



# Dark sector tensor currents contribution to lepton's anomalous magnetic moment

V. Kozhuharov, M. Naydenov\*

*Faculty of Physics, Sofia University, 5 J. Bourchier Blvd, 1164 Sofia, Bulgaria*

Received 8 July 2022; received in revised form 9 November 2022; accepted 5 December 2022

Available online 8 December 2022

Editor: Tommy Ohlsson

## Abstract

In this work we consider a model including dark sector bosons interacting through tensor currents with Standard Model leptons. We show that for certain values of the interaction constant this model has the potential of providing an explanation for the discrepancy between theory and experiment, regarding the anomalous magnetic moment of the muon. The effect on the already established measurements for the electron are small and the lepton universality violation is naturally incorporated. Possible experimental searches together with systematic approach to characteristic properties of the final states are discussed.

© 2022 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP<sup>3</sup>.

## 1. Overview

Quantum electrodynamics is the theory giving the most accurate predictions confirmed by experiment in the history of physics. One of the experimentally testable consequences of the theory is the prediction first made by Julian Schwinger for the anomalous magnetic moment of the electron [1]. The anomalous magnetic moment is a quantum property of charged leptons arising from loop corrections to the fermionic electromagnetic vertex. It is defined as  $a = \frac{g-2}{2}$ , where  $g$  is the Landé  $g$ -factor and the agreement between theoretical predictions and experiment for the case of an electron, including corrections from QED, hadron physics and electroweak theory is [2]

\* Corresponding author.

*E-mail address:* [mnaydenov@phys.uni-sofia.bg](mailto:mnaydenov@phys.uni-sofia.bg) (M. Naydenov).

$$\Delta a_e = a_e^{\text{EXP}} - a_e^{\text{SM}} = (4.8 \pm 3) \times 10^{-13}. \tag{1}$$

A problem arises when one tries to treat the muon or the tau lepton the same way as the electron. The discrepancy between theory and experiment for the case of the muon is [3]

$$\Delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}. \tag{2}$$

One sees that the application of the same theoretical treatment to leptons from different families leads to different accuracy of the prediction and the lepton universality violation is manifest. There exist independent experimental evidences supporting this difference [4,5]. This is one of the main motivations for searching for physics beyond the Standard Model which potentially compensates for the discrepancy between theory and experiment.

Various probable solutions to the  $\Delta a_\mu$  problem have been suggested - possibly within the Standard Model [6], including leptoquarks [7], using supersymmetric models [8], and various other exotic models [9,10]. Others consider high energy solutions, testable at the LHC [11,12]. So far, no generally applicable solution to the muon anomalous magnetic moment problem exists that is also supported by the experimental observations. In addition to the  $g - 2$  problem the  $^8\text{Be}$  decay anomaly observed at the ATOMKI collaboration [15] might as well be a strong sign for physics beyond the Standard Model [16–18].

In this work we are particularly interested in solutions involving dark sector particles, similar to the models presented in [13,14]. The dark sector extension was proposed by B. Holdom [19] where the U(1) gauge group is generalized to  $U'(1)$  by adding a massive dark gauge particle interacting with visible matter through an interaction constant related to the charge of the electron. Then the inclusion of dark scalar, pseudo-scalar and pseudo-vector terms in the Lagrangian is trivial. Only the pseudo-scalar and the vector particles can serve as a solution to the ATOMKI anomaly.

In the present paper a model Lagrangian will be discussed and the corresponding corrections to the photon vertex will be numerically estimated. We show that for the lightest charged lepton (electron) the correction to the anomalous magnetic moment can be negligibly small compared to the established experimental accuracy, and the heavier the lepton is, the bigger is the correction to the anomalous magnetic moment.

This paper is organized as follows: we start with a Lagrangian describing the phenomenology of fermion states interacting through tensor currents, we derive the consequent Feynman rules for the various vertices and the resulting propagators and then we calculate the resulting correction to the electromagnetic vertex. We show that dark tensor bosons can be responsible for the muon  $g - 2$  anomaly, and have the potential to be discovered at present and future experiments such as PADME [24], SeDS [25].

## 2. The model

The Lagrangian proposed in the paper by B. Holdom is an extension of the U(1) including a dark massive vector particle, the dark photon. This Lagrangian can be extended for fields described by Lorentz invariant currents corresponding to scalar, pseudo scalar, pseudo vector, tensor and pseudo tensor terms. We are especially interested in processes which have the potential of explaining the  $^8\text{Be}$  anomaly by considering the Lagrangian

$$\begin{aligned} \mathcal{L} = & -e\bar{\Psi}\gamma_\mu A^\mu\Psi - e_1\bar{\Psi}\gamma_\mu A_1^\mu\Psi - e_2\bar{\Psi}\gamma_\mu\gamma^5 A_2^\mu\Psi \\ & + ie_3\bar{\Psi}\gamma^5 A_3\Psi - ie_4\bar{\Psi}\frac{q^\mu}{|q|}\sigma_{\mu\nu}A_4^\nu\Psi + e_5\bar{\Psi}\frac{q^\mu}{|q|}\sigma_{\mu\nu}\gamma^5 A_5^\mu\Psi + \end{aligned}$$

$$+ i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\bar{\Psi}\Psi. \tag{3}$$

The first term is the lepton-photon interaction from standard QED and then we have in turn the interaction with dark sector particles - a vector, pseudo vector, pseudo scalar, tensor and pseudo tensor current. At the end we have the kinetic terms for the lepton of interest. The factors  $e_i$  are dimensionless constants of interaction. Gauge fields can be taken into account by including 1-loop corrections with four external lines. The chiral symmetry condition requires the definition of the tensor currents to include the incoming momentum of the particle  $q$ .

A tensor vertex was proposed by Nambu and Jona-Lasinio [22] for the interactions between mesons and fermions, in analogy with superconductivity. The trivial extension of vector currents to tensor currents leads to a chiral pair of terms  $(\bar{\Psi}\sigma_{\mu\nu}\Psi)^2 + (\bar{\Psi}i\gamma^5\sigma_{\mu\nu}\Psi)^2$ , which for chiral transformations is identically 0. Therefore, the Lagrangian should contain a unique momentum dependence, because the local product of two tensor currents with different chiralities vanishes identically [20,21]. The appearance of a tensor vertex in eq. (3) can be seen as an effective low-energy approximation (i.e. effective interaction) of a more fundamental theory, for example string field theories, where non-local interactions arise naturally due to the extended structure of the fundamental objects.

After leaving only terms containing vector, tensor and pseudotensor interactions as in [23] we are left with the final Lagrangian which will be used throughout the paper:

$$\begin{aligned} \mathcal{L} = & -e\bar{\Psi}\gamma_\mu A^\mu\Psi - e_1\bar{\Psi}\gamma_\mu A_1^\mu\Psi - ie_4\bar{\Psi}\frac{q^\mu}{|q|}\sigma_{\mu\nu}A_4^\nu\Psi \\ & + e_5\bar{\Psi}\frac{q^\mu}{|q|}\sigma_{\mu\nu}\gamma^5 A_5^\nu\Psi + i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\bar{\Psi}\Psi. \end{aligned} \tag{4}$$

The Feynman rules emerging from this model are the following (Figs. 1, 2):

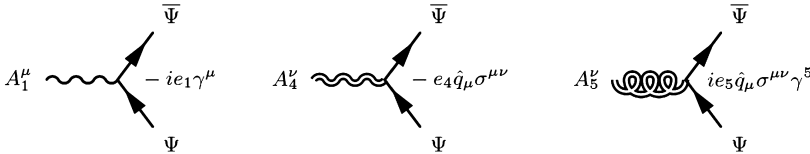


Fig. 1. Basic Feynman rules for leptonic interactions with dark sector particles. Here  $\hat{q}^\mu = \frac{q^\mu}{|q|}$ .

$$A_4^\mu \text{---} A_4^\nu = \frac{-i\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)}{q^2 - m_4^2} \quad A_5^\mu \text{---} A_5^\nu = \frac{-i\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)}{q^2 - m_5^2}$$

Fig. 2. Propagator rules for the  $A_4^\mu$  and  $A_5^\mu$  dark bosons.

The magnitude of the constants of interaction are defined as a rescaled electron charge  $e_i = \epsilon_i e$ , where  $\epsilon_i$  is a rescaling factor governing the mixing between the photon and the dark sector particles.

### 3. Anomalous magnetic moment contribution due to dark sector bosons

A possible existence of new vector particles interacting through (pseudo-)tensor currents with the fundamental leptons will modify their magnetic moment. The subsequent calculations are

performed for independent values of  $\epsilon_4$  and  $\epsilon_5$  but where applicable, a typical benchmark value is used,  $\epsilon_4 = \epsilon_5 = \epsilon = 10^{-3}$ . The diagrams influencing the anomalous magnetic moment of the leptons are the following (Fig. 3):

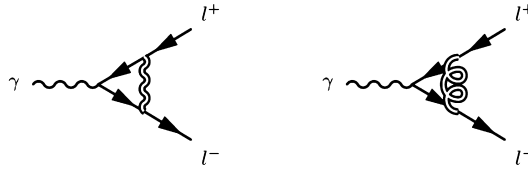


Fig. 3. Feynman diagrams for the electromagnetic correction due to  $A_4^\mu$  and  $A_5^\mu$  dark bosons.

The electromagnetic vertex function can be expanded in terms of form factors as

$$\Gamma^\mu = F_1(k^2)\gamma^\mu + F_2(k^2)\frac{i\sigma^{\mu\nu}}{2m}k_\nu, \tag{5}$$

where  $k$  is the photon momentum. At tree level the electron is a point-like particle, where  $F_1 = 1$  and  $F_2 = 0$ . Quantum corrections from 1-loop diagrams give rise to non-trivial behaviour of the form factors in which in standard QED  $F_1(k^2)$  contains infrared divergence and the quantum contribution to the anomalous magnetic moment is evaluated when taking the limit  $F_2(k^2 = 0)$ .

For the case of a virtual tensor boson  $A_4^\mu$  we obtain

$$\begin{aligned} \delta_4\Gamma^\mu(k) = & -\epsilon_4^2 e^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2} \frac{g_{\alpha\beta}}{q^2 - m_4^2} \bar{u}(p_1)\sigma^{\alpha\rho}q_\rho \\ & \times \frac{\hat{p}_1 - \hat{q} + m}{(p_1 - q)^2 - m^2} \gamma^\mu \frac{\hat{p} - \hat{q} + m}{(p - q)^2 - m^2} \sigma^{\beta\omega} q_\omega u(p) \end{aligned} \tag{6}$$

and for the case of a virtual pseudo-tensor boson  $A_5^\mu$

$$\begin{aligned} \delta_5\Gamma^\mu(k) = & -\epsilon_5^2 e^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2} \frac{g_{\alpha\beta}}{q^2 - m_5^2} \bar{u}(p_1)\sigma^{\alpha\rho}q_\rho \gamma^5 \\ & \times \frac{\hat{p}_1 - \hat{q} + m}{(p_1 - q)^2 - m^2} \gamma^\mu \frac{\hat{p} - \hat{q} + m}{(p - q)^2 - m^2} \sigma^{\beta\omega} q_\omega \gamma^5 u(p). \end{aligned} \tag{7}$$

These integrals are calculated using a *Mathematica* package [26], where the result is the contribution to the tensor part in Eq. (5). Here the dependence on  $k$  is hidden in the kinematical relation between the momenta  $p$ ,  $p_1$  and  $k$  and is made manifest by using Mandelstam variables and by the requirement for conservation of energy. Setting the condition  $k^2 = 0$  we obtain the two corrections

$$\begin{aligned} \delta_4\Gamma^\mu = & \frac{e^2\epsilon_4^2}{16\pi^2} \left( -\frac{9m^2 + 2M^2}{m^2} + \frac{(8m^4 - 3m^2M^2 - M^4) \ln\left(\frac{m^2}{M^2}\right)}{m^4} \right. \\ & \left. - \frac{2\sqrt{M^2(M^2 - 4m^2)}(16m^4 - m^2M^2 - M^4) \ln\left(\frac{\sqrt{M^2 - 4m^2} + M}{2m}\right)}{m^4(4m^2 - M^2)} \right) \end{aligned} \tag{8}$$

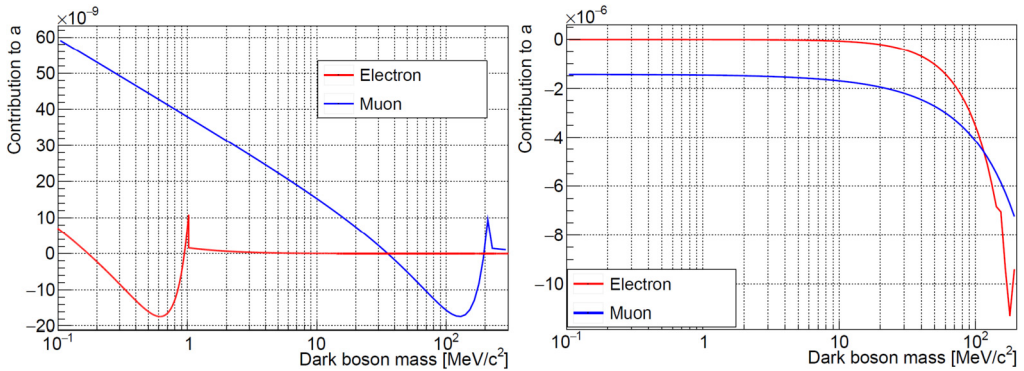


Fig. 4. Dependence of the contribution of the dark boson to  $a = (g - 2)/2$  on the mass of the tensor interacting dark boson for tensor (left) and pseudotensor (right) interaction,  $\epsilon_{4/5} = 10^{-3}$ .

$$\delta_5 \Gamma^\mu = \frac{e^2 \epsilon_5^2}{16\pi^2} \left( -\frac{3m^2 - 2M^2}{m^2} - \frac{M^2 (3m^2 - M^2) \ln\left(\frac{m^2}{M^2}\right)}{m^4} - \frac{2(m^2 - M^2) \sqrt{M^2(M^2 - 4m^2)} \ln\left(\frac{\sqrt{(M^2 - 4m^2)} + M}{2m}\right)}{m^4} \right), \tag{9}$$

where we take  $m_4 = M$  or  $m_5 = M$  and  $m$  is the lepton mass in the final state.

We obtain a total correction to the anomalous magnetic moment as a function of the lepton mass. One can note that the correction increases for bigger masses, so the influence on the electron magnetic moment is negligible, and for the muon and potentially the tau lepton is much bigger.

The dependence of the  $\Delta a_\mu$  correction as a function of the dark boson mass is shown for the muon and for the electron in Fig. 4 both for tensor (left) and pseudotensor (right) interactions. The contribution as a function of  $M$  arising from pseudotensor interaction is always negative as can be seen in Fig. 4 right, while for pure tensor interactions there exist regions with positive or negative contributions. In fact, for  $M \leq 35$  MeV the contribution to  $a$  is always positive, while it can be vanishing for  $a_e$ . For  $M > 2m$  the contribution of the tensor term is always positive and decreases with  $M$  while the pseudotensor term leads to a negative contribution to the anomalous magnetic moment.

The presented distributions indicate that with an appropriate choice of the parameters, the difference between the experimental and theoretical value for  $\Delta a_\mu$  [2,3] can be completely explained by one or more bosons of mass around 20 MeV with tensor interactions and an interaction constant  $\epsilon$  of the order of  $10^{-3} - 10^{-4}$ .

If we assume that the whole discrepancy in  $a$  is due to a tensor interaction with  $A_4$ , (i.e.  $\Delta a_\mu = \delta_4 \Gamma$ ), the central values of the parameters  $\epsilon^2$  and  $M$  are given by the blue line in Fig. 5. The allowed parameters space covers  $\epsilon^2$  of the order  $\mathcal{O}(10^{-7} - 10^{-6})$  and  $M \leq 35$  MeV. This region is consistent in mass with the observed anomaly in  ${}^8\text{Be}$ , where  $M \simeq 17$  MeV, while the preferred range of the coupling constant is still to be determined. In the presence of additional contribution to  $\Delta a_{\mu,e}$  the corresponding lines should be considered as upper limits. However, one should note that in the presence of new bosons interacting with pseudotensor currents the

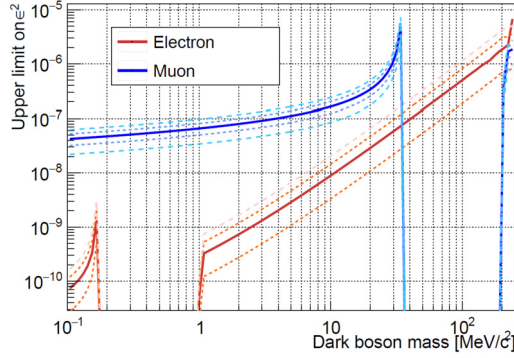


Fig. 5. Allowed region of the DB boson parameters assuming that  $A_4$  is responsible for the whole contribution to  $\Delta a$ . The solid line denotes the preferred region from the central value of  $\Delta a_{\mu,e}$  while the dashed/dot line indicates the  $\pm 1.2 \sigma$ .

parameter space is four dimensional and there are regions in which the positive contribution from the tensor interaction is compensated by a negative contribution from the pseudotensor currents.

#### 4. Sensitivity to $A_1$ and $A_4$ production in positron-on-target annihilation experiments

The preferred by the  $\Delta a_\mu$  parameter space ( $\epsilon^2 \sim \mathcal{O}(10^{-7} - 10^{-6})$  and  $M \leq 35$  MeV) makes it extremely attractive to probe the existence of new light particles with (pseudo)tensor interaction with the SM leptons in direct studies of the lepton interactions. Recently, new direction has started in precise study of the annihilation products of accelerated positrons. Such type of experiments are sensitive to  $A_i$  through the process  $e^+e^- \rightarrow \gamma A_i$ .

The cross section for the process  $e^+e^- \rightarrow \gamma A_1$  follows from the lagrangian (eq. (3)) and is

$$\frac{d\sigma_1}{dz} = \frac{4\pi\epsilon^2\alpha^2}{s} \left( \frac{s - M^2}{2s} \frac{1 + z^2}{1 - \beta^2 z^2} + \frac{2M^2}{s - M^2} \frac{1}{1 - \beta^2 z^2} \right), \tag{10}$$

$$\sigma_1 = \frac{8\pi\alpha^2\epsilon^2}{s} \left[ \left( \frac{s - M^2}{2s} + \frac{M^2}{s - M^2} \right) \log \frac{s}{m^2} - \frac{s - M^2}{2s} \right], \tag{11}$$

where  $s$  is the invariant mass squared,  $\alpha$  is the fine structure constant,  $\beta = \sqrt{1 - \frac{4m^2}{s}}$  and  $z = \cos\theta$ .

The differential cross-sections  $d\sigma_{4,5}/dz$  for the processes  $e^+e^- \rightarrow \gamma A_{4,5}$  are identical. The values were obtained using CalcHEP. For positron-on-target annihilation with positron momentum  $p_{e^+} = 550$  MeV, the dependence of the total cross-section as a function of the dark boson mass is shown in Fig. 6. The total cross-section  $\sigma_{4,5}$  increases both for small masses  $M$  and when the mass of the dark boson approaches the invariant mass limit ( $M \rightarrow \sqrt{s}$ ). The behaviour at low  $M$  differs significantly for  $A_1$  and  $A_{4/5}$  due to the extra factor  $|q|$  in the (pseudo)tensor terms.

The following studies were performed by selecting a benchmark point,  $M = 17$  MeV, motivated by the existing anomaly in  $^8\text{Be}$  and  $^4\text{He}$ . The recoil photon energy distribution is shown in Fig. 7 left. Due to its higher mass, most of the initial state energy is taken by the dark boson. In a typical experiment the recoil photon could only be detected in a limited opening angle interval [24]. The two-body kinematics of the events translates the benchmark angular interval  $10 \text{ mrad} \leq \theta_\gamma \leq 90 \text{ mrad}$  to an energy interval  $50 \text{ MeV} \leq E_\gamma \leq 254 \text{ MeV}$  of the energy of the

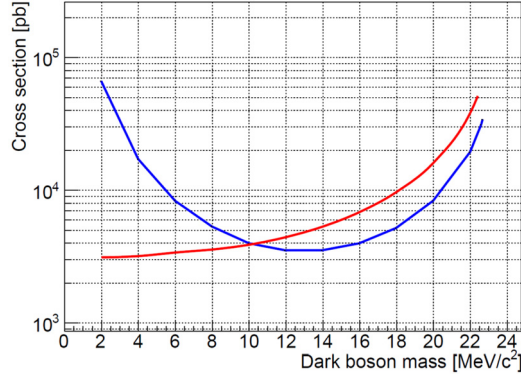


Fig. 6. Total cross-sections for the dark boson production in positron-on-target annihilation, with  $p_{e^+} = 550$  MeV and  $\epsilon = 10^{-3}$  for vector (red) and (pseudo)tensor (blue) interaction.

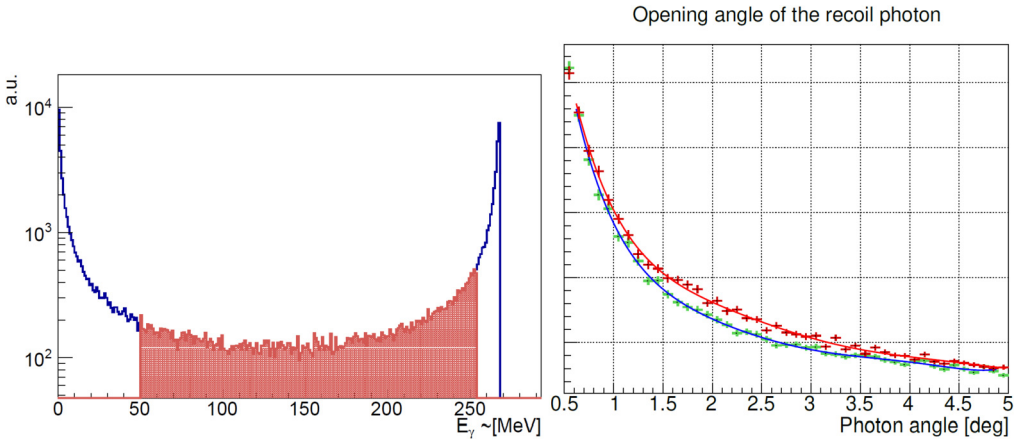


Fig. 7. Left: Gamma energy distribution for  $e^+e^- \rightarrow \gamma + A_4$  with  $M = 17$  MeV for all events (blank histogram) and for the events with the recoil photon in the acceptance of the experimental complex (filled area). Right: Angular distribution of the recoil photon in  $e^+e^- \rightarrow \gamma A_i$  for vector ( $A_1$ , green) and tensor ( $A_{4/5}$  red) interaction of the dark boson.

photon (shown with filled area). Assuming near 100% detection efficiency, the resulting geometrical acceptance  $Acc_{A_{4/5}}$  is about 33%.

The angular distributions for the vector and (pseudo)tensor case slightly differ, as can be seen in Fig. 7, right. This difference is mostly pronounced in the region  $20 \text{ mrad} \leq \theta_\gamma \leq 70 \text{ mrad}$ , which coincides with the sensitive region of the PADME experiment. This could allow a single experiment to determine both the interaction constants and the type of the interaction of the dark boson, in case a positive signal is observed.

### 5. Conclusion

In this work we consider a phenomenological model of interaction between leptons and dark sector particles. The study is motivated by the observed discrepancy in the anomalous magnetic moment of the muon which interpretation may be well achieved incorporating a new dark sector of particles. We investigate the possibility of the existence of a dark boson having a tensor and

pseudo-tensor interactions with the fermions. The proposed interaction introduces terms dependent on the lepton mass beyond just the phase space difference and may also manifest itself in the muon anomalous magnetic moment. Such a dark boson can be produced in electron-positron interactions, and be detected in positron-on-target annihilation experiments. The described results are applicable to present (e.g. PADME) and to future (e.g. SeDS in Brasil [25] and others [27]) positron-on-target annihilation experiments whose sensitivity addresses directly the possible simultaneous explanation of  $\Delta a_\mu$  and  $^8Be$  anomaly. In addition, a dark boson with mass  $M \leq 35$  MeV can even be probed at hadron and heavy ion colliders with detectors allowing access to low dilepton invariant mass region, for example at ALICE experiment at CERN LHC [28]. While the present work focuses on particular leptonic processes involving (pseudo)tensor interactions, numerous different experimental studies could potentially be sensitive to the presence of  $A_{4/5}$  and thus restrict, even significantly limit, the parameter space.

### CRediT authorship contribution statement

**V. Kozhuharov:** Visualization, Writing – review & editing. **M. Naydenov:** Conceptualization, Methodology.

### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

### Data availability

Data will be made available on request.

### Acknowledgements

The authors would like to thank Mihail Chizhov and Tsvetan Vetsov for the valuable discussions. This work was partially supported by the Bulgarian National Programme for “Young scientists and Postdocs” RD-22-1730/11.10.2022 and BNSF through COST action CA21106 - COSMIC WISPerS in the Dark Universe.

### References

- [1] J. Schwinger, On quantum-electrodynamics and the magnetic moment of the electron, *Phys. Rev.* 73 (1948) 416–417.
- [2] L. Morel, Z. Yao, P. Clade, S. Guellati-Khelifa, Determination of the fine-structure constant with an accuracy of 81 parts per trillion, *Nature* 588 (2020) 61–65.
- [3] B. Abi, et al., Measurement of the positive muon anomalous magnetic moment to 0.46 ppm, *Phys. Rev. Lett.* 126 (2021) 141801.
- [4] Muon  $g - 2$  (E989) Collaboration, Status of the Fermilab muon  $g - 2$  experiment, arXiv:2202.11391v1.
- [5] Muon  $g - 2$  Collaboration, Final report of the E821 muon anomalous magnetic moment measurement at BNL, *Phys. Rev. D* 73 (2006) 072003.
- [6] S. Borsanyi, Z. Fodor, J.N. Guenther, et al., Leading hadronic contribution to the muon magnetic moment from lattice QCD, *Nature* 593 (2021) 51–55.
- [7] K. Cheung, Muon anomalous magnetic moment and leptoquark solutions, *Phys. Rev. D* 64 (2001) 033001.



- [8] M. Davier, W. Marciano, The theoretical prediction for the muon anomalous magnetic moment, *Annu. Rev. Nucl. Part. Sci.* 54 (2004) 115–140.
- [9] H. Li, P. Wang, Solution of lepton  $g - 2$  anomalies with nonlocal QED, arXiv:2112.02971.
- [10] A. Brignole, E. Perazzi, F. Zwirner, On the muon anomalous magnetic moment in models with a superlight gravitino, *J. High Energy Phys.* 1999 (1999).
- [11] C. Chiang, R. Obuchi, K. Yagyu, Dark sector as origin of light lepton mass and its phenomenology, arXiv:2202.07784.
- [12] E. Ma, Radiative origin of all quark and lepton masses through dark matter with flavor symmetry, arXiv:2202.07784v1.
- [13] S. Gninenko, N. Krasnikov, Leptonic scalar portal: the origin of muon  $g - 2$  anomaly and dark matter?, arXiv:2202.04410.
- [14] S. Gninenko, N. Krasnikov, The muon anomalous magnetic moment and a new light gauge boson, *Phys. Lett. B* 513 (2001) 119–122.
- [15] A.J. Krasznahorkay, M. Csatlós, L. Csige, Z. Gácsi, J. Gulyás, M. Hunyadi, I. Kuti, B. Nyakó, L. Stuhl, J. Timár, T. Tornyi, Zs. Vajta, T. Ketel, A. Krasznahorkay, Observation of anomalous internal pair creation in  $^8\text{Be}$ : a possible indication of a light, neutral boson, *Phys. Rev. Lett.* 116 (2016) 042501.
- [16] L. Rose, S. Khalil, S. King, S. Moretti, New physics suggested by Atomki anomaly, *Front. Phys.* 7 (2019).
- [17] X. Zhang, G. Miller, Can a protophobic vector boson explain the ATOMKI anomaly?, *Phys. Lett. B* 813 (2021) 136061.
- [18] O. Seto, T. Shimomura, Atomki anomaly in gauged  $U(1)$  R symmetric model, *J. High Energy Phys.* 2021 (2021) 25.
- [19] B. Holdom, Two  $U(1)$ 's and epsilon charge shifts, *Phys. Lett. B* 166 (1986) 196–198.
- [20] M. Chizhov, Vector meson couplings to vector and tensor currents in extended NJL quark model, *J. Exp. Theor. Phys.* 80 (2004) 73–77.
- [21] T. Eguchi, New approach to collective phenomena in superconductivity models, *Phys. Rev. D* 14 (1976) 2755–2763.
- [22] Y. Nambu, G. Jona-Lasinio, Dynamical model of elementary particles based on an analogy with superconductivity. II, *Phys. Rev.* 124 (1961) 246–254.
- [23] M. Naydenov, V. Kozhuharov, Dark boson mediation of the  $\pi^0 \rightarrow \gamma e^+ e^-$  decay, *Nucl. Phys. B* 115723 (2022).
- [24] M. Raggi, V. Kozhuharov, Proposal to search for a dark photon in positron on target collisions at DAΦNE linac, *Adv. High Energy Phys.* 509 (2014) 959802.
- [25] L. Duarte, L. Lin, M. Lindner, V. Kozhuharov, S.V. Kuleshov, A.S. de Jesus, F.S. Queiroz, Y. Villamizar, H. Westfahl, Search for dark sector by repurposing the UVX Brazilian synchrotron, arXiv:2206.05305 [hep-ph].
- [26] H. Patel, Package-X: a Mathematica package for the analytic calculation of one-loop integrals, *Comput. Phys. Commun.* 197 (2015) 276–290.
- [27] L. Marsicano, et al., Dark photon production through positron annihilation in beam-dump experiments, *Phys. Rev. D* 98 (2018) 015031.
- [28] David d'Enterria, et al., Opportunities for new physics searches with heavy ions at colliders, arXiv:2203.05939 [hep-ph].