



# Quasielastic charged-current neutrino-nucleus scattering with nonrelativistic nuclear energy density functionals

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## ABSTRACT

Charged-current neutrino-nucleus scattering is studied in the quasielastic region with the KIDS (Korea-IBS-Daegu-SKKU) nuclear energy density functional. We focus on the uncertainties stemming from the axial mass and the in-medium effective mass of the nucleon. Comparing the result of theory to the state-of-the-art data from MiniBooNE, T2K, and MINERνA, we constrain the axial mass and the effective mass that are compatible with the data. We find that the total cross section is insensitive to the effective mass, so the axial mass could be determined independently of the uncertainty in the effective mass. Differential cross sections at different kinematics are, on the other hand, sensitive to the effective mass as well as the axial mass. Within the uncertainty of the axial mass constrained from the total cross section, dependence on the effective mass is examined. As a result we obtain the axial mass and the effective mass that are consistent with the experimental data.

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## 1. Introduction

Measurement of the neutrino-nucleus ( $\nu - A$ ) scattering cross section in the last decade at NOMAD [1], MiniBooNE [2–6], SciBooNE [7,8], MINERνA [9–12], MINOS [13], T2K [14,15], and NOνA [16] has improved the accuracy of the data dramatically, so the era of precision neutrino physics is dawning. One major purpose of the experiments is to resolve long-standing puzzles such as the neutrino mass, flavor oscillation, and CP violation in the leptonic sector. Success of the forthcoming experiments is expected to identify the limit of the standard model more stringently and lead to a new physics beyond the standard model. Interaction of the neutrino with nuclei plays a crucial role in understanding the result of the experiment. For a precise measurement of the standard model physics, uncertainties stemming from both hadronic and nuclear structures should be understood correctly, and should be reduced as much as possible. Those uncertainties also play a critical role in the interaction of neutrinos with nuclear matter at finite density and temperature, which has an essential consequence in the explosion of supernovae, and thermal evolution of the neutron star.

Axial mass  $M_A$  which is defined from the behavior of nucleon axial form factor in the zero momentum limit is one of the key issues in the uncertainty. After a new value of the axial mass was measured at NOMAD [1] with  $M_A = 1.05 \pm 0.02(\text{stat}) \pm 0.06(\text{sys})$  GeV, the axial mass from the charged-current (CC) reaction was extracted at four momentum transfer squared  $Q^2 = 0$ , which is  $M_A = 1.39 \pm 0.11$  GeV by measuring the flux-averaged differential cross section at MiniBooNE [3]. The other value of  $M_A = 0.99$  GeV was obtained at MINERνA Collaboration [9] which is in agreement with standard value, 1.032 GeV. Another value of  $M_A$  was measured  $1.23_{-0.09}^{+0.13}$  GeV at MINOS experiment [13] in the CC quasielastic scattering from  $^{56}\text{Fe}$  and was also measured  $1.26_{-0.18}^{+0.21}$  GeV at T2K experiment [14,15] from  $^{12}\text{C}$ . These experiments have performed the neutrino CC reactions in the quasielastic region to measure accurate differential cross section. However it is difficult to unambiguously define and precisely measure the CC cross section without the pions in the final state because of neutrino flux and backgrounds in detectors, so neutrino beams typically span wide energy range. Due to this wide range of incident neutrino energy, the contributions of inelastic processes like 2p-2h channel are not small although they are small in the quasielastic electron scattering.

There have been many theoretical works [17–25] for the axial form factor. In Ref. [17], the neutral-current (NC) quasielastic cross section described the experimental data with the standard

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value of  $M_A$  by using the relativistic Green function method. The authors in Ref. [18] extracted  $M_A = 1.37$  GeV with a relativistic distorted impulse approximation (RDWIA) and  $M_A = 1.36$  GeV with a relativistic Fermi gas model (RFGM) by comparing with the MiniBooNE data in the high energy region. However, the addition of the channels such as 2p-2h meson-exchange-current (MEC) into the calculations yields a better agreement with data when using the standard value of  $M_A$ . After this paper, Butkevich and Luchuk [19] obtained  $M_A = 1.2$  GeV including the contribution of 2p-2h MEC by comparing with the NOvA experimental data [16]. Martini *et al.* [20] calculated the contribution of the multinucleon (np-nh) to the  $\nu$ - $^{12}\text{C}$  cross section associated with MiniBooNE data with the standard value. In Ref. [21], the CC double differential cross section was compared with the MiniBooNE data with the inclusion of multi-nucleon processes like 2p-2h contribution and  $\pi$ -production and then the  $M_A = 1.08 \pm 0.03$  GeV was obtained from the data due to underestimating the neutrino flux. Ankowski [22] reported the values of the  $1.13 \pm 0.06 \leq M_A \leq 1.16 \pm 0.06$  GeV within the local density approximation. In Ref. [23], the values of the  $M_A$  were determined as  $1.34 \pm 0.06$  GeV with the relativistic mean field (RMF) and  $1.42 \pm 0.06$  GeV with the superscaling approach (SuSA) by calculating NC cross section but in conclusion, the large values of  $M_A$  are owing to excluding the contribution of the 2p-2h MEC. The authors in Ref. [24] analyzed the data from MiniBooNE, T2K, MINERvA, NOMAD, and SciBooNE with the standard value of  $M_A$  by including 2p-2n channel within the framework of extended SuSA (SUSAv2) and the contribution of the 2p-2h is about 15%  $\sim$  25% depending on the kinematics. Amaro and Arriola [25] described the MiniBooNE data with the RFGM model by using axial-vector-meson dominance with the perturbative QCD. In Ref. [26], the contribution of 2p-2h and 3p-3h was investigated with final-state correlations of knocked-out nucleons and the effect of final state hadron distribution was compared with the results by using NEUT generator. In the present work, we revisit the issue with a nuclear density functional theory based on a systematic expansion scheme.

KIDS (Korea-IBS-Daegu-SKKU) nuclear energy density functional (EDF) was initiated with a prospect to construct a nuclear model in which finite nuclei and infinite nuclear matter can be described to desired accuracy within a single framework. A series of works applied the model to nuclear matter and nuclei [27–29]. The results showed that a unified description of nuclei and nuclear matter is feasible by expanding nuclear EDF in the power of the Fermi momentum. Combining the nuclear data and the neutron star observations, parameters in the symmetry energy could be constrained within narrow ranges [30–32]. Extending the range of application, we considered quasielastic electron scattering off nuclei with the nuclear wave functions obtained with KIDS EDF [33,34]. Without any adjustment of the model parameters to scattering data, KIDS EDF reproduces the experimental data accurately. Uncertainties in the nuclear structure arising from the nucleon effective mass in nuclear medium and the symmetry energy have been explored in detail. Some results turn out to depend on the effective mass sensitively, so it is demonstrated that the electron scattering could be a tool to constrain the effective mass of the nucleon in nuclear medium.

Stimulated by the success in the electron scattering, we apply the KIDS EDF to the  $\nu - A$  scattering, and explore the uncertainty due to the in-medium effective mass of the nucleon and the axial mass, simultaneously. Nuclear wave functions are obtained by solving Hartree-Fock equations in which nonrelativistic nuclear potentials are imported from the KIDS EDF. Role of the effective mass is examined by using four models KIDS0, KIDS0-m\*77, KIDS0-m\*99 and SLy4, in which isoscalar and isovector effective

masses at the saturation density are  $(\mu_s, \mu_v) = (1.0, 0.8)$ ,  $(0.7, 0.7)$ ,  $(0.9, 0.9)$  and  $(0.7, 0.8)$ , respectively in the unit of free nucleon mass. Axial mass is defined in terms of the form factor slope at four-momentum transfer  $Q^2 = 0$  as  $M_A = [G'_A(0)/2G_A(0)]^{-1/2}$  where  $G_A(Q^2)$  denotes the axial form factor of the nucleon. Dependence on the axial mass is considered by employing a standard value  $M_A = 1.032$  GeV, and a large value  $M_A = 1.30$  GeV.

In the result we find that in several kinematic conditions, the effect of the effective mass appears to be clear, and the result agrees with data better when the isoscalar effective mass at the saturation density is close to the free mass. On the other hand, when the difference due to the effective mass is small, theoretical results agree well with the data regardless of the effective mass. Contribution of the axial mass is discriminated well in the total cross section of the neutrino. Large axial mass  $M_A = 1.30$  GeV reproduces the MiniBooNE data better than the standard value  $M_A = 1.032$  GeV. Interestingly the total cross section is insensitive to the effective mass, so the role of the axial mass can be singled out and probed without being interfered by other uncertainties. In the comparison of the differential cross section, large axial mass combined with large effective mass gives better agreement to data on average. However more accurate measurements are demanded to constrain the axial mass with the differential cross sections.

In the present paper, the formalism of the CC  $\nu - A$  scattering is briefly introduced in Sec. 2, and Section 3 presents the results and discussion. Finally, we summarize the work in Sec. 4.

## 2. Formalism

The  $\nu(\bar{\nu}) - A$  scattering is described by the connection of the electromagnetic interaction and weak interaction. In order to calculate the  $\nu(\bar{\nu}) - A$  scattering, we choose that the target nucleus is seated at the origin of the coordinate system.  $p_i^\mu = (E_i, \mathbf{p}_i)$ ,  $p_f^\mu = (E_f, \mathbf{p}_f)$ ,  $p_A^\mu = (E_A, \mathbf{p}_A)$ ,  $p_{A-1}^\mu = (E_{A-1}, \mathbf{p}_{A-1})$ , and  $p^\mu = (E_N, \mathbf{p})$  represent the four-momenta of the incident neutrino, outgoing neutrino, target nucleus, the residual nucleus, and the knocked-out nucleon, respectively. For the CC reaction in the laboratory frame, the inclusive cross section is given by the contraction between lepton and hadron tensors:

$$\frac{d\sigma}{dT_N} = 4\pi^2 \frac{M_N M_{A-1}}{(2\pi)^3 M_A} \times \int \sin\theta_l d\theta_l \int \sin\theta_N d\theta_N p_{f,rec}^{-1} \sigma_M^{W^\pm} [v_L R_L + v_T R_T + h v'_T R'_T], \quad (1)$$

where  $M_N$  is the nucleon mass in free space,  $\theta_l$  denotes the scattering angle of the lepton,  $\theta_N$  is the polar angle of knocked-out nucleons,  $T_N$  is the kinetic energy of the knocked-out nucleon, and  $h = -1$  ( $h = +1$ ) corresponds to the intrinsic helicity of the incident neutrino (antineutrino). The  $R_L$ ,  $R_T$  and  $R'_T$  are longitudinal, transverse, and transverse interference response functions, respectively. Detailed forms for the kinematical coefficients  $v$  and the corresponding response functions  $R$  are given in Refs. [35,36]. The squared four-momentum transfer is given by  $Q^2 = q^2 - \omega^2 = -q_\mu^2$ . For the CC reaction, the kinematic factor  $\sigma_M^{W^\pm}$  is defined by

$$\sigma_M^{W^\pm} = \sqrt{1 - \frac{M_l^2}{E_f}} \left( \frac{G_F \cos(\theta_C) E_f M_W^2}{2\pi(Q^2 + M_W^2)} \right)^2, \quad (2)$$

where  $M_W$  is the rest mass of  $W$ -boson, and  $M_l$  is the mass of an outgoing lepton.  $\theta_C$  represents the Cabibbo angle given by  $\cos^2 \theta_C \simeq 0.9749$ .  $G_F$  denotes the Fermi constant. The recoil factor  $f_{rec}$  is written as

$$f_{rec} = \frac{E_{A-1}}{M_A} \left| 1 + \frac{E_N}{E_{A-1}} \left[ 1 - \frac{\mathbf{q} \cdot \mathbf{p}}{p^2} \right] \right|. \quad (3)$$

The nucleon current  $J^\mu$  represents the Fourier transform of the nucleon current density written as

$$J^\mu = \int \bar{\psi}_p \hat{\mathbf{j}}^\mu \psi_b e^{i\mathbf{q} \cdot \mathbf{r}} d^3r, \quad (4)$$

where  $\hat{\mathbf{j}}^\mu$  is a free weak nucleon current operator, and  $\psi_p$  and  $\psi_b$  are wave functions of the knocked-out and the bound state nucleons, respectively. The wave functions are generated with the same approach as the previous work [34]. For a free nucleon, the current operator of the CC reaction consists of the weak vector and the axial vector form factors:

$$\begin{aligned} \hat{\mathbf{j}}^\mu &= F_1^V(Q^2) \gamma^\mu + F_2^V(Q^2) \frac{i}{2M_N} \sigma^{\mu\nu} q_\nu \\ &+ G_A(Q^2) \gamma^\mu \gamma^5 + \frac{1}{2M_N} G_P(Q^2) q^\mu \gamma^5. \end{aligned} \quad (5)$$

By the conservation of the vector current (CVC) hypothesis, the vector form factors for the proton (neutron),  $F_i^{V, p(n)}(Q^2)$ , are expressed as

$$F_i^V(Q^2) = F_i^p(Q^2) - F_i^n(Q^2). \quad (6)$$

The axial form factors for the CC reaction are given by

$$G_A(Q^2) = -g_A / (1 + Q^2/M_A^2)^2, \quad (7)$$

with  $g_A = 1.262$  and two values 1.032 GeV and 1.30 GeV are assumed for  $M_A$ .

The induced pseudoscalar form factor is parameterized by the Goldberger-Treiman relation

$$G_P(Q^2) = \frac{2M_N}{Q^2 + m_\pi^2} G_A(Q^2), \quad (8)$$

where  $m_\pi$  is the pion mass. But the contribution of the pseudoscalar form factor vanishes for the NC reaction because of the negligible final lepton mass participating in this reaction.

### 3. Result

With the KIDS EDF model, we calculate the various differential cross sections and total cross sections in the quasielastic CC  $\nu - A$  scattering off  $^{12}\text{C}$ , and compare the result with MiniBooNE, MINER $\nu$ A, and T2K data. In order to obtain the wave functions of bound and final nucleons from nonrelativistic nuclear model, the relativistic wave functions are generated by using the nonunitary transformation [33,34,37,38]. For the Coulomb distortion of the final lepton, the same approximation exploited by the Ohio group [39] is used. In these neutrino experiments, the energy of the incident neutrino cannot be fixed but has an energy spectrum, so the cross sections have to be averaged over the flux of the incoming neutrino beam.

Fig. 1 shows the flux-averaged double-differential cross sections in terms of polar angle or kinetic energy of the outgoing muon. Data in the given kinematic regions are available from the MiniBooNE Collaboration at Fermi Lab. [2]. The theoretical cross sections in Fig. 1 (a)  $\sim$  (d) and (e)  $\sim$  (h) are

the results for the value of the axial mass  $M_A = 1.032$  and 1.30 GeV, respectively. The theoretical results do not describe the data well at fixed  $T_\mu$  (especially  $0.2 < T_\mu < 0.3$  GeV region), but describe the data relatively well at fixed the angle. The results with large axial mass provide better agreement but the influence of the effective mass cannot be distinguished in Fig. 1.

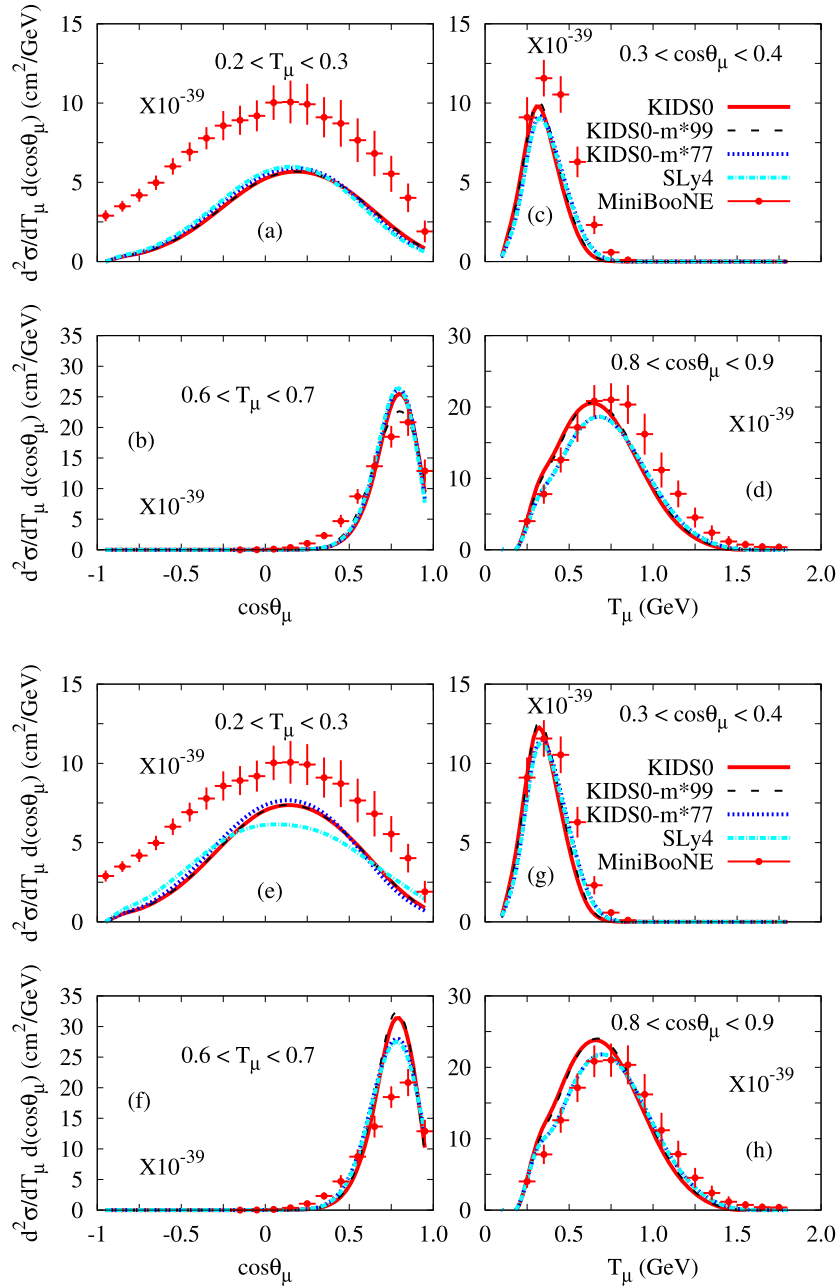
In Fig. 2, the flux-averaged double-differential cross sections with kinematics different from Fig. 1 are shown in terms of the momentum of the outgoing muon at fixed polar angle of the muon and compared with the data measured from T2K [14]. The value of  $M_A$  is 1.032 GeV in the upper panels and 1.30 GeV in lower panels. The results of  $M_A = 1.30$  GeV agree with the data better than the results of 1.032 GeV. From these results, the effect of the effective mass appears to be clear, and the results of the large effective mass agree with the data better than the small ones.

In Fig. 3, the flux-averaged double-differential cross sections are shown for the outgoing muon antineutrino in terms of  $p_T$ , where  $p_T$  and  $p_\parallel$  represent the transverse and longitudinal component of the muon momentum with respect to the incident antineutrino beam, respectively. According to Ref. [12], this kinematics was exploited to include the nuclear effects in the  $\nu - A$  scattering like the final state interaction, meson production, and so on. In this work, we show the results of low momenta because the inelastic processes like meson production are excluded and the numerical difficulty is avoided due to partial-wave expansion. The legend of the curve is the same as the Figs. 1 and 2. In this case, the theoretical results of  $M_A = 1.032$  GeV describe the data better than the results of  $M_A = 1.30$  GeV. Contrary to the results in Figs. 1, 2, the large effective mass results are suppressed to the small effective mass ones in these kinematics, but agree with the measurement better than the small effective masses.

Fig. 4 shows the flux-averaged differential cross sections in terms of the squared four-momentum transfer for the incident neutrino and antineutrino scattering. In the case of the incoming neutrinos, for both values of  $M_A$ , the theoretical results shift to the left side of data by about  $0.1$  (GeV/c) $^2$ . Axial mass tends to increase the magnitude of the cross section. As a result, the result of  $M_A = 1.30$  GeV agrees to data better than  $M_A = 1.032$  GeV. For the antineutrino, the data are reproduced well by both  $M_A = 1.032$  and 1.30 GeV. On the other hand, the effective mass hardly affects the cross section in this kinematics.

We calculate the total scaled cross sections in terms of the incident neutrino (antineutrino) energies, in which the total cross section is divided by the number of participated nucleon in the reaction. The results are shown in Fig. 5. For the neutrino, the effect of the  $M_A$  enhances the cross section about 30% but for the antineutrino it does about 15%. In the case of the antineutrino, the influence of the effective mass increases the cross section at higher incident energies although its effect is very small for the neutrino.

Recently, a new experiment was performed at MiniBooNE [6] with monoenergetic muon neutrinos at 236 MeV, which are created when a positive kaon at rest decays, called kaon-decays-at-rest (KDAR). We calculate the differential cross sections in terms of the kinetic energy of the outgoing muon and compare the result with the data in Fig. 6. The red solidus part is with shape-only  $1\sigma$  error band, where  $\sigma$  denotes total cross section and yields  $\sigma = (2.7 \pm 1.2) \times 10^{-39}$  cm $^2$ /neutron. The legend of the curve is the same as Fig. 1. In the low incident neutrino energies, the effect of the  $M_A$  is small, so gives a difference less than 10%. The cross sections of small effective mass are out



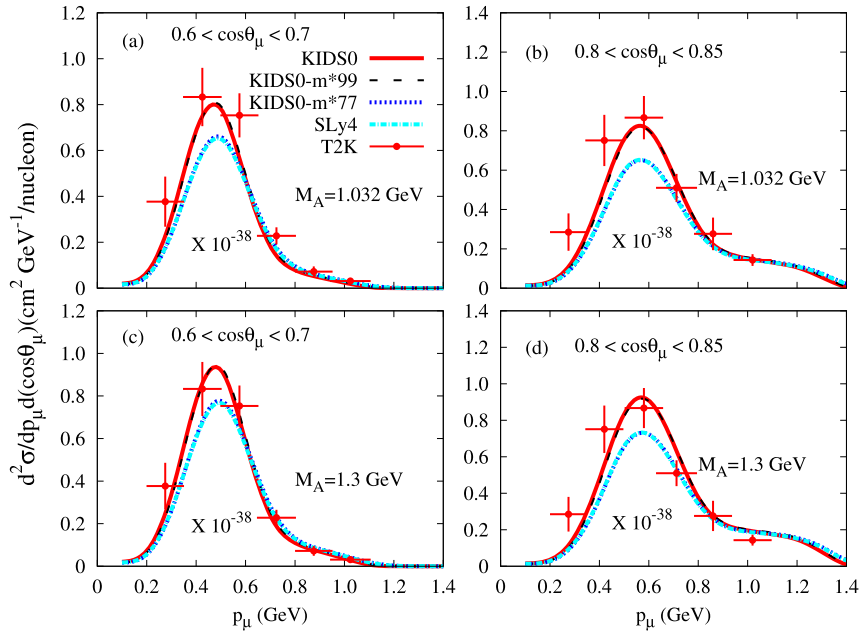
**Fig. 1.** Flux-averaged double-differential cross sections in terms of the kinetic energy and the scattering angle of the final muon from  $^{12}\text{C}$ . The data were measured by the MiniBooNE Collaboration [2]. Panels (a-d) are the result with  $M_A = 1.032$  GeV, and (e-h) with  $M_A = 1.30$  GeV.

of the data around the peak for both  $M_A = 1.032$  GeV and 1.30 GeV.

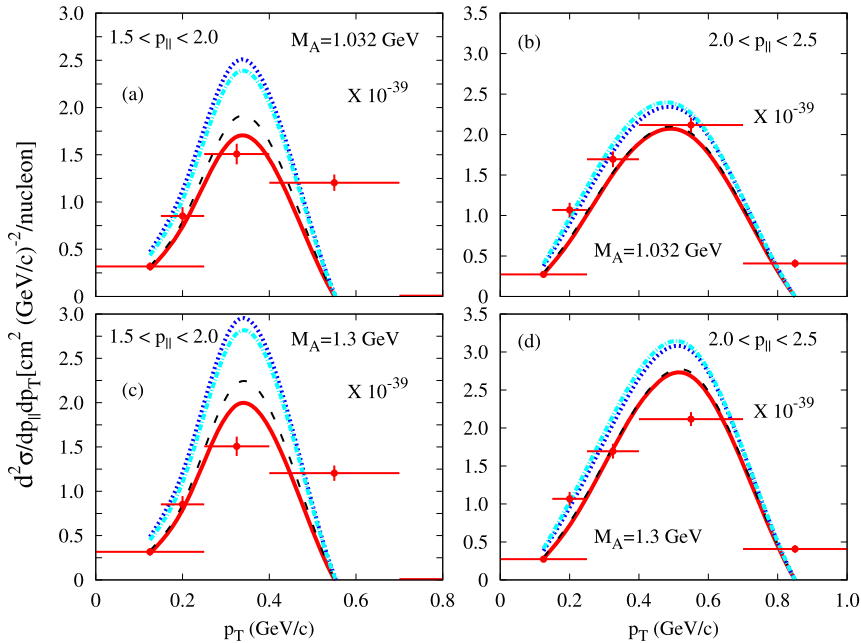
#### 4. Summary

Charged-current quasielastic scattering of the neutrino and antineutrino with  $^{12}\text{C}$  target has been considered within a nonrelativistic nuclear density functional theory. Both the wave functions of bound nucleon in  $^{12}\text{C}$  nucleus and final state interactions of the outgoing nucleons are obtained by using the effective nuclear potentials obtained from KIDS EDF. Parameters of the KIDS EDF have been fixed to satisfy well-defined nuclear matter properties and nuclear data, and there is no calibration of the model to scattering data.

We found that the model reproduces experimental data at various kinematics well even without the multi-particle-multi-hole channel. At the same time, dependence on the in-medium effective mass of the nucleon and its axial mass is identified clearly. Dependence on the effective mass is probed by using two groups of models, one group with isoscalar effective mass close to the free mass ( $\mu_s \simeq 1$ ), and the other group with  $\mu_s \simeq 0.7$ . Comparisons with the data from T2K and MINERvA Collaborations are crucial in diagnosing the effect of the effective mass. Results of the KIDS EDF are in good agreement with the T2K and MINERvA data with  $\mu_s \simeq 1$ . It is also confirmed that the dependence on the effective mass is dominated by the isoscalar effective mass, and the role of the isovector effective mass can be neglected. We observed the same behavior in the quasielastic electron scattering, in which  $\mu_s \simeq 1$



**Fig. 2.** Flux-averaged double-differential cross sections in terms of the incident muon momentum at fixed angle of muon. The data were measured by the T2K Collaboration [14]. Panels (a, b) are the result with  $M_A = 1.032$  GeV, and (c, d) with  $M_A = 1.30$  GeV.



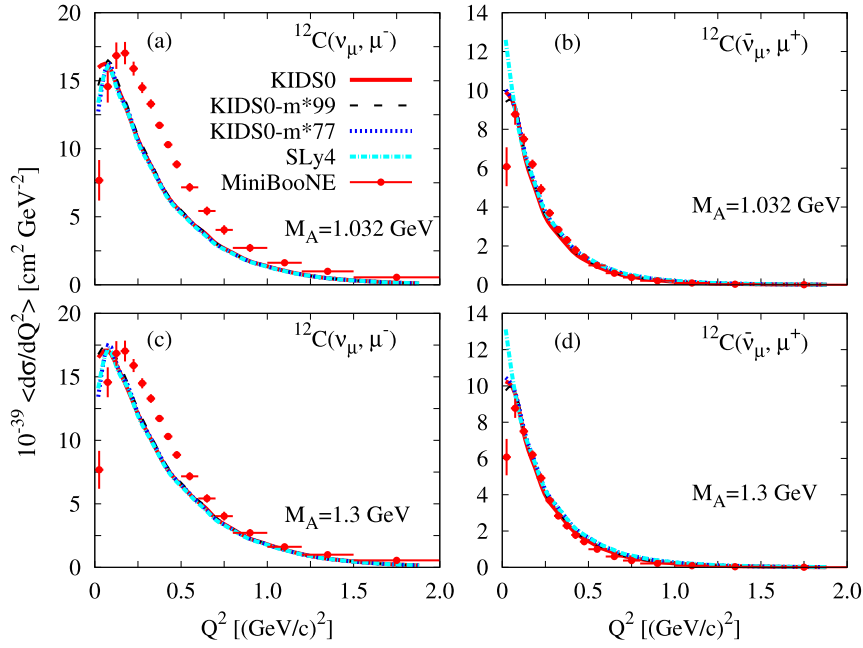
**Fig. 3.** Flux-averaged double-differential cross sections in terms of the muon transverse momentum at fixed longitudinal momentum. The data were measured by the MINERvA Collaboration [12].

models agree with the data better than the  $\mu_s \simeq 0.7$  models.

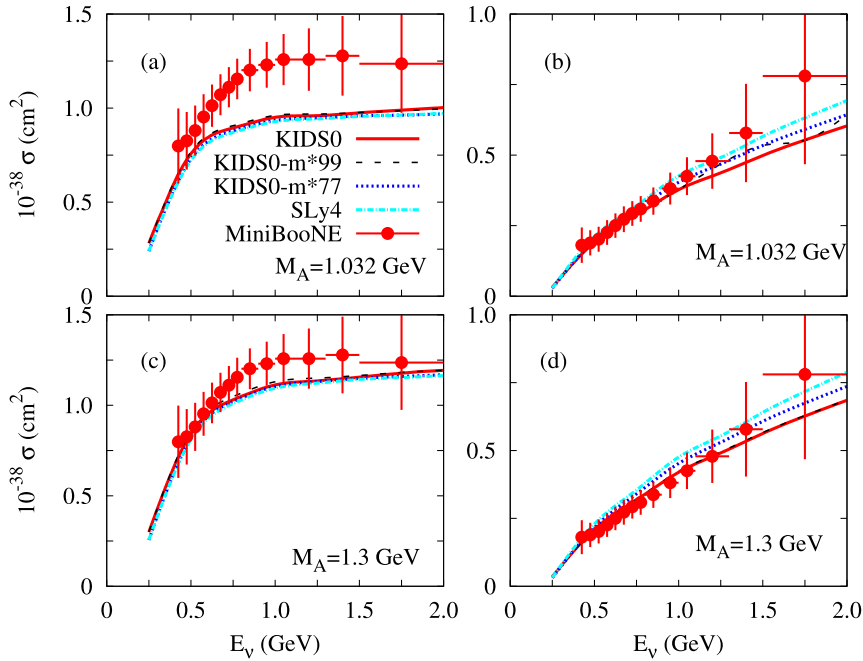
In the comparison with the MiniBooNE data, role of the effective mass becomes less dominant compared to the T2K and MINERvA data, but the effect of the axial mass becomes large by comparing with the effective mass. A highlighting result is the total cross section of the neutrino, where the standard value of the axial mass  $M_A = 1.032$  GeV fails to reproduce the neutrino data. For  $M_A = 1.30$  GeV, theory results reside within the experimental uncertainty with quasielastic contributions. In our calculation, the total cross section for the antineutrino is less sensitive to  $M_A$  than that for the neutrino. More importantly, total cross sections de-

pend on the effective mass very weakly, so they provide a unique opportunity to constrain the uncertainty of the axial mass. The effect of the axial mass  $M_A$  is small at low incident energies of the neutrino, but the effect increases with higher neutrino energies. It is necessary to determine the value of the axial mass exactly by considering the charged-current quasielastic-like processes such as the multi-meson-exchange currents, multi-particle-multi-hole contributions, and pion production processes. We will consider semi-inclusive and multi-nucleon contributions in future works to generate the precise value of the axial mass. In addition, implementing the KIDS model in the Markov chain event generators used in neutrino oscillation experiments will provide a useful





**Fig. 4.** Flux-averaged differential cross sections in terms of the four-momentum transfer squared. The data were measured by the MiniBooNE Collaboration [2].



**Fig. 5.** Total scaled cross sections in terms of the incident neutrino energy. Left panels are the result for the neutrino and the right panels for the antineutrino. Upper panels are the result for  $M_A = 1.032$  MeV, and lower panels are the result for  $M_A = 1.30$  MeV. The experimental data for the incident neutrino and antineutrino were measured from MiniBooNE [2] and [4], respectively.

tool to analyze the charged-current events in the quasielastic region.

#### Declaration of competing interest

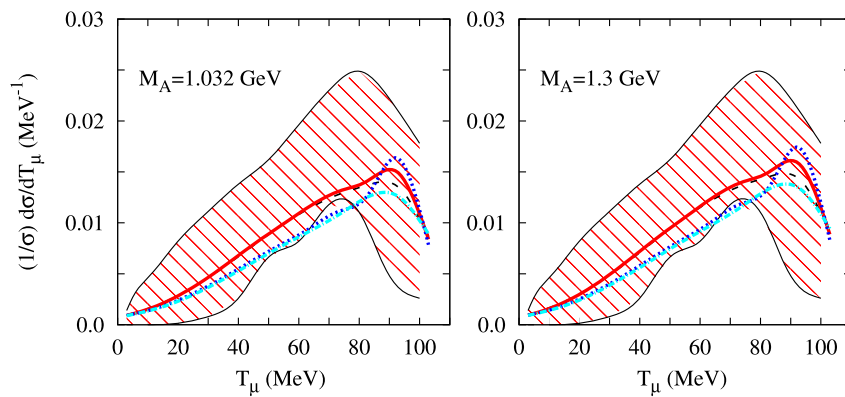
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

#### Data availability

No data was used for the research described in the article.

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**Fig. 6.** Differential cross sections in terms of the kinetic energy of the outgoing muon. Left panel is the result for  $M_A = 1.032$  GeV, and right panel for  $M_A = 1.30$  GeV. The red solidus parts are obtained from KDAR data on MiniBooNE [6].

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