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Enhancements of electron-positron pair production at very low transverse momentum in peripheral hadronic A + A collisions

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ABSTRACT

The STAR collaboration has observed an excess production of electron-positron pairs which have transverse momenta $p_\perp < 150$ MeV/c in peripheral gold-gold and uranium-uranium collisions. ALICE has also reported on an excess of $\mu^+\mu^-$ pairs at low $p_\perp$ in very peripheral (70–90% centrality) lead-lead collisions at a center-of-mass energy of 2.76 TeV/nucleon pair. These excesses cannot be explained by the QGP thermal radiation and ρ in-medium broadening calculations. This is a sign of coherent photon–photon interactions in ultra-peripheral collisions. However, the number of lepton pair production is more than the predictions from hadronic production models. In literature, there are number of explanations about the $p_\perp$ broadening, however none of them has included the multi-pair production processes in their calculations. The aim of this paper is to show that at RHIC and LHC energies and for small impact parameters unitarity is violated and lowest order calculations are not adequate to calculate electromagnetic lepton pair production. Therefore, we have to include higher order effects in the calculations. We have shown that electron-positron multi-pair production cross section is large and it cannot be ignored in explaining the excess production of electron-positron pairs for low transverse momenta.

1. Introduction

Dileptons play a crucial role in studying quark gluon plasma (QGP) created at Relativistic Heavy-Ion Collider (RHIC) and at Large Hadron Collider (LHC). Although dileptons are produced in this hot and dense medium, their interactions with this medium are very small. Therefore they carry direct information from this QGP phase of the matter.

The ultraperipheral collisions (UPCs) of heavy ions are described by an impact parameter b greater than the sum of the radius of the colliding nuclei. If $b = 0$ this is called head-on collision (0% centrality) or if $b = 2R_A$ it is called grazing collision (100% centrality) where R_A is the nuclear radius of the colliding nuclei. When two ultrarelativistic heavy ions pass near each other, the intense electromagnetic fields are very strong so that they pull lepton pairs from the vacuum ($\gamma\gamma \to \ l^+l^-$). Lepton pairs coming from the quark-gluon plasma can be mixed with the electromagnetically produced lepton pairs. Therefore, it is extremely important to investigate the electromagnetic production of lepton pairs in detail.

Electromagnetic lepton production has been studied in UPCs by the STAR [1,2], ATLAS [3], ALICE [4], and CMS [5] collaborations. The STAR collaboration has observed an excess production of electron-positron pairs which have transverse momenta $p_\perp < 150$ MeV/c in peripheral gold-gold and uranium-uranium collisions. The number of lepton pair production is more than the predictions from hadronic production models. The enhancement factors of the $p_\perp$ distributions of electron-positron pairs are the largest in 0.4-0.76 and 1.2-2.6 GeV/c^2 invariant mass regions for 60-80% Au+Au and U+U collisions [6]. The observed excess is found to concentrate below $p_\perp \approx 0.15$ GeV/c, on the other hand hadronic cocktail, are found for $p_\perp > 0.15$ GeV/c in both mass regions of Au + Au and U + U collisions. These excesses can not be explained by the QGP thermal radiation and ρ in-medium broadening calculations. This is a sign of coherent photon–photon interactions in ultra-peripheral collisions [7].

There are number of papers published to explain this excess of electron-positron pairs. S. R. Klein [8] calculated the cross sections and kinematics for two-photon production of electron-positron pairs. The calculation is based on the STARlight simulation code, which is based on the Weizsäcker-Williams virtual photon approach. His results for the STAR continuum observations are compatible with two-photon production of e^+e^- pairs. Klein et al.

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[9–11] further derive an all order QED re-summation for the soft photon radiation to investigate the electromagnetic production of lepton pairs with low total transverse momentum in relativistic heavy-ion collisions. In this work, authors discussed the additional $p_\perp$ broadening effects from multiple interactions with the medium and the magnetic field contributions from the quark-gluon plasma for peripheral and central collisions.

In reference [12,13], authors calculated the cross section of lepton pair production in the classical field approximation with equivalent photon approximation (EPA) as well as the corrections beyond EPA in the Born approximation. On the other hand, in ref. [14–17], authors discussed the advantages and inadequacies of EPA methods. Although EPA methods describe the $\gamma\gamma$ process relatively well, it has problems to describe various experimental results, such as the recently observed broadening of the pair transverse momentum with impact parameter.

There are number of studies about the multiple lepton pair productions in the literature [18–23]. Almost all of them have a prediction that unitary is violated in these high energies so that probability of producing electron-positron pairs is greater than one. In Fig. 1, it is clear that at RHIC and LHC energies, probabilities of producing electron-positron pairs for impact parameters smaller than Compton-wavelength of electron is greater than one. This means that for small impact parameters where the electric field is strongest multiple electron-positron pair productions is inescapable and it must be included in calculations. In literature, there are number of explanations about the $p_\perp$ broadening however none of them has included the multi-pair production processes in their calculations. The aim of this paper is to show that electron-positron multi-pair production cross section is large and it can not be ignored in calculations.

Ultra-relativistic peripheral collisions of heavy ions at RHIC and at LHC can produce copies of numbers of lepton pairs via the two-photon process. Since the energies of the heavy ions are 200 GeV and 2760 GeV per nucleon in these colliders, multi-pair production cross sections of electron-positron pairs, are quite large so that it is possible to measure them experimentally. To calculate the multi-pair production probabilities, first, we should have an impact parameter dependence cross section. We have obtained a well-behaved impact parameter dependence cross section and by using the Monte Carlo methods, we have calculated multipair production cross sections of electron-positron pairs in Au-Au and in Pb-Pb heavy-ion collisions at RHIC and at LHC energies, respectively. We have also used some experimental restrictions in our calculation to compare our findings with the experimental results.

2. Formalism

Two heavy ions move with relativistic velocities u along the $z-$axis in opposite directions to each other, and undergo the Lorentz contraction due to their velocities. The distance between the radii of the nuclei is written by the impact parameter b. Throughout this paper, we use the natural units with $\hbar = c = m = e = 1$, where these constants are reduced Planck constant, speed of light, mass of the electron and charge of the electron, respectively.

Since the physical processes such as lepton-pair production or annihilation occur with the interaction of fields, the lepton pair production from the electromagnetic field is expressed by the Interaction-lagrangian density depended on the classical 4-vector potential A^μ:

$$\mathcal{L}_{int}(x) = -\bar{\psi}(x)\gamma_\mu \psi(x) A^\mu(x) \tag{1}$$

where A^μ can be written separately for two colliding nuclei $\mathcal{A}$ and $\mathcal{B}$,

$$A^\mu(q,b) = A^\mu_{\mathcal{A}}(q,b) + A^\mu_{\mathcal{B}}(q,b) \tag{2}$$

We can write the terms of 4-vector potential in momentum space as,

$$A^0_{(\mathcal{A},\mathcal{B})}(q) = -8\pi^2\gamma^2(Ze)G_E(q^2)f_z(q^2)\frac{\delta(q_0 \mp \beta q_z)}{q_z^2+\gamma_\perp^2} \times \exp\left(\pm i\mathbf{q}_\perp \cdot \frac{\mathbf{b}}{2}\right), \tag{3}$$

$$A^z_{(\mathcal{A},\mathcal{B})}(q) = \pm\beta A^0_{(\mathcal{A},\mathcal{B})}(q), \tag{4}$$

$$\mathbf{A}^\perp_{(\mathcal{A},\mathcal{B})}(q) = 0 \tag{5}$$

where $f_z(q^2)$ is the form factor of a nucleus which gives the momentum distribution of a proton in the nucleus, and $G_E(q^2)$ is the form factor of a proton which represents electric distribution of the proton. Form factors are crucial for the cross-section calculation of lepton-pair production in heavy-ion collisions. In our calculations, we have used the Fermi (Woods-Saxon) distribution for the charge distribution of the nucleus,

$$\rho(r) = \frac{\rho_0}{1+\exp\left(\frac{r-R}{a}\right)} \tag{6}$$

where R is the radius of the nucleus, and a is a quantity related to the thickness of nucleus shell. Also, ρ_0 can be calculated by normalization. The Fourier transform of this charge distribution gives us the form factor of the nucleus

$$f_z(q^2) = \frac{4\pi\hbar}{Zeq}\int r\rho(r)\sin(qr)dr. \tag{7}$$

On the other hand, for the electric distribution function of the proton, we can write the dipole form factor for the proton as

$$G_E(q^2) = (\frac{\Lambda^2}{\Lambda^2+q^2})^2 \tag{8}$$

where the value of Λ^2 is 0.71 GeV^2 [24]. We have observed that the effect of this distribution is negligible.

Using the expressions given so far, the cross-section depending on the impact parameter b for the lepton pair production can be written

$$\begin{aligned}\frac{d\sigma}{db} = \frac{\pi}{8\beta^2}\sum_{\sigma_k}\sum_{\sigma_q}\int_0^\infty qdq\, b\, J_0(qb)\int_0^{2\pi} d\phi_q \int \frac{dk_z dq_z d^2k_\perp d^2K d^2Q}{(2\pi)^{10}} \\ \times\left\{F\left(\frac{\mathbf{Q}-\mathbf{q}}{2};\omega_{\mathcal{A}}\right)F(-\mathbf{K};\omega_{\mathcal{B}})\mathcal{T}_{kq}\left(\mathbf{k}_\perp - \frac{\mathbf{Q}-\mathbf{q}}{2};\beta\right)\right. \\ \left.+F\left(\frac{\mathbf{Q}-\mathbf{q}}{2};\omega_{\mathcal{A}}\right)F(-\mathbf{K};\omega_{\mathcal{B}})\mathcal{T}_{kq}(\mathbf{k}_\perp-\mathbf{K};-\beta)\right\} \\ \times\left\{F\left(\frac{\mathbf{Q}+\mathbf{q}}{2};\omega_{\mathcal{A}}\right)F(-\mathbf{q}-\mathbf{K};\omega_{\mathcal{B}})\mathcal{T}_{kq}\left(\mathbf{k}_\perp - \frac{\mathbf{Q}+\mathbf{q}}{2};\beta\right)\right. \\ \left.+F\left(\frac{\mathbf{Q}+\mathbf{q}}{2};\omega_{\mathcal{A}}\right)F(-\mathbf{q}-\mathbf{K};\omega_{\mathcal{B}})\mathcal{T}_{kq}(\mathbf{k}_\perp+\mathbf{q}-\mathbf{K};-\beta)\right\}\end{aligned} \tag{9}$$

and this 10-dimensional integral can be calculated with Monte-Carlo integration method. The quantity $\mathcal{T}_{kq}$ contains the propagator of the intermediate lepton and the matrix elements for the coupling of the photon to the leptons. The quantity $F(q,\omega)$ is the scalar part of the electromagnetic field of the moving heavy ions in momentum space. The frequencies $\omega_{\mathcal{A}}$ and $\omega_{\mathcal{B}}$ of the virtual photons that are fixed by energy conservation at the vertex where the photon is absorbed. The variables $\mathbf{k}_\perp, \mathbf{k}_z$ perpendicular and longitudinal momenta of positive energy leptons, $\mathbf{q} = \mathbf{q}_\perp + \mathbf{q_z}$, perpendicular and longitudinal momenta of negative energy leptons, finally **Q** and **K** are the intermediate lepton momentum. Details of the calculations can be found in these articles [19,22,23]. Then, we can easily calculate the impact parameter b dependence probability of producing lepton-pairs

$$P(b) = \frac{1}{2\pi b}\frac{d\sigma}{db}. \tag{10}$$

However, for small impact parameters, the probabilities of lepton pair productions are greater than one, therefore unitary is violated. The equation (10) is the probability of producing lepton pairs as a function of impact parameters. This equation is obtained from the equation (9). For large colliding energies of heavy ions and for small impact parameters the equation becomes greater than 1 which means that unitary is violated. In Tables 1 and 2, equation (10) is used for unitary violated cross sections. Unitary violation indicate that there are more than 1 lepton pairs are created. This means that second order Feynman diagrams are not adequate to describe the lepton pair production process, therefore we have to include the higher order diagrams in the calculations. After including the higher order diagrams, multi-pair lepton production probability becomes the Poisson distribution [18–23],

$$P_N(b) = \frac{P(b)^N e^{-P(b)}}{N!} \tag{11}$$

where N is the number of pairs. Therefore by using the Poisson distribution unitary is now restored. For one-pair cross section ($N = 1$), we just need to integrate the one-pair probability $P_1(b)$,

$$\sigma_{1_{pair}} = \int d^2b\, P_1(b) \tag{12}$$

In above derivations, we applied Feynman rules for the S-matrix. We first work with the operator form of the S-matrix and then take the matrix element with the initial (here the vacuum) and final (here N pairs) states. The Poisson form of the results follows from the summation of a subset of all the diagrams that would occur in the external-field model [22].

3. Results

We have calculated impact parameter dependence cross sections for $b > 2R_A$ and found that for small impact parameters unitary is violated. This clearly can be seen in Fig. 1 so that for small impact parameters probabilities are greater than one. This means that more than one electron-positron pair is produced in single collisions of ions. Therefore we must consider this higher order corrections in our calculations. In Table 1, total cross sections are shown for various collisions of ions at RHIC and LHC energies. We have done our calculations within the STAR acceptance: invariant mass of the pairs $M_{e^+e^-} > 0.4$ GeV/c^2, pair rapidity $\mid y_{e^+e^-} \mid < 1$, individual lepton $p_{\perp,e} > 200$ MeV/c and pseudo-rapidity $\mid \eta_e \mid < 1$. Unitary violated cross sections are considerably greater than the one pair production cross sections and these unitary violated cross sections do not represent the real single pair cross sections. We can also clearly see that at these high energies and high charges of the colliding ions, two, three and even four pair productions probabilities are not negligible. It would be significant error without including these higher order effects in the calculations.

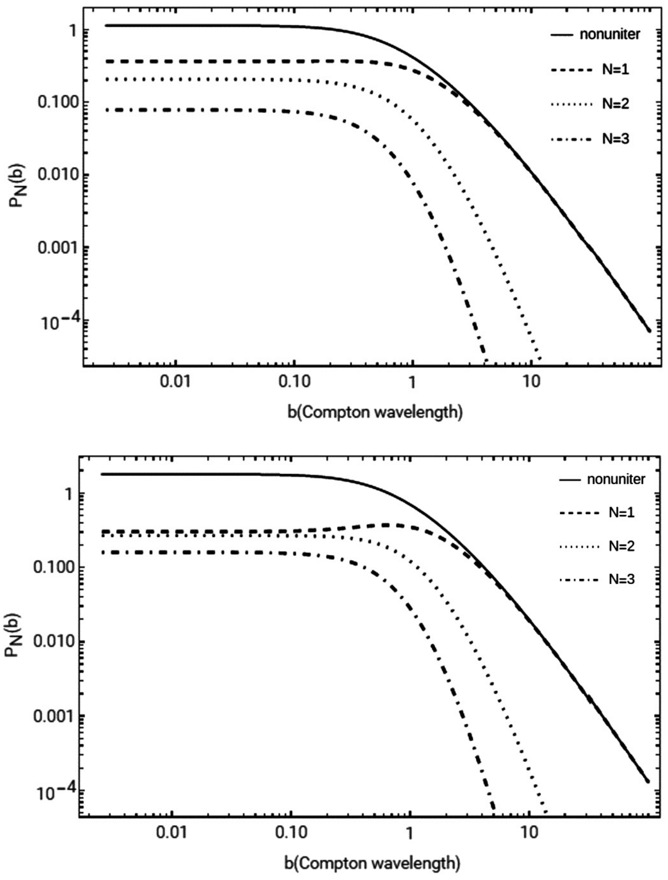

Fig. 1. Probabilities of producing electron-positron pairs as functions of impact parameters b (in unit of Compton wavelength of electron). Top graph is for the Au + Au collisions, middle one is the for U + U collisions and bottom one is for the Pb + Pb collisions. In all graphs, solid lines shows the nonuniter plots and it is clear that for small impact parameters they are greater than one. When we include the higher order effects, unitary is restored and for small impact parameters one-pair, two-pair and three-pair producing probabilities are significant.

In Fig. 1 pair production probabilities of $N = 1$, $N = 2$ and $N = 3$ pairs can be seen for $Au + Au$, $U + U$ and $Pb + Pb$ collisions for different energies, respectively. In each graph, first line represents the single pair production probabilities. These probability values are greater than 1 that shows the contribution of more than one pair production probability can not be ignored especially for small impact parameters. We have plotted the differential cross section as a function of transverse momentum of the produced pairs in Fig. 2. We have also calculated the integrated cross sections over the transverse momentum of the produced lepton $p_\perp$ and we have found that ratio of the integrals between $0 - 0.15$ GeV/c and between 0 - infinity is nearly 1

$$\frac{\int_0^{0.15GeV/c} dp_\perp \frac{d\sigma}{dp_\perp}}{\int_0^\infty dp_\perp \frac{d\sigma}{dp_\perp}} \approx 1. \tag{13}$$

This means that transverse momentum of the produced leptons mostly in the range $0 - 0.15$ GeV/c. In fact, the maximum produced leptons transverse momentum are between $20 - 50$ MeV/c range. On the other hand, the ratio of the invariant mass $M_{e^+e^-}$ integrals between $0.4 \leq M_{e^+e^-} \leq 2.6\ GeV/c^2$ and between $1.02\ MeV/c^2$ and ∞

$$\frac{\int_{0.4GeV/c^2}^{2.6GeV/c^2} dM \frac{d\sigma}{dM}}{\int_{1.02MeV/c^2}^{\infty} dM \frac{d\sigma}{dM}} \approx 0.24 - 0.27, \tag{14}$$

where the ratio 0.24 is for Au + Au collisions at 200 GeV energies, and the ratio 0.27 is for Pb + Pb collisions at 2.760 TeV energies. For the U + U collisions at 193 GeV energies this ratio is about 0.245. Here 0.24 means that 24% of the produced electron-positron pairs are in the range of invariant mass $0.4 \leq M_{e^+e^-} \leq 2.6\ GeV/c^2$. These calculations clearly show that the ratios of different charges and energies are almost constant, however for the Pb + Pb collisions, the ratio is slightly higher.

Table 1

By using Eq. (10), unitary violated total cross sections are obtained. With Poisson distribution multi-pair cross sections, total pair production cross sections results for different ions, energies are tabulated in unit of barn. It is important that unitary restored total pair cross sections are slightly smaller than the unitary violated cross sections and multi-pair cross sections are not negligible.

Ion	Unitary violated	N=1	N=2	N=3	N=4	Unitary restored total pair
RHIC Au+Au (200 GeV)	54400	51276	1274	158	23	52672
RHIC U+U (193 GeV)	98300	90290	2964	508	109	93830
LHC Pb+Pb (2.76 TeV)	226000	210700	5974	861	151	217493

Table 2

Multipair cross sections, total pair production cross sections and unitary violated cross sections results for different ions, energies and impact parameter ranges.

Ion	b range	N=1 (mb)	N=2 (mb)	N=3 (mb)	N=4 (mb)	Total pair (mb)	Unitary violated (mb)
RHIC Au+Au	11.4-13.2 fm	0.508	0.287	0.108	0.031	0.942	1.57
RHIC Au+Au	9.4-11.6 fm	0.530	0.300	0.113	0.032	0.983	1.64
RHIC Au+Au	4.8-9.4 fm	0.749	0.424	0.160	0.045	1.39	2.33
RHIC U+U	14.1-15.8 fm	0.481	0.426	0.252	0.111	1.33	2.83
LHC Pb+Pb	13-14.7 fm	0.513	0.356	0.164	0.057	1.11	2.05

In Table 2, we have calculated impact parameter range in terms of collision centrality. We have used a black-disk model [8] to convert from centrality to impact parameter range. We have done our calculations within the STAR acceptance: invariant mass of the pairs $M_{e^+e^-} > 0.4$ GeV/c^2, pair rapidity $\mid y_{e^+e^-} \mid < 1$, individual lepton $p_{\perp,e} > 200$ MeV/c and pseudo-rapidity $\mid \eta_e \mid < 1$. The collision energies of heavy ions are for Au + Au at 200 GeV, for Pb + Pb at 2.760 TeV and for the U + U collisions at 193 GeV. Our calculations show that unitary is violated for high energies and small impact parameters. For centralities smaller than the radius of the colliding nuclei, the unitary violation would be even greater. In Table 2, on the right column we have tabulated the cross sections for certain impact parameter regions for various heavy ion collisions. In order to restore the unitary, we include the higher order corrections and calculated the one-pair, two-pair, three-pair and four-pair productions. It is clear that more than one-pair production cross sections are quite large and compatible with single-pair production. When we add all multiple pair productions, we can obtain the total pair production cross sections and these values are tabulated next to the far right column in the table.

As expected, unitary violated cross sections are almost two times greater than total pair cross sections. Klein [8] has calculated the same calculations without including the higher order effects by using the STARlight Monte Carlo method. His calculations are 4 or 5 times larger than our total pair cross section calculations, 2 or 3 times greater than our unitary violated cross sections. One reason is that Weizsäcker-Williams method works well for impact parameter greater than one Compton wavelength of electron, not less than one Compton wavelength of electron. On the other hand, Monte Carlo QED calculations give well behaved impact parameter dependence cross section, hence we can calculate the related observables directly without making further assumptions and approximations.

4. Conclusion

We have calculated impact parameter dependence cross sections of lepton pair production from electromagnetic fields of relativistic heavy ions. At these energies and small impact parameters it is clear that unitary is violated. This means that, second order perturbation theory is not sufficient alone to describe the lepton pair production. When we include the higher order effects in the calculations, Tables 1 and 2 shows that multiple pair production cross sections are not negligible.

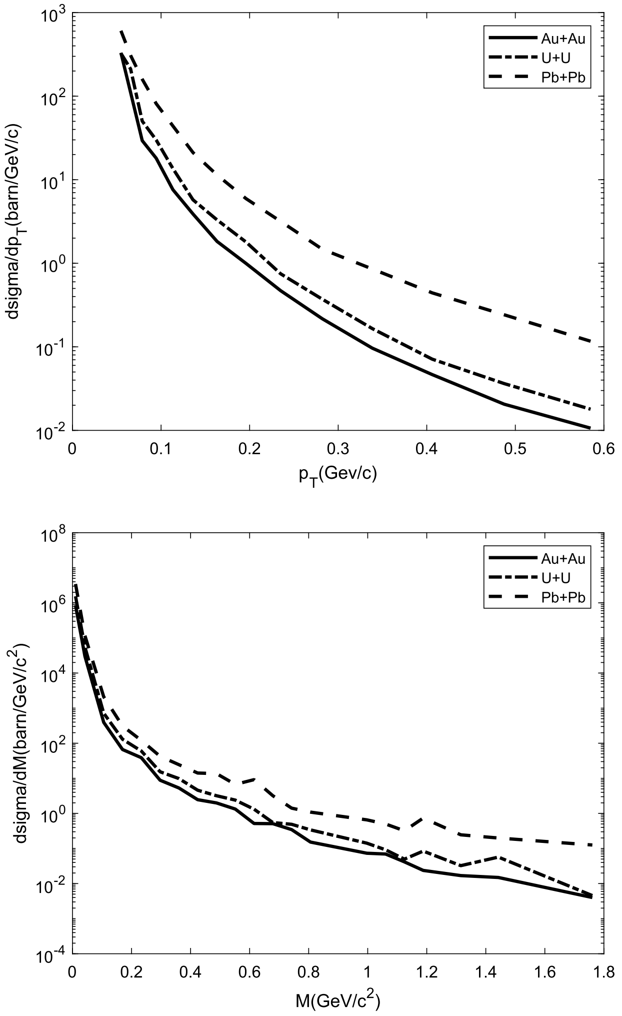

Fig. 2. Differential cross sections as functions of transverse momentum $p_\perp$, energy p_0 and invariant mass M of the produced electron-positron pairs. All three plots show that when the colliding energies and charges of the heavy ions are large, the values of the differential cross sections are also large. In equations (12) and (13), the ratios of some certain integral ranges to the whole integral ranges are shown. Especially for the $p_\perp$ plot, it is interesting that almost all lepton pairs are produced below $p_\perp < 0.15$ GeV/c.

In Table 1, we can see that unrestricted multi-pair cross sections makes about ~ 2 or 3 percent of the total pair cross sections, however for the restricted case in Table 2, multi-pair production cross sections makes about more than half of total pair cross sections. For the case of U + U collision, one pair production cross section is almost equal to the two pair production cross section, since the charge of the uranium is larger than gold nuclei and cross section is proportional to $\sim Z^4$. We can also explain why multi-pair cross sections are compatible with the single pair production cross sections at 70 - 90% centrality collisions because at this impact parameters electric fields of the heavy ions reaches the maximum values.

We have also calculated differential cross sections as a function of transverse momentum of produced lepton pairs. Electron-positron pairs are created mostly in the transverse momentum domain of less than 0.15 MeV/c and maximum probability of the pairs are produced between $20 - 50$ MeV/c transverse momentum region. When we include multi-pairs in our calculations this can explain the excess production of electron-positron pairs which have transverse momenta $p_\perp < 150$ MeV/c in peripheral Au + Au and U + U collisions.

In references [25,26] the authors study the invariant-mass distributions of dileptons produced in ultrarelativistic heavy-ion collisions at very low pair transverse momenta, $p_\perp < 150$ MeV/c. They worked for the impact parameter region both for $b > 2R_A$ and $b < 2R_A$, where R_A is the nuclear radius. In their calculations, they show that the combination of photon fusion, thermal radiation and final-state hadron decays gives a fair description of the low-$p_\perp$ invariant-mass as well as $p_\perp$ spectra as recently measured by the STAR collaboration for different centrality classes. The coherent contribution dominates in peripheral collisions, while thermal radiation shows stronger increase with centrality.

Recent study [27–29] about the impact parameter dependence of the $\cos 4\phi$ azimuthal asymmetry for purely electromagnetic lepton pair production in heavy ion collisions at low $p_\perp$ shows that azimuthal asymmetry has a strong $b_\perp$ dependence, specifically, the asymmetry decreases with increasing impact parameter.

As a final remark, there was an experimental attempt [30] to measure double-pair production. Electron-positron pairs created by fully stripped 200 A GeV sulfur ions incident on thin targets of Al, Pd, and Au. Since the colliding energies of heavy ions are not as large as nowadays, multiple pairs were not seen, however upper limits on the total cross section were set. We have produced a new Monte Carlo technique for calculating the impact parameter-dependence of the cross section which also allows a calculation of the multi-pair production cross section. We find our results are consistent with the upper limits which experiments [30] put on this process. On the other hand, recently the authors [31,32] have calculated the two positron-electron pairs and two muon-antimuon pair cross sections for the ATLAS, CMS and ALICE detectors and these calculations could be tested at these detectors. With the advent of higher-energy heavy-ion colliders, the study of the physics of the two-photon process emerges as an exciting new field. The process may be used as a means of production of exotic particles perhaps the study of nonperturbative effects in QED. The ability to calculate the impact parameter dependence of the cross section and the related ability to calculate multiple-pair production provides an additional piece in the understanding of the process.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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References

[1] J. Adams, et al., Phys. Rev. C 70 (2004) 031902(R).
[2] J. Adam, et al., Phys. Rev. Lett. 121 (2018) 132301.
[3] M. Dyndal, ATLAS Collaboration, Nucl. Phys. A 967 (2017) 281.
[4] E. Abbas, et al., ALICE Collaboration, Eur. Phys. J. C 73 (2013) 2617.
[5] V. Khachatryan, et al., CMS Collaboration, Phys. Lett. B 772 (2017) 489.
[6] S. Yang, Int. J. Mod. Phys. Conf. Ser. 46 (2018) 1860013.
[7] W. Zha, L. Ruan, Z. Tang, Z. Xu, S. Yang, Phys. Lett. B 781 (2018) 182.
[8] S.R. Klein, Phys. Rev. C 97 (2018) 054903.
[9] S.R. Klein, P. Steinberg, Annu. Rev. Nucl. Part. Sci. 70 (2020) 323.
[10] S. Klein, A.H. Mueller, B. Xiao, F. Yuan, Phys. Rev. Lett. 122 (2019) 132301.
[11] S. Klein, A.H. Mueller, B. Xiao, F. Yuan, Phys. Rev. D 102 (2020) 094013.
[12] R. Wang, S. Pu, Q. Wang, Phys. Rev. D 104 (2021) 056011.
[13] W. Zha, J.D. Brandenburg, Z. Tang, Z. Xu, Phys. Lett. B 800 (2020) 135089.
[14] J.D. Brandenburg, W. Zha, Z. Xu, Eur. Phys. J. A 57 (2021) 299.
[15] Z. Ma, Z. Lu, J. Zhu, L. Zhang, Phys. Rev. D 104 (2021) 074023.
[16] R. Wang, S. Lin, S. Pu, Y. Zhang, Q. Wang, Phys. Rev. D 106 (2022) 034025.
[17] W. Schäfer, Eur. Phys. J. A 56 (2020) 231.
[18] G. Baur, K. Hencken, D. Trautmann, Phys. Rep. 453 (2007) 1.
[19] S. Karadağ, M.C. Güçlü, Phys. Rev. C 102 (2020) 014904.
[20] A. Alscher, K. Hencken, D. Trautmann, G. Baur, Phys. Rev. A 55 (1997) 396.
[21] G. Baur, Phys. Rev. A 42 (1990) 5736.
[22] M.C. Güçlü, J. Li, A.S. Umar, D.J. Ernst, M.R. Strayer, Ann. Phys. 272 (1999) 7.
[23] M.C. Güçlü, Nucl. Phys. A 668 (2000) 149.
[24] C. Bottcher, M.R. Strayer, J. Phys. G, Nucl. Part. Phys. 16 (1990) 975.
[25] M. Kłusek-Gawenda, R. Rapp, W. Schäfer, A. Szczurek, Phys. Lett. B 790 (2019) 339.
[26] M. Kłusek-Gawenda, W. Schäfer, A. Szczurek, Phys. Lett. B 814 (2021) 136114.
[27] C. Li, J. Zhou, Y. Zhou, Phys. Rev. D 101 (2020) 034015.
[28] J. Adam, et al., Phys. Rev. Lett. 127 (2021) 052302.
[29] A.M. Sirunyan, et al., Phys. Rev. Lett. 127 (2021) 122001.
[30] C.R. Vane, S. Datz, E.F. Deveney, P.F. Dittner, H.F. Krause, R. Schuch, H. Gao, R. Hutton, Phys. Rev. A 56 (1997) 5.
[31] M. Kłusek-Gawenda, A. Szczurek, Phys. Lett. B 763 (2016) 416.
[32] A. van Hameren, M. Kłusek-Gawenda, A. Szczurek, Phys. Lett. B 776 (2018) 84.