CP phases in 2HDM and effective potential: A geometrical view

Qing-Hong Cao,[1,2,3,*] Kun Cheng,[1,†] and Changlong Xu[1,‡]

[1]*Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*
[2]*Collaborative Innovation Center of Quantum Matter, Beijing 100871, China*
[3]*Center for High Energy Physics, Peking University, Beijing 100871, China*

(Received 17 June 2022; accepted 4 January 2023; published 20 January 2023)

Using a geometric description of 2HDM, the *CP*-odd invariants are classified into three independent sectors: scalar potential, Yukawa interaction, and CKM matrix. We calculate the effective thermal potential of 2HDM in a basis invariant way and show that the *CP* phases in the CKM matrix cannot leak to the effective potential at all orders. In the 2HDM with a softly broken Z_2 symmetry, the leading thermal correction to the effective potential tends to restore the *CP* symmetry at high temperatures.

DOI: 10.1103/PhysRevD.107.015016

I. INTRODUCTION

Two-Higgs-Doublet-Model (2HDM) is one of the most straightforward extensions of Standard Model (SM) that can provide both new sources of *CP* violation and strong first-order phase transition [1–6]. It suffers, unfortunately, from the arbitrariness of the scalar basis choice, e.g., a unitary transformation between the two Higgs doublets does not have any physical consequence. In addition to the *CP* phase in Cabbibo-Kobayashi-Maskawa (CKM) matrix [7], the 2HDM can also have *CP* phases from the scalar potential and the Yukawa interactions of an additional scalar [8]. As a global symmetry, the *CP* symmetry is best studied in basis invariant methods, such as the bilinear notation [9–13] or the tensor notation [8,14,15].

The *CP* property of the 2HDM effective potential in the early universe is attractive as it relates to baryogenesis. It is shown that the bilinear notation is also convenient in handling global symmetries of 2HDM effective potential [16], but a complete analysis, including the contributions from the scalar self-interactions, the Yukawa interactions, and the gauge interactions, still needs to be done. In this work, we adopt the bilinear notation to categorize all the independent *CP*-violating sources and extend the bilinear notation to the Yukawa couplings so that the effective potential from the fermion loop contribution can be expressed in the bilinear notation. For the first time, the one-loop 2HDM effective potential is calculated fully by basis invariant form, which enables us to identify various *CP*-violating sources in the loop correction.

The paper is organized as follows. In Sec. II, we briefly review the bilinear notation and provide a geometric picture of the *CP*-odd invariant. In Sec. III, we discuss the *CP* phases in the CKM and show that the phase in the CKM matrix cannot enter the effective scalar potential. In Sec. IV, we present the one-loop effective potential in the bilinear notation. Finally, we conclude in Sec. V.

II. *CP* SYMMETRIES IN THE BILINEAR NOTATION

We start by reviewing the bilinear notation of the 2HDM potential. A general scalar potential is

$$\begin{aligned}V(\Phi_1,\Phi_2) &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - m_{12}^2\Phi_1^\dagger\Phi_2\\ &\quad+\frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2+\frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2\\ &\quad+\lambda_3(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_1)+\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\\ &\quad+\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2+\lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2)\\ &\quad+\lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2)+\text{H.c.},\end{aligned}\tag{1}$$

in which $(m_{12}^2, \lambda_{5,6,7})$ are generally complex while all the other parameters are real. The scalar potential can be reparametrized by an $SU(2)_\Phi$ rotation $\Phi_i' = U_{ij}\Phi_j$ ($i, j = 1, 2$). Making use of the relation between $SU(2)$ and $SO(3)$ groups, the basis transformation can be viewed explicitly in the so-called K-space, in which the $SU(2)_\Phi$ basis transformation $\Phi_i' = U_{ij}\Phi_j$ corresponds to an $SO(3)_K$ rotation. Define a four-vector $K^\mu \equiv \Phi_i^\dagger\sigma_{ij}^\mu\Phi_j = (K_0, \vec{K})^T$, where

*qinghongcao@pku.edu.cn
†chengkun@pku.edu.cn
‡changlongxu@pku.edu.cn

$$\vec{K} = (\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1, i(\Phi_2^\dagger\Phi_1 - \Phi_1^\dagger\Phi_2), \Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2)^T,$$
$$K_0 = \Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2. \quad (2)$$

Under an $SO(3)_K$ rotation $R_{ab}(U) = \frac{1}{2}\mathrm{tr}[U^\dagger\sigma_a U\sigma_b]$, K_0 behaves as a scalar while $\vec{K}$ transforms as a vector, i.e.,

$$(K_0', \vec{K}') = (K_0, R(U)\vec{K}). \quad (3)$$

The scalar potential in the K-space is written as [9,11]

$$V = \xi_0 K_0 + \eta_{00}K_0^2 + \vec{\xi}\cdot\vec{K} + 2K_0\vec{\eta}\cdot\vec{K} + \vec{K}^T E\vec{K}, \quad (4)$$

where

$$\xi_0 \equiv \frac{1}{2}(m_{11}^2 + m_{22}^2), \qquad \eta_{00} = (\lambda_1 + \lambda_2 + 2\lambda_3)/8,$$
$$\vec{\xi} = \left(-\Re(m_{12}^2), \Im(m_{12}^2), \frac{1}{2}(m_{11}^2 - m_{22}^2)\right)^T,$$
$$\vec{\eta} = (\Re(\lambda_6+\lambda_7)/4, -\Im(\lambda_6+\lambda_7)/4, (\lambda_1-\lambda_2)/8)^T,$$
$$E = \frac{1}{4}\begin{pmatrix} \lambda_4 + \Re(\lambda_5) & -\Im(\lambda_5) & \Re(\lambda_6-\lambda_7) \\ -\Im(\lambda_5) & \lambda_4 - \Re(\lambda_5) & \Im(\lambda_7-\lambda_6) \\ \Re(\lambda_6-\lambda_7) & \Im(\lambda_7-\lambda_6) & (\lambda_1+\lambda_2-2\lambda_3)/2 \end{pmatrix}.$$

As the scalar potential is invariant under the $SO(3)_K$ rotation, it demands the coefficients transform covariantly in the dual space of K^μ, i.e., $\xi_0' = \xi_0$, $\eta_{00}' = \eta_{00}$, $\vec{\xi}' = R(U)\vec{\xi}$, $\vec{\eta}\prime = R(U)\vec{\eta}$, and $E' = R(U)ER(U)^T$.

The conventional CP transformation $\Phi_i \to \Phi_i^*$ corresponds to a mirror reflection $K_2 \to -K_2$ in the K-space [9,10,12]. The CP conserving potential satisfies

$$V(\Phi_1, \Phi_2) = V(\Phi_1, \Phi_2)|_{\Phi_i\to\Phi_i^*}, \quad (5)$$

while in the K-space it becomes

$$V(K_0, \vec{K}) = V(K_0, \vec{K})|_{K_2\to -K_2}. \quad (6)$$

It requires that m_{12} and $\lambda_{5,6,7}$ are all real. Denote $\hat{\Pi}$ as the mirror reflection operator that reflects K_2 when acting on $\vec{K}$, i.e., $\hat{\Pi}(K_1, K_2, K_3) = (K_1, -K_2, K_3)$. The potential is invariant under the mirror reflection on the condition that

$$\vec{\xi} = \hat{\Pi}\,\vec{\xi}, \qquad \vec{\eta} = \hat{\Pi}\,\vec{\eta}, \qquad E = \hat{\Pi}E\hat{\Pi}^T, \quad (7)$$

which can be easily understood from a geometrical view. The 3×3 real symmetric tensor E possesses at least three C_2 axes (principal axis) and three symmetry planes. To respect the CP conserving condition in Eq. (7), $(E, \vec{\xi}, \vec{\eta})$ are all invariant under the mirror reflection. Therefore, $\vec{\xi}$ and $\vec{\eta}$ must lie on the same symmetry plane of E; see Fig. 1.

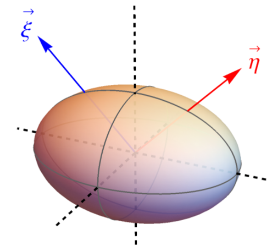

FIG. 1. Configuration of parameter vectors in a CP symmetric potential, where the black dashed line denotes the three principal axes of the ellipsoid.

The symmetric tensor E can be visualized either as an ellipsoid or a hyperboloid when not positive definite.

We can construct independent CP-odd invariants from the geometrical view. For example, two independent conditions for CP conserving scalar potential can be constructed from $\vec{\xi}$, $\vec{\eta}$, and E [12],

$$I_1 = (\vec{\xi}\times\vec{\eta})\cdot E\vec{\xi} = 0, \qquad I_2 = (\vec{\xi}\times\vec{\eta})\cdot E\vec{\eta} = 0. \quad (8)$$

The CP-conserving condition requires all the CP-odd invariants one can construct to vanish. Therefore, I_i are CP-odd invariants that parametrize irremovable CP phases in the scalar potential. In case of $\vec{\xi}$ is collinear with $\vec{\eta}$ along direction $\vec{l}$, another invariant $(\vec{l}\times E\vec{l})\cdot E^2\vec{l}$ can be constructed; see Ref. [12] for details.

Yukawa interaction. The Yukawa couplings of the additional Higgs doublet lead to additional CP phases [8]. The CP phases from Yukawa interactions have yet to be studied in the bilinear notation because the Yukawa coupling terms in the Lagrangian cannot be expressed in bilinear notation. However, we can project the Yukawa couplings coefficients to the bilinear space to study whether the Yukawa couplings spoil the global symmetries in the bilinear space.

We start with Yukawa coupling terms of the u-quark for illustration

$$-\mathcal{L}_{\rm Yuk} \supset \bar{Q}_L y_i \tilde{\Phi}_i u_R + \text{H.c.}, \quad i = 1, 2, \quad (9)$$

where $Q_L = (u_L, d_L)^T$ and y_i are the coefficients of the Yukawa couplings. Consider a scalar basis transformation $U_{ij} \in SU(2)_\Phi$,

$$\Phi_i' = U_{ij}\Phi_j. \quad (10)$$

It is required that the couplings y_i transform as

$$y_i' = U_{ij}y_j, \quad (11)$$

so that the Yukawa coupling terms are also invariant under the Higgs basis transformation,

$$\bar{Q}_L y_i \tilde{\Phi}_i u_R = \bar{Q}_L y'_i \tilde{\Phi}'_i u_R. \tag{12}$$

Since y_i transform covariantly with Φ_i, the vector $Y_\mu \equiv y_i(\sigma_\mu)_{ij} y_j^* = (Y_0, \vec{Y})$ also transforms covariantly with K_μ, i.e., Y_0 is invariant and $\vec{Y}$ transforms as

$$\vec{Y}' = y_i'^*(\vec{\sigma})_{ij} y'_j = y_l^* U_{li}^*(\vec{\sigma})_{ij} U_{jk} y_k = R(U)\vec{Y}. \tag{13}$$

Thus $\vec{Y}$ is a vector in the $SO(3)_K$ space, and any $SO(3)_K$ invariant constructed by $\vec{Y}$ is a weak basis invariant in the Higgs family space. If the Lagrangian preserves the *CP* symmetry, the vector $\vec{Y}$ should be invariant under the mirror reflection $\hat{\Pi}$ and lie on the same reflection plane of $\vec{\xi}$ and $\vec{\eta}$ shown in Fig. 1. For a *CP* conserving potential satisfying Eq. (8), the u-quark Yukawa coupling term in Eq. (9) does not spoil this symmetry only when

$$J_u = (\vec{\xi} \times \vec{\eta}) \cdot \vec{Y} = 0. \tag{14}$$

When $\vec{Y}$ is not in the same plane of $\vec{\xi}$ and $\vec{\eta}$, a nonzero *CP*-odd invariant J_u is generated, and its magnitude is the "volume" of the cross product $(\vec{\xi} \times \vec{\eta}) \cdot \vec{Y}$.

If we include both up and down-type quarks with N generations, the Yukawa couplings for quarks in the 2HDM are

$$\mathcal{L}_{\mathrm{Yuk}} = -\bar{Q}_L^m y_{u,i}^{mn} \tilde{\Phi}_i u_R^n - \bar{Q}_L^m y_{d,i}^{mn} \Phi_i d_R^n + \mathrm{H.c.}, \tag{15}$$

where the superscripts m and n sum over N generations. Under the $SU(2)_\Phi$ basis transformation, $y_{u,i}$ transform like Φ_i while $y_{d,i}$ transform like Φ_i^*. Then we define $2N^2$ covariant vectors in the dual space of K^μ in terms of the Yukawa couplings:

$$Y_{u,\mu}^{mn} \equiv y_{u,i}^{mn*}(\sigma_\mu)_{ij} y_{u,j}^{mn}, \qquad Y_{d,\mu}^{mn} \equiv y_{d,i}^{mn}(\sigma_\mu)_{ij} y_{d,j}^{mn*}, \tag{16}$$

where i, j sum over all Higgs family and m, n do not sum. And Y_μ^{mn} transform in the same way as ξ_μ and η_μ, i.e.,

$$(Y_0, \vec{Y})_{u/d}^{mn} \to (Y_0, R(U)\vec{Y})_{u/d}^{mn}. \tag{17}$$

For the Yukawa interactions of N-generation quarks, we obtain $2N^2$ *CP*-odd invariants J_u^{mn} and J_d^{mn} where $m, n = 1, 2, \ldots, N$.

$$J_{u/d}^{mn} = (\vec{\xi} \times \vec{\eta}) \cdot \vec{Y}_{u/d}^{mn}. \tag{18}$$

To conserve the *CP* symmetry, the vectors $\vec{\xi}$, $\vec{\eta}$, and $\vec{Y}_{u/d}^{mn}$ have to lie on the same reflection plane of E. Each vector acts as an independent *CP* violation source if it departs from the reflection plane.

Counting the numbers of independent *CP*-odd invariants in the 2HDM, there are two from the scalar potential, $2N^2$ from the additional Yukawa interactions, and $(N-1)(N-2)/2$ from the CKM matrix.

III. *CP* PHASES IN THE CKM MATRIX AND EFFECTIVE POTENTIAL

Before calculating the effective potential, we would like to emphasize the difference between the *CP* phases in the additional Yukawa couplings and the *CP* phase of the CKM matrix and discuss the question asked in Ref. [17] that whether a *CP*-conserving 2HDM potential will receive *CP*-violating phases from the CKM matrix through quantum corrections of quarks inside loops. It is pointed out in Ref. [17] that the *CP* violation effect from the Jarlskog invariant may contribute the effective potential through Feynman diagrams with at least three loops, e.g., Fig. 2. However, their calculation of three-loop tadpole diagrams of the *CP*-odd scalar field shows no impact from the Jarlskog invariant. And we will be able to prove that the *CP*-violation effect in the CKM matrix never contributes to the effective potential after discriminating *CP*-violating sources in the Yukawa interactions and the CKM matrix.

The irremovable *CP* phase of the CKM matrix origins from the complex phases in the quark mass matrix, which can be described by the so-called Jarlskog invariant [18,19], a basis invariant quantity under the basis transformations in the quark family space,

$$J = \det[M_u M_u^\dagger, M_d M_d^\dagger]. \tag{19}$$

Next, we write down all the components of the Yukawa couplings and the quark mass matrices to discuss the independence between Eqs. (14) and (19). Because y_1 and y_2 in Eq. (9) are two complex numbers, we can always parametrize them with three angles (β, γ, δ) and a magnitude $|\mathcal{Y}| = \sqrt{|y_1|^2 + |y_2|^2}$. Then (y_1, y_2) and $Y_\mu \equiv y_i^*(\sigma_\mu)_{ij} y_j$ are written as

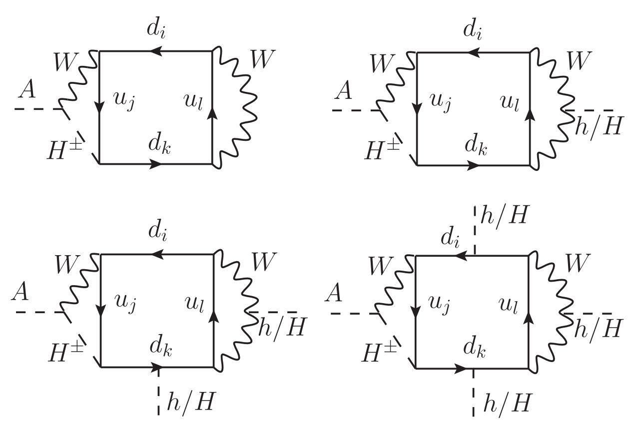

FIG. 2. An example set of the diagrams of the effective potential may receive a contribution from the Jarlskog invariant.

$$(y_1, y_2) = e^{i\delta}|\mathcal{Y}|(s_\beta, c_\beta e^{-i\gamma}), \tag{20}$$

$$(Y_0, \vec{Y}) = |\mathcal{Y}|^2(1, s_{2\beta}c_\gamma, -s_{2\beta}s_\gamma, -c_{2\beta}). \tag{21}$$

Note that the global phase δ yields no information in vector $\vec{Y}$ and is therefore independent of the CP violations in Eq. (18).

We add the quark flavor index and include all three generations of quarks to write down the quark mass matrix and CKM matrix. After the electro-weak symmetry breaking (EWSB), the Higgs doublet field have non-zero expectation value $\langle\Phi_i\rangle = (0, v_i)^T$, $K_v^\mu \equiv K^\mu|_{\Phi_i\to\langle\Phi_i\rangle}$. Each quark mass matrix is an $SU(2)_\Phi$ basis invariant quantity:

$$M_u^{mn} = y_{u,i}^{mn} v_i^*, \qquad M_d^{mn} = y_{d,i}^{mn} v_i. \tag{22}$$

The complex phase of each element of the quark mass matrix, which is related to the global phases δ^{mn} of the Yukawa couplings, is not included in $\vec{Y}$. To show the independence between the CP phases in the quark mass matrix and additional Yukawa couplings, it is most convenient to do the decomposition Eq. (20) in the Higgs basis, where $v_1 = v$ and $v_2 = 0$. Then the complex phases in the quark mass matrices are only determined by δ,

$$M_u^{mn} = e^{i\delta_u^{mn}}\sqrt{\frac{1}{2}(Y_{u,0}^{mn}K_{v,0} + \vec{Y}_u^{mn}\cdot\vec{K}_v)},$$
$$M_d^{mn} = e^{i\delta_d^{mn}}\sqrt{\frac{1}{2}(Y_{d,0}^{mn}K_{v,0} + \vec{Y}_d^{mn}\cdot\vec{K}_v)}. \tag{23}$$

Here m, n in the right-hand side (rhs) do not sum, and the quark mass matrix can be diagonalized as

$$\mathrm{diag}(m_u, m_c, m_t) = V_u^L M_u V_u^R$$
$$\mathrm{diag}(m_d, m_s, m_b) = V_d^L M_d V_d^R,$$
$$V_{\mathrm{CKM}} = (V_u^L)^\dagger V_d^L. \tag{24}$$

We can see that the CP phase of the CKM matrix is determined only by the phases $\delta_{u/d}^{mn}$. The Jarlskog invariant parametrizes the phase $\delta_{u/d}^{mn}$ that cannot be removed by the rotations in the quark family space, while the CP-odd invariants constructed by $\vec{Y}_{u/d}^{mn}$ parametrize the CP phases that the rotations in the Higgs family space cannot remove. Therefore the CP-odd invariants constructed by $\vec{Y}_{u/d}^{mn}$ are independent of the Jarlskog invariant.

In other words, the CP phases in the CKM matrix are from the quark family space, but the CP phases in the scalar potential and the additional Yukawa couplings are from the Higgs family space.

The above arguments help to answer the question of Ref. [17]. We notice that the CP violation effect appears in the effective potential only in the form of $SO(3)_K$ tensors. As the Jarlskog invariant cannot be described by the tensor structure of the $SO(3)_K$ group, we conjecture that *the CP phase in the CKM matrix would not leak into the effective potential through quantum corrections*.

Without loss of generality, consider a CP-conserving Type-I 2HDM in which the $SO(3)_K$ covariant parameter tensors are E, $\vec{\xi}$, $\vec{\eta}$, and $\vec{Y}$. CP conserving conditions require all the tensors invariant under a mirror symmetry $\hat{\Pi}$, i.e.,

$$E_{ab} = E_{cd}\Pi_{ac}\Pi_{bd}, \qquad \xi_a = \xi_c\Pi_{ac}, \qquad \eta_a = \eta_c\Pi_{ac}. \tag{25}$$

Besides, the vectors $\vec{Y}$ also conserve CP symmetry $Y_a = Y_c\Pi_{ac}$, and the only CP-violating source left is the CP phases in the CKM matrix. Any global $SU(2)_L$ invariant effective potential can be written as

$$V_{\mathrm{eff}}(\Phi_i^\dagger\Phi_j)$$
$$= V_{\mathrm{eff}}(K_0, \vec{K})$$
$$= V_{\mathrm{eff}}(K_0, T_a^{(1)}K_a, T_{ab}^{(2)}K_aK_b, T_{abc}^{(3)}K_aK_bK_c, \cdots), \tag{26}$$

where $T_{a_1\cdots a_q}^{(q)}$ transforms as a rank-q tensor under the $SO(3)_K$ rotation. In the K-space, the tensor $T_{a_1\cdots a_q}^{(q)}$ can be constructed by tensor products of tree-level parameter tensors. Moreover, the tensor constructed by $E, \vec{\eta}, \vec{\xi}, \vec{Y}$ is also invariant under the mirror reflection,

$$T_{a_1\cdots a_q}^{(q)} = T_{b_1\cdots b_q}^{(q)}\Pi_{a_1b_1}\cdots\Pi_{a_qb_q}. \tag{27}$$

It guarantees the CP invariance of the effective potential,

$$V_{\mathrm{eff}}(K_0, \vec{K}) = V_{\mathrm{eff}}(K_0, \hat{\Pi}\vec{K}). \tag{28}$$

Even though the quark mass matrix, whose elements are $SU(2)_\Phi$ singlets, may enter $T_{a_1\cdots a_q}^{(q)}$ as a factor, tensor structures of $T_{a_1\cdots a_q}^{(q)}$ remain unchanged and the result of Eq. (28) still holds.

On the other hand, when the additional Yukawa couplings break the CP symmetry, i.e., $Y_a \neq Y_c\Pi_{ac}$, the effective potential may break the CP symmetry because tensors in Eq. (26) that are constructed by $\vec{Y}$ will break the CP symmetry. To summarize, the CP phases of an electroweak global invariant 2HDM effective potential will not receive a contribution from the Jarlskog invariant. Still, it may receive contributions from the CP phases of additional Yukawa interactions.

IV. ONE-LOOP EFFECTIVE POTENTIAL

Next, we calculate the one-loop effective potential and analyze its global symmetries. Using the background method, the one-loop effective potential at zero temperature is [20]

$$V_{\rm eff}(\phi_c) = V_{\rm tree}(\phi_c) + V_{\rm CW}(\phi_c), \tag{29}$$

where $V_{\rm tree}$ denotes the tree level potential and

$$V_{\rm CW}(\phi_c) = \frac{1}{2}\mathbf{Tr}\int\frac{d^4p}{2\pi^4}\ln\left[p^2+\mathbf{M}^2(\phi_c)\right] = \frac{1}{64\pi^2}\sum_i n_i m_i^4(\phi_c)\left[\ln\frac{m_i^2(\phi_c)}{\mu^2}-c_i\right] \tag{30}$$

is the Coleman-Weinberg (CW) potential [21] calculated in the Landau gauge under $\overline{\rm MS}$ scheme; $\mathbf{M}^2$ is the mass matrix with eigenvalues m_i^2; n_i denotes the degree of freedom of the field; and c_i is equal to $5/6$ for gauge bosons and $3/2$ for others.

The effective potential of the 2HDM has been discussed extensively [22–27]. As a usual practice, only neutral or *CP* even components of Higgs boson doublets are treated as background fields, which breaks the $SU(2)_L$ invariance explicitly such that our previous discussions cannot apply to $V_{\rm eff}(\phi_c)$. Therefore the mass matrices need to be evaluated in a global $SU(2)_L$ invariant way to analyze the *CP* property of effective potential in bilinear notation. For that, we take all the components of the Higgs boson doublets to be background fields, and $\Phi_i = (\phi_{i\uparrow}, \phi_{i\downarrow})^T$ should be understood as background fields hereafter.

(1) Contributions from the scalar loops: *In a scenario that the scalar potential preserves the CP symmetry at the tree level, the scalar self-interaction cannot induce CP violation effects in the effective potential*. However, it is hitherto verified only in a specific choice of the scalar basis. Below we provide basis-independent proof.

The mass matrices of the scalar sector cannot be analytically diagonalized; therefore, we use the method in Ref. [28] to derive the bilinear form of the potential. We first expand the trace of logarithm in Eq. (30) as Taylor series and calculate the trace for each term. For example, the first-term yields

$$\mathbf{Tr}(m_S^2) = 8R_\mu A^\mu + 4S_{\mu\nu}\eta^{\nu\mu} = (20\eta_{00}+4{\rm Tr}(E))K_0 + 24\vec{K}\cdot\vec{\eta} + 8\xi_0, \tag{31}$$

where

$$\begin{aligned} S^{\mu\nu} &= R^\mu K^\nu + R^\nu K^\mu - g^{\mu\nu}(RK),\\ S_A^{\mu\nu} &= A^\mu K^\nu + A^\nu K^\mu - g^{\mu\nu}(AK),\\ A_\mu &= 2\eta_{\mu\nu}K^\nu + \xi_\mu,\\ R^\mu &= (1,0,0,0). \end{aligned} \tag{32}$$

This term was also calculated in Refs. [16,28]. Here we finish the calculation of the total Taylor series, and the final result can be expressed as

$$V_{CW}^{(S)} = \mathcal{F}(S^{\mu\nu}\eta_{\nu\rho}, S_A^{\mu\nu}\eta_{\nu\rho}), \tag{33}$$

where $\mathcal{F}$ is a function of the traces of $S^{\mu\nu}\eta_{\nu\rho}, S_A^{\mu\nu}\eta_{\nu\rho}$ and their combinations [29]. If the tree level potential is invariant under the mirror reflection $\hat{\Pi}$ operation, i.e.,

$$V_{\rm tree}(K_0,\vec{K}) = V_{\rm tree}(K_0,\hat{\Pi}\vec{K}), \tag{34}$$

then any combination of the tensors given in Eq. (32) is invariant too. As a result, the $V_{\rm CW}^{(S)}$ is also *CP* invariant, i.e.,

$$V_{\rm CW}^{(S)}(K_0,\vec{K}) = V_{\rm CW}^{(S)}(K_0,\hat{\Pi}\vec{K}), \tag{35}$$

as it should be.

(2) Contributions from the gauge boson loops: The mass matrix of gauge bosons can be diagonalized and written in gauge invariant form directly. The eigenvalues of gauge boson masses obtained from the kinetic terms are

$$\begin{aligned} m_Z^2 &= \frac{g^2}{8}\left((1+t_W^2)K_0 + \sqrt{4t_W^2|\vec{K}|^2+(t_W^2-1)^2K_0^2}\right),\\ m_\gamma^2 &= \frac{g^2}{8}\left((1+t_W^2)K_0 - \sqrt{4t_W^2|\vec{K}|^2+(t_W^2-1)^2K_0^2}\right),\\ m_{W^\pm}^2 &= \frac{g^2}{4}K_0, \end{aligned} \tag{36}$$

where $t_W = \tan\theta_W$, and a massless photon is ensured by the neutral vacuum condition $K_0 = |\vec{K}|$ [9,11]. Therefore, the CW potential from gauge boson loop contributions $V_{\rm CW}^{(G)} = V_{\rm CW}^{(G)}(K_0, |\vec{K}|)$ is spherically symmetric in the K-space,

$$V_{\rm CW}^{(G)}(K_0,\vec{K}) = V_{\rm CW}^{(G)}(K_0,R\vec{K}), \qquad R\in O(3).$$

Furthermore, *any global symmetry exhibited by the tree-level potential cannot be broken by quantum corrections from gauge bosons*.

(3) Contributions from the quark loops: Usually, the dominant correction of quark loops to the effective potential is from the heaviest quark. Nevertheless, we include top and bottom quarks to ensure our calculation is $SU(2)_L$ invariant. The top and bottom quark masses can mix as there are charged background fields. The fermion mass matrix derived from $-\partial^2\mathcal{L}/\partial\bar{\psi}_L^i\partial\psi_R^j$ reads as

$$(\bar{t}_L, \bar{b}_L)\begin{pmatrix} y_{it}\phi_{i\downarrow}^* & y_{ib}\phi_{i\uparrow} \\ -y_{it}\phi_{i\uparrow}^* & y_{ib}\phi_{i\downarrow}\end{pmatrix}\begin{pmatrix} t_R \\ b_R\end{pmatrix}. \tag{37}$$

After singular decomposition $M_{\rm diag} = L^{-1}MR$, two elements of the diagonalized mass matrix are

$$m_{t/b}^2 = \frac{B \pm \sqrt{B^2 + C}}{2}, \tag{38}$$

where B and C, in terms of $Y_\mu = (Y_0, \vec{Y})$ defined in Eq. (16), are

$$B = \frac{1}{2}(Y_{t0} + Y_{b0})K_0 + \frac{1}{2}(\vec{Y}_t + \vec{Y}_b) \cdot \vec{K},$$
$$C = -\frac{1}{2}(Y_t \cdot Y_b)K_0^2 - (Y_{t0}\vec{Y}_b + Y_{b0}\vec{Y}_t)K_0\vec{K} + \frac{1}{2}\vec{K} \cdot (\vec{Y}_t \cdot \vec{Y}_b - Y_{t0}Y_{b0} - \vec{Y}_t \otimes \vec{Y}_b - \vec{Y}_b \otimes \vec{Y}_t) \cdot \vec{K}.$$

The symbol "⊗" means the direct product of two vectors. For illustration, we consider the special case of $y_t \gg y_b$, in which the top quark plays the leading role. The mass square of the top quark is

$$m_t^2 = \frac{1}{4}(Y_{t0}K_0 + \vec{Y}_t \cdot \vec{K}). \tag{39}$$

Consider a scalar potential conserving the CP symmetry at the tree level, i.e., $V_{\text{tree}}(K_0, \vec{K}) = V_{\text{tree}}(K_0, \hat{\Pi}\vec{K})$. When the Yukawa couplings break the CP symmetry, or equivalently, $\vec{Y}_{t/b} \neq \hat{\Pi}\vec{Y}_{t/b}$, m_t^2 is no longer invariant under the mirror reflection and introduces the CP violation effect to the effective potential at one-loop level,

$$V_{\text{CW}}^{(F)}(K_0, \vec{K}) \neq V_{\text{CW}}^{(F)}(K_0, \hat{\Pi}\vec{K}). \tag{40}$$

The CP violation effect is related to $|J_t|$.

Thermal corrections. Previous discussions show that the bilinear notation is convenient for analyzing the effective potential's global symmetries. Next, we include the thermal corrections into consideration. At finite temperature, the effective potential should be written as [20],

$$V_{\text{eff}} = V_{\text{tree}} + V_{\text{CW}} + V_T + V_{\text{daisy}},$$
$$V_T = \sum_i n_i \frac{T^4}{2\pi^2} J_{B/F}(m_i^2/T^2), \tag{41}$$

where V_T is the one-loop approximation result and V_{daisy} is the contribution from Daisy diagrams [30,31]. At a high temperature, the leading contributions of the thermal bosonic function J_B and fermionic function J_F yield

$$V_T^{(G)} \approx \frac{g^2T^2}{32}(3 + t_W^2)K_0, \tag{42}$$

$$V_T^{(F)} \approx -\frac{T^2}{8}\left[(Y_{t0} + Y_{b0})K_0 + (\vec{Y}_t + \vec{Y}_b) \cdot \vec{K}\right], \tag{43}$$

$$V_T^{(S)} \approx \frac{T^2}{6}\left[(5\eta_{00} + tr(E))K_0 + 6\vec{\eta} \cdot \vec{K}\right], \tag{44}$$

which shift the tree-level parameters $\xi_\mu \to \xi_\mu + \Delta\xi_\mu$,

$$\Delta\xi_0 = T^2\left(\frac{g^2}{32}(3 + t_W^2) - \frac{Y_{t0} + Y_{b0}}{8} + \frac{5\eta_{00} + \text{Tr}(E)}{6}\right),$$
$$\Delta\vec{\xi} = \frac{T^2}{8}(8\vec{\eta} - \vec{Y}_t - \vec{Y}_b). \tag{45}$$

And only the contributions from quark and scalar loops can modify global symmetries of the potential in the K-space by shifting the direction of $\vec{\xi}$. The Daisy resummation result is

$$V_{\text{daisy}} = -\frac{T}{12\pi}\sum_{i=\text{bosons}} n_i[\mathcal{M}_i^3(\phi_c, T) - m_i^3(\phi_c)], \tag{46}$$

where $\mathcal{M}_i(\phi_c, T)$ denotes the masses after the ξ_μ shifting given in Eq. (45). Notice that the T-dependent tensor structures of the $\mathcal{M}_i$'s are related only to $\Delta\xi_\mu$ in Eq. (45). Therefore we can conclude that leading thermal corrections only affect the global symmetries of scalar potential by shifting the direction of $\vec{\xi}$.

Softly broken Z_2 symmetry. Finally, we examine the 2HDM with a softly broken Z symmetry which is often studied in literature. The 2HDM often induces a flavor-changing neutral current, which is prohibited by precision measurements. A Z_2 symmetry,

$$\Phi_1 \to -\Phi_1, \qquad \Phi_2 \to \Phi_2, \tag{47}$$

is therefore introduced to forbid the flavor-changing neutral current [32]. The Z_2 symmetry demands $m_{12} = \lambda_6 = \lambda_7 = 0$. The parameter tensors are of the patterns

$$\vec{\xi} = \begin{pmatrix} 0 \\ 0 \\ \# \end{pmatrix}, \qquad \vec{\eta} = \begin{pmatrix} 0 \\ 0 \\ \# \end{pmatrix}, \qquad E = \frac{1}{4}\begin{pmatrix} \# & \# & 0 \\ \# & \# & 0 \\ 0 & 0 & \# \end{pmatrix}, \tag{48}$$

where the symbol "#" denotes combinations of other coefficients. From geometrical perspective, $\vec{\xi}$ and $\vec{\eta}$ are on the third primary axis of the symmetric tensor E, and the Z_2 symmetry is nothing but a 180° rotation (C_2) around the third primary axis, i.e., $C_2\vec{K} = C_2(K_1, K_2, K_3) = (-K_1, -K_2, K_3)$. A Z_2 invariant 2HDM satisfies

$$\vec{\xi} = C_2\vec{\xi}, \qquad \vec{\eta} = C_2\vec{\eta}, \qquad E = C_2EC_2^T. \tag{49}$$

The vectors $\vec{Y}_{u/d}^{mn}$ defined in Eq. (17) are covariant under an $SO(3)_K$ rotation, therefore also satisfy

$$\vec{Y}_{u/d}^{mn} = C_2\vec{Y}_{u/d}^{mn}, \tag{50}$$

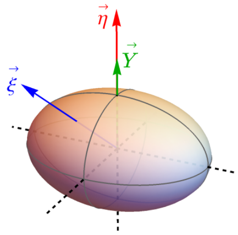

FIG. 3. Parameter vectors for a broken Z_2 symmetry.

and they are parallel to $\vec{\xi}, \vec{\eta}$ and the C_2 axis of E likewise. $\vec{Y}^{mn}_{u/d}$ point to the same direction in Type-I 2HDM, while $\vec{Y}^{mn}_{u}$ and $\vec{Y}^{mn}_{d}$ are in opposite directions in Type-II case. We thus use $\vec{Y}$ to label the direction of $\vec{Y}^{mn}_{u/d}$ for simplicity. A Z_2 symmetric 2HDM Lagrangian is always CP invariant.

The Z_2 symmetry can be softly broken when $m_{12}^2 \neq 0$. The vector $\vec{\eta}$ is not changed, but $\vec{\xi} = (\#, 0, \#)^T$ points to an arbitrary direction; see Fig. 3. Hence, the scalar potential exhibits only one CP invariant, $(\vec{\xi} \times \vec{\eta}) \cdot E\vec{\xi}$, and no CP invariant arises from the Yukawa interactions as $\vec{Y} \| \vec{\eta}$.

As a result, for a tree-level CP-conserving 2HDM with a softly broken Z_2 symmetry, the scalar potential maintains the CP invariance at the loop level. In addition, for a tree-level CP-violating 2HDM with a softly broken Z_2 symmetry, $\vec{\xi}$ is the only CP-violating source. But the leading thermal correction to $\vec{\xi}$ in Eq. (45) is CP conserving as both $\vec{\eta}$ and $\vec{Y}$ lie on the principal axis of E. Therefore, as long as the length of $\vec{\eta}$ and $\vec{Y}$ are not fine-tuned, $\vec{\xi}$ tends to be bent toward the principal axis at sufficiently high temperatures, restoring the Z_2 and CP symmetries. The leading T-dependent contributions from Daisy diagrams, only related to the quartic and Yukawa couplings, do not spoil the CP restoration.

V. CONCLUSION

We generalized the bilinear notation of 2HDM scalar potential to Yukawa couplings by defining dual vectors $\vec{Y}^{mn}_{u/d}$ in bilinear space. By doing so, we obtained all the independent CP-odd invariants in the 2HDM from a geometrical view. The separation of CP phases from the Yukawa interactions and the CKM matrix is made intuitively evident for the first time.

We calculated the Coleman-Weinberg potential in a basis invariant manner. The scalar potential that preserves the CP symmetry at the tree level can receive CP violation corrections *only* from the Yukawa interactions at the one-loop level. We proved that the CP phase in the CKM matrix could not leak to the effective potential at all orders based on the basis invariant form. We further showed that the leading thermal corrections shift the scalar quadratic couplings only with Yukawa and scalar quartic couplings. When a softly broken Z_2 symmetry is imposed, the scalar quadratic terms $\vec{\xi}$ is the only source that breaks Z_2 and CP symmetries. Still, its effect tends to be suppressed by Yukawa couplings and scalar quartic couplings at a large temperature such that the Z_2 and the CP symmetries tend to restore.

ACKNOWLEDGMENTS

We thank Yandong Liu, Jiang-Hao Yu, and Hao Zhang for their valuable suggestions. The work is supported by the National Science Foundation of China under Grants No. 12235001, No. 11725520, No. 11675002, and No. 11635001.

[1] T. D. Lee, Phys. Rev. D **8**, 1226 (1973).
[2] J. F. Gunion and H. E. Haber, Phys. Rev. D **72**, 095002 (2005).
[3] G. C. Branco, M. N. Rebelo, and J. I. Silva-Marcos, Phys. Lett. B **614**, 187 (2005).
[4] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva, Phys. Rep. **516**, 1 (2012).
[5] R. A. Battye, G. D. Brawn, and A. Pilaftsis, J. High Energy Phys. 08 (2011) 020.
[6] A. Pilaftsis, Phys. Lett. B **706**, 465 (2012).
[7] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[8] F. J. Botella and J. P. Silva, Phys. Rev. D **51**, 3870 (1995).
[9] I. P. Ivanov, Phys. Rev. D **75**, 035001 (2007); **76**, 039902(E) (2007).
[10] I. P. Ivanov, Phys. Rev. D **77**, 015017 (2008).
[11] M. Maniatis, A. von Manteuffel, O. Nachtmann, and F. Nagel, Eur. Phys. J. C **48**, 805 (2006).
[12] M. Maniatis, A. von Manteuffel, and O. Nachtmann, Eur. Phys. J. C **57**, 719 (2008).
[13] C. C. Nishi, Phys. Rev. D **74**, 036003 (2006); **76**, 119901(E) (2007).
[14] S. Davidson and H. E. Haber, Phys. Rev. D **72**, 035004 (2005); **72**, 099902(E) (2005).
[15] A. Trautner, J. High Energy Phys. 05 (2019) 208.
[16] I. P. Ivanov, Acta Phys. Pol. B **40**, 2789 (2009), https://www.actaphys.uj.edu.pl/R/40/10/2789.
[17] D. Fontes, M. Löschner, J. C. Romão, and J. a. P. Silva, Eur. Phys. J. C **81**, 541 (2021).
[18] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985).
[19] I. Dunietz, O. W. Greenberg, and D.-d. Wu, Phys. Rev. Lett. **55**, 2935 (1985).

[20] M. Quiros, in ICTP Summer School in High-Energy Physics and Cosmology (1999), pp. 187–259, arXiv:hep-ph/9901312.
[21] S. R. Coleman and E. J. Weinberg, Phys. Rev. D **7**, 1888 (1973).
[22] J. M. Cline, K. Kainulainen, and M. Trott, J. High Energy Phys. 11 (2011) 089.
[23] P. Basler, M. Mühlleitner, and J. Müller, J. High Energy Phys. 05 (2020) 016.
[24] P. Basler, M. Krause, M. Muhlleitner, J. Wittbrodt, and A. Wlotzka, J. High Energy Phys. 02 (2017) 121.
[25] P. M. Ferreira, L. A. Morrison, and S. Profumo, J. High Energy Phys. 04 (2020) 125.
[26] P. Basler and M. Mühlleitner, Comput. Phys. Commun. **237**, 62 (2019).
[27] J. Bernon, L. Bian, and Y. Jiang, J. High Energy Phys. 05 (2018) 151.
[28] A. Degee and I. P. Ivanov, Phys. Rev. D **81**, 015012 (2010).
[29] Q.-H. Cao, K. Cheng, and C. Xu (to be published).
[30] M. E. Carrington, Phys. Rev. D **45**, 2933 (1992).
[31] P. B. Arnold and O. Espinosa, Phys. Rev. D **47**, 3546 (1993); **50**, 6662(E) (1994).
[32] S. L. Glashow and S. Weinberg, Phys. Rev. D **15**, 1958 (1977).