# Study of the $\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$decay with hidden local symmetry model ${ }^{*}$ 

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#### Abstract

Within the hidden local symmetry framework，the Dalitz decay $\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$is studied with the vector meson dominance model．It is found that the partial width $\Gamma\left(\eta^{\prime} \rightarrow \omega \mathrm{e}^{+} \mathrm{e}^{-}\right) \approx 40 \mathrm{eV}$ and branching ratio $\mathcal{B}\left(\eta^{\prime} \rightarrow \omega \mathrm{e}^{+} \mathrm{e}^{-}\right)$ $\approx 2 \times 10^{-4}$ ，and $\Gamma\left(\eta^{\prime} \rightarrow \rho \mathrm{e}^{+} \mathrm{e}^{-}\right) \approx 10 \Gamma\left(\eta^{\prime} \rightarrow \omega \mathrm{e}^{+} \mathrm{e}^{-}\right)$and $\mathcal{B}\left(\eta^{\prime} \rightarrow \rho \mathrm{e}^{+} \mathrm{e}^{-}\right) \approx 10 \mathcal{B}\left(\eta^{\prime} \rightarrow \omega \mathrm{e}^{+} \mathrm{e}^{-}\right)$．The maximum position of the dilepton distribution is $m_{\mathrm{e}^{+} \mathrm{e}^{-}} \approx 1.33 \mathrm{MeV}$ ．These decays are measurable with the advent of high statistic $\eta^{\prime}$ experiments．


Key words：$\eta^{\prime}$ meson，VMD model，decay width，branching ratio
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## 1 Introduction

The $\eta$ and $\eta^{\prime}$ mesons play an important role in un－ derstanding the low energy QCD．They provide a valu－ able place for studying three distinct symmetry breaking patterns simultaneously（the explicit symmetry breaking due to finite quark mass，dynamical spontaneous sym－ metry breaking，and the axial $U(1)$ anomaly）［1］．In addition，they are the eigenstates of G，C，P，namely， $I^{G} J^{P C}=0^{+} 0^{-+}[2]$ ，so their decays are also suitable to test the conservation or breaking of these discrete sym－ metries（such as charge conjugation invariance ${ }^{3)}$ ）in the strong and electromagnetic interactions［3－5］．

The $\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$decay（where $\left.\mathrm{V}=\rho^{0}, \boldsymbol{\omega}\right)^{4)}$ is interest－ ing in several respects．（1）The $\eta^{\prime}$ is the most esoteric meson of the light pseudoscalar nonet because it is closely related to the axial $U(1)$ anomaly of the strong interac－ tions［7］，which is manifested in its heaviest mass and largest width among the pseudoscalar nonet［2］．The ef－ fect of the QCD anomaly should be manifested in the $\eta^{\prime}$ decay modes，besides the $\eta$ and $\pi^{0}$ decay modes ${ }^{5}$ ．From the $\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$decay，we may get some phenomenologi－ cal implication of the anomaly at low energy．（2）Many
model－dependent approaches of low energy mesonic in－ teraction，e．g．whether the vector meson dominance （VMD）is valid in nature，especially，the applicability of the chiral perturbation theory，can be tested via $\eta^{\prime}$ decays．On one hand，the influence of the light vector mesons on the $\eta^{\prime} \rightarrow V$ transition form factor and branch－ ing ratios of $\eta^{\prime}$ decay can be investigated；on the other hand，the electron－positron invariant mass distribution will provide us with some information about the intrin－ sic structure of the $\eta^{\prime}$ meson and momentum dependence of the transition form factor．（3）The two－body strong decays of $\eta^{\prime} \rightarrow \pi \pi, \pi \eta$ are forbidden by $P$ invariance．The electromagnetic decays of $\eta^{\prime} \rightarrow \gamma \pi, \gamma \eta$ are forbidden by $C$ invariance．The main decays of $\eta^{\prime}$ meson fall into two distinctive classes．One class is the hadronic decay into three pseudoscalar mesons due to isospin conservation， such as $\eta^{\prime} \rightarrow \eta \pi \pi$ ．The other is the radiative decay into particles with quantum number $J^{P C}=1^{--}$，such as $\eta^{\prime} \rightarrow \rho \gamma$ ，$\omega \gamma$ ，where $\eta^{\prime} \rightarrow \gamma \gamma$ decay is the second order electromagnetic transition．It is interesting to study the $\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$decay which is related to the two－body ra－ diative decay into the vector meson and photon，but in this case the photon is off－shell．To our knowledge，only

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branching ratio of $\eta^{\prime} \rightarrow \boldsymbol{w e}^{+} \mathrm{e}^{-}$decay is estimated to be $\sim 2 \times 10^{-4}$ [8] or $(1.69 \pm 0.56) \times 10^{-4}$ [9] with the phenomenological VMD model, where many resonance parameters are used to fit the data. In this paper, the relativistic Breit-Wigner form is taken to describe the resonance with the hidden local symmetry (HLS) model [10] which has been tested in great detail [11-14]. The measurement about branching ratios of $\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$decay is not given by the Particle Data Group (PDG) [2] because available data on $\eta^{\prime}$ is relatively scarce for the moment. However, this situation will be ameliorated with the advent of high statistics $\eta^{\prime}$ experiments, such as WASA at COSY, Crystal Ball at MAMI, BESIII at $B E P C$ II, KLOE-2 at DA $\Phi$ NE, and so on [15]. There is a necessity to provide a consistent and uniform theoretical description for the decay $\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$.

## 2 Theoretical framework and decay amplitudes

The HLS model [10] provides a convenient and constraining QCD-inspired framework for studying the phenomenology of light mesons in the low energy regime of strong interactions. In this approach, the pseudoscalar mesons are the Nambu-Goldstone bosons, and the vector
mesons are gauge bosons of a spontaneously broken hidden local symmetry that generates their Higgs-Kibble masses. The anomalous sector (also called WZW [16] and FKTUY [17] Lagrangian) based on HLS allows one to describe the coupling of the form AAP, AVP, VVP, APPP, and VPPP, where A, V, and P denote the electromagnetic field, vector meson, and pseudoscalar, respectively. The explicit expression of the corresponding Lagrangian can be found in [11, 12], e.g. the triangle anomaly Lagrangians can be written as

$$
\begin{align*}
\mathcal{L}_{\mathrm{AAP}} & =-\frac{N_{\mathrm{c}} e^{2}}{4 \pi^{2} f_{\pi}}\left(1-c_{4}\right) \varepsilon^{\mu \nu \alpha \beta} \partial_{\mu} A_{\nu} \partial_{\alpha} A_{\beta} \operatorname{Tr}\left[Q^{2} P\right]  \tag{1}\\
\mathcal{L}_{\mathrm{AVP}} & =-\frac{N_{\mathrm{c}} g e}{8 \pi^{2} f_{\pi}}\left(c_{3}-c_{4}\right) \varepsilon^{\mu \nu \alpha \beta} \partial_{\mu} A_{\nu} \operatorname{Tr}\left[\left\{\partial_{\alpha} V_{\beta}, Q\right\} P\right]  \tag{2}\\
\mathcal{L}_{\mathrm{VVP}} & =-\frac{N_{\mathrm{c}} g^{2}}{8 \pi^{2} f_{\pi}} c_{3} \varepsilon^{\mu \nu \alpha \beta} \operatorname{Tr}\left[\partial_{\mu} V_{\nu} \partial_{\alpha} V_{\beta} P\right] \tag{3}
\end{align*}
$$

where $N_{\mathrm{c}}=3$ is the number of colors; $e^{2}=4 \pi \alpha ; g$ is the universal vector meson coupling constant; $f_{\pi} \approx 92.4 \mathrm{MeV}$ is the pion decay constant [12]; $Q=\operatorname{diag}(2 / 3,-1 / 3,-1 / 3)$ is the quark charge matrix; $P$ is the matrix of the pseudoscalar meson - the Goldstone bosons associated with the spontaneous breakdown of $G_{\text {global }}=U(3)_{\mathrm{L}} \otimes U(3)_{\mathrm{R}}$; and $V$ is the matrix of vector meson - the gauge bosons of the hidden local $U(3)_{\mathrm{V}}$ symmetry [13],

$$
P=\frac{1}{\sqrt{2}}\left(\begin{array}{cccc}
\frac{1}{\sqrt{3}} \eta_{0}+\frac{1}{\sqrt{6}} \eta_{8}+\frac{1}{\sqrt{2}} \pi^{0} & \pi^{+} & K^{+}  \tag{4}\\
\pi^{-} & \frac{1}{\sqrt{3}} \eta_{0}+\frac{1}{\sqrt{6}} \eta_{8}-\frac{1}{\sqrt{2}} \pi^{0} & K^{0} \\
K^{-} & \bar{K}^{0} & \frac{1}{\sqrt{3}} \eta_{0}-\frac{2}{\sqrt{6}} \eta_{8}
\end{array}\right),
$$

$$
V=\frac{1}{\sqrt{2}}\left(\begin{array}{ccc}
\frac{1}{\sqrt{2}}\left(\omega+\rho^{0}\right) & \rho^{+} & K^{*+}  \tag{5}\\
\rho^{-} & \frac{1}{\sqrt{2}}\left(\omega-\rho^{0}\right) & K^{* 0} \\
K^{*-} & \bar{K}^{* 0} & \phi
\end{array}\right) .
$$

The triangle anomaly Lagrangian Eqs. (1)-(3) depend on two parameters $c_{3}$ and $c_{4}$. In our calculation, $c_{3}=c_{4}=1[11,12]^{1)}$, so one can obtain the same predictions as the VMD models in the triangle anomalous sector, i.e. $\mathcal{L}_{\mathrm{AAP}}$ and $\mathcal{L}_{\mathrm{AVP}}$ vanish, and photons can only couple to pseudoscalar mesons via the $V-\gamma$ transitions.

The physical states $\eta$ and $\eta^{\prime}$ are mixtures of the octet $\eta_{8}=\frac{u \bar{u}+d \bar{d}-2 s \bar{s}}{\sqrt{6}}$ and singlet $\eta_{0}=\frac{u \bar{u}+d \bar{d}+s \bar{s}}{\sqrt{3}}$ states.

$$
\binom{\eta}{\eta^{\prime}}=\left(\begin{array}{cc}
\cos \theta & -\sin \theta  \tag{6}\\
\sin \theta & \cos \theta
\end{array}\right)\binom{\eta_{8}}{\eta_{0}},
$$

where $\theta$ is the mixing angle, with $\sin \theta \approx-1 / 3, \cos \theta \approx$ $2 \sqrt{2} / 3$ [18].

Phenomenologically, the Dalitz decay $\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$is regarded as a sequential two-body decay chain, i.e. $\eta^{\prime} \rightarrow \mathrm{V} \gamma^{*} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$. In the triangle anomaly Lagrangians, terms of $\mathcal{L}_{\mathrm{AVP}}$ and $\mathcal{L}_{\mathrm{VVP}}$ contribute to decay $\eta^{\prime} \rightarrow \mathrm{V} \gamma$. The decay amplitude can be written as

$$
\begin{equation*}
\mathcal{A}\left(\eta^{\prime} \rightarrow V \gamma\right)=C_{\eta^{\prime} \vee \gamma} \varepsilon_{\mu \nu \alpha \beta} p_{\gamma}^{\mu} \epsilon_{\gamma}^{\nu} p_{\mathrm{V}}^{\alpha} \epsilon_{\mathrm{V}}^{\beta} \times\left\{\left(c_{3}-c_{4}\right)+2 c_{3}\right\}, \tag{7}
\end{equation*}
$$

where $\epsilon_{\gamma}\left(\epsilon_{\mathrm{V}}\right)$ and $p_{\gamma}\left(p_{\mathrm{V}}\right)$ are the polarization vector and the four-momentum of the on-shell photon (vector meson $\rho^{0}$ and $\omega$ ), respectively; the coefficient $C_{\eta^{\prime}{ }^{\gamma} \gamma}$ contains the information of the $\eta^{\prime} \rightarrow \mathrm{V}$ mesonic transition form factor,

$$
\begin{align*}
C_{\eta^{\prime} \omega \gamma} & =\frac{-N_{\mathrm{c}} g e}{48 \sqrt{3} \pi^{2} f_{\pi}}\left[\frac{f_{\pi}}{f_{8}} \sin \theta+\sqrt{2} \frac{f_{\pi}}{f_{0}} \cos \theta\right],  \tag{8}\\
C_{\eta^{\prime} \rho \gamma} & =3 C_{\eta^{\prime} \omega \gamma}, \tag{9}
\end{align*}
$$

1) As the statement given in [11], the relation $c_{3}=c_{4}=1$ cannot be considered as firmly established without a fully comprehensive fit of all relevant measurements. For example, various fits with different data sets are presented in [11], where one global fit with relatively large probability using ND and CMD data samples prefers $c_{3}=0.927 \pm 0.010$.
with the singlet $\eta_{0}$ and octet $\eta_{8}$ pseudoscalar meson decay constant $f_{0} \approx 1.04 f_{\pi}$ and $f_{8} \approx 1.30 f_{\pi}$ [18]. By the same token, diagrams contributing to the $\eta^{\prime} \rightarrow \mathrm{V} \gamma^{*} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$decay are shown in Fig. 1.


Fig. 1. Diagrams contributing to the $\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$ decay, where (a) is the direct contribution from $\mathcal{L}_{\text {AVP }}$ term, and (b) is the VMD contribution from $\mathcal{L}_{\text {VVP }}$ term.

The corresponding decay amplitude can be written as

$$
\begin{align*}
\mathcal{A}\left(\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}\right)= & C_{\eta^{\prime} \mathrm{V} \gamma} \varepsilon_{\mu \nu \alpha \beta} p_{\gamma^{*}}^{\mu} \frac{\bar{u}_{\mathrm{e}}-\left(-\mathrm{ie} \gamma^{\nu}\right) v_{\mathrm{e}}+}{p_{\gamma^{*}}^{2}+\mathrm{i} \epsilon} p_{\mathrm{V}}^{\alpha} \epsilon_{V}^{\beta} \\
& \times\left\{\left(c_{3}-c_{4}\right)+\frac{2 c_{3}}{1-\frac{p_{\gamma^{*}}^{2}}{m_{\mathrm{V}}^{2}}-\mathrm{i} \frac{\Gamma_{V}}{m_{V}}}\right\} . \tag{10}
\end{align*}
$$

Compared to the two-body decay amplitude Eq. (7), the polarization of the off-shell photon turned into the electromagnetic current $j^{\nu}=\bar{u}_{\mathrm{e}^{-}}\left(-\mathrm{ie} \gamma^{\nu}\right) v_{\mathrm{e}^{+}}$, and the VMD factor is dependent on the invariant momentum $p_{\gamma^{*}}^{2}=m_{\gamma^{*}}^{2}=\left(p_{\mathrm{e}^{+}}+p_{\mathrm{e}^{-}}\right)^{2}=m_{\mathrm{e}^{+} \mathrm{e}^{-}}^{2}$.

## 3 Decay rate and discussion

The partial width of the two-body $\eta^{\prime} \rightarrow \mathrm{V} \gamma$ decay is

$$
\begin{equation*}
\Gamma\left(\eta^{\prime} \rightarrow \mathrm{V} \gamma\right)=\frac{1}{32 \pi}\left(\frac{m_{\eta^{\prime}}^{2}-m_{V}^{2}}{m_{\eta^{\prime}}}\right)^{3}\left|C_{\eta^{\prime} \mathrm{V} \gamma}\right|^{2} \tag{11}
\end{equation*}
$$

The differential width of the three-body $\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$ decay is

$$
\begin{align*}
\mathrm{d} \Gamma\left(\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}\right)= & \frac{1}{(2 \pi)^{5}} \frac{1}{16 m_{\eta^{\prime}}^{2}}\left|\mathcal{A}\left(\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}\right)\right|^{2}\left|\vec{p}_{\mathrm{e}^{+}}^{*}\right| \\
& \cdot\left|\vec{p}_{\mathrm{V}}\right| \mathrm{d} m_{\gamma^{*}} \mathrm{~d} \Omega_{\mathrm{e}^{+}}^{*} \mathrm{~d} \Omega_{\mathrm{V}}, \tag{12}
\end{align*}
$$

where $\left(\left|\vec{p}_{\mathrm{e}^{+}}^{*}\right|, \Omega_{\mathrm{e}^{+}}^{*}\right)$ is the momentum of the lepton $\mathrm{e}^{+}$in the rest frame of the off-shell photon $\gamma^{*}$; and ( $\left.\left|\vec{p}_{\mathrm{V}}\right|, \Omega_{\mathrm{V}}\right)$ is the momentum of the vector meson in the rest frame of the decaying $\eta^{\prime}$ meson,

$$
\begin{align*}
\left|\vec{p}_{\mathrm{e}^{+}}^{*}\right| & =\frac{\lambda^{1 / 2}\left(m_{\gamma^{*}}^{2}, m_{\mathrm{e}}^{2}, m_{\mathrm{e}}^{2}\right)}{2 m_{\gamma^{*}}}=\frac{1}{2} m_{\gamma^{*}} \sqrt{1-\frac{4 m_{\mathrm{e}}^{2}}{m_{\gamma^{*}}^{2}}}=\frac{1}{2} m_{\gamma^{*}} \beta_{\mathrm{e}} \\
\left|\vec{p}_{\mathrm{V}}\right| & =\frac{\lambda^{1 / 2}\left(m_{\eta^{\prime}}^{2}, m_{\gamma^{*}}^{2}, m_{\mathrm{V}}^{2}\right)}{2 m_{\mathfrak{\eta}^{\prime}}} \tag{13}
\end{align*}
$$

where $\lambda(a, b, c)=a^{2}+b^{2}+c^{2}-2 a b-2 b c-2 a c$. Finally, the differential width in terms of the electron-positron invariant mass $m_{\mathrm{e}^{+} \mathrm{e}^{-}}=m_{\gamma^{*}}$ can be written as

$$
\begin{align*}
\mathrm{d} \Gamma\left(\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}\right)= & \frac{\alpha}{96 \pi^{2} m_{\gamma^{*}} m_{\eta^{\prime}}^{3}}\left|C_{\eta^{\prime} \mathrm{V} \gamma}\right|^{2} \lambda^{3 / 2} \\
& \times\left(m_{\eta^{\prime}}^{2}, m_{\gamma^{*}}^{2}, m_{\mathrm{V}}^{2}\right) \beta_{\mathrm{e}}\left(3-\beta_{\mathrm{e}}^{2}\right) \mathrm{d} m_{\gamma^{*}} . \tag{14}
\end{align*}
$$

With the input parameters collected in Table 1 (if not specified explicitly, their central values are taken as the default input), we can get the integrated partial width and the corresponding branching ratio of the two-body electromagnetic radiative $\eta^{\prime}$ decays as follows

$$
\begin{gather*}
\Gamma\left(\eta^{\prime} \rightarrow \omega \gamma\right)=5.426 \pm 0.005_{m_{\eta^{\prime}}} \pm 0.010_{m_{\omega}} \pm 0.021_{g} \mathrm{keV}  \tag{15}\\
\Gamma\left(\eta^{\prime} \rightarrow \rho \gamma\right)=54.397 \pm 0.049_{m_{\eta^{\prime}}} \pm 0.273_{m_{\rho}} \pm 0.215_{g} \mathrm{keV}  \tag{16}\\
\mathcal{B}\left(\eta^{\prime} \rightarrow \omega \gamma\right)=\left(2.727 \pm 0.003_{m_{\eta^{\prime}}} \pm 0.005_{m_{\omega}}\right. \\
\left. \pm 0.011_{g_{-0.118}}^{+0.129} \Gamma_{\eta^{\prime}}\right) \%  \tag{17}\\
\mathcal{B}\left(\eta^{\prime} \rightarrow \rho \gamma\right)= \\
\left(27.335 \pm 0.025_{m_{\eta^{\prime}}} \pm 0.137_{m_{\rho}}\right.  \tag{18}\\
\left. \pm 0.108_{g-1.183 \Gamma_{\eta^{\prime}}}^{+1.295}\right) \%
\end{gather*}
$$

where the uncertainties come from $m_{\eta^{\prime}}, m_{\mathrm{V}}, g$ and $\Gamma_{\eta^{\prime}}$, respectively. It is clear that (1) there are two proportions, $\Gamma\left(\eta^{\prime} \rightarrow \rho \gamma\right) \approx 10 \Gamma\left(\eta^{\prime} \rightarrow \omega \gamma\right)$ and $\mathcal{B}\left(\eta^{\prime} \rightarrow \rho \gamma\right) \approx$ $10 \mathcal{B}\left(\eta^{\prime} \rightarrow \omega \gamma\right)$, due to the Eq. (9) relationship. (2) The largest uncertainty of the predicted branching ratio is from the measurement $\Gamma_{\eta^{\prime}}$. (3) These branching ratios are in agreement with the measurements $\mathcal{B}\left(\eta^{\prime} \rightarrow \omega \gamma\right)=$ $(2.75 \pm 0.22) \%$ and $\mathcal{B}\left(\eta^{\prime} \rightarrow \rho \gamma\right)=(29.3 \pm 0.6) \%$ (including non-resonant $\pi \pi \gamma$ ) [2] within one standard deviation.

Table 1. Input parameters for $\eta^{\prime} \rightarrow \omega \mathrm{e}^{+} \mathrm{e}^{-}$decay.

| parameter | value | Ref. |
| :---: | :---: | :---: |
| mass of $\eta^{\prime}$ meson | $m_{\eta^{\prime}}=957.78 \pm 0.06 \mathrm{MeV}$ | $[2]$ |
| mass of $\omega$ meson | $m_{\omega}=782.65 \pm 0.12 \mathrm{MeV}$ | $[2]$ |
| mass of $\rho$ meson | $m_{\rho}=775.49 \pm 0.34 \mathrm{MeV}$ | $[2]$ |
| width of $\eta^{\prime}$ meson | $\Gamma_{\eta^{\prime}}=199 \pm 9 \mathrm{keV}$ | $[2]$ |
| width of $\omega$ meson | $\Gamma_{\omega}=8.49 \pm 0.08 \mathrm{MeV}$ | $[2]$ |
| width of $\rho$ meson | $\Gamma_{\rho}=149.1 \pm 0.8 \mathrm{MeV}$ | $[2]$ |
| vector coupling constant | $g=5.568 \pm 0.011$ | $[11]$ |

The integrated partial width and the corresponding branching ratio of the Dalitz $\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$decays are

$$
\begin{gather*}
\Gamma\left(\eta^{\prime} \rightarrow \boldsymbol{\omega} \mathrm{e}^{+} \mathrm{e}^{-}\right)=39.401 \pm 0.040_{m_{\eta^{\prime}}} \pm 0.079_{m_{\omega}} \pm 0.156_{g} \mathrm{eV},  \tag{19}\\
\Gamma\left(\eta^{\prime} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}\right)=384.525 \pm 0.377_{m_{\eta^{\prime}}}{ }_{-2.080 m_{\rho}}^{+2.087}{ }_{-1.518 g}^{+1.521} \mathrm{eV}, \tag{20}
\end{gather*}
$$

$$
\begin{align*}
\mathcal{B}\left(\eta^{\prime} \rightarrow \omega \mathrm{e}^{+} \mathrm{e}^{-}\right)= & \left(1.980 \pm 0.002_{m_{\eta^{\prime}}} \pm 0.004_{m_{\omega}}\right. \\
& \left. \pm 0.008_{g-0.086 \Gamma_{\eta^{\prime}}}^{+0.094}\right) \times 10^{-4},  \tag{21}\\
\mathcal{B}\left(\eta^{\prime} \rightarrow \rho \mathrm{e}^{+} \mathrm{e}^{-}\right)= & \left(1.932 \pm 0.002_{m_{\eta^{\prime}}} \pm 0.010_{m_{\rho}}\right. \\
& \left. \pm 0.008_{g_{-0.087}^{+0.095} \Gamma_{\eta^{\prime}}}^{+0 .}\right) \times 10^{-3} . \tag{22}
\end{align*}
$$

Similarly, the largest uncertainty of the predicted branching ratio comes from the measurement $\Gamma_{\eta^{\prime}}$. Our estimation of the $\eta^{\prime} \rightarrow \boldsymbol{\omega} \mathrm{e}^{+} \mathrm{e}^{-}$decay is in good agreement with the previous work $[8,9]$ within the uncertainties. In [8], the branching ratio is estimated to be about $\mathcal{B}\left(\eta^{\prime} \rightarrow \omega \mathrm{e}^{+} \mathrm{e}^{-}\right) \sim 2 \times 10^{-4}$ with the effective meson theory ${ }^{1)}$. In [9], the branching ratio is given by $\mathcal{B}\left(\eta^{\prime} \rightarrow \omega \mathrm{e}^{+} \mathrm{e}^{-}\right)=(1.69 \pm 0.56) \times 10^{-4}$ with the effective chiral Lagrangian ${ }^{2)}$. Our results about $\eta^{\prime} \rightarrow$ $\rho \mathrm{e}^{+} \mathrm{e}^{-}$decay agree basically with the prediction of $\Gamma\left(\eta^{\prime} \rightarrow \pi^{+} \pi^{-} \mathrm{e}^{+} \mathrm{e}^{-}\right)=431_{-64}^{+38} \mathrm{eV}$ and $\mathcal{B}\left(\eta^{\prime} \rightarrow \pi^{+} \pi^{-} \mathrm{e}^{+} \mathrm{e}^{-}\right)=$ $\left(2.13_{-0.32}^{+0.19}\right) \times 10^{-3}$ [19] within the uncertainties, and accord with the recent measurement $\mathcal{B}\left(\eta^{\prime} \rightarrow \pi^{+} \pi^{-} \mathrm{e}^{+} \mathrm{e}^{-}\right)=$ $(2.11 \pm 0.12 \pm 0.14) \times 10^{-3}$ reported by BESIII [20] within one standard deviation, where almost all of the final states $\pi^{+} \pi^{-}$will probably come from the resonant $\rho^{0}$ meson. There are some difficulties in the measurement of $\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$experimentally due to the fact that the masses of the $\omega$ and $\rho$ meson are close to each other, and for $\eta^{\prime} \rightarrow \omega \mathrm{e}^{+} \mathrm{e}^{-}$decay, the large background might come from $\eta^{\prime} \rightarrow \rho \mathrm{e}^{+} \mathrm{e}^{-}$, and vice versa.

Although no available measurement of the $\eta^{\prime} \rightarrow$
$\mathrm{Ve}^{+} \mathrm{e}^{-}$decays is enumerated by PDG so far [2], there is renewed experimental interest in $\eta^{\prime}$ decays with the advent of high statistic $\eta^{\prime}$ experiments. For example, some $10^{5}$ fully reconstructed $\eta^{\prime}$ events per day can be reached with WASA at COSY [21]; Approximately $15 \times 10^{3} \eta^{\prime}$ events per hour are expected with Crystal Ball at MAMI [22]; With one year's luminosity at J/ $\psi$ peak, some 60 million $\eta^{\prime}$ events could be collected by BESIII at BEPC II [23]; KLOE-2 at DA $\Phi$ NE experiment expects to increase this sample up to about a few $\mathrm{fb}^{-1}$ integral luminosity per year at the next running [24]. We take BESIII as an example to estimate the production rate of $\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$ decays. It is estimated that there are more than $5 \times 10^{6}$ $\eta^{\prime}$ sample, corresponding to the radiative decay $\mathrm{J} / \psi \rightarrow$ $\gamma \eta^{\prime}$ with some $10^{9} \mathrm{~J} / \psi$ dataset accumulated at BESIII [23]. Given the detection efficiency for $\eta^{\prime} \rightarrow \pi^{+} \pi^{-} \mathrm{e}^{+} \mathrm{e}^{-}$is about $17 \%$ [20], some $2000 \eta^{\prime} \rightarrow \rho \mathrm{e}^{+} \mathrm{e}^{-}$and some $100 \eta^{\prime} \rightarrow$ $\omega \mathrm{e}^{+} \mathrm{e}^{-}$events could be observed at BESIII ${ }^{3)}$. The corresponding distribution of the dilepton spectra are displayed in Fig. 2. Our studies also show that (1) the influence of the mass and width of vector mesons on the normalization distribution of $\mathrm{d} \Gamma\left(\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}\right) / \mathrm{d} m_{\mathrm{e}^{+} \mathrm{e}^{-}}$is small ${ }^{4)}$. (2) The maximum position in the distribution is near the dilepton threshold ${ }^{5)}$, i.e. $m_{\mathrm{e}^{+} \mathrm{e}^{-}} \approx 1.33 \mathrm{MeV}$, the corresponding common momentum of vector mesons $\omega$ and $\rho$ in the $\eta^{\prime}$ rest frame are 159.11 MeV and 164.94 MeV , respectively. This distinctive feature will be helpful in distinguishing the signals from the backgrounds.

1) In [8], the multiplicative representation of the transition form factor is used to fit the data. That is to say, many vector and/or excited vector mesons are taken into account. For example, the transition form factor $F_{\omega \gamma \pi}$ is written as:

$$
\begin{equation*}
F_{\omega \gamma \pi}(t)=\frac{(1+C t) m_{\rho}^{2} m_{X}^{2} m_{\rho^{\prime \prime}}^{2}}{\left(m_{\rho}^{2}-t\right)\left(m_{\mathrm{X}}^{2}-t\right)\left(m_{\rho^{\prime \prime}}^{2}-t\right)} \tag{23}
\end{equation*}
$$

where the satisfactory fit quality is achieved at the price of introducing much more resonance parameters related to the corresponding vector mesons.
2) In [9], the form factor of process $\eta^{\prime} \rightarrow \omega \gamma$ is written as a function of six parameters (see Eqs. (44) and (45) in [9] for more detail) many inputs may cause large uncertainty.

In addition, we would like to point out that if the parameters $c_{3} \neq 1$ in the triangle anomaly Lagrangians Eqs. (2) and (3), then the partial width should be $|c|^{2} \Gamma\left(\eta^{\prime} \rightarrow \mathrm{V} \gamma\right)$ and $|c|^{2} \Gamma\left(\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}\right)$for the $\eta^{\prime} \rightarrow \mathrm{V} \gamma$ and $\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$decays, respectively. For example, if the fitted value of $c_{3}=0.927 \pm 0.010$ [11] is used, then the partial widths and branching ratios should be (85.9 $\pm 2.2$ ) $\%$ of those given in Eqs. (15)- (22), i.e. the central value of the branching ratio of $\eta^{\prime} \rightarrow \omega \mathrm{e}^{+} \mathrm{e}^{-}$decay will be $1.70 \times 10^{-4}$, which is fairly consistent with that predicted in [9].
3) In fact, the $\omega$ meson decays mainly into $3 \pi$ and the neutral pion decays mostly into two photons. So the detection efficiency will be less than $17 \%$ for $\eta^{\prime} \rightarrow \omega \mathrm{e}^{+} \mathrm{e}^{-}$decays due to much more final states. According to the referee's suggestion, the reconstruction efficiency of each photon is $\sim 80 \%$, so the approximate 100 events for $\eta^{\prime} \rightarrow \omega \mathrm{e}^{+} \mathrm{e}^{-}$could be observed at BESIII.
4) Here, the maximum invariant mass of the electron-positron pair is $m_{\mathrm{e}^{+} \mathrm{e}^{-} \max }=\left(m_{\eta^{\prime}}-m_{\mathrm{V}}\right)$, so $p_{\gamma^{*} \max }^{2} / m_{\mathrm{V}}^{2}=m_{\gamma^{*} \max }^{2} / m_{\mathrm{V}}^{2}=$ $m_{\mathrm{e}^{+} \mathrm{e}^{-} \max }^{2} / m_{\mathrm{V}}^{2}$ is less than $6 \%$. In addition, the ratio $\Gamma_{\mathrm{V}} / m_{\mathrm{V}} \sim 1 \%$ for $\omega$ meson and $\sim 20 \%$ for $\rho$ meson, so with $m_{\gamma^{*} \max }$, the factor $\left|1 /\left(1-\frac{p_{\gamma^{*}}^{2}}{m_{\mathrm{V}}^{2}}-\mathrm{i} \frac{\Gamma_{\mathrm{V}}}{m_{\mathrm{V}}}\right)\right|^{2} \approx 1.11$ for $\omega$ meson and $\approx 1.07$ for $\rho$ meson, that is to say, the effects of the mass and width of the vector meson are about $10 \%$. At the maximum position in the dilepton distribution $m_{\mathrm{e}^{+} \mathrm{e}^{-}} \approx 1.33 \mathrm{MeV}$, the factor $\left|1 /\left(1-\frac{p_{\gamma^{*}}^{2}}{m_{\mathrm{V}}^{2}}-\mathrm{i} \frac{\Gamma_{\mathrm{V}}}{m_{\mathrm{V}}}\right)\right|^{2} \sim 99.99 \%$ for $\omega$ meson and $\sim 96.44 \%$ for $\rho$ meson, that is to say, the effects of the mass and width of the vector meson are negligible for the $\omega$ meson and less than $4 \%$ for the $\rho$ meson.
5) It is shown from Eq. (14) that the differential width is proportional to $1 / m_{\gamma^{*}}$, so the spectra lineshape tends to the maximum with $m_{\gamma^{*}}$ moving to the dilepton threshold. In addition, because $m_{\gamma^{*}}$ is far away from the mass of the vector resonance, there is no peak near the tail of the dilepton distribution and the shapeline is falling down smoothly.


Fig. 2. The dilepton spectra of the $\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$decay, where the horizontal axis denotes the dilepton invariant mass $m_{\mathrm{e}^{+} \mathrm{e}^{-}}$in the unit of MeV , and the vertical axis denotes the normalization distribution of $\mathrm{d} \Gamma\left(\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}\right) / \mathrm{d} m_{\mathrm{e}^{+} \mathrm{e}^{-}}$ in (a) (the area below the line is one), some $100 \eta^{\prime} \rightarrow \omega \mathrm{e}^{+} \mathrm{e}^{-}$events in (b) and some $2000 \eta^{\prime} \rightarrow \rho \mathrm{e}^{+} \mathrm{e}^{-}$events in (c).

## 4 Summary

Based on the triangle anomaly HLS effective Lagrangian, the interesting $\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$decay is studied with the VDM model. Our study shows that the partial width $\Gamma\left(\eta^{\prime} \rightarrow \omega \mathrm{e}^{+} \mathrm{e}^{-}\right) \approx 40 \mathrm{eV}$ and branching ratio $\mathcal{B}\left(\eta^{\prime} \rightarrow \omega \mathrm{e}^{+} \mathrm{e}^{-}\right) \approx 2 \times 10^{-4}$, and $\Gamma\left(\eta^{\prime} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}\right) \approx$ $10 \Gamma\left(\eta^{\prime} \rightarrow \omega \mathrm{e}^{+} \mathrm{e}^{-}\right)$and $\mathcal{B}\left(\eta^{\prime} \rightarrow \rho \mathrm{e}^{+} \mathrm{e}^{-}\right) \approx 10 \mathcal{B}\left(\eta^{\prime} \rightarrow \omega \mathrm{e}^{+} \mathrm{e}^{-}\right)$, which are basically consistent with previous estimations and measurements within uncertainties. Compared with the radiative decay $\eta^{\prime} \rightarrow \mathrm{V} \gamma$, the electron-positron pair,
splitting away from the off-shell photon, its invariant mass is momentum dependent, which could provide us with much more information about the intrinsic structure of the $\eta^{\prime}$ meson and form factor for electromagnetic transition $\eta^{\prime} \rightarrow \mathrm{V}$. It is well known that the charged electron and positron are easily identified by the detector saturated with a magnetic field. In addition, there is distinctive maximum position $m_{\mathrm{e}^{+} \mathrm{e}^{-}} \approx 1.33 \mathrm{MeV}$. It can be expected that the era of accurate measurements on the $\eta^{\prime} \rightarrow \mathrm{Ve}^{+} \mathrm{e}^{-}$decay is imminent with the advent of high statistic $\eta^{\prime}$ experiments.

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    1）E－mail：yangyueling＠htu．cn
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    3）There are 19 tests of $C$ invariance listed in［2］，including $8 \eta$ decays and $6 \eta^{\prime}$ decays．
    4）The $\eta^{\prime} \rightarrow \phi$ transition is forbidden by the kinematic constraint．The $\eta^{\prime} \rightarrow K^{0 *} e^{+} e^{-}$decay is a weak process，and the permitted weak decays of $\eta^{\prime}$ mesons in the standard model are expected to occur at the level of $10^{-11}$ and below［6］．

    5）It is well known that all possible strong and electromagnetic decays of $\eta$ are highly suppressed by various constraints（such as $P$ ，C， G parity）$[3,4]$ ．

