KK wave function in Ref. [47]), and the index n represents the nth KK mode. By using the KK decomposition (16) and the orthonormality condition

$$\int_{-\infty}^{\infty} \tilde{\rho}_n(y) \tilde{\rho}_m(y) = \delta_{mn}, \tag{17}$$

we can get an effective action including the four-dimensional massless vector field (the zero mode $a_\mu^{(0)}$) and a set of massive vector fields $a_\mu^{(n)}$ with $n > 0$:

$$S_V = -\frac{1}{4} \sum_n \int d^4 x (f_{\mu\lambda}^{(n)} f_{(n)}^{\mu\lambda} + 2m_n^2 a_\mu^{(n)} a_\nu^{(n)} \eta^{\mu\nu}), \tag{18}$$

where $f_{\mu\nu}^{(n)} = \partial_\mu a_\nu^{(n)} - \partial_\nu a_\mu^{(n)}$ is the four-dimensional vector field strength tensor. In addition, the extradimensional part $\tilde{\rho}_n(y)$ should satisfy the following equation:

$$-\partial_y (e^{2A(y)} \partial_y \tilde{\rho}_n) + \tilde{\rho}_n e^{2A(y)} \mathcal{W} = m_n^2 \tilde{\rho}_n. \tag{19}$$

To solve Eq. (19), we make a coordinate transformation $dz = e^{-A(y)} dy$, for which the metric can be expressed as

$$ds^2 = e^{2A(z)} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2), \tag{20}$$

and Eq. (19) is then rewritten as

$$-\partial_z(e^{A(z)}\partial_z\tilde{\rho}_n) + \tilde{\rho}_n e^{3A(z)}\mathcal{W} = e^{A(z)}m_n^2\tilde{\rho}_n, \tag{21}$$

with $\mathcal{W} = e^{-2A}(-12\alpha - 3\beta)(\partial_z A)^2 + e^{-2A}(-8\alpha - \beta)\partial_z^2 A$. After the field transformation $\tilde{\rho}_n = e^{-\frac{1}{2}A(z)}\rho_z(z)$, Eq. (21) can be rewritten as a Schrödinger-like equation,

$$(-\partial_z^2 + V_v(z))\rho_n = m_n^2\rho_n, \tag{22}$$

where the explicit expression of the effective potential $V_v(z)$ is

$$V_v(z) = \left(\frac{1}{4} - 12\alpha - 3\beta\right)(\partial_z A)^2 + \left(\frac{1}{2} - 8\alpha - \beta\right)\partial_z^2 A. \tag{23}$$

To exclude the tachyonic vector modes, the eigenvalues of Schrödinger-like equation (22) should be non-negative, i.e., $m_n^2 \geq 0$. So, Eq. (22) should be written in the form of $Q^+Q\rho_n = m_n^2\rho_n$ with $Q = -\partial_z + \gamma\partial_z A$. That is, the effective potential should be in the form

$$V_v(z) = \gamma^2(\partial_z A)^2 + \gamma\partial_z^2 A. \tag{24}$$

To this end, the two parameters α and β should satisfy the following relation,

$$\beta = -1 - 8\alpha \pm \sqrt{1 + 12\alpha}, \tag{25}$$

so parameter γ in Eq. (24) is given by

$$\gamma = \frac{3}{2} \pm \sqrt{1 + 12\alpha}. \tag{26}$$

With the relation (25) and the expression (26), the Schrödinger-like equation (22) can be further rewritten as

$$(-\partial_z^2 + \gamma^2(\partial_z A)^2 + \gamma\partial_z^2 A)\rho_n = m_n^2\rho_n. \tag{27}$$

Now, we investigate the localization of the zero mode of $\hat{A}_\mu$, for which $m_0 = 0$ and the solution is given by

$$\rho_0(z) = c_0 e^{\gamma A(z)}, \tag{28}$$

where c_0 is the normalization constant. According to the orthonormality condition (17), the integration of ρ_0^2 should be finite—namely,

$$\int_{-\infty}^{+\infty}\rho_0^2 dz = c_0^2\int_{-\infty}^{+\infty} e^{2\gamma A(z)}dz = c_0^2\int_{-\infty}^{+\infty} e^{(2\gamma-1)A(y)}dy = 1. \tag{29}$$

For the RS-like braneworld scenarios, the warp factor has the following asymptotic behavior:

$$A(y)|_{y\to\pm\infty} \to -k|y|, \tag{30}$$

where k is the scale parameter of the brane with mass dimension. Plugging it into Eq. (29), we obtain that

$$e^{(2\gamma-1)A(y)}|_{y\to\pm\infty} \to e^{-(2\gamma-1)k|y|}. \tag{31}$$

To ensure that the integration (29) is convergent, the parameter should satisfy $\gamma > 1/2$, i.e., $1 \pm \sqrt{1 + 12\alpha} > 0$. So, the range of the parameter α for different concrete expressions of β is

$$\alpha > -1/12, \qquad \beta = -1 - 8\alpha - \sqrt{1 + 12\alpha}, \tag{32}$$

$$0 > \alpha > -1/12, \qquad \beta = -1 - 8\alpha + \sqrt{1 + 12\alpha}. \tag{33}$$

IV. THE RESONANT CHARACTER OF $\hat{A}_\mu$

In this section, we would like to investigate the massive KK states of the transverse vector part for the vector field. We will mainly look for resonant KK states of the vector field, which are quasilocalized on the brane but propagate into extra dimensions eventually. The resonance spectrum of these KK states is one of the typical characteristics of RS-like brane world models. They can interact with four-dimensional particles, which may lead to the nonconservation of energy and momentum since the KK resonances can escape out of the brane. So, it is possible to probe extra dimensions by detecting resonant states [48]. Besides, some physicists regard those massive KK particles as a candidate for dark matter (see Refs. [49–51] for details). The appearance of these resonances is related to the structure of the brane. Thus, it is important and interesting to study the resonant KK modes on the thick brane with different structures. References [52–57] have considered resonances of gravitons and fermions. Besides, Arakawa *et al.* considered a massive vector field as a candidate for dark matter to explain the strong *CP* problem [58]. So, we will study the resonances of the five-dimensional vector field.

To study the resonant states, Almeida *et al.* proposed the large peaks of the wave function as the resonance method for studying fermion resonances [52]. Then, Landim and co-workers researched the resonant states with the transfer matrix method [56,57]. Here we will choose the relative probability method proposed by Liu *et al.* [54] to calculate the resonant KK modes of the vector part $\hat{A}_\mu$ since the method is effective for both odd and even KK states. The relative probability is defined as [54]

$$P = \frac{\int_{-z_b}^{z_b}|\rho_n(z)|^2 dz}{\int_{-z_{\max}}^{z_{\max}}|\rho_n(z)|^2 dz}, \tag{34}$$

where $2z_b$ is approximately the width of the thick brane and $z_{\max} = 10z_b$. Since the potentials considered in this paper are symmetric, the wave functions are either even or odd. Hence, we can use the following boundary conditions to solve the differential equation (27) numerically:

$$\rho_n(0) = 0, \qquad \rho_n'(0) = 1 \quad \text{for odd KK modes,}$$
$$\rho_n(0) = 1, \qquad \rho_n'(0) = 0 \quad \text{for even KK modes.} \tag{35}$$

We solve the Schrödinger-like equation (25) with the general RS-like warp factor $A(z) = -\ln(1 + k^2z^2)/2$. According to supersymmetric quantum mechanics, the supersymmetric partner potentials will share the same spectrum of massive excited states. So, we can judge whether there are resonances by analyzing the shape of the supersymmetric partner potential (we call it the dual potential). In our case, the dual potential corresponding to Eq. (24) is $V_v^{(\mathrm{dual})}(z) = \gamma^2(\partial_z A)^2 - \gamma\partial_z^2 A$. If there is no well or quasiwell in the dual potential, then there are no resonances. Thus, only for $\gamma > 3$ might there exist resonances. We solve the KK states numerically. The result shows that the parameters k and γ both will affect the properties of the resonant states.

Figure 1 shows the influence of the combination parameter γ on the effective potential $V_v(z)$ and the resonant KK modes of the vector field $\hat{A}_\mu$. Figure 1(a) shows that the height of the potential barrier increases with the combination parameter γ, which indicates that there are more resonant KK modes for larger γ, and this can be confirmed from Figs. 1(b)–1(d). Combining Figs. 1(b), 1(c), and 1(d), we can see that the mass of the first resonant KK modes, the number of resonant states, and the mass gap of the resonant KK modes increase with the parameter γ.

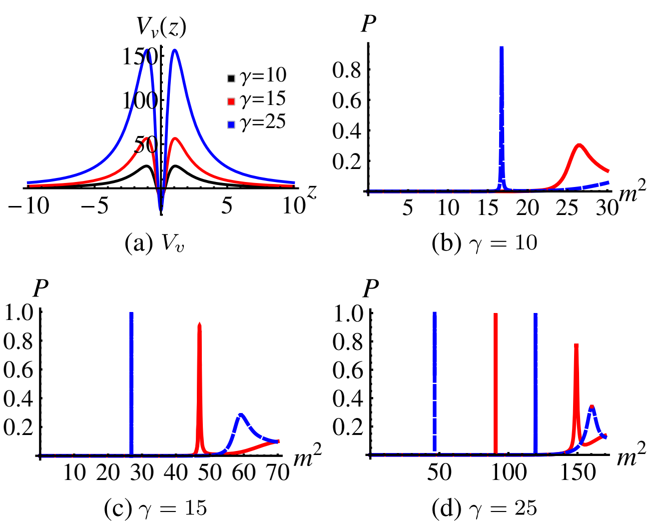


FIG. 1. The influence of the combination parameter γ on the effective potential V_v and the probabilities P (as a function of m^2) for the odd-parity (blue dashed lines) and even-parity (red lines) massive KK modes. (a) The effective potential V_v. (b–d) The probabilities P. The scale parameter is set at $k = 1$.

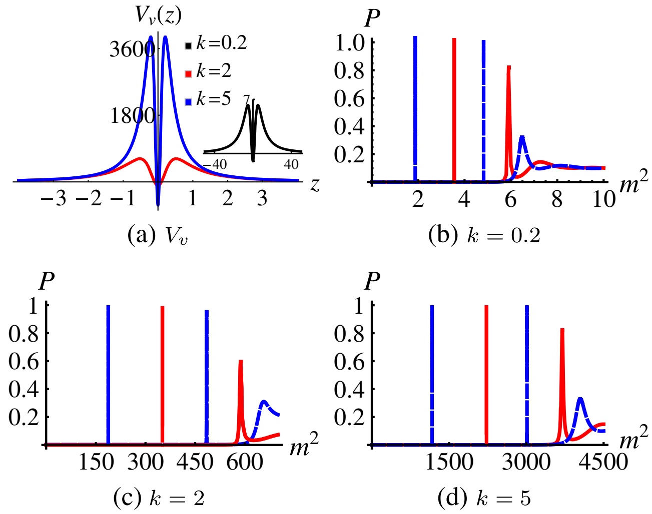


FIG. 2. The influence of the scale parameter k on the effective potential V_v and the probabilities P (as a function of m^2) for both the odd-parity (blue dashed lines) and even-parity (red lines) massive KK modes. (a) The effective potential V_v. (b–d) The probabilities P. The combination parameter is set at $\gamma = 25$.

The effect of the scale parameter k is shown in Fig. 2. From Fig. 2(a), we can see that the scale parameter k can influence not only the width of the potential well but also its height. With the increasing of the scale parameter k, the potential well becomes narrower and higher. From Figs. 2(b)–2(d), we can see that the mass of the first resonant KK mode and the mass gap of the resonant KK modes increase with the parameter k. However, the number of resonances does not change with k for a fixed γ.

Tables I and II are the specific values of the mass m_n, the relative probability P, the width Γ, and the lifetime τ for different parameters. Here we define the width $\Gamma = \Delta m_n$ at half maximum of the peak and $\tau = 1/\Gamma$. Table I shows that with the increasing of the parameter γ, the relative probability P of the corresponding nth resonant state becomes

TABLE I. The influence of combination parameter γ on the mass spectrum m_n, the relative probability P, the width of mass Γ, and the lifetime τ of the KK resonances. The scale parameter k is set at $k = 1$.

γ	n	m_n	P	Γ	τ
10	1	4.0917	0.9474	1.0998×10^{-7}	9.0924×10^6
	2	5.1461	0.3027	2.911×10^{-2}	34.3351
15	1	5.1822	0.9973	1.5778×10^{-8}	8.6372×10^7
	2	6.8525	0.9032	1.4593×10^{-6}	6.8523×10^5
	3	7.6902	0.2839	6.5012×10^{-3}	1.5382×10^2
25	1	6.8457	0.9997	1.09557×10^{-9}	9.1277×10^8
	2	9.3884	0.9468	1.598×10^{-7}	6.2589×10^6
	3	10.9936	0.9189	9.0939×10^{-6}	1.0996×10^5
	4	12.1288	0.7774	8.2448×10^{-4}	1.2129×10^3
	5	12.7800	0.2456	1.9736×10^{-3}	5.0668×10^2

TABLE II. The influence of scale parameter k on the mass spectrum m_n, the relative value m_n/k, the relative probability P, the width of mass Γ, the relative width as Γ/k, the lifetime τ, and the relative $\tau * k$ of the KK resonances. The combination parameter γ is set at $\gamma = 25$.

k	n	m_n	m_n/k	P	Γ	Γ/k	τ	$\tau * k$
0.2	1	1.3681	6.8405	0.9999	2.1928×10^{-10}	1.0964×10^{-9}	4.5602×10^{9}	9.1204×10^{8}
	2	1.8852	9.4262	0.9987	2.6522×10^{-8}	1.3261×10^{-7}	3.7704×10^{7}	7.5408×10^{7}
	3	2.1962	10.9813	0.9872	1.8213×10^{-6}	9.1065×10^{-6}	5.4904×10^{5}	1.0981×10^{5}
	4	2.4306	12.1529	0.8212	1.6546×10^{-4}	8.2731×10^{-4}	6.0767×10^{3}	1.2153×10^{3}
	5	2.5470	12.7350	0.3192	3.5343×10^{-3}	1.7672×10^{-3}	2.8294×10^{3}	5.6588×10^{2}
	1	13.6982	6.8493	0.9993	2.1902×10^{-9}	1.0951×10^{-9}	4.5657×10^{8}	9.1314×10^{8}
	2	18.7361	9.3680	0.9923	2.6686×10^{-7}	1.3343×10^{-7}	3.7472×10^{6}	7.4944×10^{6}
	3	22.0077	11.0039	0.9361	1.8175×10^{-5}	9.0875×10^{-6}	5.5021×10^{4}	1.1004×10^{5}
	4	24.2305	12.1141	0.8164	1.6507×10^{-3}	8.2535×10^{-4}	6.0578×10^{2}	1.2116×10^{3}
	5	25.6334	12.8167	0.3082	3.5109×10^{-3}	1.7554×10^{-3}	2.8482×10^{2}	5.6964×10^{2}
5	1	34.1950	6.8391	0.9991	5.4417×10^{-9}	1.0883×10^{-9}	1.8377×10^{8}	9.1883×10^{8}
	2	47.2038	9.4404	0.9986	7.4164×10^{-7}	1.4833×10^{-7}	1.3484×10^{6}	6.7419×10^{6}
	3	54.8917	10.9784	0.9822	4.3735×10^{-5}	8.7471×10^{-6}	2.8654×10^{4}	1.4327×10^{5}
	4	60.8046	12.1611	0.8203	4.1115×10^{-3}	8.2234×10^{-4}	2.43×10^{2}	1.2166×10^{3}
	5	63.6192	12.7238	0.3290	7.8591×10^{-3}	1.5718×10^{-3}	1.2724×10^{2}	6.3227×10^{2}

larger, and the lifetime of the resonant state becomes longer. From Table II, we can see that when the parameter γ is fixed, the mass m_n, the width Γ, and the lifetime τ for the corresponding nth resonant state are all influenced by the parameter k. However, the values of m_n/k are basically the same for different values of k, and so are the relative probability P, the relative width Γ/k, and the relative lifetime $\tau * k$. So, we can make a coordinate transformation as $\bar{z} = kz$ to offset the effect of k. Combining these two tables, we can see that the lifetime τ increases with γ, while it decreases with k, which means that if the parameter γ is large enough or the parameter k is small enough, the lifetime of the resonant states can be as long as our Universe's. So, in this case, we can consider the resonant states one of the candidates for dark matter.

Then we calculate the lifetime of the first resonant state to check to see whether or not it can be a candidate for dark matter. For convenience, we make a coordinate transformation $\bar{z} = kz$ and define the scaled mass $\bar{m}_1 = m_1/k$ and the scaled lifetime $\bar{\tau} = 1/\bar{\Gamma}$ for the first resonant state in the Natural System of Units, where $\bar{\Gamma} = \Delta\bar{m}_1$ is the half maximum of the peak for the first resonant state. Note that both $\bar{m}_1$ and $\bar{\tau}$ are dimensionless.

Figure 3 shows that both the scaled mass $\bar{m}_1$ and the scaled lifetime $\log(\bar{\tau})$ depend linearly on the parameter γ, and the fit functions can be expressed as

$$\bar{m}_1 = -3.2 + 2.0\gamma, \tag{36}$$

$$\log(\bar{\tau}) = 4.7 + 0.2\gamma. \tag{37}$$

It is known that the age of our Universe is of about 13.8×10^9 yr, i.e., 4.35×10^{17} s. So, if we consider the first resonant state to be one of the candidates for dark matter, its lifetime should be longer than the Universe's, i.e., $\tau \gtrsim 4.35 \times 10^{17}$ s, or, in the Natural System of Units,

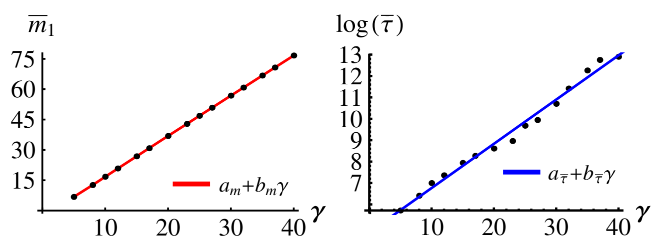
$$\tau = 1/(k\bar{\Gamma}) = \bar{\tau}/k \gtrsim 6.6 \times 10^{32}\ \text{eV}^{-1}. \tag{38}$$

Thus, the restriction of the scale parameter k can be expressed as

$$k \lesssim 1.5 \times 10^{-33}\bar{\tau}\ \text{eV} \simeq 7.5 \times 10^{-29+0.2\gamma}\ \text{eV}. \tag{39}$$

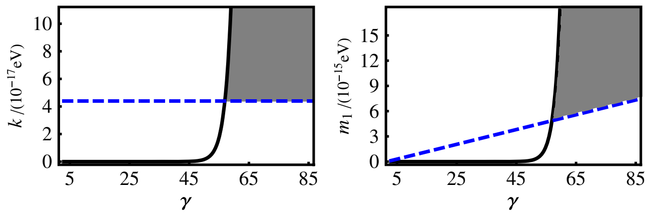
In addition, in the brane world theory considered in this paper, the relation between the four-dimensional effective Planck scale M_{Pl} and the five-dimensional fundamental scale M_* is given by [11]

$$M_{\text{Pl}}^2 = M_*^3 \int_{-\infty}^{\infty} dz e^{3A(z)} = 2M_*^3/k. \tag{40}$$



(a) $\bar{m}_1$ (b) $\log(\bar{\tau})$

FIG. 3. The influence of the combination parameter γ on the scaled mass $\bar{m}_1$ and the scaled lifetime $\log(\bar{\tau})$ of the first resonant state. (a) The combination parameter γ. (b) The scaled lifetime $\log(\bar{\tau})$. The black dots are numerical results, the red solid line is the fit function for $\bar{m}_1$ with $a_m = -3.2$ and $b_m = 2.0$, and the blue solid line is the fit function for $\log(\bar{\tau})$ with $a_{\bar{\tau}} = 4.7$, $b_{\bar{\tau}} = 0.2$.



(a) k (b) m_1

FIG. 4. The limit range of the scale parameter k and the mass of the first resonant state m_1. (a) The scale parameter k. (b) The mass of the first resonant state m_1. The blue lines are restrictions from the five-dimensional fundamental scale M_* should larger than 13 TeV and the black lines are restrictions from the lifetime of the first resonant state should be on the magnitude of the universe life.

Theoretically, if the energy scale reaches the five-dimensional fundamental scale M_*, the quantum effect of gravity cannot be ignored. Experimentally, in a recent experiment of the LHC, the collision energy is 13 TeV, and this result shows that the quantum effect of gravity can be ignored, which means that the five-dimensional fundamental scale $M_* > 13$ TeV. Thus, the constraint on the parameter k is

$$k > 4.4 \times 10^{-17}\ \text{eV}. \tag{41}$$

By combining the two conditions (39) and (41) and the fit function (36), we can get the restricted expressions of the mass of the first resonant state m_1 with the combination parameter γ as

$$m_1 > (8.8\gamma - 14.1) \times 10^{-17}\ \text{eV}, \tag{42}$$

$$m_1 \lesssim (1.5\gamma - 2.4) \times 10^{-28+0.2\gamma}\ \text{eV}. \tag{43}$$

The shadow regions of Fig. 4 show the available ranges of the parameters k and m_1, respectively. From Fig. 4(a), we can see that the two restricted conditions (39) and (41) of k can be satisfied only if $\gamma > 57$, which means that the parameter γ has a lower limit. Correspondingly, Fig. 4(b) shows that there is a lower limit for the first resonant state mass m_1, i.e., $m_1 \gtrsim 10^{-15}$ eV.

V. CONCLUSION

We generalized the geometrical coupling mechanism to localize a five-dimensional vector field on RS-like thick branes. The key feature of the mechanism was the introduction of two mass terms of the vector field which are proportional to the five-dimensional Ricci scalar and the Ricci tensor, respectively. We decomposed the vector field A_M into three parts: the vector part $\hat{A}_\mu$ and the scalar parts ϕ and A_5. With the transverse condition $\partial_\mu \hat{A}^\mu = 0$, we got a decoupled action of $\hat{A}_\mu$. We found that when the two parameters α and β in the action (3) satisfied the relation $\beta = -1 - 8\alpha \pm \sqrt{1 + 12\alpha}$, the effective potential $V_v(z)$ of the vector KK modes could be expressed as $V_v(z) = \gamma^2 (\partial_z A)^2 + \gamma \partial_z^2 A$, with $\gamma = \frac{3}{2} \pm \sqrt{1 + 12\alpha}$ and where the tachyonic KK modes of $\hat{A}_\mu$ could be excluded. For $\gamma > 1/2$, the zero mode of $\hat{A}_\mu$ could be localized on the brane.

Then we investigated the resonances of the vector field by using the relative probability method and considered the possibility of these resonances being one of the candidates for dark matter. We analyzed the influence of the parameters k and γ on the resonant behavior. We found that the massive resonant KK modes could exist only for $\gamma > 3$. Both of the two parameters affected the height of the potential and hence the vector resonances. The number of the resonant states increased only with the parameter γ. We also considered the scaled lifetime $\bar{\tau}$ and the scaled mass $\bar{m}_1$ of the first resonant state. We found that both the scaled mass $\bar{m}_1$ and the scaled lifetime $\log(\bar{\tau})$ could be fitted by a linear function of γ approximately. To view the first resonant vector KK state as dark matter, its lifetime should be as long as the Universe's. This would introduce some constraints on the parameters k and γ as well as the mass of the first resonance, i.e., $k \gtrsim 10^{-17}$ eV, $\gamma > 57$, and $m_1 \gtrsim 10^{-15}$ eV.

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Note added.—Recently, we found another work [46] that also considered the same localization mechanism (3) for the vector field.

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