



Do the small numbers in the quark mixing arise from new physics?

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Abstract We put forward the conjecture that the small numbers in the V_{CKM} matrix, are generated by physics beyond the Standard Model. We identify as small numbers V_{ub} and the strength of CP violation, measured by $|\text{Im}Q|$, where Q stands for a rephasing invariant quartet of V_{CKM} . We illustrate how the conjecture can be realised in the context of an extension of the Standard Model where an up-type vector-like quark is introduced leading to a realistic spectrum of quark masses and an effective V_{CKM} in agreement with experiment.

1 Introduction

In the Standard Model (SM) the pattern of fermion masses and mixing is dictated by the flavour structure of Yukawa couplings. This flavour structure is not constrained by gauge invariance and, as a result, understanding the pattern of fermion masses and mixing in the SM continues being at present an open, fundamental question. In a bottom-up approach to the Yukawa puzzle, one usually tries to infer about the possible presence of a family symmetry, from the observed pattern of quark masses and mixing. This may be a specially difficult task if New Physics also contributes to the effective quark mass matrices at low energies.

In this paper, we put forward the conjecture that the small numbers in V_{CKM} namely $|V_{ub}|$ and $I_{CP} = |\text{Im}Q|$ (Q stands for a rephasing invariant quartet of V_{CKM}) arise from New Physics.

The paper is organised as follows: in the next section we explain in detail how to implement our conjecture concerning the origin of the small numbers in V_{CKM} . In Sect. 3, we analyse some of the phenomenological consequences of the model, in the particular the mass of the new heavy-top quark and the size of flavour-changing-neutral-currents (FCNC). Finally, we present a specific realistic model with a vector-like up-type quark that realises our conjecture.

2 The origin of the smallness of $|V_{ub}|$ and CP violation

Quark mixing within the framework of the Standard Model (SM) is generated by the Yukawa couplings which lead to quark mass matrices $M_d M_d^\dagger$, $M_u M_u^\dagger$, with different flavour structures. As a result, these two matrices are diagonalised by two different unitary matrices, usually denoted V_L^d , V_L^u . Physically, only $V_{CKM} = V_L^{u\dagger} V_L^d$ is measured. For a review see [1]. We have described the scenario of quark mixing in the SM. But the SM leaves open many fundamental questions, so there is motivation to consider New Physics (NP) beyond the SM. Then it is likely that NP also contributes to the flavour structure of the effective quark mass matrices at low energies, leading to an effective CKM which also reflects the presence of New Physics.

As mentioned in the introduction, in this paper we put forward the conjecture that the small numbers in the CKM matrix are generated by NP. We adopt the standard parametrization of the CKM matrix [2,3] and identify as the small numbers $|V_{ub}| \approx 4 \times 10^{-3}$, and $I_{CP} = |\text{Im}Q| \equiv |\text{Im}(V_{us} V_{cb} V_{ub}^* V_{cs}^*)| \simeq 3 \times 10^{-5}$. We further propose that there is a weak basis (WB) where the dominant contributions to the orthogonal matrices O_{23} and O_{12} entering in the standard parametrization arise from the up and down sectors, respectively. So altogether we have

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$$V_{\text{CKM}}^{\text{eff}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}_{\text{up}} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{12} \end{pmatrix}_{\text{NP}} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{13} & 0 \\ 0 & 0 & 1 \end{pmatrix}_{\text{down}} \tag{1}$$

where the subscripts indicate the origin of the contributions to the mixing and NP stands for the contribution coming from New Physics. In the limit where NP is not present, both $|V_{ub}|$ and $\text{Im}Q$ vanish.

2.1 Framework

In order to implement the structure in Eq. (1), we assume that there is a basis where the down and up quark matrices take the form:

$$M_u = \begin{pmatrix} m_{11}^u & 0 & 0 \\ 0 & m_{22}^u & m_{23}^u \\ 0 & m_{32}^u & m_{33}^u \end{pmatrix}, \quad M_d = \begin{pmatrix} m_{11}^d & m_{12}^d & 0 \\ m_{21}^d & m_{22}^d & 0 \\ 0 & 0 & m_{33}^d \end{pmatrix}. \tag{2}$$

Without the introduction of NP, one simply obtains a simplified and reduced CKM mixing, where

$$V_{\text{CKM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}_{\text{up}} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}_{\text{down}} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}c_{23} & c_{23}c_{12} & -s_{23} \\ -s_{23}s_{12} & s_{23}c_{12} & c_{23} \end{pmatrix}. \tag{3}$$

Note however that this limit already indicates that $|V_{31}| = |V_{12}||V_{23}|$ is much larger than $|V_{13}|$, which vanishes at this stage, and that $V_{13} = 0$ also leads to vanishing CP violation.

The search for meaningful relations between the different CKM matrix elements (and/or mass ratios) has been part of the quest and the relentless effort of trying to unravel the flavour puzzle. For instance, recently Grossman and Ruderman studied in [4] the hypothesis of whether the CKM matrix has a substructure “that goes beyond the single small parameter of the Wolfenstein parameterization”.

In the case studied here, it is important to notice, that our specific limit already points to a realistic physical content. By this we mean that, if on the contrary, we were to interchange the structure forms of M_u and M_d in Eq. (2), then we would obtain the result where $|V_{13}| = |V_{12}||V_{23}|$ and $V_{31} = 0$, which would clearly be in contradiction with experiment. Indeed, when arguing that (some of) the elements of V_{CKM} might be connected by some kind of relationship, what we consider most striking is that experimentally one already

finds that $|V_{31}| \approx |V_{12}||V_{23}|$, and this is exactly what occurs in models where the substructures in Eq. (2) are satisfied.

Now, when one introduces New Physics and extends the up quark sector with one isosinglet VLQ, one obtains a 4×4 extended up-quark mass matrix with new elements. Let us then assume that this new mass matrix structure is near to M_u in Eq. (2) and given by

$$\mathcal{M}_u = \begin{pmatrix} 0 & 0 & 0 & m_{14} \\ 0 & m_{22} & m_{23} & m_{24}e^{i\beta} \\ 0 & m_{32}e^{i\alpha} & m_{33} & 0 \\ m_{41} & 0 & -m_{43}e^{i\delta} & M \end{pmatrix}, \tag{4}$$

where the (m_{ij}, M) are real, $\alpha, \beta, \delta \in [0, 2\pi]$ and for simplicity, we have taken $m_{11}^u = 0$ and $m_{34}^u = 0$. In addition one may also choose a WB where $m_{42}^u = 0$. These structures for the up and down quark mass matrices may also be obtained by imposing a discrete Z_4 symmetry on the Lagrangian (see Appendix A).

However, the crucial point here is that, in this new scenario, $|V_{13}|$ is now effectively different from zero, and generated by the mixing with the heavy extra vector-like particle. Consequently, one may now have significant CP violation. To see this, it is useful to re-write \mathcal{M}_u in a different WB where some right-handed fields have been transformed, such as to obtain $(\mathcal{M}'_u)_{4i} = 0, i = 1, 2, 3$. One finds, in leading order

$$\mathcal{M}_u \xrightarrow{\text{WB}} \mathcal{M}'_u = \mathcal{M}_u \cdot \mathcal{W}_u, \quad \mathcal{M}'_u = \begin{pmatrix} -\frac{m_{14}m_{41}}{M} & 0 & \frac{m_{14}m_{43}}{M}e^{i\delta} & m_{14} \\ -\frac{m_{24}m_{41}}{M}e^{i\beta} & m_{22} & m_{23} & m_{24}e^{i\beta} \\ 0 & m_{32}e^{i\alpha} & m_{33} & -\frac{m_{33}m_{43}}{M}e^{-i\delta} \\ 0 & 0 & 0 & M \end{pmatrix}, \tag{5}$$

where we assume that $|m_{ij}| \leq |m_{33}| \ll |M|$.

Several results can then be, immediately, derived from Eq. (5). The first one is that the effective up-quark mass matrix for the three lightest quarks is given by

$$M_u^{\text{eff}} = \begin{pmatrix} -\frac{m_{41}m_{14}}{M} & 0 & \frac{m_{14}m_{43}}{M}e^{i\delta} \\ -\frac{m_{41}m_{24}}{M}e^{i\beta} & m_{22} & m_{23} \\ 0 & m_{32}e^{i\alpha} & m_{33} \end{pmatrix}. \tag{6}$$

Then, taking here the limit $m_{41} = 0$, corresponding to $m_u = 0$, and assuming that $M \approx m_T$, the mass of the heavy vector-like quark, and $m_{33} \approx m_t$ while the other parameters are (much) smaller, one already finds, in a rough approximation, for the diagonalization matrix V^u of the up-quarks that

$$V_{23}^u \approx \frac{m_{23}}{m_t}, \quad V_{13}^u \approx \frac{m_{14}m_{43}}{m_T m_t} e^{i\delta}. \tag{7}$$

From from Eq. (5), one also obtains rough estimates for the extra mixing angles involving the New Physics coming from the extra heavy up quark, the heavy top. It easy to see that

$$V_{14}^u \approx \frac{m_{14}}{m_T}, \quad V_{24}^u \approx \frac{m_{24}}{m_T}, \quad V_{34}^u \approx -\frac{m_t m_{43}}{m_T^2} e^{-i\delta}. \quad (8)$$

These estimates correspond to what one also finds when doing a rigorous calculation of the 4×4 unitary matrix \mathcal{V}^u which diagonalizes $\mathcal{M}_u \mathcal{M}_u^\dagger$.

Finally, taking into account the O_{12} mixing coming from the down quarks,¹ and this \mathcal{V}^u , we obtain for the full quark mixing $\mathcal{V} = \mathcal{V}^{u\dagger} O_{12}$, which is presented in leading order in Eq. (9).

$$\mathcal{V} \approx \begin{pmatrix} c_{12} & s_{12} & \frac{m_{14} m_{43}}{m_t m_T} e^{-i\delta} & \frac{m_{14}}{m_T} \\ -s_{12} & c_{12} & \frac{m_{23}}{m_t} & \frac{m_{24}}{m_T} e^{-i\beta} - \frac{m_{23}}{m_t} \frac{m_t m_{43}}{m_T^2} e^{i\delta} \\ -c_{12} \frac{m_{14} m_{43}}{m_t m_T} e^{i\delta} + s_{12} \frac{m_{23}}{m_t} & -c_{12} \frac{m_{23}}{m_t} & 1 & -\frac{m_t m_{43}}{m_T^2} e^{i\delta} \\ -c_{12} \frac{m_{14}}{m_T} & -c_{12} \frac{m_{24}}{m_T} e^{i\beta} - s_{12} \frac{m_{14}}{m_T} & \frac{m_t m_{43}}{m_T^2} e^{-i\delta} & 1 \end{pmatrix} \quad (9)$$

It is understood that the CKM mixing matrix V_{CKM} is the 4×3 submatrix of (the left part of) \mathcal{V} . Note that the phase α plays no role at leading order.

2.2 The CKM unitarity problem and CP violation

Violation of 3×3 unitarity is a prominent feature of VLQ models, which in turn makes them one of the most promising extensions of the SM when addressing the CKM unitarity problem.

Evidence for the CKM unitarity problem stems from significant tensions between current determinations of $|V_{ud}|$ and $|V_{us}|$ and the assumption of 3×3 unitarity of the SM [5–14]. Presently, the values for these mixings disfavour CKM unitarity at $2 - 3\sigma$ and the best-fit value for the deviation to the unitarity of the first row is $\Delta = 0.0387 \pm 0.0090$ [3].

With regard to our model, the deviation from unitarity is, primarily, expressed in the normalisation of the first row of the CKM matrix, which yields

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \Delta^2, \quad (10)$$

where, in leading order, we have

$$\Delta = |\mathcal{V}_{14}| \approx \left| \frac{m_{14}}{m_T} \right|. \quad (11)$$

¹ Here for convenience, we embed the O_{12} in a 4×4 context.

Now, when considering CP violation, induced by the mixing with the heavy extra vector-like quark, and the CP-odd invariant quartet $|\text{Im}Q| \equiv |\text{Im}(V_{us} V_{cb} V_{ub}^* V_{cs}^*)|$, we find, from Eq. (9), that this invariant (in leading order) is given by

$$|\text{Im}Q| \approx |\mathcal{V}_{12}| |\mathcal{V}_{23}| |\mathcal{V}_{13}| |\sin \delta| \quad (12)$$

where (again, in leading order)

$$|\mathcal{V}_{12}| \simeq s_{12}, \quad |\mathcal{V}_{23}| \approx \left| \frac{m_{23}}{m_t} \right|, \quad |\mathcal{V}_{13}| \approx \left| \frac{m_{43}}{m_t} \right| \left| \frac{m_{14}}{m_T} \right|. \quad (13)$$

As one sees, from Eqs. (11, 12, 13) in our model, the size of $|V_{ub}| = |\mathcal{V}_{13}|$, CP violation and the deviation from unitarity

in the first row of the CKM matrix are intrinsically connected. As an example, let us consider a unitarity deviation of $\Delta \approx 0.04$, then with $\left| \frac{m_{43}}{m_t} \right| \approx 0.1$, one obtains a value for $|\mathcal{V}_{13}|$ which is in agreement with experiment, $|\mathcal{V}_{13}| \approx 0.004$, and with this we also recover the current measured value for $|\text{Im}Q| \approx 3 \times 10^{-5}$.

Thus, by proposing a common origin for both questions, our framework is able to explain (the smallness of) V_{ub} , and the CP violation observed in the quark sector and, simultaneously, provide a simple solution to the CKM unitarity problem.

3 Phenomenology

The introduction of a heavy-top implies changes to the SM electroweak Lagrangian, which will then lead to new contributions to various processes and consequently to the electroweak precision measurements (EWPM) associated with these. Some processes are stringently constrained by experiment and it is therefore crucial to analyse the respective phenomenological quantities.

With the addition of the extra quark, the mixing matrix is larger and the three standard down quarks now have an extra W -mediated mixing with the heavy-top. These contributions are relevant for processes such as $K^0 - \bar{K}^0$ and $B_{d,s}^0 - \bar{B}_{d,s}^0$ and correspond to a simple generalisation of the SM loop diagrams, but with the heavy-top also participating [15–17].

Other contributions arise from flavour changing neutral current (FCNC) interactions with the Z and Higgs h boson [18–22]. In the physical basis, one finds for the neutral currents,

$$\mathcal{L}_Z = \frac{g}{c_W} Z_\mu \left[\frac{1}{2} (\bar{u}_L F^u \gamma^\mu u_L - \bar{d}_L \gamma^\mu d_L) - s_W^2 \left(\frac{2}{3} \bar{u} \gamma^\mu u - \frac{1}{3} \bar{d} \gamma^\mu d \right) \right], \tag{14}$$

$$-\mathcal{L}_h = \bar{u}_L \frac{h}{v} F^u \mathcal{D}_u u_R + \bar{d}_L \frac{h}{v} D_d d_R + \text{h.c.},$$

where \mathcal{D}_u stands for diagonal matrix containing all four up-type masses and D_d the three down quarks. The FCNC's are controlled by

$$F^u = V_{\text{CKM}} V_{\text{CKM}}^\dagger, \tag{15}$$

which in general is non-diagonal.

In our framework, one obtains (in leading order) for F^u which is indeed explicitly non-diagonal. This means that mixings between different up-type quark flavours are now allowed, leading, most notably, to NP contributions to $D^0 - \bar{D}^0$ at tree level [23]. These dangerous effects are, nonetheless, suppressed by the size of the mass of the heavy-top m_T .

Other processes, such as the golden modes $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ or the CP related parameter ε'/ε will also be affected by both types of contributions.

In the following, we briefly describe the viability of our model with respect to the most important EWPM's.

parameter x_D , which will lead to

$$|F_{12}^u| < 1.22 \times 10^{-3}. \tag{17}$$

Taking e.g. a central value for $\Delta \simeq 0.0387$, one obtains an upper-bound for

$$|\mathcal{V}_{24}| \lesssim 3.14 \times 10^{-3}. \tag{18}$$

Similarly, the ratio m_{43}/m_T is constrained by the requirement[3]

$$|\mathcal{V}_{13}| \simeq |V_{ub}|^{\text{exp}} = (3.82 \pm 0.20) \times 10^{-3}, \tag{19}$$

leading to $m_{43} \sim 15$ GeV. Moreover, currently, the least stringent lower-bound for the mass of the heavy-top is $m_T \simeq 0.685$ TeV, which originates from searches that assume dominant mixing of the heavy-top with the first generation [29]. Using this result one has

$$|\mathcal{V}_{34}| \simeq \frac{m_t m_{43}}{m_T^2} \lesssim 6.27 \times 10^{-3}. \tag{20}$$

These upper-bounds for $|\mathcal{V}_{24}|$ and $|\mathcal{V}_{34}|$ are well within the allowed regions presented in [25], where the parameter regions of a general model with a heavy-top are scanned while imposing the experimental constraints coming from $K^0 - \bar{K}^0$ and $B_{d,s}^0 - \bar{B}_{d,s}^0$. Nonetheless, as it was pointed out in [24], the NP contributions to ε_K are currently much more constrained, roughly 10% of the standard model contribution which is very near to the experimental bound. Therefore, when introducing a VLQ, one should be very careful as not to exceed this bound. As it was shown, there exists a significant

$$F^u \approx \begin{pmatrix} 1 - \frac{m_{14}^2}{m_T^2} & -\frac{m_{14}m_{24}}{m_T^2} e^{i\beta} & \frac{m_{14}m_{43}m_t}{m_T^3} e^{-i\delta} & -\frac{m_{14}}{m_T} \\ -\frac{m_{14}m_{24}}{m_T^2} e^{-i\beta} & 1 - \frac{m_{24}^2}{m_T^2} & \frac{m_{24}m_{43}m_t}{m_T^3} e^{-i(\beta+\delta)} & -\frac{m_{24}}{m_T} e^{-i\beta} \\ \frac{m_{14}m_{43}m_t}{m_T^3} e^{i\delta} & \frac{m_{24}m_{43}m_t}{m_T^3} e^{i(\beta+\delta)} & 1 - \frac{m_{43}^2 m_t^2}{m_T^4} & \frac{m_{43}m_t}{m_T^2} e^{i\delta} \\ -\frac{m_{14}}{m_T} & -\frac{m_{24}}{m_T} e^{i\beta} & \frac{m_{43}m_t}{m_T^2} e^{-i\delta} & \frac{m_{14}^2}{m_T^2} \end{pmatrix} \tag{16}$$

In Appendix B, we provide more detail. An extensive analyses of the NP contributions to these phenomena was done in [24–26], where, in the context of an up-type VLQ, a solution to the CKM unitarity problem was proposed with $\Delta \approx 0.04$.

For instance, the NP tree-level contribution to $D^0 - \bar{D}^0$ is controlled by the size of $|F_{12}^u|$. Since, the SM contribution is negligible, one can use the upper-bound $x_D^{\text{NP}} < x_D^{\text{exp}} = 0.39_{-0.12}^{+0.11}\%$ [27,28] for the NP contribution to the mixing

region of parameter space where the NP contribution to ε_K are small (and $|\varepsilon_K^{\text{NP}}| \lesssim |\varepsilon_K^{\text{exp}}|/10$), and where

$$|\mathcal{V}_{24}|, |\mathcal{V}_{34}| \ll |\mathcal{V}_{14}| \sim \lambda^2. \tag{21}$$

In this region, the NP contributions to the golden mode kaon decays and ε'/ε are also within the experimental bounds (see Appendix B).

3.1 Numerical example

Next, we present a benchmark numerical example of our model and compute the NP contributions to the most relevant EWPM quantities.

We consider the following mass matrices (in GeV, at the M_Z scale) for the down and up-sectors, respectively

$$M_d = \begin{pmatrix} 0.00292338 & -0.0134741 & 0 \\ 0.000673705 & 0.0584675 & 0 \\ 0 & 0 & 2.9 \end{pmatrix},$$

$$\mathcal{M}_u = \begin{pmatrix} 0 & 0 & 0 & 53.7334 \\ 0 & 0.59952 & -6.91815 & 1.250e^{-0.285i} \\ 0 & -0.0239936 & 172.862 & 0 \\ 0.046526 & 0 & 14.886e^{-1.190i} & 1250 \end{pmatrix}. \tag{22}$$

From these one obtains the following mass spectrum (also in GeV, at the M_Z scale)

$$m_d = 0.003, \quad m_s = 0.060, \quad m_b = 2.9,$$

$$m_u = 0.002, \quad m_c = 0.60, \quad m_t = 173, \quad m_T = 1251. \tag{23}$$

The CKM matrix is the 4×3 left-sub-matrix of the following full 4×4 mixing matrix

$$|\mathcal{V}| = \begin{pmatrix} 0.97354 & 0.224413 & 0.00370431 & 0.0429468 \\ 0.224536 & 0.973644 & 0.0399975 & 0.000996211 \\ 0.00833917 & 0.0393001 & 0.999192 & 0.00151171 \\ 0.0416344 & 0.0105585 & 0.001674 & 0.999076 \end{pmatrix}, \tag{24}$$

and the resulting CP violation rephasing invariant phases are

$$\gamma \equiv \arg(-V_{ud}V_{cb}V_{ub}^*V_{cd}^*) \simeq 68.0^\circ,$$

$$\sin(2\beta) \equiv \sin[2 \arg(-V_{cd}V_{tb}V_{cb}^*V_{td}^*)] \simeq 0.746,$$

$$\chi \equiv \arg(-V_{ts}V_{cb}V_{cs}^*V_{tb}^*) \simeq 0.020,$$

$$\chi' \equiv \arg(-V_{cd}V_{us}V_{cs}^*V_{ud}^*) \simeq 5.71 \times 10^{-4}.$$

with the CP-odd invariant quantity $I_{CP} = |\text{Im } Q| \equiv |\text{Im}(V_{ub}V_{cd}V_{ud}^*V_{cb}^*)|$,

$$I_{CP} \simeq 3.00 \times 10^{-5}. \tag{26}$$

Table 1 NP contributions to various processes for the numerical case in Eq. (22)

Observable	NP prediction	Observable	NP prediction
Δm_{B_d}	1.50×10^{-11} MeV	x_D	0.048%
Δm_{B_s}	2.79×10^{-11} MeV	ε'/ε	-6.28×10^{-5}
Δm_K	7.96×10^{-13} MeV	$\frac{\text{Br}(K^+ \rightarrow \pi^+ \bar{\nu} \nu)}{\text{Br}(K^+ \rightarrow \pi^+ \bar{\nu} \nu)_{\text{SM}}}$	0.429
ε_K	2.25×10^{-5}	$\frac{\text{Br}(K^0 \rightarrow \pi^0 \bar{\nu} \nu)}{\text{Br}(K^0 \rightarrow \pi^0 \bar{\nu} \nu)_{\text{SM}}}$	0.636

Finally, in Table 1 we present the results for the most relevant EWPM quantities.

4 Conclusions

We have put forward the conjecture that the small numbers in V_{CKM} originate from Physics Beyond the SM. As small numbers, we identify $|V_{ub}|$ and the strength of CP violation, namely $I_{CP} = |\text{Im } Q|$, with Q denoting a rephasing invariant quartet of V_{CKM} .

We further propose that there is a weak basis where the effective V_{CKM} matrix arises from a rotation in the 2 – 3 quark sector and a 1 – 2 rotation in the down quark sector.

Within this framework, in the SM both V_{ub} and I_{CP} vanish, and from quark mixing generated by SM Yukawa couplings, the exact relation $V_{31} = V_{12}V_{23}$ holds. In the framework of our conjecture one finds an explanation why V_{31} is larger than V_{13} and near to its experimental value. We propose a specific extension of the SM consisting of the addition of an up type vector like quark. It is also shown that the specific textures of Yukawa couplings result from a Z_4 symmetry imposed on the full Lagrangian.

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A Mass matrix structure with a Z₄ symmetry

Bellow, we describe how to obtain the structures for the down quark mass matrix M_d and the 4×4 up-quark mass matrix \mathcal{M}_u in a model with a discrete symmetry Z_4 . To achieve this, we also introduce an extra scalar doublet ϕ_2 , and a singlet S .

We give a transformation table for the left-hand fields Q_{L_i} , the up and down-quark fields u_{R_i} , d_{R_i} and and their possible couplings to the doublets ϕ , ϕ_2 , and the singlet S . These fields transform as

$$\begin{aligned} \phi &\longrightarrow \phi, & \phi_2 &\longrightarrow i\phi_2, & S &\longrightarrow -iS, \\ \bar{Q}_{L1} &\longrightarrow -\bar{Q}_{L1}, & \bar{Q}_{L2} &\longrightarrow i\bar{Q}_{L2}, & \bar{Q}_{L3} &\longrightarrow \bar{Q}_{L3}, \\ U_{R,L} &\longrightarrow iU_{R,L} & d_{R1} &\longrightarrow -d_{R1}, & d_{R2} &\longrightarrow -d_{R2}, \end{aligned} \tag{27}$$

where the remaining RH quark fields (u_{R_j} with $j = 1, 2, 3$ and d_{R3}) do not transform.

In the following transformation tables, we position next to each field, its respective Z_4 property, and use zeros to denote a forbidden entry (by the symmetry) in the quark mass matrices.

Up-type quarks	u_{R1} (1)	u_{R2} (1)	u_{R3} (1)	U_R (-i)
\bar{Q}_{L1} (-1)	0	0	0	$y_{13}^u \tilde{\phi}_2$
\bar{Q}_{L2} (i)	$y_{21}^u \tilde{\phi}_2$	$y_{22}^u \tilde{\phi}_2$	$y_{23}^u \tilde{\phi}_2$	$y_{24}^u \tilde{\phi}$
\bar{Q}_{L3} (1)	$y_{31}^u \tilde{\phi}$	$y_{32}^u \tilde{\phi}$	$y_{33}^u \tilde{\phi}$	0
\bar{U}_L (i)	$y_{41}^u S$	$y_{42}^u S$	$y_{43}^u S$	M

Down-type quarks	d_{R1} (-1)	d_{R2} (-1)	d_{R3} (1)
\bar{Q}_{L1} (-1)	$y_{11}^d \phi$	$y_{12}^d \phi$	0
\bar{Q}_{L2} (i)	$y_{21}^d \phi_2$	$y_{22}^d \phi_2$	0
\bar{Q}_{L3} (1)	0	0	$y_{33}^d \phi$

After a WB transformations of the up right-handed fields, we obtain the 4×4 up-quark mass matrix \mathcal{M}_u in Eq. (4), as well as the 3×3 down-quark mass matrix in Eq. (2).

B Recovering the realistic s₁₄-dominance scenario

Phenomenological constraints impose restrictions on the region of parameters and limits the size of the mixing of the heavy-top and the standard quarks. Our region of parameter space is such that $|\mathcal{V}_{24}|, |\mathcal{V}_{34}| \ll |\mathcal{V}_{14}| \sim \lambda^2$. More precisely, in terms of the mixing angles in the Botella–Chau parametrization [30], we have in approximation

$$s_{14} \sim \lambda^2, \quad s_{24} \lesssim \lambda^4, \quad s_{34} \lesssim \lambda^3. \tag{28}$$

Comparing Eq. (9) with the BC parametrization, in our framework one has that the angles and phases in the BC parametrization correspond, in approximation, to

$$s_{14} \approx \frac{m_{14}}{m_T}, \quad s_{24} \approx \frac{m_{24}}{m_T}, \quad s_{34} \approx \frac{m_{14} m_{43}}{m_T^2}, \tag{29}$$

$$\delta' \equiv \delta_{24} - \delta_{14} \approx \beta, \tag{30}$$

$$\delta_{14} \approx \delta + \pi.$$

Only two phases are relevant at leading order, contrary to the three phases that typically participate in the general framework.

With this in mind, one finds that our model is compatible with the stringent experimental constraints coming from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, ϵ'/ϵ and ϵ_K .

With respect to ϵ_K^{NP} , it was argued [24,31], that, since the SM-value and the experimental value of $|\epsilon_K|$,

$$|\epsilon_K^{\text{SM}}| = (2.16 \pm 0.18) \times 10^{-3}, \tag{31}$$

$$|\epsilon_K^{\text{exp}}| = (2.228 \pm 0.011) \times 10^{-3},$$

are very close to each other, having $|\epsilon_K^{\text{exp}}| - |\epsilon_K^{\text{SM}}| \simeq (0.68 \pm 1.80) \times 10^{-4}$, the NP contribution to this EWPM parameter should be limited. At 1σ a new upper-bound for the NP contribution to $|\epsilon_K|$ was established, such that $|\epsilon_K^{\text{NP}}| \lesssim 0.1 |\epsilon_K^{\text{exp}}|$, or more concretely

$$|\epsilon_K^{\text{NP}}|^{1\sigma} \lesssim \Delta = 2.48 \times 10^{-4}, \tag{32}$$

which severely restricts various models.

In the case studied here, this crucial EWPM quantity now receives a new contribution, which was absent, in leading order, in the realistic s_{14} -dominance case presented in [24]. We now have roughly (with the same phase convention) that,

$$|\varepsilon_K^{\text{NP}}| \propto s_{12}s_{14}^2 |\eta_{iT}^K S(x_i, x_T) s_{13}s_{23} \sin \delta - \eta_{TT}^K S(x_T) s_{14} (s_{24} \sin \delta' - s_{23}s_{34} \sin \delta)|, \quad (33)$$

with the QCD corrections η_{ij}^K , the Inami-Lim functions $S(x_i, x_j)$, and $x_i \equiv m_i^2/M_W^2$ [32]. The expression for $\varepsilon_K^{\text{NP}}$ is very similar to the previously one obtained for the realistic s_{14} -dominance case. The point is that, now we have an extra term proportional to s_{34} and that within the region of parameters considered here, enough cancellation can also be easily achieved, allowing for small ($|\varepsilon_K^{\text{NP}}| \sim |\varepsilon_K^{\text{exp}}|/10$) contributions to ε_K . Thus, even here in this particular framework, we can still recover the main features of the realistic s_{14} -dominance case.

Furthermore, the NP contributions to $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ are controlled by $\lambda_T^K \equiv \mathcal{V}_{42}^* \mathcal{V}_{41}$. In our case, we now have

$$\lambda_T^K \approx s_{12}s_{14}^2 + s_{14}s_{24}e^{i\delta'}, \quad (34)$$

which has a similar form to the one obtained for realistic s_{14} -dominance case. Even, for values of s_{24} closer to the upper-bound in Eq. (18), the first term will dominate while the second will still correspond to a smaller correction, as was the case in the realistic s_{14} -dominance scenario. Hence, one also finds that $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ is compatible with current experimental results.

For the NP contributions to $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and ε'/ε , we have that these are controlled by $\text{Im}(\lambda_T^K)$,

$$\text{Im}(\lambda_T^K) \approx s_{14} (s_{24} \sin \delta' - s_{23}s_{34} \sin \delta). \quad (35)$$

where now we have also written the next order term. This expression differs from the realistic s_{14} -dominance by a term in s_{34} , which is not necessarily negligible. Typically, this extra term will correspond to a correction to the dominant term with s_{24} , which was the only relevant term in the realistic s_{14} -dominance limit. However, for smaller values of s_{24} , we must have $\sin \delta' \approx \sin \delta$ to ensure enough cancellation in (Eq. 33), so that these terms can nearly cancel each other, and subsequently, also suppress $(\varepsilon'/\varepsilon)_{\text{NP}}$ sufficiently enough.

For the remaining golden mode, these changes are somewhat irrelevant as in our case case, $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ is of the same order as the SM prediction, which is still well below the current experimental bound [3].

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