



Corrections to the Hawking tunneling radiation in extra dimensions



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ABSTRACT

Although the tunneling approach is fully established for black hole radiation, much work has been done to support the extension of this approach to more general settings. In this paper the Parikh–Kraus–Wilczek tunneling proposal of black hole tunneling radiation is considered. The thermodynamics of the higher dimensional Schwarzschild black hole is studied based on the generalized uncertainty principle (GUP) and the modified dispersion relation (MDR) analysis, separately. It is shown that entropy and the rate of the higher dimensional Schwarzschild black hole tunneling radiation receive some corrections. The leading-order corrections does not contain the logarithmic term of the entropy, if the dimensions of the space–time is an odd number. Through the comparison, it is found that the results of these two alternative approaches are identical if one uses the suitable expansion coefficients.

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1. Introduction

Classically, black holes are perfect absorbers; do not emit anything and their physical temperature is absolute zero. In 1974, Hawking proved that the physical temperature of a black hole is not absolute zero. However, in quantum theory a black hole radiates all species of particles with a perfect black body spectrum, at temperature T proportional to the horizon surface gravity. The entropy is a geometrical object relating to the horizon area through the well-known entropy–area relation [1].

Recently, Parikh, Kraus and Wilczek have provided an alternative method to drive the Hawking radiation of the black holes. Their method was based on the semi-classical tunneling [2], and received much attention [3,5–7]. In this method, derivation of the Hawking radiation is related to the imaginary part of action for classically forbidden process of emission across the horizon. The imaginary part of the action for emitted particles is calculated using different methods. In the null-geodesic method, the contribution to the imaginary part of the action comes from the integration of radial momentum of the emitted particles. Other approaches satisfy the relativistic Hamilton–Jacobi equation of the emitted particles to obtain the imaginary part of action [2,3,8].

Now the GUP and MDR have been the subject of many interesting works and a lot of papers have appeared in which the usual uncertainty principle is generalized at the framework

of microphysics [9,10]. The GUP corrections to the entropy of black holes have been obtained by several authors based on the Cardy–Verlinde formula [11–13]. Also the modification of energy–momentum relations and its applications have been investigated extensively [14]. Furthermore, the extra dimensional version of the MDR has been proposed in Ref. [15], by direct comparison with the extra dimensional form of GUP. The proposed extra dimensional MDR has been applied to obtain the first order corrections to the entropy of d -dimensional Schwarzschild black hole through the Cardy–Verlinde formula [13]. To study the quantum gravitational effects to the higher dimensional black holes tunneling rate, it is interesting to relate the entropy of the black holes with a minimal length quantum gravity scale.

The main goal of this paper is to compare the results of different approaches to quantum gravitational corrections on the Hawking quantum tunneling rate in extra dimensions. For this purpose, the corrections are calculated in the framework of (1) the generalized uncertainty principle and (2) the modified dispersion relation, separately. We believe that it can essentially lead to a deeper insight into the ultimate quantum gravity proposals as well as the physical properties of the higher dimensional black holes.

The paper is organized on the following order. Section 2 is devoted to a brief review of Hawking radiation via tunneling through the horizon of the d -dimensional Schwarzschild black hole. In Section 3, making use of the GUP, we calculate the quantum gravitational effects on the entropy and the quantum tunneling radiation through the horizon of the higher dimensional Schwarzschild black hole up to the sixth order in the Planck length. In Section 4, we

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examine the MDR to calculate the corrections to the entropy and tunneling radiation, up to the same order in the Planck length, through horizon of the black hole. In Section 5, we compare the results of these two alternative approaches and show that a suitable choice of the expansion coefficients leads to the same results for the entropy and tunneling radiation of the black holes in extra dimensions. Some concluding remarks and discussions are given in Section 6.

2. Review of d-dimensional Schwarzschild black hole and Hawking tunneling radiation

The metric of d-dimensional Schwarzschild black hole in $(t, r, \theta_1, \theta_2, \dots, \theta_{d-2})$ coordinates is

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_{d-2}^2, \quad (2.1)$$

where $f(r) = 1 - \frac{m}{r^{d-3}}$. The mass of the black hole is $M = \frac{m(d-2)\Omega_{d-2}}{16\pi G_d}$ and $\Omega_{d-2} = \frac{\pi^{(d-1)/2}}{\Gamma[(d-1)/2]}$ is the area of a unit $(d-2)$ -sphere. $d\Omega_{d-2}^2$ is the line element on the sphere S^{d-2} and G_d is d-dimensional gravitational constant.

According to tunneling picture, the radiation arises by a process similar to electron-positron pair production in a constant electric field. The idea is that the energy of a particle changes sign as it crosses the horizon, so that the pair created just inside or outside the horizon can materialize with zero total energy, after each one of the pairs has tunneled to the opposite sides [2]. This suggests that it should be possible to describe the black hole emission process in a semiclassical fashion as quantum tunneling. In the WKB approximation, the tunneling probability is a function of the imaginary part of the action [16]

$$\Gamma \sim e^{-2\mathcal{I}_m(I)}, \quad (2.2)$$

where \mathcal{I}_m means the imaginary part and I is the classical action of the trajectory.

To describe the tunneling phenomena, we need the coordinates which, unlike the coordinates given in Eq. (2.1), are regular on the horizon. For this purpose, we use the following coordinate transformations in (2.1),

$$dt \rightarrow dt - \frac{\sqrt{1-f(r)}}{f(r)} dr. \quad (2.3)$$

This coordinate transformation leads to the non-singular nature of the space-time with the following line element [17]

$$ds^2 = -f(r)dt^2 + 2\sqrt{1-f(r)}drdt + dr^2 + r^2 d\Omega_{d-2}^2. \quad (2.4)$$

In the null geodesic method, the imaginary part of the action for an outgoing positive energy particle which crosses the horizon outward from r_{in} to r_{out} comes from the radial part of the momentum

$$\begin{aligned} \mathcal{I}_m(I) &= \mathcal{I}_m \int_{r_{in}}^{r_{out}} p_r dr = \mathcal{I}_m \int_{r_{in}}^{r_{out}} \int_0^{p_r} dp_r dr \\ &= \mathcal{I}_m \int_M^{M-\omega} \int_{r_{in}}^{r_{out}} \frac{dr}{\dot{r}} dH, \end{aligned} \quad (2.5)$$

where the Hamilton's equation $\dot{r} = dH/dp_r$ is used. The radial null geodesic for an outgoing massless particle is given by [4]

$$\dot{r} = 1 - \sqrt{1-f(r)}. \quad (2.6)$$

If we fix the total mass and let the black hole mass to fluctuate, a shell of energy ω travels on the geodesic given by line element (2.4) with $M \rightarrow M - \omega$. Using Eq. (2.6) in Eq. (2.5) and switch the order of integration we have

$$\mathcal{I}_m(I) = \int_M^{M-\omega} \int_{r_{in}}^{r_{out}} \frac{dr}{1 - \sqrt{1-f(r)}} d\omega' \quad (2.7)$$

which can be calculated by deforming the contour according to Feynman's $\omega' \rightarrow \omega' + i\epsilon$ prescription. This results the black hole radiation probability as

$$\Gamma \sim e^{-2\mathcal{I}_m(I)} = e^{\Delta S}, \quad (2.8)$$

where $\Delta S = S(M - \omega) - S(M)$ is the difference between the initial and the final values of the Beckenstein–Hawking entropy of the black hole.

The horizon has an associated entropy (S) and Hawking temperature (T) as

$$S = \frac{A}{4G_d}, \quad \text{and} \quad A = d r_h^{d-2} \Omega_{d-2}, \quad (2.9)$$

$$T = \frac{d-3}{4\pi r_h}, \quad \text{and} \quad r_h = m^{\frac{1}{d-3}}. \quad (2.10)$$

The mass is related to horizon radius as

$$M = \frac{(d-2)\Omega_{d-2} r_h^{d-3}}{16\pi G_d}. \quad (2.11)$$

In the following, we calculate the corrections to the tunneling rate (2.8) using the quantum corrected entropy of the black hole based on GUP and MDR analysis. Finally we compare the results obtained from these two alternative approaches.

3. The GUP corrections to the tunneling rate

To study the quantum gravity effects on the thermodynamics and tunneling probability, we employ the GUP. It is shown that usual uncertainty principle receives a modification at the microphysics regime [18],

$$\delta x \geq \frac{\hbar}{\delta p} + \alpha L_p^2 \frac{\delta p}{\hbar}, \quad (3.1)$$

where $L_p = (\hbar G_d/c^3)^{1/(d-2)}$ is the Planck length. The term $\alpha L_p^2 \delta p/\hbar$ in Eq. (3.1) shows the gravitational effects on the usual uncertainty principle. Inverting Eq. (3.1) we obtain

$$\frac{\delta x}{2\alpha L_p^2} \left(1 - \sqrt{1 - \frac{4\alpha L_p^2}{(\delta x)^2}} \right) \leq \frac{\delta p}{\hbar} \leq \frac{\delta x}{2\alpha L_p^2} \left(1 + \sqrt{1 - \frac{4\alpha L_p^2}{(\delta x)^2}} \right). \quad (3.2)$$

From Eq. (3.2), one can write

$$\begin{aligned} \left(\frac{\delta p}{\hbar} \right)_{min} &= \frac{1}{\delta x} \left[\frac{(\delta x)^2}{2\alpha L_p^2} \left(1 - \sqrt{1 - \frac{4\alpha L_p^2}{(\delta x)^2}} \right) \right] \\ &= \frac{1}{\delta x} F_{(GUP)}((\delta x)^2), \end{aligned} \quad (3.3)$$

where

$$F_{(GUP)}((\delta x)^2) = \frac{(\delta x)^2}{2\alpha L_p^2} \left(1 - \sqrt{1 - \frac{4\alpha L_p^2}{(\delta x)^2}} \right), \quad (3.4)$$

characterizes the departure of GUP from the usual uncertainty principle.

Now we consider the impact of GUP on the Hawking tunneling rate from the horizon of the black hole. One can identify the energy of the absorbed or radiated particle as the uncertainty of momentum,

$$dE \simeq \delta p. \tag{3.5}$$

The increase or decrease in the area of the black hole horizon can be written as

$$dA = \frac{4G_d}{T} dE \simeq \frac{4G_d}{T} \frac{1}{\delta x}. \tag{3.6}$$

When the gravitation is turned on Eq. (3.6) generalizes to

$$dA_{(GUP)} = \frac{4G_d}{T} dE \simeq \frac{4G_d}{T} \frac{1}{\delta x} F_{(GUP)}((\delta x)^2). \tag{3.7}$$

Combining Eqs. (3.6) and (3.7) we have

$$dA_{(GUP)} = F_{(GUP)}((\delta x)^2) dA. \tag{3.8}$$

By modeling the black hole as a cube of size $2r_h$, the uncertainty in the position of a Hawking emitted particle is

$$\delta x = 2r_h = 2 \left(\frac{A}{d \Omega_{d-2}} \right)^{\frac{1}{d-2}}, \tag{3.9}$$

from which $F_{(GUP)}$ can be rewritten as the function of A

$$F_{(GUP)}(A) = \frac{2}{\alpha L_p^2} \left(\frac{A}{d \Omega_{d-2}} \right)^{\frac{2}{d-2}} \times \left[1 - \sqrt{1 - \alpha L_p^2 \left(\frac{d \Omega_{d-2}}{A} \right)^{\frac{2}{d-2}}} \right]. \tag{3.10}$$

Expanding (3.10) around $L_p = 0$ gives,

$$F_{(GUP)}(A) = 1 + \frac{\alpha L_p^2}{4} \left(\frac{d \Omega_{d-2}}{A} \right)^{\frac{2}{d-2}} + \frac{(\alpha L_p^2)^2}{8} \left(\frac{d \Omega_{d-2}}{A} \right)^{\frac{4}{d-2}} + \frac{5(\alpha L_p^2)^3}{64} \left(\frac{d \Omega_{d-2}}{A} \right)^{\frac{6}{d-2}} + \dots \tag{3.11}$$

If we substitute Eq. (3.11) into Eq. (3.8) and integrating, we can get the generalized horizon area of the black hole from the GUP. Then we can also get the correction to the entropy and tunneling rate of the black hole. But integrating Eq. (3.8) might be complicated and dimensional dependent. Therefore, we consider the following cases separately

- $d = 4$;

$$A_{(GUP)}^{(4)} = A + \alpha \Omega_2 L_p^2 \ln A - \frac{(4\alpha \Omega_2)^2}{8} L_p^4 (A)^{-1} - \frac{5(4\alpha \Omega_2)^3}{128} L_p^6 (A)^{-2} + const., \tag{3.12}$$

up to the sixth power of the Planck length and

$$S_{(GUP)}^{(4)} = S + \frac{\alpha \Omega_2 L_p^2}{4G_4} \ln S - \frac{1}{8} \left(\frac{\alpha \Omega_2}{G_4} \right)^2 L_p^4 (S)^{-1} - \frac{5}{128} \left(\frac{\alpha \Omega_2}{G_4} \right)^3 L_p^6 (S)^{-2} + const., \tag{3.13}$$

where the relation $S^{(d)} = \frac{A^{(d)}}{4G_d}$ is used. This relation has the same form of the entropy area as given by other approaches in black

holes thermodynamics. The leading order corrections contain the logarithmic, inverse and inverse square terms of the entropy which are consistent with numerous other studies that have devoted to this subject.

- $d = 6$ ($d > 4$ and even);

$$A_{(GUP)}^{(6)} = A + \frac{\alpha L_p^2}{2} (6\Omega_4 A)^{\frac{1}{2}} + \frac{6\Omega_4}{8} (\alpha L_p^2)^2 \ln A - \frac{5(6\Omega_4)^{\frac{3}{2}}}{32} (\alpha L_p^2)^3 (A)^{-\frac{1}{2}} + const., \tag{3.14}$$

$$S_{(GUP)}^{(6)} = S + \frac{\alpha L_p^2}{2} \left(\frac{3\Omega_4}{2G_6} \right)^{\frac{1}{2}} (S)^{\frac{1}{2}} + \frac{(\alpha L_p^2)^2}{8} \left(\frac{3\Omega_4}{2G_6} \right) \ln S - \frac{5(\alpha L_p^2)^3}{32} \left(\frac{3\Omega_4}{2G_6} \right)^{\frac{3}{2}} (S)^{-\frac{1}{2}} + const. \tag{3.15}$$

It is obvious that the logarithmic correction term exist when the spacetime dimension is an even number.

- $d = 5$ ($d > 4$ and odd);

$$A_{(GUP)}^{(5)} = A + \frac{3(5\Omega_3)^{\frac{2}{3}}}{4} (\alpha L_p^2) (A)^{\frac{1}{3}} - \frac{3(5\Omega_3)^{\frac{4}{3}}}{8} (\alpha L_p^2)^2 (A)^{-\frac{1}{3}} - \frac{5(5\Omega_3)^2}{64} (\alpha L_p^2)^3 (A)^{-1} + const., \tag{3.16}$$

$$S_{(GUP)}^{(5)} = S + \frac{3\alpha L_p^2}{4} \left(\frac{5\Omega_3}{4G_5} \right)^{\frac{2}{3}} (S)^{\frac{1}{3}} - \frac{3(\alpha L_p^2)^2}{8} \left(\frac{5\Omega_3}{4G_5} \right)^{\frac{4}{3}} (S)^{-\frac{1}{3}} - \frac{5(\alpha L_p^2)^3}{64} \left(\frac{5\Omega_3}{4G_5} \right)^2 (S)^{-1} + const. \tag{3.17}$$

It means that when the spacetime dimension is an odd number the logarithmic correction term does not exist. This implies that the logarithmic correction term depends on the spacetime dimensions. Using Eqs. (3.13), (3.15) and (3.17) in Eq. (2.8) we can calculate the GUP corrections to the Hawking tunneling radiation in extra dimensions

$$\Gamma_{(GUP)}^{(d)} = \Gamma^{(d)} \exp \left(\Delta S_{(GUP)}^{(d)} - \Delta S^{(d)} \right), \tag{3.18}$$

where $\Gamma_{(GUP)}^{(d)}$ and $\Gamma^{(d)}$ are d-dimensional quantum tunneling rate in the presence and absence of GUP effects respectively. Also $\Delta S_{(GUP)}^{(d)}$ and $\Delta S^{(d)}$ are the entropy difference of the d-dimensional black hole when the gravitational effects are and are not considered respectively.

4. The MDR corrections to the tunneling rate

In the study of loop quantum gravity and of models based on noncommutative geometry, there has been a strong interest in some candidate modifications of the energy-momentum dispersion relation. In this section, we consider the effects of MDR on the black hole tunneling radiation in the extra dimensions.

It is interesting that the usual relation between energy and momentum that characterizes the special theory of relativity, $p^2 = E^2 - m^2$, may be modified in the Planck scale regime. Anomalies in ultra high cosmic ray photons, and possibly TeV photons, may be explained by modification of the dispersion relation as [15,19]

$$\vec{p} \cdot \vec{p} \equiv p^2 = f(E, m; L_p) = E^2 - \mu^2 + \alpha_1 L_p^2 E^4 + \alpha_2 L_p^4 E^6 + \alpha_3 L_p^6 E^8 + \mathcal{O} \left(L_p^8 E^{10} \right), \tag{4.1}$$

where f is the function that gives the exact dispersion relation, and on the right-hand side we have assumed a Taylor-series expansion for $E \ll \frac{1}{L_p}$. The coefficients α_i can take different values in

different quantum-gravity proposals. Note that m is the rest energy of the particle and the mass parameter μ on the right-hand side is directly related to the rest energy, but $\mu \neq m$, if the α_i do not all vanish. Now differentiation of Eq. (4.1) and taking the inverse of the result gives

$$dE = dp \left[1 - \frac{3\alpha_1}{2} L_p^2 E^2 - \left(\frac{5\alpha_2}{2} - \frac{23\alpha_1^2}{8} \right) L_p^4 E^4 + \left(\frac{37}{4} \alpha_1 \alpha_2 - \frac{91}{16} \alpha_1^3 - \frac{7}{2} \alpha_3 \right) L_p^6 E^6 + \dots \right]. \quad (4.2)$$

Within quantum field theory, the relation between particle localization and its energy is given by $E \geq \frac{1}{\delta x}$, where δx is particle position uncertainty. Now, it is obvious that within MDR, this relation should be modified. To the first order, assuming $dE \simeq \delta E$, making use of the usual uncertainty principle $\delta E \simeq \delta p \simeq \frac{1}{\delta x}$ we have

$$dE_{(MDR)} \simeq \frac{1}{\delta x} \left[1 - \frac{3\alpha_1}{2} \frac{L_p^2}{(\delta x)^2} - \left(\frac{5\alpha_2}{2} - \frac{23\alpha_1^2}{8} \right) \frac{L_p^4}{(\delta x)^4} + \left(\frac{37}{4} \alpha_1 \alpha_2 - \frac{91}{16} \alpha_1^3 - \frac{7}{2} \alpha_3 \right) \frac{L_p^6}{(\delta x)^6} + \dots \right]. \quad (4.3)$$

Eq. (4.3) can be rewritten as

$$dE_{(MDR)} = \frac{1}{\delta x} F_{(MDR)} \left((\delta x)^2 \right), \quad (4.4)$$

where

$$F_{(MDR)} \left((\delta x)^2 \right) = 1 - \frac{3\alpha_1}{2} \frac{L_p^2}{(\delta x)^2} - \left(\frac{5\alpha_2}{2} - \frac{23\alpha_1^2}{8} \right) \frac{L_p^4}{(\delta x)^4} + \left(\frac{37}{4} \alpha_1 \alpha_2 - \frac{91}{16} \alpha_1^3 - \frac{7}{2} \alpha_3 \right) \frac{L_p^6}{(\delta x)^6} + \dots \quad (4.5)$$

Now Eq. (3.6) modifies as

$$dA_{(MDR)} = F_{(MDR)} \left((\delta x)^2 \right) dA, \quad (4.6)$$

making use of (3.9) in Eq. (4.6) we have

$$dA_{(MDR)} = dA \left[1 - \frac{3\alpha_1}{8} \left(\frac{d \Omega_{d-2}}{A} \right)^{\frac{2}{d-2}} L_p^2 - \left(\frac{5\alpha_2}{32} - \frac{23\alpha_1^2}{128} \right) \left(\frac{d \Omega_{d-2}}{A} \right)^{\frac{4}{d-2}} L_p^4 + \left(\frac{37}{256} \alpha_1 \alpha_2 - \frac{91}{1024} \alpha_1^3 - \frac{7}{128} \alpha_3 \right) \left(\frac{d \Omega_{d-2}}{A} \right)^{\frac{6}{d-2}} L_p^6 + \dots \right]. \quad (4.7)$$

By integrating, we can get the modified horizon area of the black hole from the MDR. Then we can also get the correction to the entropy and tunneling rate of the black hole. But integrating Eq. (4.7) might be complicated and dimensional dependent. Therefore, we consider the following cases separately

• $d = 4$;

$$A_{(MDR)}^{(4)} = A - \frac{3\alpha_1}{8} \left(4\Omega_2 L_p^2 \right) \ln A + \left(\frac{5\alpha_2}{32} - \frac{23\alpha_1^2}{128} \right) \left(4\Omega_2 L_p^2 \right)^2 (A)^{-1}$$

$$- \left(\frac{37}{512} \alpha_1 \alpha_2 - \frac{91}{2048} \alpha_1^3 - \frac{7}{256} \alpha_3 \right) \left(4\Omega_2 L_p^2 \right)^3 (A)^{-2} + \text{const.}, \quad (4.8)$$

$$S_{(MDR)}^{(4)} = S - \frac{3\alpha_1}{8} \left(\frac{4\Omega_2}{4G_4} \right) L_p^2 \ln S + \left(\frac{5\alpha_2}{32} - \frac{23\alpha_1^2}{128} \right) \left(\frac{4\Omega_2}{4G_4} \right)^2 L_p^4 (S)^{-1} - \left(\frac{37}{512} \alpha_1 \alpha_2 - \frac{91}{2048} \alpha_1^3 - \frac{7}{256} \alpha_3 \right) \left(\frac{4\Omega_2}{4G_4} \right)^3 L_p^6 (S)^{-2} + \text{const.} \quad (4.9)$$

This relation has the same form of the entropy area as given by other approaches in black holes thermodynamics. The leading order corrections contain the logarithmic, inverse and inverse square terms of the entropy which are consistent with numerous other studies that have devoted to this subject.

• $d = 6$ ($d > 4$ and even);

$$A_{(MDR)}^{(6)} = A - \frac{3\alpha_1}{4} L_p^2 (6\Omega_4)^{\frac{1}{2}} (A)^{\frac{1}{2}} - \left(\frac{5\alpha_2}{32} - \frac{23\alpha_1^2}{128} \right) L_p^4 (6\Omega_4) \ln A - \left(\frac{37}{128} \alpha_1 \alpha_2 - \frac{91}{512} \alpha_1^3 - \frac{7}{64} \alpha_3 \right) L_p^6 (6\Omega_4)^{\frac{3}{2}} (A)^{-\frac{1}{2}} + \text{const.}, \quad (4.10)$$

$$S_{(MDR)}^{(6)} = S - \frac{3\alpha_1}{4} L_p^2 \left(\frac{6\Omega_4}{4G_6} \right)^{\frac{1}{2}} (S)^{\frac{1}{2}} - \left(\frac{5\alpha_2}{32} - \frac{23\alpha_1^2}{128} \right) L_p^4 \left(\frac{6\Omega_4}{4G_6} \right) \ln S - \left(\frac{37}{128} \alpha_1 \alpha_2 - \frac{91}{512} \alpha_1^3 - \frac{7}{64} \alpha_3 \right) L_p^6 \left(\frac{6\Omega_4}{4G_6} \right)^{\frac{3}{2}} (S)^{-\frac{1}{2}} + \text{const.} \quad (4.11)$$

It is obvious that the logarithmic correction term exist when the spacetime dimension is an even number.

• $d = 5$ ($d > 4$ and odd);

$$A_{(MDR)}^{(5)} = A - \frac{9\alpha_1}{8} L_p^2 (5\Omega_3)^{\frac{2}{3}} (A)^{\frac{1}{3}} + \left(\frac{15\alpha_2}{32} - \frac{69\alpha_1^2}{128} \right) L_p^4 (5\Omega_3)^{\frac{4}{3}} (A)^{-\frac{1}{3}} - \left(\frac{37}{256} \alpha_1 \alpha_2 - \frac{91}{1024} \alpha_1^3 - \frac{7}{128} \alpha_3 \right) L_p^6 (5\Omega_3)^2 (A)^{-1} + \text{const.}, \quad (4.12)$$

$$S_{(MDR)}^{(5)} = S - \frac{9\alpha_1}{8} L_p^2 \left(\frac{5\Omega_3}{4G_5} \right)^{\frac{2}{3}} (S)^{\frac{1}{3}} + \left(\frac{15\alpha_2}{32} - \frac{69\alpha_1^2}{128} \right) L_p^4 \left(\frac{5\Omega_3}{4G_5} \right)^{\frac{4}{3}} (S)^{-\frac{1}{3}} - \left(\frac{37}{256} \alpha_1 \alpha_2 - \frac{91}{1024} \alpha_1^3 - \frac{7}{128} \alpha_3 \right) L_p^6 \left(\frac{5\Omega_3}{4G_5} \right)^2 (S)^{-1} + \text{const.} \quad (4.13)$$

It means that when the spacetime dimension is an odd number the logarithmic correction term does not exist. This implies

that the logarithmic correction term depends on the spacetime dimension. Note that the corrections have been calculated up to the sixth order in the Planck length. Now making use of Eqs. (4.9), (4.11) and (4.13) in Eq. (2.8) one can obtain the MDR corrections to the quantum tunneling rate from the horizon of the black hole in extra dimensions

$$\Gamma_{(MDR)}^{(d)} = \Gamma^{(d)} \exp\left(\Delta S_{(MDR)}^{(d)} - \Delta S^{(d)}\right), \quad (4.14)$$

where $\Gamma_{(MDR)}^{(d)}$ and $\Gamma^{(d)}$ are extra dimensional quantum tunneling rate when the MDR impacts are and are not taken into account respectively. Also $\Delta S_{(MDR)}^{(d)}$ and $\Delta S^{(d)}$ are the d-dimensional entropy difference in the presence and absence of MDR impacts respectively.

5. Comparison of the results

Since GUP and NDR are different manifestations of the same physical concept (existence of a minimal length scale of the same order of Planck length) we expect the results of applications of these two alternative approaches to the physical systems should be identical.

In the two previous sections we examined the GUP and MDR separately and obtained the corrections to the thermodynamics quantities as well as the tunneling rate from the black hole horizon. Now it is evident to expect the results of these two alternative approaches to be consistent. The assumption behind this expectation is that GUP and MDR are two faces of an underlying quantum gravitational proposal.

Through direct comparison of the results obtained we found that by using the suitable choice of the coefficients, that is

$$\alpha_1 = -\frac{2}{3}\alpha, \quad \text{and} \quad \alpha_2 = -\frac{13}{45}\alpha^2, \quad \text{and} \quad \alpha_3 = -\frac{46}{105}\alpha^3, \quad (5.1)$$

in Eqs. (3.13), (3.15), (3.17) and (3.18), also Eqs. (4.9), (4.11), (4.13) and (4.14) one can show that

$$S_{(GUP)}^{(d)} = S_{(MDR)}^{(d)}, \quad \text{and} \quad \Gamma_{(GUP)}^{(d)} = \Gamma_{(MDR)}^{(d)}. \quad (5.2)$$

As a mathematical result it seems that GUP and MDR approaches lead to the same corrections for the thermodynamical properties of the black holes and tunneling rate in extra dimensions. In other words, consistency of results in these two alternative approaches constrains the expansion coefficients in the MDR.

6. Conclusion

Recent studies in perturbative string theory and quantum gravity predict the existence of a fundamental measurable length which is of the order of Planck length. The essence of this fundamental length can be captured by generalizing usual uncertainty principle known as GUP or by modifying the usual energy momentum relation conventionally named as MDR.

In this work, the thermodynamics and quantum tunneling radiation probability through the horizon of the extra dimensional black holes are analyzed in the presence of 1) the generalized uncertainty principle and 2) the modified dispersion relation, separately. We showed that the black holes entropy and quantum tunneling radiation receive corrections at the Planck scale as Eqs. (3.13), (3.15), (3.17) and (3.18) also Eqs. (4.9), (4.11), (4.13) and (4.14). The corrections are worked out up to the sixth order in the Planck length. The leading-order corrections contain the logarithmic term of the entropy, if the dimensions of the spacetime is an

even number. The logarithmic correction term does not exist when the dimensions of the spacetime is an odd number. This implies that the logarithmic correction term depends on the spacetime dimensions. The results consistent with numerous other studies that have delved into the subject of this paper. It is significant that one can obtain the GUP or MDR by starting from modified momentum operator. From this point of view, it is evident to expect the results of these two alternative approaches to be consistent. Through the comparison of the corrected results obtained from these two alternative approaches, we found that a suitable choice of the expansion coefficients in the MDR (Eq. (5.1)) leads to the same results in both approaches.

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