Radiative neutrino mass via fermion kinetic mixing

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We propose that the radiative generation of the neutrino mass can be achieved by incorporating the kinetic mixing of fermion fields which arises radiatively at one-loop level. As a demonstrative example of the application of the mechanism, we present the particular case of the Standard Model extension by $U(1)_D$ symmetry. As a result, we show how neutrino masses can be generated via a kinetic mixing portal instead of a mass matrix with residual symmetries responsible for the stability of multicomponent dark matter.

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In the Standard Model (SM) of electroweak (EW) interactions, neutrinos are predicted to be massless. There are many extensions of the SM that generate Majorana neutrino masses via the Weinberg dimension-five operator [1] $LHLH/\Lambda$ realized at tree level [2–10], and one-loop level [11–16], as well as Dirac neutrino masses [17,18].

In this paper, we introduce a new possibility that kinetic mixing of fermions occurs radiatively and it leads to the radiative generation of neutrino masses. In order to present the idea of kinetic mixing of fermions that induces the radiative generation of neutrino masses, we introduce the minimal set of fields in the context of $U(1)_D$ dark gauge symmetry extension of the SM. The field content for the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_D$ gauge symmetry case is shown in Table I, where all fermions are presented by left-handed fields. The last column in Table I shows the number of copies beside the flavor count. The quantum number assignments for these fields are presented in Table I.

First of all, let us show how the kinetic mixing between two new fermions A_L and C_L^i can be generated. In order for the kinetic mixing to occur radiatively, we need mediators which are a massive fermion Ψ_L and new scalars $s_{7,11}$. In addition, a scalar ϕ is introduced so as to spontaneously break $U(1)_D$ and then $\langle\phi\rangle = v_\phi/\sqrt{2}$. The new interaction terms leading to the kinetic mixing between A_L and C_L^i are given by

$$-\mathcal{L}_{\rm KM} = \Psi_{La} Y_A^{ab} A_{Lb} s_{11} + \Psi_{La} Y_{C^i}^{ab} C_{Lb}^i s_7^* + \mu_3 \phi s_{11} s_7 + \text{H.c.} \qquad (1)$$

Note that all terms which could give A_L a mass, even at one-loop order, are forbidden by symmetry, whereas the mass terms for C_L^i are allowed. The Feynman diagram representing the fermion kinetic mixing mechanism is shown in Fig. 1. In general, all five copies of C_L can kinetically mix with A_L, but without loss of generality and for the sake of simplicity, we take the bases of C_L fermions

TABLE I. Particle content for the $\mathcal{G}_{\rm SM} \otimes U(1)_D$ model. The bold values indicate that the numbers given are irreducible representations of the non-Abelian groups.

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_D$	Flavors	Copies
Q	**3**	**2**	$\frac{1}{6}$	0	3	1
u^c	$\bar{\mathbf{3}}$	**1**	$-\frac{2}{3}$	0	3	1
d^c	$\bar{\mathbf{3}}$	**1**	$\frac{1}{3}$	0	3	1
L	**1**	**2**	$-\frac{1}{2}$	0	3	1
e^c	**1**	**1**	1	0	3	1
H	**1**	**2**	$\frac{1}{2}$	0	1	1
A_L	**1**	**1**	0	3	3	1
C_L	**1**	**1**	0	1	3	5
N_L	**1**	**1**	0	-4	3	1
N_R^c	**1**	**1**	0	4	3	1
S_L	**1**	**1**	0	-2	3	4
Ψ_L	**1**	**1**	0	$\frac{5}{2}$	3	1
Ψ_R^c	**1**	**1**	0	$-\frac{5}{2}$	3	1
η_L	**1**	**2**	$-\frac{1}{2}$	3	1	1
η_D	**1**	**1**	0	-1	1	1
ϕ	**1**	**1**	0	2	1	1
s_7	**1**	**1**	0	$\frac{7}{2}$	1	1
s_{11}	**1**	**1**	0	$-\frac{11}{2}$	1	1

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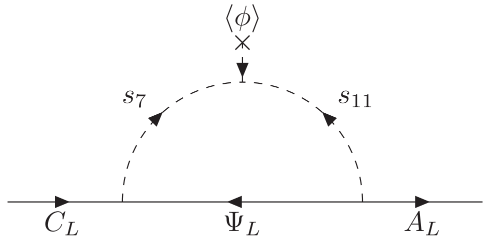

FIG. 1. Radiative kinetic mixing between C_L and A_L in the $G_{\mathrm{SM}} \times U(1)_D$ gauge symmetry case.

in which only one particular C_L^i mixes with A_L. It is worthwhile to notice that ϕ carries even Q_D charge in order to generate a residual $\mathbb{Z}_2$ symmetry upon breaking of $U(1)_D$, which stabilizes the dark matter (DM) candidate. This residual symmetry is desirable to prevent the collapsing of the loop down to tree level, i.e., to prevent the s_7 and s_{11} from obtaining the vacuum expectation values (VEVs). In our case, it is achieved by choosing the specific Q_D charge assignments for the fields in the loop. The other new fields are required to generate neutrino masses and for cancellation of chiral anomalies, as will be shown later. The result of Fig. 1 produces the effective kinetic mixing between two fermion fields which leads to the Lagrangian kinetic term $\imath a \bar{A}_L \partial\!\!\!/ C_L^i + \mathrm{H.c.}$, where a represents the loop structure and will be given in Eq. (5). All relevant kinetic terms are presented by

$$\mathcal{L} = \imath (\bar{A}_L \quad \bar{C}_L^i) \partial\!\!\!/ \begin{pmatrix} 1 & a \\ a^* & 1 \end{pmatrix} \begin{pmatrix} A_L \\ C_L^i \end{pmatrix}. \tag{2}$$

In order to bring the kinetic terms into canonical form, the first step is to rotate by $\pi/4$ so that the kinetic matrix becomes diagonalized. Next, renormalization of the corresponding fermion fields is required for the kinetic terms to be properly normalized. Finally, we need to diagonalize back the mass matrix of the fermions A_L and C_L^i in the new basis of properly normalized mass eigenstates F_{1L} and F_{2L}. This rotation will differ from $\pi/4$ due to the presence of rescaling. The relation between (A_L, C_L^i) and (F_{1L}, F_{2L}) is given by

$$\begin{pmatrix} A_L \\ C_L^i \end{pmatrix} = U(\pi/4, \delta)^\dagger R^{-1} U(\alpha, 0)^\dagger \begin{pmatrix} F_{1L} \\ F_{2L} \end{pmatrix} \tag{3}$$

$$= \begin{pmatrix} 1 & -\frac{\epsilon}{\sqrt{1-\epsilon^2}} \\ 0 & \frac{\epsilon}{\sqrt{1-\epsilon^2}} \end{pmatrix} \begin{pmatrix} F_{1L} \\ F_{2L} \end{pmatrix}. \tag{4}$$

Here, $U(\theta, \delta)$ is the unitary 2×2 transformation with mixing angle θ and relative phase change δ, $\mathrm{Im}[\epsilon] = \mathrm{Im}[a e^{-\imath 2\delta}] = 0$, $\tan 2\alpha = -\sqrt{1-|\epsilon|^2}/|\epsilon|$, and $R = \mathrm{Diag}(\sqrt{1-|\epsilon|}, \sqrt{1+|\epsilon|})$. a is defined as

$$a = \frac{1}{16\pi^2} Y_A^* [s_{sR} c_{sR} G(y_{s2R}, y_{s1R}) + s_{sI} c_{sI} G(y_{s1I}, y_{s2I})] Y_C, \tag{5}$$

$$G(y_i, y_j) = \frac{[y_i(y_i/2 - 1)\ln y_i - y_j(y_j/2 - 1)\ln y_j}{(1-y_i)^2(1-y_j)^2} + y_i y_j (y_i y_j + 4) \ln\left(\frac{y_i}{y_j}\right) \Big/ 2 + \frac{1}{2}\{y_i(y_i - 1) - y_j(y_j - 1) + y_i y_j (y_j - y_i)\}], \tag{6}$$

where $y_i = m_{s_i}^2 / M_\Psi^2$, and Yukawa couplings are given with the flavor indices suppressed. Here, M_Ψ is the Dirac mass of Ψ_L, and $s_{sR(sI)}$ and $c_{sR(sI)}$ stand for the sinus and cosinus, respectively, corresponding to the mixing between the real (imaginary) components of s_7 and s_{11}, which is proportional to μ_3 in Eq. (1). The final form of the relevant Lagrangian is $\imath F_{1L} \partial\!\!\!/ F_{1L} + \imath \bar{F}_{2L} \partial\!\!\!/ F_{2L} - M_{F_2} F_{2L} F_{2L}$, and the mass eigenvalues are $M_{F_1} = 0$ and $M_{F_2} = m_C \exp[\imath 2\delta]/(1 - |\epsilon|^2)$, where m_C is the mass of C_L^i.

Now, let us consider how Dirac neutrino mass can be radiatively generated. For our aim, lepton doublet L couples only to A_L, and N_L is needed as a Dirac mass partner for ν_L. A fermion field Ψ_R is added to produce a Dirac mass for Ψ_L. Also, new scalar fields η_L and η_D are added for the generation of Dirac *radiative* neutrino mass (see Fig. 2). The new interaction terms leading to the radiative generation of Dirac neutrino mass are given by

$$-\mathcal{L}_{\mathrm{DM}} = L_{ai} Y_L^{ab} A_{Lb} \eta_L^{\dagger i} + A_{La} Y_N^{ab} N_{Lb} \eta_D^* + C_{La}^i Y_{C^i C^j}^{ab} C_{Lb}^j \phi^* + \mu_D \eta_D^2 \phi + \lambda_{H\eta\phi} H_i \eta_{Lj} \eta_D \phi^* \epsilon^{ij} + \lambda_{s\eta} \eta_D^2 s_7^* s_{11}^* + \mathrm{H.c.}, \tag{7}$$

where H develops a nonzero VEV, $\langle H^0 \rangle = v/\sqrt{2}$. Note that the $\lambda_{H\eta\phi}$ quartic scalar term, which mixes η_L^0 with η_D, is needed to generate Dirac radiative neutrino mass. Figure 2

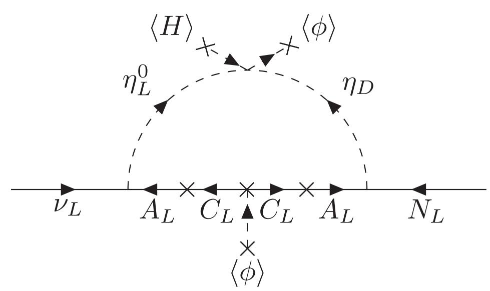

FIG. 2. Radiative neutrino mass generation via fermion kinetic mixing in the $G_{\mathrm{SM}} \otimes U(1)_D$ gauge symmetry case. Crosses between the A_L and C_L fields correspond to kinetic mixing given in Fig. 1.

represents the radiative neutrino mass generation via kinetic mixing for $G_{\rm SM}\times U(1)_D$ gauge symmetry. Thus, neutrino masses are generated via the effective dimension-eight LH $N_L\phi^2(\phi^*\phi)$ operator at three-loop order after H and ϕ develop nonzero vacuum expectation values after spontaneous symmetry breaking.

The neutrino Dirac mass corresponding to the mixing between ν_L and N_L [see Eq. (12)] is given by

$$m_{\rm loop}=\frac{Y_N M_{F_2} Y_L \epsilon^2}{16\pi^2}[s_{\eta R}c_{\eta R}F(x_{1R},x_{2R})+s_{\eta I}c_{\eta I}F(x_{2I},x_{1I})], \tag{8}$$

where $s_{\eta R(\eta I)}$, $c_{\eta R(\eta I)}$ stand for the sinus and cosinus, respectively, corresponding to the mixing between the real (imaginary) components of η_L^0 and η_D. $F(x_i,x_j)$ is defined as

$$F(x_i,x_j)=\frac{x_i\ln x_i - x_j\ln x_j + x_i x_j\ln\frac{x_j}{x_i}}{(1-x_i)(1-x_j)}, \tag{9}$$

with $x_i=\frac{m^2_{\eta_i}}{M^2_{F_2}}$. The extra suppression from ϵ in $m_{\rm loop}$ allows for a wider range of masses, mass splittings, and Yukawa couplings.

For completeness of the model with $\mathcal{G}_{\rm SM}\otimes U(1)_D$ gauge symmetry, we require that $SU(3)_C\times SU(2)_L\times U(1)_Y$ triangular anomalies are canceled in the same way as in the canonical SM case. Since there are no fermions that transform nontrivially under the SM and dark sector simultaneously, any cross anomalies between the SM and $U(1)_D$ are trivially absent. The only anomalies to consider for cancellation are $U(1)_{D\rm Grav}$ and $[U(1)_D]^3$. For this purpose, multiple copies of C_L as well as new fermions $S_L^i (i=1,..,4)$, N_R^c, and Ψ_R^c are introduced. Considering $U(1)_D$ sector anomalies, they cancel in the following way: $\sum_i Q_{Di}=1\times(3)+5\times(1)+4\times(-2)+1\times(-4)+1\times(4)+1\times(\frac{5}{2})+1\times(-\frac{5}{2})=0$, $\sum_i Q^3_{Di}=1\times(3)^3+5\times(1)^3+4\times(-2)^3+1\times(-4)^3+1\times(4)^3+1\times(\frac{5}{2})^3+1\times(-\frac{5}{2})^3=0$. In addition to $\mathcal{L}_{\rm SM}$, $\mathcal{L}_{\rm KM}$, and $\mathcal{L}_{\rm DM}$, we introduce invariant mass terms of $\Psi_{L(R)}$ and $N_{L(R)}$ and extra new interaction terms given by

$$\begin{aligned}-\mathcal{L}_{\rm extra}&=A_{La}Y^{ab}_{AS^i}S^i_{Lb}\eta_D+C^i_{La}Y^{ab}_{C^iS^j}S^j_{Lb}\eta_D^*\\&\quad+\bar{S}^i_{La}Y^{ab}_{NS^i}N_{Rb}\phi+\text{H.c.},\end{aligned} \tag{10}$$

where C_L^i and S_L^j run over 1–5 and 1–4, respectively.

After EW and dark symmetry breaking due to the symmetry and field content of the model, we obtain two residual dark $\mathbb{Z}_2$ symmetries which are *not ad hoc*. The first $\mathbb{Z}_2$ symmetry is analogous to the one from the canonical scotogenic model, but here is it obtained from $U(1)_D$ spontaneous symmetry breaking. The other $\mathbb{Z}_2$ symmetry is new and present here due to fractional charge assignments of the particles involved in the kinetic mixing. This gives us the opportunity for the multicomponent DM case. SM fields, $N_{L,R}$, S_L, and ϕ fields transform trivially, fields with $(-1)^{Q_D(=\rm odd)}$ which are η_L, η_D, A_L, C_L^i transform as $(-,+)$, scalar fields with fractional Q_D charges, i.e., s_7, s_{11}, transform as $(+,-)$, and $\Psi_{L,R}$ transform as $(-,-)$ under both $\mathbb{Z}_2$ symmetries, respectively.

Let us present the mass spectrum of the new fields we introduced.

Fermions: We need to consider three different sectors that do not mix with each another. First, the (ν_L,N_L,N_R^c,S_L) sector, which is $(+,+)$ under $\mathbb{Z}_2^{1,2}$. Next, the $\mathbb{Z}_2^1\sim(-)$ odd sector similar to the one present in the canonical scotogenic paper [12], (A_L,C_L^i) fields. Lastly, the $\mathbb{Z}_2^2\sim(-)$ odd fermions, special for this model, due to the presence of kinetic mixing, i.e., $\Psi_{L(R)}$. Starting with the $\mathbb{Z}_2$ even fermions we have and considering the Lagrangian given in Eqs. (7) and (10), we get the following mass matrix for these new fermions and neutrinos

$$\begin{pmatrix} 0 & m_{\rm loop} & 0 & 0\\ m_{\rm loop} & 0 & M_N & 0\\ 0 & M_N & 0 & Y_{NS^i}v_\phi\\ 0 & 0 & Y_{NS^i}v_\phi & 0\end{pmatrix} \tag{11}$$

in the (ν_L,N_L,N_R^c,S_L^i) basis. We choose the basis for S_L^i in which the linear combination of the four S_L^i's that couple to N_R appears in the mass matrix, and the other three orthonormal combinations do not couple to N_R. Before $U(1)_D$ symmetry breaking, $N_{L(R)}$ is a vectorlike fermion with mass M_N, and the neutrinos together with S_L^i are massless. After $U(1)_D$ symmetry breaking, we get one heavy Dirac fermion, mostly $N_{L(R)}$, and most importantly, the neutrino combines with S_L^i to become a Dirac fermion. The eigenvalues, to the leading order in the $m_{\rm loop}\ll M_N$, $Y_{NS^i}v_\phi$ limit, are approximately given by

$$m_\nu\approx\frac{m_{\rm loop}}{\sqrt{1+M_N^2Y^{-2}_{NS^i}v_\phi^{-2}}},\qquad m_H\approx\sqrt{M_N^2+Y^2_{NS^i}v_\phi^2}. \tag{12}$$

Neutrinos are Dirac at leading order. Since the lepton number is violated softly by two units by the $\mu_D\eta_D^2\phi$ trilinear term, neutrinos become pseudo-Dirac when higher-loop corrections are considered. In the case where the $m_{\rm loop}$ does not provide enough suppression for the neutrino masses, the ratio of M_N/v_ϕ can provide extra suppression for the neutrino masses. For example, if $m_{\rm loop}\sim10^{-4}$ GeV, then $M_N/v_\phi\sim10^6$ would give $m_\nu\sim O(0.1\text{ eV})$. The other three S_L^i states orthonormal to the S_L^j state coupled to the N_R fermion obtain their masses radiatively through C_L^i Majorana masses.

It is worthwhile to note that in the $\mathbb{Z}_2^1 \sim (-)$ odd sector, A_L remains massless till neutrinos get their masses, such that A_L mass is generated through Dirac neutrino mass ($m_{\rm loop}$). To generate radiative neutrino masses, we need a fermion mass in the loop, which is M_{F_2} the mass of F_{2L} mass eigenstate. F_{2L} is mixed state of A_L and C_L interaction eigenstates, mostly C_L, whereas the other mass eigenstate, F_{1L} mostly A_L, is still massless. Radiative neutrino masses are generated without M_{F_1}. Therefore, the dominant contribution to the $m_A(M_{F_1})$ comes from the effective four-loop diagram proportional to $m_{\rm loop}$, and m_A is then given by

$$\frac{Y_N m_{\rm loop} Y_L}{16\pi^2}[s_{\eta R} c_{\eta R} F(x_{1R}, x_{2R}) + s_{\eta I} c_{\eta I} F(x_{2I}, x_{1I})], \qquad (13)$$

where $F(x_i, x_j)$ and $x_{i(j)}$ are given in Eq. (9). This is an important point because if A_L obtained its mass in some other way, the neutrinos would generate their masses through A_L's mass, and the kinetic mixing would contribute in the subleading order and be unnecessary. Also, it is important to mention that this predicts one dark fermion to be naturally lighter than the $m_{\rm loop}$ since its mass is one-loop suppressed with respect to the $m_{\rm loop}$ and overall four-loop suppressed. To avoid $m_A < m_\nu$, we can use the freedom of M_N. For instance, setting $M_N/v_\phi \sim 10^6$, $m_{\rm loop} \sim 10^{-4}$ GeV, and Yukawa's of $O(1)$ would give $m_\nu \sim O(0.1\text{ eV})$ and $m_A \sim O(\text{keV})$. The five copies of C_L^i dark fermions obtain their masses through $\langle\phi\rangle$ at tree level by incorporating the diagonalization of the 5×5 mass matrix in the C_L^i basis. The $\mathbb{Z}_2^2 \sim (-)$ odd sector vectorlike fermion Ψ has an invariant mass of M_Ψ.

Gauge bosons: The $U(1)_D$ dark gauge boson gets its mass through a canonical Higgs mechanism during spontaneous symmetry breaking of $U(1)_D$ gauge symmetry in the dark sector. Mass of the dark $U(1)_D$ gauge boson is given by $m_{A_D}^2 = 2g_D^2 v_\phi^2$, and the corresponding would-be Nambu-Goldstone boson is Im$[\phi]$. Because of the absence of scalars with nonzero VEV that simultaneously transform under $G_{\rm SM}$ and dark $U(1)_D$ gauge symmetry, there is no tree level mixing between A_D^μ and SM neutral gauge bosons. Mixing will appear at one-loop order through $\eta_L^{\pm,0}$ running in the loop, but it is loop suppressed and we will ignore the mixing here. The rest of the gauge bosons obtain their masses just like in the SM.

Scalars: The charged Higgs scalar from the SM $H^\pm$ corresponds to the would-be Nambu-Goldstone boson and gets eaten up by $W^\pm$. The other electrically charged scalar $\eta_L^\pm$, which is part of the η_L doublet needed for the neutrino mass generation, does not mix with $H^\pm$ due to the presence of $\mathbb{Z}_2^1$ under which $\eta_L \sim -$ and $H \sim +$. For electrically neutral scalar components, we have three sectors: the mixing of real components of H^0 and ϕ, and the real and imaginary components of (η_L^0, η_D) and (s_7, s_{11}). (H_I^0, ϕ_I) correspond to the longitudinal degrees of freedom of the Z and A_D gauge bosons. The three separate sectors arise due to the $2 \times \mathbb{Z}_2$ symmetries present; the first one is analogous to the canonical scotogenic model and plays the same role here, whereas the second $\mathbb{Z}_2$ symmetry in this case is unique and is present here due to the kinetic mixing mechanism and the fractional charges of the particles involved. It can be thought of as a dark stabilizing symmetry for the dark sector within dark sector of the scotogenic model. In this way, the corresponding neutral scalars can be categorized as $(H^0, \phi) \in \{+,+\}$, $(\eta_L^0, \eta_D) \in \{-,+\}$, and $(s_7, s_{11}) \in \{+,-\}$ under the two $\mathbb{Z}_2$ symmetries. All 2×2 scalar mass matrices can be easily diagonalized using unitary rotations with one angle.

The $G_{\rm SM} \otimes U(1)_D$ model can accommodate the multicomponent DM scenario. The lightest of the particles that transform as $\mathbb{Z}_2^{1,2} \sim (-,+)$ is one component, and the stability is provided by the $\mathbb{Z}_2^1$ symmetry, which is exact. Assuming $M_\Psi > m_{s_{7,11}}$, the second component is the lightest eigenstate of the s_7, s_{11} sector, which transforms as $\mathbb{Z}_2^{1,2} \sim (+,-)$. In this case, $\Psi \sim (-,-)$ under $\mathbb{Z}_2^{1,2}$ would decay into lighter $s_{7,11}$ eigenstates $\sim (+,-)$ and $A_L \sim (-,+)$ through $Y_{A,C}$ Yukawa couplings.

In this paper, we have presented neutrino mass generation via a fermion kinetic mixing mechanism in the context of the anomaly free $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_D$ gauge symmetry. In this case, neutrino Dirac mass is generated after EW and $U(1)_D$ symmetry breaking via the kinetic mixing of two fermions in the dark sector. As a consequence, the neutrino mass is naturally suppressed by the radiative nature of the generation mechanism. This model includes two dark sectors: The first one is similar to the scotogenic scenario, and the second is unique to realize the kinetic mixing mechanism, which allows for the multiparticle DM scenario. Despite presenting the particular example with $G_{\rm SM} \otimes U(1)_D$ gauge symmetry, the kinetic mixing idea is more general and can be realized in cases with other gauge symmetries as well. In principle, the kinetic mixing of fermions does not need to be carried out in the dark sector, and we could kinetically mix neutrinos with other fermions, but this would require us to include sterile neutrinos in the model. Furthermore, in this case, in order to increase the neutral fermion mass matrix rank (to give the neutrino mass), one would need to follow the following scenario: The neutrino mixes with another neutral fermion leading to mass generation of the dark sector. Then using the same diagram, the neutrino would get mass through the mediation of the same dark sector. These and other prospects and phenomenology are among the further possible research directions of this work.

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