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Primordial black holes and lepton flavor violation with scotogenic dark matter

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We show that if the lepton flavor-violating $\mu \to e\gamma$ process is observed in the MEG II experiment, the initial density of primordial black holes (PBHs) can be constrained with scotogenic dark matter. As a benchmark case, if PBH evaporation occurs in the radiation-dominated era, the initial density may be $2 \times 10^{-17} \lesssim \beta \lesssim 3 \times 10^{-16}$ for the $\mathcal{O}$(TeV)-scale dark sector in the scotogenic model, where β is the ratio of the PBH density $\rho_{\rm PBH}$ to the radiation density $\rho_{\rm rad}$ at the time of PBH formation. As another benchmark case, if PBHs evaporate in the PBH-dominated era, the initial density may be $1 \times 10^{-8} \lesssim \beta \lesssim 3 \times 10^{-7}$ for $\mathcal{O}$(GeV)-scale dark matter, with other $\mathcal{O}$(TeV)-scale particles in the scotogenic model.

Subject Index B71, B77, E75

1. Introduction

Primordial black holes (PBHs) are a type of black hole produced in the early Universe [1–4]. They are produced via a number of mechanisms, such as the collapse of large density perturbations generated from inflation [5–13], a sudden reduction in pressure [14,15], bubble collisions [16–20], a curvaton [21–24] or collapse of a cosmic string [25].

A PBH emits particles via Hawking radiation [26]. Since this is induced by gravity, PBHs evaporate into all particle species. Thus, the study of PBHs is important not only for cosmology but also particle physics. For example, since particle dark matter is also produced by Hawking radiation from PBHs, the correlation between the initial density of PBHs and dark matter mass has been extensively studied in the literature [27–48]. For another example, the influence of lepton flavor asymmetries on the mass spectrum of PBHs have recently been studied [49,50].

On the other hand, lepton flavor-violating phenomena, such as the $\mu \to e\gamma$ process, are directly related to the new physics beyond the standard model of particle physics [51]. Most new physics models predict some lepton flavor violation effects. To test the availability of these new models, theoretical predictions of the branching ratios of lepton flavor-violating processes within these models are important. In addition, since the physics run of the MEG II experiment to search for $\mu \to e\gamma$ processes with ten times better sensitivity than the MEG experiment will be started in the very near future [52,53], a study related to the $\mu \to e\gamma$ process is interesting and timely.

In this paper we show the correlation between the initial density of PBHs and the branching ratio of $\mu \to e\gamma$ with scotogenic dark matter. The scotogenic dark matter [54] in the scotogenic model is one of the most successful and well-studied dark matter candidates [55–105]. Since the scotogenic model can account for dark matter candidates and predict the lepton

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flavor-violating processes simultaneously [55,56], and scotogenic dark matter can also be produced by Hawking radiation from PBHs [45], the initial density of PBHs and the branching ratio of $\mu \to e\gamma$ are related via scotogenic dark matter.

Dark matter and PBHs in the scotogenic model has already been discussed in Ref. [45], where the constraints from lepton flavor-violating processes were also studied. We have to be clear about the differences between Ref. [45] and this paper. Here, we perform more advanced analysis by:

- taking into account the expected sensitivity of the branching ratio of the lepton flavor-violating $\mu \to e\gamma$ process from the future MEG II experiment (only the current MEG constraint is taken into account in Ref. [45]);
- including the entropy production effect via PBH evaporation into the numerical calculations in the PBH-dominant case (there are only some comments about the effect of entropy production in Ref. [45]);
- searching a wider parameter region of the scotogenic mode (only TeV-scale dark matter is considered as a typical value in the scotogenic model in Ref. [45]).

Thanks to these new ingredients, especially including the expected results from the future MEG II experiment, the following new scientific findings are obtained in this paper:

- not only an upper limit but also a lower limit, an allowed band, for the initial density of PBHs in the radiation-dominant case (only an upper limit for the initial density of PBHs was shown in the radiation-dominant case in Ref. [45]);
- constraints on the initial density of PBHs in the PBH-dominant case (there is no significant discussion of the constraint on the initial density of PBHs in the PBH-dominant case in Ref .[45]).

This paper is organized as follows. In Sect. 2 we present a review of the scotogenic model. In Sect. 3 we show the correlation between the initial density of PBHs and the branching ratio of $\mu \to e\gamma$ with scotogenic dark matter. Section 4 is devoted to a summary.

2. Scotogenic model

The scotogenic model [54] is an extension of the standard model in particle physics. In this model, three new Majorana $SU(2)_L$ singlets N_k ($k = 1, 2, 3$) with mass M_k and one new scalar $SU(2)_L$ doublet (η^+, η^0) are introduced. These new particles are odd under exact Z_2 symmetry. The relevant Lagrangian and scalar potential for this paper are given by

$$\mathcal{L} = Y_{\alpha k}(\bar{\nu}_{\alpha L}\eta^0 - \bar{\ell}_{\alpha L}\eta^+)N_k + \frac{1}{2}M_k\bar{N}_k N_k^C + \text{h.c.},$$

$$V = \frac{1}{2}\lambda(\phi^\dagger\eta)^2 + \text{h.c.}, \tag{1}$$

where $L_\alpha = (\nu_\alpha, \ell_\alpha)$ ($\alpha = e, \mu, \tau$) is the left-handed lepton doublet and $\phi = (\phi^+, \phi^0)$ is the standard Higgs doublet.

Owing to the Z_2 symmetry the tree-level neutrino mass should vanish, but they acquire masses via one-loop interactions. The flavor neutrino mass matrix M is obtained as

$$M_{\alpha\beta} = \sum_{k=1}^{3} \frac{\lambda v^2 Y_{\alpha k} Y_{\beta k} M_k}{16\pi^2\left(m_0^2 - M_k^2\right)}\left(1 - \frac{M_k^2}{m_0^2 - M_k^2}\ln\frac{m_0^2}{M_k^2}\right), \tag{2}$$

where $m_0^2 = \frac{1}{2}(m_R^2 + m_I^2)$, and v, m_R, and m_I denote vacuum expectation value of the Higgs field and the masses of $\sqrt{2}\,\mathrm{Re}[\eta^0]$ and $\sqrt{2}\,\mathrm{Im}[\eta^0]$, respectively.

Since the lightest Z_2 odd particle is stable, it becomes a dark matter candidate. We assume that the lightest Majorana singlet fermion, N_1, is the dark matter particle. It is known that if the lightest singlet fermion is almost degenerate with the next-to-lightest singlet fermions, the observed relic abundance of dark matter $\Omega_{\rm DM}h^2 = 0.12 \pm 0.0009$ [106] and the observed upper limit of the branching ratio of the $\mu \to e\gamma$ process $\mathrm{BR}(\mu \to e\gamma) \leq 4.2 \times 10^{-13}$ [107] can be simultaneously consistent with the prediction from the scotogenic model [55,56]. Thus, we set $M_1 \sim M_2 = 1.0001M_1 < M_3 < m_0$.

We would like to comment that if the DM and the new scalar particle are degenerate in mass, $M_1 \sim m_0$, their coannihilation processes become significant for the DM relic density. In this case, to compensate for DM relic density, the dark matter particle should be heavier. Heavier dark matter yields smaller $\mathrm{BR}(\mu \to e\gamma)$ (see, for example, Ref. [45]). Thus, if we include a parameter region with $M_1 \sim m_0$ in our numerical calculations, more stringent constraints of the parameters may be obtained. In this paper we would like to keep the requirement of $M_1 < m_0$ and omit the possibility of the coannihilation of DM and the new scalar particle.

The relic abundance of cold dark matter which is produced by the freeze-out mechanism is estimated to be [108]

$$\Omega_{\rm FO}h^2 = \frac{1.07 \times 10^9 x_{\rm FO}}{g_*^{1/2} M_{\rm Pl}(a_{\rm eff} + 3b_{\rm eff}/x_{\rm FO})}, \tag{3}$$

where

$$a_{\rm eff} = \frac{a_{11}}{4} + \frac{a_{12}}{2} + \frac{a_{22}}{4}, \qquad b_{\rm eff} = \frac{b_{11}}{4} + \frac{b_{12}}{2} + \frac{b_{22}}{4}, \tag{4}$$

with

$$a_{ij} = \frac{1}{8\pi}\frac{M_1^2}{\left(M_1^2 + m_0^2\right)^2}\sum_{\alpha\beta}\left(Y_{\alpha i}Y_{\beta j} - Y_{\alpha j}Y_{\beta i}\right)^2,$$

$$b_{ij} = \frac{m_0^4 - 3m_0^2M_1^2 - M_1^4}{3\left(M_1^2 + m_0^2\right)^2}a_{ij} + \frac{1}{12\pi}\frac{M_1^2\left(M_1^4 + m_0^4\right)}{\left(M_1^2 + m_0^2\right)^4}\sum_{\alpha\beta}Y_{\alpha i}Y_{\alpha j}Y_{\beta i}Y_{\beta j}, \tag{5}$$

and

$$x_{\rm FO} = \frac{m_{\rm DM}}{T_{\rm FO}} \simeq 25 \tag{6}$$

is the freeze-out temperature [109].

In this model, flavor-violating processes such as $\mu \to e\gamma$ are induced at the one-loop level. The branching ratio of $\mu \to e\gamma$ is given by [83]

$$\mathrm{BR}(\mu \to e\gamma) = \frac{3\alpha_{\rm em}}{64\pi\left(G_F m_0^2\right)^2}\left|\sum_{k=1}^{3}Y_{\mu k}Y_{ek}^*F\left(\frac{M_k}{m_0}\right)\right|^2, \tag{7}$$

where $\alpha_{\rm em}$ denotes the fine-structure constant, G_F denotes the Fermi coupling constan,t and $F(x)$ is defined by $F(x) = \frac{1-6x^2+3x^4+2x^6-6x^4\ln x^2}{6(1-x^2)^4}$.

The Yukawa coupling Y can be express in terms of λ, M_k, m_0, the neutrino masses m_i ($i = 1$, 2, 3), the mixing angles θ_{ij} ($ij = 12$, 23, 13), the Dirac CP-violating phase δ, and the Majorana CP phases α_i ($i = 1$, 2) by Casas–Ibarra parametrization [110]. Since the relic density of scotogenic dark matter depends only weakly on CP-violating phases, the Majorana CP phases are neglected [105].

We would like to comment that the contributions of the Majorana phases would be small for the DM relic density but not for neutrino phenomena. Since neutrino oscillation experiments are not sensitive to the Majorana phases, neutrino phenomena with Majorana phases are often studied in cosmological discussions. For example, the so-called leptogenesis scenarios for the origin of the baryon asymmetry of the Universe [111] may depend on the Majorana phases of neutrinos. In this paper, although we have attempted to obtain some connection between Majorana phases of neutrinos and PBHs, since we just connect PBHs and DM relic abundance, and DM relic abundance is not sensitive to the Majorana phases, we cannot give any prediction for Majorana phases with PBHs at present. Up to now, leptogenesis with scotogenic dark matter with PBHs [48] and the interplay between thermal and PBH-induced leptogenesis [112] have been studied; however, there is no significant prediction for Majorana phases with PBHs. Studies about the relation between Majorana phases of neutrinos and PBHs are required in the future.

We use the best-fit values of the neutrino parameters in Ref. [113]. For simplicity, we assume the normal mass ordering for the neutrinos, $m_1 < m_2 < m_3$. According to the constraint $\sum m_i < 0.12 - 0.69\,\mathrm{eV}$ from observation of cosmic microwave background radiation [106,114], we require

$$\sum m_i < 0.12\,\mathrm{eV} \tag{8}$$

and

$$m_1 = 0.001 - 0.1\,\mathrm{eV}. \tag{9}$$

In addition, we have an observed upper limit $|M_{ee}| < 0.066 - 0.155\,\mathrm{eV}$ from neutrinoless double beta decay experiments [114,115]. We require the condition

$$|M_{ee}| < 0.066\,\mathrm{eV}, \tag{10}$$

in our numerical calculations.

For the remaining four model parameters, $\{M_1, M_3, m_0, \lambda\}$, we set the commonly used ranges [83,92,93]

$$\begin{aligned} &1.0\,\mathrm{GeV} \le M_1, M_3, m_0 \le 1.0 \times 10^6\,\mathrm{GeV}, \\ &1.0 \times 10^{-11} \le \lambda \le 1.0 \times 10^{-6} \end{aligned} \tag{11}$$

in our numerical calculations. Since we assume that the lightest Majorana fermion N_1 is the dark matter particle, hereafter we use the notation $m_{\mathrm{DM}} = M_1$.

3. Initial density of primordial black holes

3.1 *Primordial black holes*

We assume that PBHs are produced in the early Universe by large density perturbations generated from an inflation [5–13], that a PBH's mass is proportional to a horizon mass, and that PBHs have the same masses at their formation time. In addition, we assume that PBHs form during the radiation-dominated era, with a monochromatic mass function.

The temperature of the Universe at PBH formation time is obtained as

$$T_{\mathrm{in}} = \frac{\sqrt{3}5^{1/4}}{2\pi^{3/4}} \frac{\gamma^{1/2}}{g_*(T_{\mathrm{in}})^{1/4}} \left(\frac{M_{\mathrm{Pl}}^3}{M_{\mathrm{in}}}\right)^{1/2}, \tag{12}$$

where $\gamma \sim 0.2$ [1], g_* is the relativistic effective degrees of freedom for the radiation energy density, $M_{\mathrm{Pl}} \simeq 1.221 \times 10^{19}$ GeV is the Planck mass, and M_{in} is the initial mass of the PBH.

We introduce the dimensionless parameter

$$\beta = \frac{\rho_{\rm PBH}(T_{\rm in})}{\rho_{\rm rad}(T_{\rm in})} \tag{13}$$

to represent the initial energy density of PBHs at the time of its formation, $\rho_{\rm PBH}(T_{\rm in})$, where $\rho_{\rm rad}(T_{\rm in})$ is the radiation energy density.

A black hole loses its mass by producing particles with masses below the Hawking temperature

$$T_{\rm BH} = \frac{M_{\rm Pl}^2}{8\pi M_{\rm BH}} \tag{14}$$

via Hawking radiation [26]. Ignoring gray-body factors, the energy spectrum of the Hawking radiation is similar to the Planck distribution (the effects of the gray-body factor in the high-energy geometrical optics limit are shown in Refs. [2,34,38,42]). The temperature of the Universe right after PBH evaporation is

$$T_{\rm evap} = \frac{\sqrt{3}g_*(T_{\rm BH})^{1/4}}{64\sqrt{2}5^{1/4}\pi^{5/4}}\left(\frac{M_{\rm Pl}^5}{M_{\rm in}^3}\right)^{1/2}. \tag{15}$$

The PBHs emit scotogenic dark matter via Hawking radiation [45]. Reference [40] showed that if PBHs evaporate after the freeze-out of dark matter, $T_{\rm FO} > T_{\rm evap}$, then the dark matter particles produced from PBHs may contribute to the final relic abundance of dark matter. The criteria $T_{\rm FO} > T_{\rm evap}$ is translated into

$$\frac{M_{\rm in}}{M_{\rm Pl}} \gtrsim 2\times 10^{12}\left(\frac{\rm GeV}{m_{\rm DM}}\right)^{2/3} \tag{16}$$

by Eqs. (6) and (15). Since PBHs should be evaporated before big bang nucleosynthesis [32,116], the upper limit $M_{\rm in} \lesssim 1\times 10^9$ g ($M_{\rm in}/M_{\rm Pl} \lesssim 4.6\times 10^{13}$) is obtained [4]. We conservatively set the upper and lower bounds of the initial PBH mass as

$$2\times 10^{12}\left(\frac{\rm GeV}{m_{\rm DM}}\right)^{2/3} \le \frac{M_{\rm in}}{M_{\rm Pl}} \le 2\times 10^{13}. \tag{17}$$

3.2 *PBH evaporation in the radiation-dominated era*

Since $\rho_{\rm PBH} \propto a^{-3}$ and $\rho_{\rm rad} \propto a^{-4}$, where a denotes the scale factor, $\rho_{\rm PBH}(t_{\rm early\text{-}eq}) \simeq \rho_{\rm rad}(t_{\rm early\text{-}eq})$ may happen at the early equality time $t_{\rm early\text{-}eq}$. In order for PBH evaporation to occur before the early equality time (radiation-dominated era), $t_{\rm evap} < t_{\rm early\text{-}eq}$, the initial density of PBHs should be less than the following critical density ($\beta < \beta_{\rm c}$) [32,37,38,117]:

$$\beta_{\rm c} = \frac{T_{\rm evap}}{T_{\rm in}} = \sqrt{\frac{g_*(T_{\rm BH})}{10240\pi\gamma}}\left(\frac{M_{\rm in}}{M_{\rm Pl}}\right)^{-1} = 0.129\left(\frac{g_*(T_{\rm BH})}{106.75}\right)^{1/2}\left(\frac{0.2}{\gamma}\right)^{1/2}\left(\frac{M_{\rm in}}{M_{\rm Pl}}\right)^{-1}. \tag{18}$$

In this case, the final relic abundance of the scotogenic dark matter is

$$\Omega_{\rm DM}h^2 = \Omega_{\rm FO}h^2 + \Omega_{\rm PBH}h^2, \tag{19}$$

where $\Omega_{\rm PBH}h^2$ denotes the relic abundance of the scotogenic dark matter generated by PBH evaporation.

The relic abundance via the freeze-out mechanism, $\Omega_{\rm FO}h^2$, should be at least less than the observed relic abundance of dark matter: $\Omega_{\rm DM}h^2 = 0.12 \pm 0.0009$.

First, we estimate the allowed region of the dark matter mass $m_{\rm DM}$ without the effects of PBH evaporation. Figure 1 shows the prediction of BR($\mu \to e\gamma$) for $\Omega_{\rm FO}h^2 \le 0.12 \pm 0.0009$ in the scotogenic model. The upper horizontal line shows the current observed upper limit of

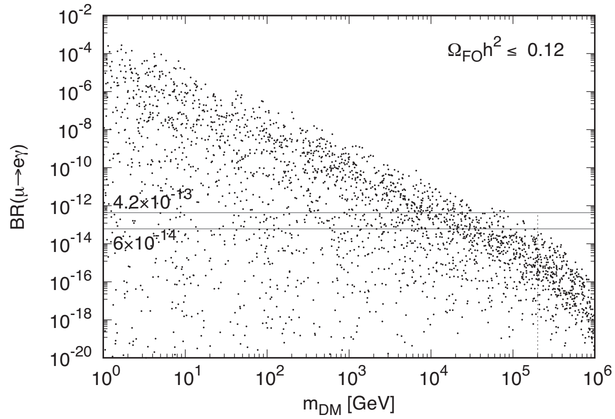

Fig. 1. Prediction of $\mathrm{BR}(\mu \to e\gamma)$ for $\Omega_{\mathrm{FO}}h^2 \leq 0.12$ in the scotogenic model. The upper horizontal line shows the current observed upper limit of $\mathrm{BR}(\mu \to e\gamma) \leq 4.2 \times 10^{-13}$ from the MEG experiment. The lower horizontal line shows the expected sensitivity of the future MEG II experiment: $\mathrm{BR}(\mu \to e\gamma) \simeq 6 \times 10^{-14}$.

$\mathrm{BR}(\mu \to e\gamma) \leq 4.2 \times 10^{-13}$ from the MEG experiment [107]. The lower horizontal line shows the expected sensitivity of the future MEG II experiment, $\mathrm{BR}(\mu \to e\gamma) \simeq 6 \times 10^{-14}$ [52,53]. We observe that the allowed mass of scotogenic dark matter is constrained by the observed upper limits of $\mathrm{BR}(\mu \to e\gamma)$ from the MEG and MEG II experiments in the case of $\beta < \beta_{\mathrm{c}}$. From Fig. 1, we perform our numerical studies in the mass region

$$1\,\mathrm{GeV} \leq m_{\mathrm{DM}} \leq 2 \times 10^5\,\mathrm{GeV} \tag{20}$$

in the case of $\beta < \beta_{\mathrm{c}}$.

We note that the upper limit of lepton flavor-violating $\tau \to \mu\gamma$ and $\tau \to e\gamma$ processes are also measured as $\mathrm{BR}(\tau \to \mu\gamma) \leq 4.4 \times 10^{-8}$ and $\mathrm{BR}(\tau \to e\gamma) \leq 3.3 \times 10^{-8}$ [118]; however, we only account for $\mathrm{Br}(\mu \to e\gamma)$ since it is the most stringent constraint.

Now, we include the effect of the PBH evaporation in our analysis. For $1\,\mathrm{GeV} \leq m_{\mathrm{DM}} \leq 2 \times 10^5\,\mathrm{GeV}$, the initial PBH mass should be

$$\frac{M_{\mathrm{in}}}{M_{\mathrm{Pl}}} \gtrsim \begin{cases} 5.84 \times 10^8 & (m_{\mathrm{DM}} = 2 \times 10^5\,\mathrm{GeV}), \\ 2 \times 10^{12} & (m_{\mathrm{DM}} = 1\,\mathrm{GeV}) \end{cases} \tag{21}$$

for $T_{\mathrm{FO}} > T_{\mathrm{evap}}$. The corresponding Hawking temperature at the PBH formation time would be

$$T_{\mathrm{BH}}^{\mathrm{in}} = \begin{cases} 8.3 \times 10^8\,\mathrm{GeV} & \left(\frac{M_{\mathrm{in}}}{M_{\mathrm{Pl}}} = 5.84 \times 10^8\right), \\ 2.4 \times 10^5\,\mathrm{GeV} & \left(\frac{M_{\mathrm{in}}}{M_{\mathrm{Pl}}} = 2 \times 10^{12}\right). \end{cases} \tag{22}$$

Thus, the relation $T_{\mathrm{BH}}^{\mathrm{in}} > m_{\mathrm{DM}}$ is satisfied in our setup. The relic abundance of scotogenic dark matter generated by PBH evaporation is obtained as

$$\Omega_{\mathrm{PBH}}h^2 \simeq 7.31 \times 10^7 \left(\frac{g_*(T_{\mathrm{in}})}{106.75}\right)^{-1/4} \beta \frac{3}{4} \frac{g_{\mathrm{DM}}}{g_*(T_{\mathrm{BH}})} \left(\frac{m_{\mathrm{DM}}}{\mathrm{GeV}}\right) \left(\frac{M_{\mathrm{in}}}{M_{\mathrm{Pl}}}\right)^{1/2} \tag{23}$$

for $T_{\mathrm{BH}}^{\mathrm{in}} > m_{\mathrm{DM}}$ [32,37,38,117].

Figure 2 shows the correlation between the branching ratio $\mathrm{BR}(\mu \to e\gamma)$ and the initial density of PBHs β for the observed relic abundance of dark matter $\Omega_{\mathrm{DM}}h^2 = 0.12 \pm 0.0009$ [106] in the case of $\beta < \beta_{\mathrm{c}}$. The upper horizontal line shows the current observed upper limit of $\mathrm{BR}(\mu \to e\gamma)$. The lower horizontal line shows the expected sensitivity of the future MEG II

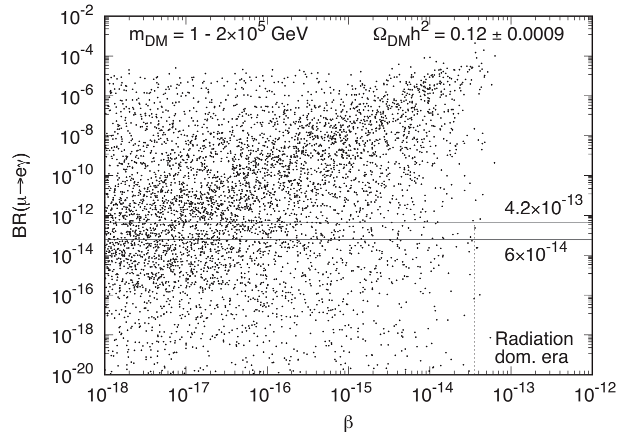

Fig. 2. Correlation between the branching ratio $\mathrm{BR}(\mu \to e\gamma)$ and the initial density of PBHs β for $\Omega_{\mathrm{DM}}h^2 = 0.12 \pm 0.0009$ in the case of $\beta < \beta_{\mathrm{c}}$ (radiation-dominated era). The upper horizontal line shows the current observed upper limit of $\mathrm{BR}(\mu \to e\gamma)$. The lower horizontal line shows the expected sensitivity of the future MEG II experiment.

experiment. From Fig. 2, the initial density of PBHs should be

$$\beta \lesssim 3.5 \times 10^{-14} \tag{24}$$

for $\Omega_{\mathrm{DM}}h^2 = 0.12 \pm 0.0009$ in the case of $\beta < \beta_{\mathrm{c}}$ with scotogenic dark matter.

Since we have four free parameters $\{m_{\mathrm{DM}}(= M_1), M_3, m_0, \lambda\}$ within a wide range, a deeper numerical study around these four parameters is necessary.

Figure 3 shows the correlations between the initial density of PBHs β and the four free parameters $\{m_{\mathrm{DM}}(= M_1), M_3, m_0, \lambda\}$ in the scotogenic model for $\Omega_{\mathrm{DM}}h^2 = 0.12 \pm 0.0009$ and $6 \times 10^{-14} \leq \mathrm{BR}(\mu \to e\gamma) \leq 4.2 \times 10^{-13}$ in the case of $\beta < \beta_{\mathrm{c}}$. From Fig. 3, if a $\mu \to e\gamma$ process is to be observed in the MEG II experiment, the allowed regions of the four free parameters are:

$$\begin{aligned} 1.0 \lesssim\ & m_{\mathrm{DM}}\ [\mathrm{GeV}] \lesssim 1 \times 10^5, \\ 5.5 \lesssim\ & M_3\ [\mathrm{GeV}] \lesssim 1 \times 10^6, \\ 3.5 \times 10^2 \lesssim\ & m_0\ [\mathrm{GeV}] \lesssim 1 \times 10^6, \\ 2.5 \times 10^{-11} \lesssim\ & \lambda \lesssim 3.8 \times 10^{-10}, \end{aligned} \tag{25}$$

in the case of $\beta < \beta_{\mathrm{c}}$. The magnitude of λ is constrained around 10^{-10} for $\beta < \beta_{\mathrm{c}}$. In the case of $\beta < \beta_{\mathrm{c}}$ (radiation-dominated era), the relic abundance via the freeze-out mechanism within the scotogenic model may be more dominant than the relic abundance via PBH evaporation in the observed relic abundance [40]. Thus, this characteristic constraint of λ is needed to satisfy the experimental constraint, especially $\mathrm{BR}(\mu \to e\gamma)$, in the scotogenic model. On the other hand, as we show later, the relic abundance via PBH evaporation becomes more dominant in the observed relic abundance in the case of $\beta > \beta_{\mathrm{c}}$ (PBH-dominated era) and a wider parameter region of λ becomes possible.

Here we present an additional discussion on the structure of Yukawa couplings to understand the phenomenology in the model. As shown in Refs. [80,105], a small λ_5 yields relatively large Yukawa couplings, e.g. $|y_1| = (0.078) \pm 0.021\sqrt{m_{\mathrm{DM}}/\mathrm{GeV}}$ with $\lambda_5 \sim 10^{-10}$ for the normal neutrino mass ordering may be expected, where y_1 denotes an eigenvalue of the Yukawa matrix [105], and the typical magnitude of the Yukawa couplings $|Y_{\alpha k}|$ may be $\mathcal{O}(0.1\text{–}1)$ [80]. Since the

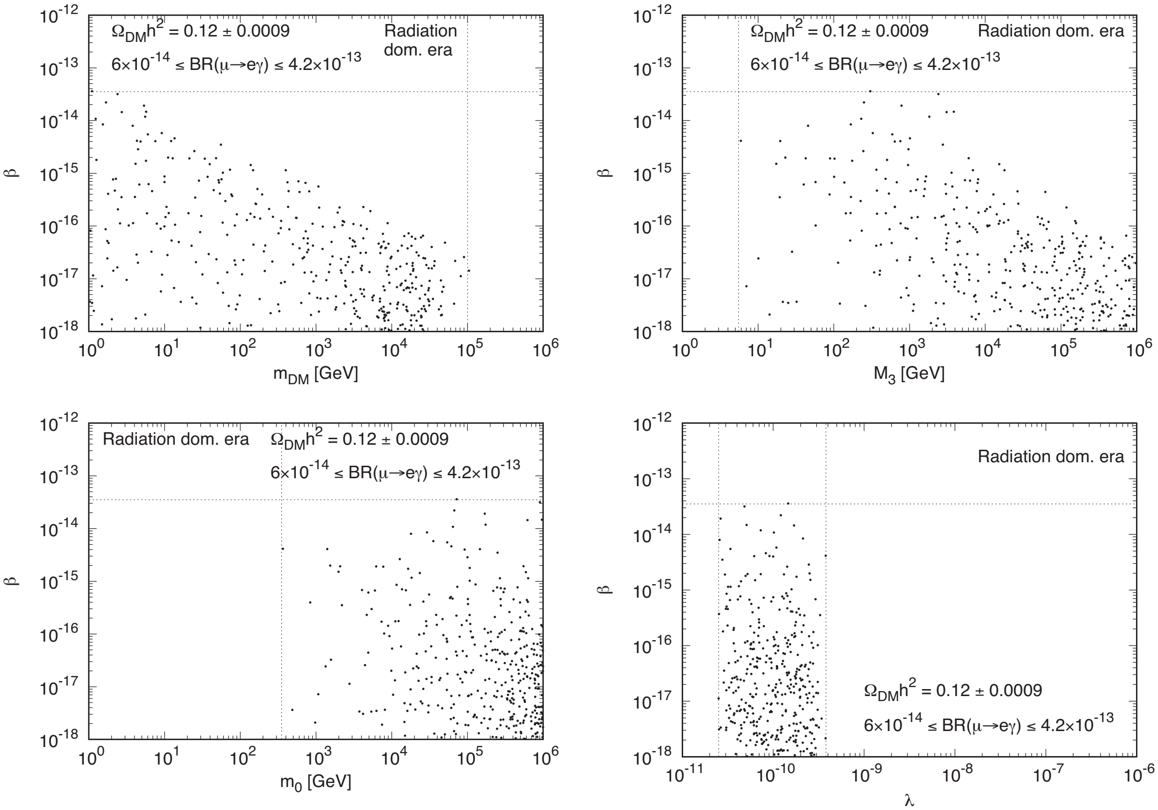

Fig. 3. Correlation between the initial density of PBHs β and the four free parameters $\{m_{\rm DM}(=M_1), M_3, m_0, \lambda\}$ in the scotogenic model for $\Omega_{\rm DM}h^2 = 0.12 \pm 0.0009$ and $6\times10^{-14} \le {\rm BR}(\mu\to e\gamma) \le 4.2\times10^{-13}$ in the case of $\beta < \beta_{\rm c}$ (radiation-dominated era).

Yukawa couplings become relatively large, cancelation among the Yukawa couplings is needed to satisfy the experimental constraints. For example, the relatively large Yukawa couplings $Y_{e1} = 0.77$, $Y_{e2} = -0.25 - 0.026i$, $Y_{e3} = 0.46 - 0.022i$, $Y_{\mu1} = 0.52$, $Y_{\mu2} = 0.58 - 0.017i$, $Y_{\mu3} = -0.55 - 0.015i$, $Y_{\tau1} = -0.18 - 0.056i$, $Y_{\tau2} = 0.97$, and $Y_{\tau3} = 0.84$ are obtained for $\lambda = 10^{-10}$ in our numerical calculations; however, the cancelation among these Yukawa couplings yields $|\sum_{k=1}^{3} Y_{\mu k}Y_{ek}^{*}F(M_k/m_0)|^2 = 4.4\times10^{-7}$, and we can obtain an acceptable small magnitude of ${\rm BR}(\mu\to e\gamma) = 1.2\times10^{-13}$.

Figure 4 shows the correlations between the allowed regions of the four free parameters $\{m_{\rm DM}(=M_1), M_3, m_0, \lambda\}$ in the scotogenic model in the case of $\beta < \beta_{\rm c}$. We pick up the following four benchmark cases in the case of $\beta < \beta_{\rm c}$ (radiation-dominated (RD) era):

RD1: $\{m_{\rm DM}, M_3, m_0\} = \{10, 300, 3000\}$ GeV as a set of light masses.
RD2: $\{m_{\rm DM}, M_3, m_0\} = \{1500, 2000, 3500\}$ GeV as a set of middle masses. The energy scale in this benchmark case could be a target of research in the next-generation experiments [83,92,93].
RD3: $\{m_{\rm DM}, M_3, m_0\} = \{1500, 5\times10^4, 8\times10^5\}$ GeV as another set of middle masses.
RD4: $\{m_{\rm DM}, M_3, m_0\} = \{3\times10^4, 3\times10^5, 8\times10^5\}$ GeV as a set of heavy masses.

The benchmark cases are marked with a $\star$ in Fig. 4. To avoid a large number of benchmark cases we have distinguished the benchmark cases by the three parameters $\{m_{\rm DM}, M_3, m_0\}$, without λ. We take any value of λ in its allowed region.

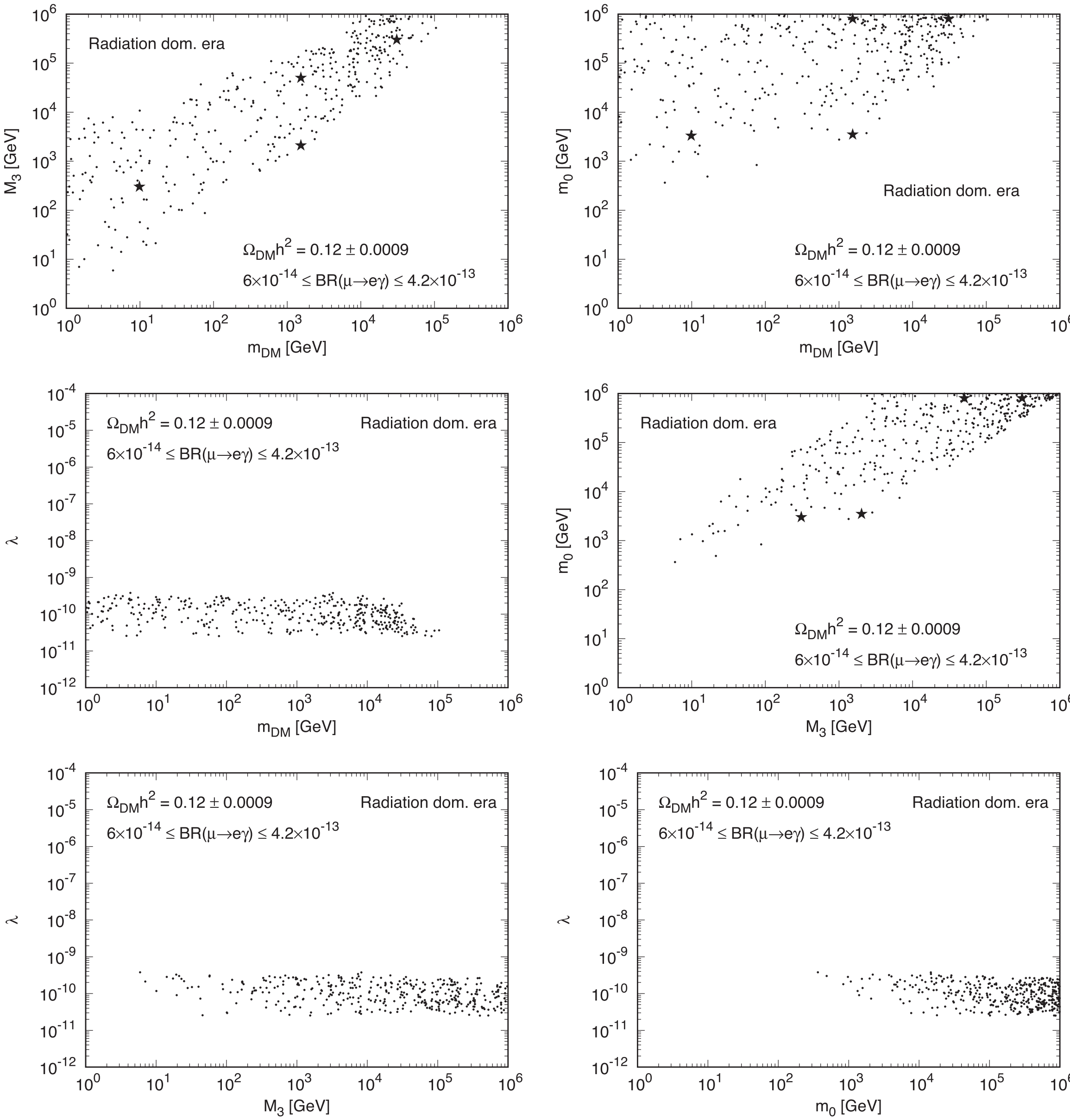

Fig. 4. Correlations between the allowed regions of the four free parameters $\{m_{\rm DM}(=M_1), M_3, m_0, \lambda\}$ in the scotogenic model for $\Omega_{\rm DM}h^2 = 0.12 \pm 0.0009$ and $6 \times 10^{-14} \leq {\rm BR}(\mu \to e\gamma) \leq 4.2 \times 10^{-13}$ in the case of $\beta < \beta_{\rm c}$ (radiation-dominated era). A $\star$ denotes a benchmark case.

Figure 5 shows the same as Fig. 2 but for the benchmark cases. We observe that if the lepton flavor-violating $\mu \to e\gamma$ process is observed in the MEG II experiment, $4.2 \times 10^{-13} \lesssim {\rm BR}(\mu \to e\gamma) \lesssim 6 \times 10^{-14}$, the initial density of PBHs should be constrained for each benchmark case as follows:

RD1: No constraint.
RD2: $\beta \lesssim 3 \times 10^{-16}$.
RD3: $2 \times 10^{-17} \lesssim \beta \lesssim 5 \times 10^{-16}$.
RD4: $\beta \lesssim 6 \times 10^{-17}$.

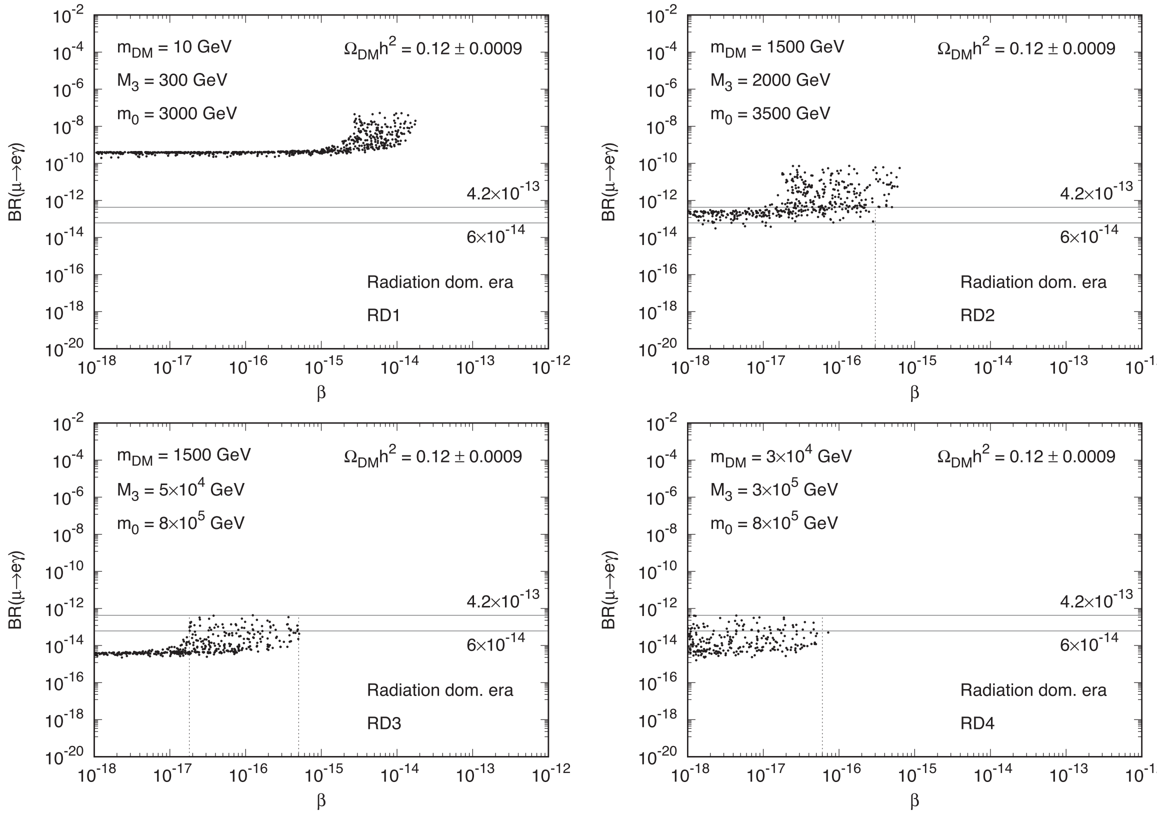

Fig. 5. As Fig. 2 but for the benchmark cases.

3.3 *PBH evaporation in the PBH-dominated era*

If the condition $\beta > \beta_c$ is satisfied, PBH evaporation occurs after the early equality time (PBH-dominated era). In the PBH-dominated era, the entropy production via the evaporation of PBHs leads to a dilution of the scotogenic dark matter originating from freeze-out [32,37,38,109,117]. The final relic abundance of scotogenic dark matter is obtained as

$$\Omega_{\rm DM}h^2 = \alpha^{-1}\Omega_{\rm FO}h^2 + \Omega_{\rm PBH}h^2, \tag{26}$$

where α denotes the entropy boost factor. The factor α is the ratio of the entropy prior to, $S_{\rm before}$, and after, $S_{\rm after}$, the PBH evaporation, $\alpha(s_{\rm before}a^3_{\rm before}) = s_{\rm after}a^3_{\rm after}$, and is given by

$$\alpha = \frac{S_{\rm after}}{S_{\rm before}} = \frac{Y_{\rm in}}{Y_{\rm evap}}, \tag{27}$$

where

$$Y_{\rm in} = \frac{n_{\rm BH}(t_{\rm in})}{s(t_{\rm in})} = \beta\frac{\rho_{\rm rad}(t_{\rm in})}{M_{\rm in}s(t_{\rm in})}, \qquad Y_{\rm evap} = \frac{n_{\rm BH}(t_{\rm evap})}{s(t_{\rm evap})} = \frac{\rho_{\rm rad}(t_{\rm evap})}{M_{\rm in}s(t_{\rm evap})}, \tag{28}$$

with

$$\rho_{\rm rad}(T) = \frac{\pi^2}{30}g_*(T)T^4, \qquad s(T) = \frac{2\pi^2}{45}g_{*s}(T)T^3, \tag{29}$$

where g_{*s} is the relativistic effective degrees of freedom for the entropy density. By combining Eqs. (27)– (29), we obtain

$$\alpha = \beta\frac{g_*(t_{\rm in})}{g_*(t_{\rm evap})}\frac{g_{*s}(t_{\rm evap})}{g_{*s}(t_{\rm in})}\frac{T_{\rm in}}{T_{\rm evap}}. \tag{30}$$

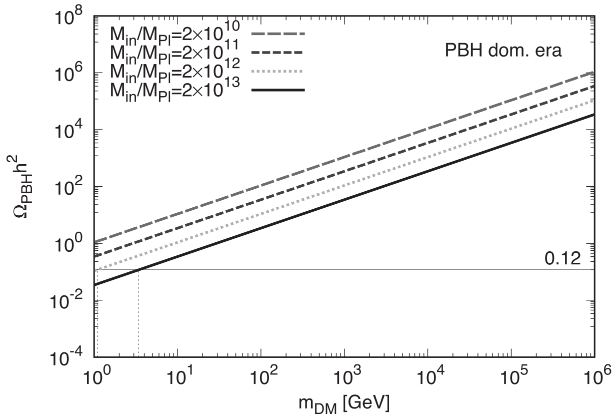

Fig. 6. Dependence of relic abundance of PBH-origin scotogenic dark matter $\Omega_{\rm PBH}h^2$ on dark matter mass $m_{\rm DM}$ in the case of $\beta > \beta_{\rm c}$ (PBH-dominated era). The horizontal line shows the observed relic abundance of dark matter.

According to the relation $g_*(T) \simeq g_{*s}(T)$ for high temperatures, the entropy boost factor becomes the ratio of the initial PBH density, β, and the critical density, $\beta_{\rm c}$:

$$\alpha = \frac{\beta}{\beta_{\rm c}}. \tag{31}$$

From the relation $\Omega_{\rm DM}h^2 = \alpha^{-1}\Omega_{\rm FO}h^2 + \Omega_{\rm PBH}h^2$, the relic abundance of dark matter via PBH evaporation, $\Omega_{\rm PBH}h^2$, should be at least less than the observed relic abundance $\Omega_{\rm DM}h^2 = 0.12 \pm 0.0009$.

First, we estimate the allowed region of the dark matter mass $m_{\rm DM}$ without relic abundance via the freeze-out mechanism. Figure 6 shows the dependence of the relic abundance of PBH-origin scotogenic dark matter $\Omega_{\rm PBH}h^2$ on the dark matter mass $m_{\rm DM}$ in the case of $\beta > \beta_{\rm c}$. The horizontal line shows the observed relic abundance of dark matter. We require $M_{\rm in}/M_{\rm Pl} \gtrsim 1 \times 10^{10}$ for $m_{\rm DM} \simeq 1$ GeV. This requirement will be commented on later. With this requirement, the following regions of scotogenic dark matter mass,

$$1.1\,{\rm GeV} \le m_{\rm DM} \le 3.4\,{\rm GeV}, \tag{32}$$

and initial PBH mass,

$$2 \times 10^{12} \lesssim M_{\rm in}/M_{\rm Pl} \lesssim 2 \times 10^{13}, \tag{33}$$

are relevant for $\beta > \beta_{\rm c}$.

For $m_{\rm DM} = 1.1$–3.4 GeV we have

$$\frac{M_{\rm in}}{M_{\rm Pl}} \gtrsim \begin{cases} 1.88 \times 10^{12} & (m_{\rm DM} = 1.1\,{\rm GeV}), \\ 8.85 \times 10^{11} & (m_{\rm DM} = 3.4\,{\rm GeV}). \end{cases} \tag{34}$$

The Hawking temperature at the PBH formation time would be

$$T_{\rm BH}^{\rm in} = \begin{cases} 2.6 \times 10^5\,{\rm GeV} & \left(\frac{M_{\rm in}}{M_{\rm Pl}} = 1.88 \times 10^{12}\right), \\ 5.5 \times 10^5\,{\rm GeV} & \left(\frac{M_{\rm in}}{M_{\rm Pl}} = 8.85 \times 10^{11}\right). \end{cases} \tag{35}$$

Thus, the relation $T_{\rm BH}^{\rm in} > m_{\rm DM}$ is satisfied in our setup. The relic abundance of PBH-origin scotogenic dark matter is obtained as

$$\Omega_{\rm PBH}h^2 \simeq 1.09 \times 10^7 \left(\frac{g_*(T_{\rm BH})}{106.75}\right)^{1/4} \frac{3}{4}\frac{g_{\rm DM}}{g_*(T_{\rm BH})}\left(\frac{m_{\rm DM}}{\rm GeV}\right)\left(\frac{M_{\rm Pl}}{M_{\rm in}}\right)^{1/2} \tag{36}$$

for $T_{\rm BH}^{\rm in} > m_{\rm DM}$ [32,37,38,117].

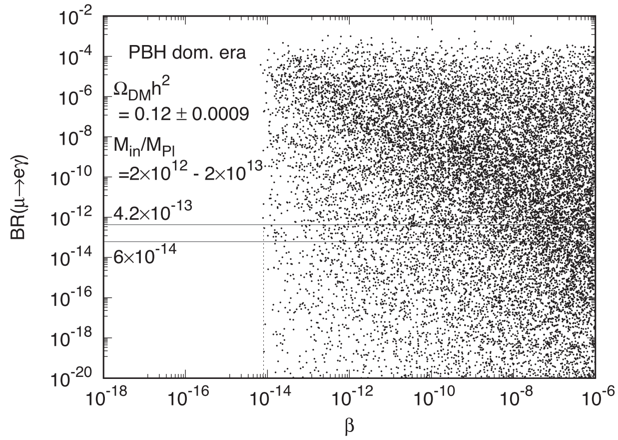

Fig. 7. Correlation between the branching ratio BR($\mu \to e\gamma$) and the initial PBH density β for $\Omega_{\rm DM}h^2 = 0.12 \pm 0.0009$ in the case of $\beta > \beta_{\rm c}$ (PBH-dominated era). The upper horizontal line shows the current observed upper limit of BR($\mu \to e\gamma$). The lower horizontal line shows the expected sensitivity in the future MEG II experiment.

We now include the relic abundance of dark matter via the freeze-out mechanism in our analysis. Figure 7 shows the correlation between the branching ratio BR($\mu \to e\gamma$) and the initial PBH density β for $\Omega_{\rm DM}h^2 = 0.12 \pm 0.0009$ and $1.1\,{\rm GeV} \le m_{\rm DM} \le 3.4\,{\rm GeV}$ in the case of $\beta > \beta_{\rm c}$. The upper horizontal line shows the current observed upper limit of BR($\mu \to e\gamma$). The lower horizontal line shows the expected sensitivity of the future MEG II experiment. From Fig. 7, the initial PBH density should be

$$\beta \gtrsim 8 \times 10^{-15} \tag{37}$$

for $\Omega_{\rm DM}h^2 = 0.12 \pm 0.0009$ in the case of $\beta > \beta_{\rm c}$ with scotogenic dark matter.

Figure 8 shows the correlations between the initial PBH density β and the four free parameters $\{m_{\rm DM}(=M_1), M_3, m_0, \lambda\}$ in the scotogenic model for $\Omega_{\rm DM}h^2 = 0.12 \pm 0.0009$ and $6 \times 10^{-14} \le {\rm BR}(\mu \to e\gamma) \le 4.2 \times 10^{-13}$ in the case of $\beta > \beta_{\rm c}$. From Fig. 8, if a $\mu \to e\gamma$ process is observed in the MEG II experiment, the allowed regions of the four free parameters are

$$\begin{aligned} 1.1 \lesssim & \; m_{\rm DM}\,[{\rm GeV}] \lesssim 3.4, \\ 1.3 \lesssim & \; M_3\,[{\rm GeV}] \lesssim 1 \times 10^6, \\ 3.6 \lesssim & \; m_0\,[{\rm GeV}] \lesssim 1 \times 10^6, \\ 2.5 \times 10^{-11} \lesssim & \; \lambda \lesssim 1 \times 10^{-6} \end{aligned} \tag{38}$$

in the case of $\beta > \beta_{\rm c}$. The allowed region of the DM mass is narrow for $\beta > \beta_{\rm c}$. In the case of $\beta > \beta_{\rm c}$ (PBH-dominated era), the relic abundance via PBH evaporation may be more dominant than the relic abundance via the freeze-out mechanism in observed relic abundance [40]. In this case, the allowed region of dark matter mass should be narrow to satisfy the condition $\Omega_{\rm PBH}h^2 \le \Omega_{\rm DM}h^2 = 0.12$ for $M_{\rm in}/M_{\rm Pl} \le 2 \times 10^{10}$, as shown in Fig. 6.

Figure 9 shows the correlations between the allowed regions of the four free parameters $\{m_{\rm DM}(=M_1), M_3, m_0, \lambda\}$ in the scotogenic model in the case of $\beta > \beta_{\rm c}$. A $\star$ denotes a benchmark case. We pick up the following four benchmark cases in the case of $\beta > \beta_{\rm c}$ (PBH-dominated (PBHD) era) for $M_{\rm in}/M_{\rm Pl} = 2 \times 10^{12}$:

PBHD1: $\{M_3, m_0\} = \{10, 300\}$ GeV as a set of light masses.
PBHD2: $\{M_3, m_0\} = \{2000, 3500\}$ GeV as a set of middle masses.

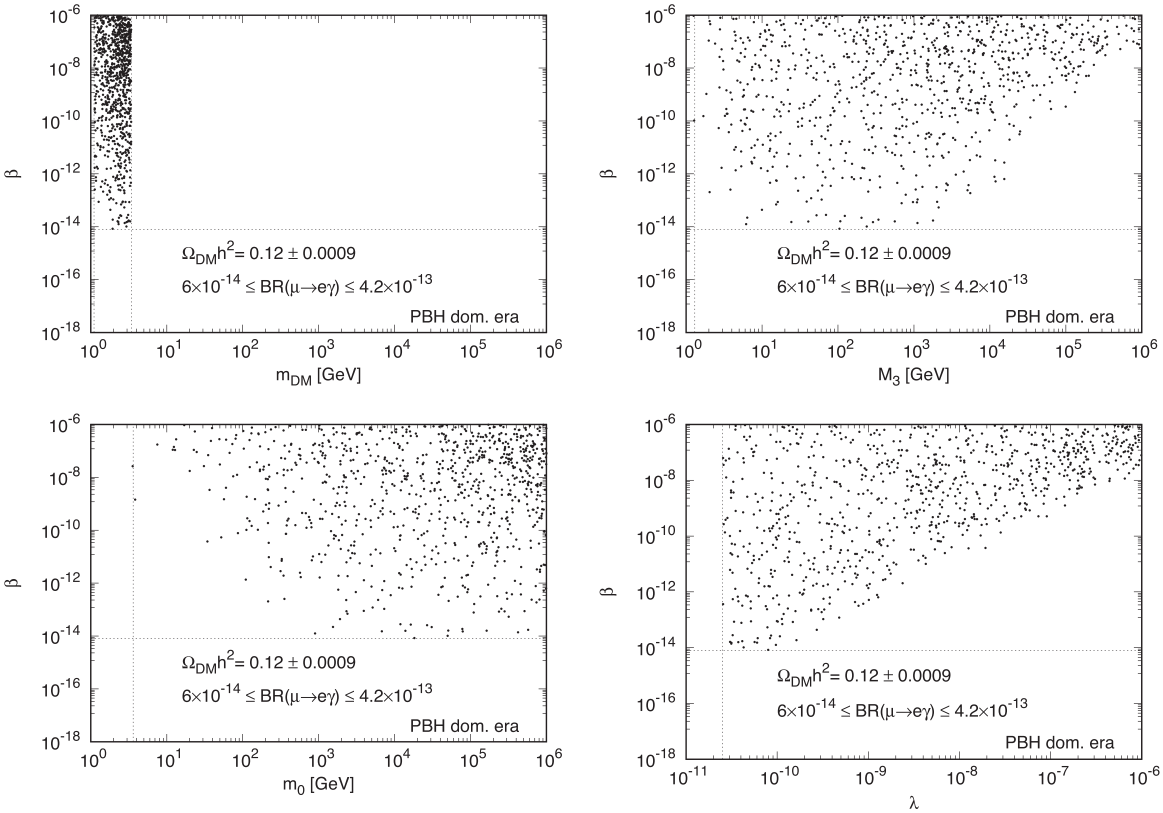

Fig. 8. Correlations between the initial PBH density β and the four free parameters $\{m_{\rm DM}(=M_1), M_3, m_0, \lambda\}$ in the scotogenic model for $\Omega_{\rm DM}h^2 = 0.12 \pm 0.0009$ and $6 \times 10^{-14} \le {\rm BR}(\mu \to e\gamma) \le 4.2 \times 10^{-13}$ in the case of $\beta > \beta_{\rm c}$ (PBH-dominated era).

PBHD3: $\{M_3, m_0\} = \{2000, 5 \times 10^5\}$ GeV as another set of middle masses.
PBHD4: $\{M_3, m_0\} = \{2 \times 10^5, 5 \times 10^5\}$ GeV as a set of heavy masses.

In addition, we consider four more benchmark cases, PBHD5, PBHD6, PBHD7, and PBHD8, with the same $\{M_3, m_0\}$ sets as PBHD1, PBHD2, PBHD3, and PBHD4, respectively, for $M_{\rm in}/M_{\rm Pl} = 2 \times 10^{13}$. Since the allowed region of the dark matter mass is narrow, we vary the dark matter mass as $m_{\rm DM} = 1.1$–3.4 GeV in these eight benchmark cases. In addition, to avoid a large number of benchmark cases, we have distinguished the benchmark cases by the three parameters $\{M_3, m_0\}$ and $M_{\rm in}/M_{\rm Pl}$ without λ.

Figure 10 shows the same as Fig. 7 but for the benchmark cases. We observe that if the lepton flavor-violating $\mu \to e\gamma$ processes is observed in the MEG II experiment, $4.2 \times 10^{-13} \lesssim {\rm BR}(\mu \to e\gamma) \lesssim 6 \times 10^{-14}$, the initial PBH density should be constrained for each benchmark case as follows:

PBHD1: $2 \times 10^{-9} \lesssim \beta$.
PBHD2: No constraint.
PBHD3: $1 \times 10^{-12} \lesssim \beta$.
PBHD4: No constraint.
PBHD5: $1 \times 10^{-12} \lesssim \beta \lesssim 3 \times 10^{-11}$.
PBHD6: $1 \times 10^{-8} \lesssim \beta \lesssim 3 \times 10^{-7}$.

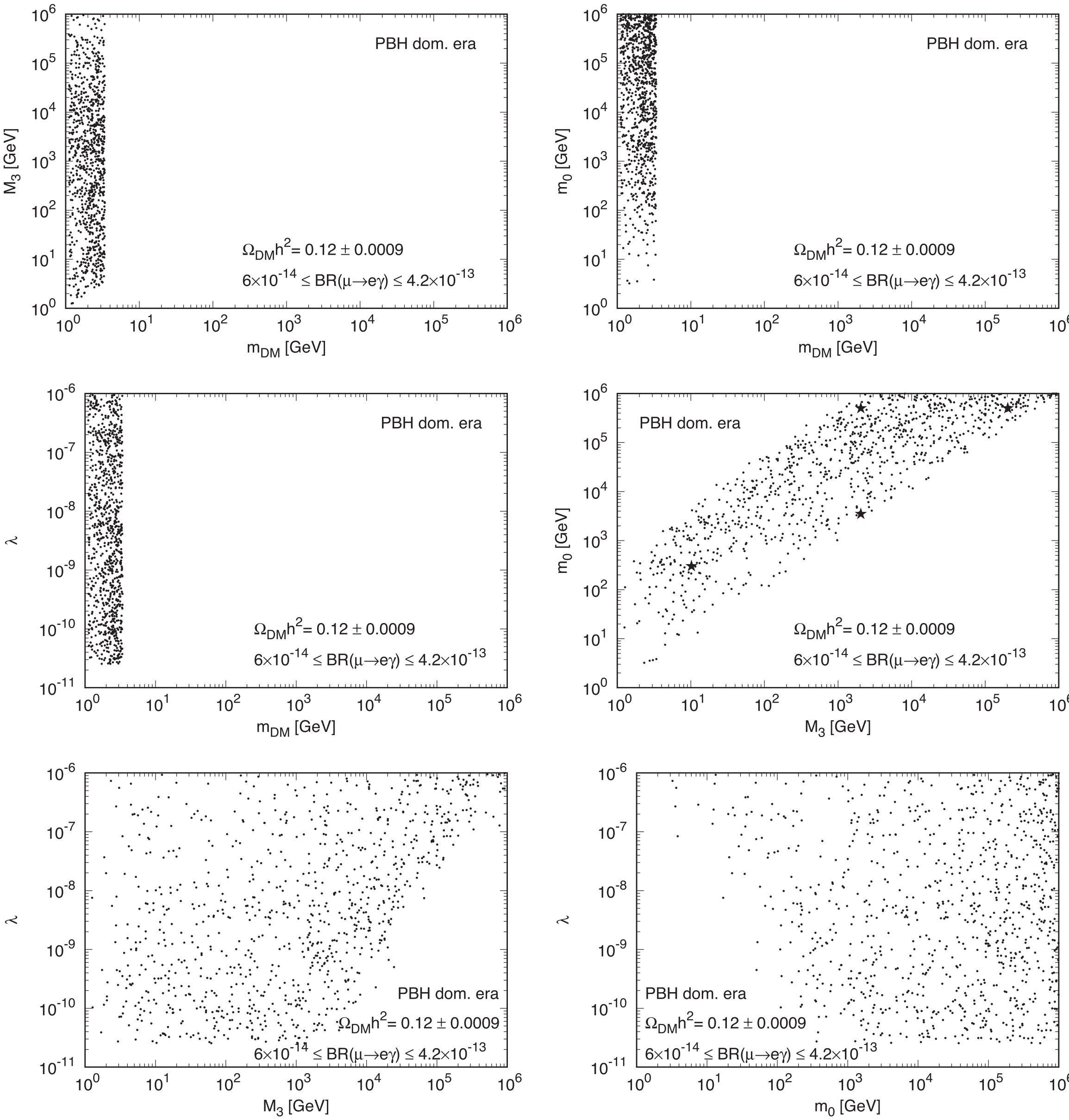

Fig. 9. Correlations between the allowed regions of the four free parameters $\{m_{\rm DM}(=M_1), M_3, m_0, \lambda\}$ in the scotogenic model for $\Omega_{\rm DM}h^2 = 0.12 \pm 0.0009$ and $6 \times 10^{-14} \le {\rm BR}(\mu \to e\gamma) \le 4.2 \times 10^{-13}$ in the case of $\beta > \beta_{\rm c}$ (PBH-dominated era). A $\star$ denotes a benchmark case.

PBHD7: $7 \times 10^{-15} \lesssim \beta \lesssim 2 \times 10^{-14}$.
PBHD8: $6 \times 10^{-9} \lesssim \beta \lesssim 1 \times 10^{-7}$.

We would now like to comment on our requirement of $M_{\rm in}/M_{\rm Pl} \gtrsim 1 \times 10^{10}$ for $m_{\rm DM} \simeq 1$ GeV. Baldes et al. showed that if all the relic abundance comes from PBH evaporation, $\Omega_{\rm PBH}h^2 = \Omega_{\rm DM}h^2$, for $m_{\rm DM} \simeq 1$ GeV, PBHs with mass $M_{\rm in}/M_{\rm Pl} \lesssim 1 \times 10^{10}$ are not allowed by the conservative Lyman-α bound for warm dark matter masses $m_{\rm WDM} > 3$ keV [38]. Figure 11 shows the portion of dark matter particles coming from PBHs for $\Omega_{\rm DM}h^2 = 0.12 \pm 0.0009$ and $6 \times 10^{-14} \le {\rm BR}(\mu \to e\gamma) \le 4.2 \times 10^{-13}$ in the case of $\beta > \beta_{\rm c}$. From Fig. 11, we observe that almost all the relic abundance may be caused by PBH evaporation in some specific parameter sets in the scoogenic model. In this case, PBHs with mass $M_{\rm in}/M_{\rm Pl} \lesssim 1 \times 10^{10}$ for $m_{\rm DM} \simeq 1$ GeV are

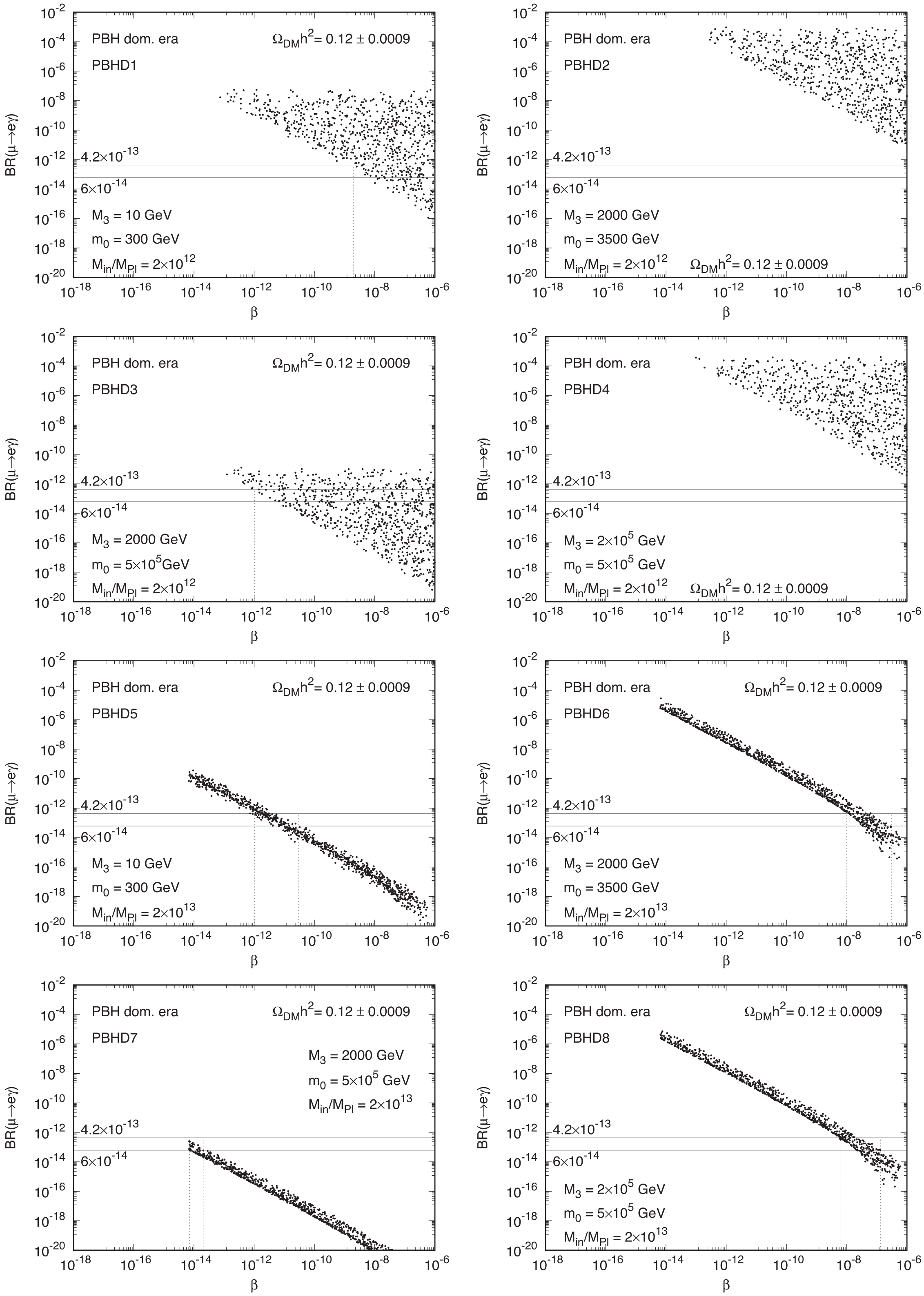

Fig. 10. As Fig. 7 but for the benchmark cases.

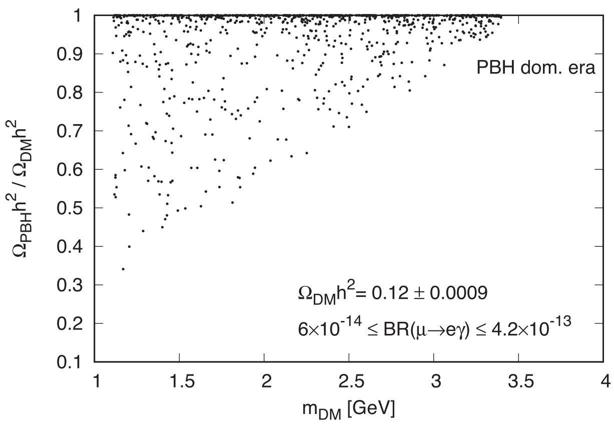

Fig. 11. Portion of dark matter particles coming from PBHs for $\Omega_{\rm DM}h^2 = 0.12 \pm 0.0009$ and $6 \times 10^{-14} \leq {\rm BR}(\mu \to e\gamma) \leq 4.2 \times 10^{-13}$ in the case of $\beta > \beta_{\rm c}$ (PBH-dominated era).

not allowed by the warm dark matter constraints. Thus, we have conservatively required the condition $M_{\rm in}/M_{\rm Pl} \gtrsim 1 \times 10^{10}$ for $m_{\rm DM} \simeq 1$ GeV in our analysis.

Finally, we would like to address the effect that having an extended dark sector may have on the contribution to the final relic abundance of dark matter. Not only the standard-model particles and the dark matter particles, but also the heavier Majorana particles, N_2 and N_3, and the new scalar η will be created via PBH evaporation. In our setup, the Hawking temperature of the PBHs is enough to be able to produce such particles. These particles will decay into dark matter N_1 via $N_3 \to \ell^\pm\ell^\mp N_{1,2}$, $N_2 \to \ell^\pm\ell^\mp N_1$, and $\eta \to \ell N_{1,2,3}$ [54], and increase the contribution on the relic density of the dark matter. This remarkable effect of the decaying PBH-origin heavy particles on the relic abundance was studied very recently in a general scheme with a simple and predictive particle model in Ref. [47]. If we include this effect, the results in this paper may be modified.

For example, N_2 decays into dark matter N_1 and a pair of leptons with the decay rate [119,120]

$$\Gamma(N_2 \to \ell_\alpha \ell_\beta N_1) = \frac{m_{N_2}^5}{6144\pi^3 M^4}\left(|Y_{1\beta}|^2|Y_{2\alpha}|^2 + |Y_{1\alpha}|^2|Y_{2\beta}|^2\right), \tag{39}$$

where M denotes the mass of the scalar particle that is exchanged in the process, and α and β denote the flavors of the final-state leptons. This decay gives a contribution to the dark matter relic abundance of the form $\Omega_{N_2 \to N_1}h^2 = m_{N_1}/m_{N_2}\Omega_{N_2}h^2$, where $\Omega_{N_2}h^2$ may be interpreted as the amount of N_2 via PBH evaporation [120]. This contribution can be constrained from the effective number of neutrinos in the early Universe, and the condition

$$\frac{\Omega_{N_2 \to N_1}h^2}{\Omega h^2} \lesssim 0.2\% \tag{40}$$

should be satisfied for consistency with cosmological observations with typical masses of the particle in the scotogenic model [120]. From this knowledge, we may expect that the effect of the decaying PBH-origin heavy particles on the dark matter final abundance does not drastically disturb the conclusion of this paper; however, including this effect for more precise analysis may give us valuable results, as shown in Ref. [47]. In this paper we have reported our results as a quasi-precise analysis, allowing us to ignore this effect. We intend to examine this important effect in a future study.

4. Summary

In this paper we have shown the correlations between the initial PBH density and the branching ratio of $\mu \to e\gamma$ with scotogenic dark matter. Since the scotogenic model can account for dark matter candidates and predict the lepton flavor-violating processes simultaneously, and scotogenic dark matter can also be produced by Hawking radiation from PBHs, the initial PBH density and the branching ratio of $\mu \to e\gamma$ are related via scotogenic dark matter.

It turns out that if the lepton flavor-violating $\mu \to e\gamma$ process is observed in the MEG II experiment, the initial density of primordial black holes can be constrained by the scotogenic dark matter. As a benchmark case, if PBH evaporation occurs in the radiation-dominated era, the initial density may be $\beta \lesssim 3 \times 10^{-16}$ for an $\mathcal{O}$(TeV)-scale dark sector in the scotogenic model. As another benchmark case, if PBHs evaporate in the PBH-dominated era, the initial density may be $1 \times 10^{-8} \lesssim \beta \lesssim 3 \times 10^{-7}$ for $\mathcal{O}$(GeV)-scale dark matter with other $\mathcal{O}$(TeV)-scale particles in the scotogenic model.

Since the physics run in the MEG II experiment is about to be started, the predictions in this study may be tested within five years.

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