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Research Article

On BPS World Volume, RR Couplings, and Their α' Corrections in Type IIB

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We compute the asymmetric and symmetric correlation functions of a four-point amplitude of a gauge field, a scalar field, and a closed string Ramond-Ramond (RR) for different nonvanishing BPS branes. All world volume, the Taylor and pull-back couplings, and their all-order α' corrections have also been explored. Due to various symmetry structures, different restricted BPS Bianchi identities have also been constructed. The prescription of exploring all the corrections of two closed string RR couplings in type IIB is given. We obtain the closed form of the entire S-matrix elements of two closed string RRs and a gauge field on the world volume of BPS branes in type IIB. All the correlation functions of $V_{A^0(x_1)}V_{C^{-1}(z_1,\bar{z}_1)}V_{C^{-1}(z_2,\bar{z}_2)}$ are also revealed accordingly. The algebraic forms for the most general case of the integrations $\int d^2z|z-i|^a|z+i|^b(z-\bar{z})^c(z+\bar{z})^d$ on the upper half plane are derived in terms of Pochhammer and some analytic functions. Lastly, we generate various singularity structures in both effective field theory and IIB string theory, producing different contact interactions as well as their α' higher derivative corrections.

1. Introduction

The fundamental objects are called D_p-branes that are long known to be existed. The late Joe Polchinski has given life to the D-branes as dynamical objects, and they are assumed to be sources for Ramond-Ramond (RR) closed string for all sorts of the established BPS branes [1, 2].

These RR couplings have various contributions in many different areas of theoretical high energy physics, ranging from pure string theory to mathematics, K-theory as well as phenomenology. For example, one may point out to the dissolving branes [3, 4], K-theory [5, 6], and the well-known dielectric or Myers effect [7] where some of their α' corrections are also derived in detail in [8].

To proceed with their dynamics, one needs to know all sorts of effective actions where various potentially interesting references are given in [9]. We would like to take advantage of conformal field theory (CFT) and try to release more information about the structures of the BPS effective actions. Indeed, one of our aims is to work out with CFT to get more data and increase our knowledge of deriving various string theory couplings, and likewise, various techniques to effective field theory (EFT) couplings along the way can also be explored.

One can just mention different applications to some of the known couplings, like the famous N^3 phenomenon for $M5$ branes, dS solutions, and entropy growth [10–12]. It is also known that RR plays the key role for all kinds of BPS and unstable branes [13, 14] where one can study some of its analysis as well as its dynamics in [15–21]. All the standard approaches to get to EFT couplings were explained in detail in [22, 23], where S-matrix computations play the most fundamental role in getting the exact form of string couplings, and their precise coefficients are even computed in the presence of higher derivative α' corrections. We would like to illustrate just some of the BPS string calculations as given in [24–33]. For the sake of comprehensiveness and for a review of open strings as well as their properties, we just highlight the original papers that are known and appeared in [34–42].

In the first part of the paper, we calculate both asymmetric and symmetric S-matrices of a four-point amplitude of a gauge, a scalar field and a closed string RR for different nonvanishing traces of all different BPS branes. All world volume, the Taylor and pull-back couplings, and their α' corrections have also been figured out. Due to various symmetry structures, various restricted BPS Bianchi identities can also be derived. We then try to demonstrate a prescription of exploring all the corrections of two closed string RR couplings in type IIB as well. To proceed further, we go ahead to obtain the closed form of the entire S-matrix elements of two closed string RRs and a gauge field on the world volume of BPS branes just in the type IIB string theory.

We first derive all the correlation functions of $V_{A^0(x_1)} V_{C^{-1}(z_1,\bar{z}_1)} V_{C^{-1}(z_2,\bar{z}_2)}$ and then reveal the algebraic forms of the integrals for the most general integrations on the upper half plane that are of the sort of $\int d^2z|z-i|^a|z+i|^b(z-\bar{z})^c(z+\bar{z})^d$, and the outcome is written down in terms of the Pochhammer and some analytic functions. Finally, we try to reconstruct various singularity structures in both the effective field theory (EFT) and IIB string theory. We also work out with different contact term analysis and reproduce different contact interactions of the same S-matrix as well as their α' higher derivative corrections and lastly point out to different remarks as well.

The lower order supersymmetric generalisation of the Wess-Zumino (WZ) action is found in [43] in both IIB and IIA. It was also highlighted that all structures as well as the coefficients of the α' corrections in type IIB are different from their type IIA couplings.

It is worth taking into account the reference [44] that deals with potentially different areas where important remarks on perturbative string amplitude calculations have been given. Indeed to do so, a systematic setup was revealed. It is highlighted that to ignore some spurious singularities, one needs to employ the vertical integration formalism with great care. Although this conjecture has potential overlaps with string calculations of IIB analysis, however, the role of the world volume couplings is not clarified in detail, nor are the bulk singularity structures given. In this paper, we would like to show the method of deriving algebraic forms of all the integrals in terms of Pochhammer and consequently argue about the role of the world volume couplings just in IIB as well as their explicit corrections.

2. All-Order Corrections to $V_{C^{-2}}V_{\phi^0}V_{A^0}$

In this section, using direct CFT methods [45], we would like to explore the entire S-matrix elements of an asymmetric RR, a scalar field, and a gauge field where its all-order α' higher derivative corrections can also be examined accordingly. It will be given by exploring all its correlation function as

$$\mathcal{A}^{C^{-2}\phi^0A^0} \sim \int dx_1 dx_2 d^2z \left\langle V_\phi^{(0)}(x_1) V_A^{(0)}(x_2) V_{RR}^{(-2)}(z,\bar{z})\right\rangle. \quad (1)$$

The related vertex operators are read off from [46, 47] as follows:

$$\begin{aligned}
V_\phi^{(-1)}(x) &= e^{-\phi(x)}\xi_{1i}\psi^i(x)e^{\alpha' iq\cdot X(x)},\\
V_A^{(-1)}(x) &= e^{-\phi(x)}\xi_a\psi^a(x)e^{\alpha' iq\cdot X(x)},\\
V_\phi^{(0)}(x) &= \xi_{1i}\left(\partial^i X(x) + i\alpha' q.\psi\psi^i(x)\right)e^{\alpha' iq.X(x)},\\
V_A^{(0)}(x) &= \xi_{1a}\left(\partial^a X(x) + i\alpha' q.\psi\psi^a(x)\right)e^{\alpha' iq.X(x)},\\
V_C^{\left(-\frac{3}{2},-\frac{1}{2}\right)}(z,\bar{z}) &= \left(P_-C_{(n-1)}M_p\right)^{\alpha\beta}e^{-3\phi(z)/2}S_\alpha(z)e^{i\frac{\alpha'}{2}p\cdot X(z)}\\
&\quad\cdot e^{-\phi(\bar{z})/2}S_\beta(\bar{z})e^{i\frac{\alpha'}{2}p\cdot D\cdot X(\bar{z})},\\
V_C^{\left(-\frac{1}{2},-\frac{1}{2}\right)}(z,\bar{z}) &= \left(P_-H_{(n)}M_p\right)^{\alpha\beta}e^{-\phi(z)/2}S_\alpha(z)e^{i\frac{\alpha'}{2}p\cdot X(z)}\\
&\quad\cdot e^{-\phi(\bar{z})/2}S_\beta(\bar{z})e^{i\frac{\alpha'}{2}p\cdot D\cdot X(\bar{z})}.
\end{aligned} \quad (2)$$

Note that the total background charge of the world-sheet with topology of a disk must be -2; hence, one needs to consider the symmetric and asymmetric picture of RR as illustrated in (2). For our notation, we use $\mu,\nu=0,1,..,9$ where world volume indices run by $a,b,c=0,1,\cdots,p$ and finally transverse indices are represented by $i,j=p+1,\cdots,9$. Other notations for spinors and projector are given by the following formulae:

$$\begin{aligned}
P_- &= \frac{1}{2}\left(1-\gamma^{11}\right), H_{(n)}\\
&= \frac{a_n}{n!}H_{\mu_1\cdots\mu_n}\gamma^{\mu_1}\cdots\gamma^{\mu_n}, \left(P_-H_{(n)}\right)^{\alpha\beta}\\
&= C^{\alpha\delta}\left(P_-H_{(n)}\right)_\delta{}^\beta,
\end{aligned} \quad (3)$$

where in type IIA (type IIB), the field strength of RR takes the value of $n=2,4$, $a_n=i$ ($n=1,3,5$, $a_n=1$). In order to use the holomorphic parts of the world-sheet fields, we apply the doubling trick which means that some change of variables is taken into account:

$$\begin{aligned}
&\tilde{X}^\mu(\bar{z}) \longrightarrow D^\mu_\nu X^\nu(\bar{z}), \tilde{\psi}^\mu(\bar{z}) \longrightarrow D^\mu_\nu\psi^\nu(\bar{z}), \tilde{\phi}(\bar{z}) \longrightarrow \phi(\bar{z}),\\
&\tilde{S}_\alpha(\bar{z}) \longrightarrow M_\alpha{}^\beta S_\beta(\bar{z}).
\end{aligned} \quad (4)$$

And the following matrices are needed:

$$D = \begin{pmatrix} -1_{9-p} & 0 \\ 0 & 1_{p+1} \end{pmatrix},$$
$$M_p = \begin{cases} \dfrac{\pm i}{(p+1)!}\gamma^{i_1}\gamma^{i_2}\cdots\gamma^{i_{p+1}}\varepsilon_{i_1\cdots i_{p+1}} \text{ for } p \text{ even} \\ \dfrac{\pm 1}{(p+1)!}\gamma^{i_1}\gamma^{i_2}\cdots\gamma^{i_{p+1}}\gamma_{11}\varepsilon_{i_1\cdots i_{p+1}} \text{ for } p \text{ odd.} \end{cases} \tag{5}$$

Now one is able to pick up just the following propagators for the whole world-sheet fields of the kind of X^μ, ψ^μ, ϕ, as

$$\begin{aligned} \langle X^\mu(z)X^\nu(w)\rangle &= -\eta^{\mu\nu}\log(z-w), \\ \langle \psi^\mu(z)\psi^\nu(w)\rangle &= -\eta^{\mu\nu}(z-w)^{-1}, \\ \langle \phi(z)\phi(w)\rangle &= -\log(z-w). \end{aligned} \tag{6}$$

Replacing the related vertex operators inside (1), exploring correlation functions, fixing the $SL(2,\mathbb{R})$ symmetry by gauge fixing as $(x_1, x_2, z, \bar{z}) = (x, -x, i, -i)$, and introducing $t = -(\alpha'/2)(k_1+k_2)^2$, one finds out the nonzero part of the asymmetric amplitude as follows:

$$\begin{aligned} \mathscr{A}^{\phi^0 A^0 C^{-2}} = &\Big(-2ik_{2c}p^i \mathrm{Tr}\left(P_- C_{(n-1)} M_p \Gamma^{ac}\right) \\ &+ 2ik_{1b}k_{2c}\mathrm{Tr}\left(P_- C_{(n-1)} M_p \Gamma^{acib}\right)\Big) \\ &\times \mu_p \xi_{1i}\xi_{2a}\pi^{1/2}\frac{\Gamma[-t+1/2]}{\Gamma[1-t]}, \end{aligned} \tag{7}$$

where the traces can be explored, note that

$$p > 3, H_n = *H_{10-n}, n \geq 5. \tag{8}$$

And the compact form of the asymmetric amplitude is derived to be

$$\begin{aligned} \mathscr{A}^{\phi^0 A^0 C^{-2}} = &\left(-k_{2c}p^i \varepsilon^{a_0\cdots a_{p-2}ac}C_{a_0\cdots a_{p-2}} + k_{1b}k_{2c}\varepsilon^{a_0\cdots a_{p-3}acb}C^i_{a_0\cdots a_{p-3}}\right) \\ &\times \mu_p \frac{32}{p!}\xi_{1i}\xi_{2a}\pi^{1/2}\frac{\Gamma[-t+1/2]}{\Gamma[1-t]}. \end{aligned} \tag{9}$$

We are dealing with massless strings; hence, the expansion energy is low (note that we removed the overall factor $(2i)^{-2t-1}$), that is, $t = -p^a p_a \longrightarrow 0$, and the expansion is found as follows:

$$\begin{aligned} \pi^{1/2}\frac{\Gamma[-t+1/2]}{\Gamma[1-t]} &= \pi\sum_{m=-1}^{\infty} c_m t^{m+1}, c_{-1} = 1, c_0 \\ &= 2\ln 2, c_1 = \frac{1}{6}\pi^2 + 2\ln 2. \end{aligned} \tag{10}$$

In order to produce the first term (9) in an EFT, one needs to consider the mixed Chern-Simons effective action and the Taylor expansion of the scalar field as below:

$$S_1 = \frac{\mu_p}{p!}\left(2\pi\alpha'\right)^2 \int_{\Sigma_{p+1}} \partial_i C_{p-1}\wedge F\phi^i \tag{11}$$

Now, if we consider the covariant derivative of the scalar field from pull-back of brane and employ the following new effective action

$$S_2 = \frac{\mu_p}{p!}\left(2\pi\alpha'\right)^2 \int_{\Sigma_{p+1}} C^i_{p-2}\wedge F \wedge D\phi_i, \tag{12}$$

then one can show that S_2 precisely produces the second term of (9).

However, as can be observed, the expansion of the amplitude consists of many contact interaction terms, and one reconstructs all the contact terms of the S-matrix in an EFT by imposing an infinite higher derivative corrections to the above S_1 and S_2 effective actions. Therefore, all contact terms for the first term of the asymmetric amplitude can be reconstructed by applying all-order corrections to S_1 as follows:

$$\frac{\mu_p}{p!}\left(2\pi\alpha'\right)^2 \int_{\Sigma_{p+1}} \partial_i C_{p-1}\wedge \mathrm{Tr}\left(\sum_{n=-1}^{\infty} c_n\left(\alpha'\right)^{n+1} D_{a_1}\cdots D_{a_{n+1}} F D^{a_1}\cdots D^{a_{n+1}}\phi^i\right). \tag{13}$$

Likewise, all-order extensions of S_2 are read off:

$$\frac{\mu_p}{p!}\left(2\pi\alpha'\right)^2 \int_{\Sigma_{p+1}} C^i_{p-2}\wedge \mathrm{Tr}\left(\sum_{n=-1}^{\infty} c_n\left(\alpha'\right)^{n+1} D_{a_1}\cdots D_{a_{n+1}} F\wedge D^{a_1}\cdots D^{a_{n+1}} D\phi_i\right). \tag{14}$$

On the other hand, the symmetric result of the amplitude $\langle V_{C^{-1}} V_{\phi^{-1}} V_{A^0}\rangle$ can be revealed as follows:

$$\begin{aligned} \mathscr{A}^{\phi^{-1}A^0C^{-1}} = &2^{1/2} i\xi_{1i}\xi_{2a}k_{2b}\int_{-\infty}^{\infty} dx\left(1+x^2\right)^{2t-1}(2x)^{-2t}\mathrm{Tr} \\ &\cdot\left(P_- H_{(n)} M_p \Gamma^{abi}\right). \end{aligned} \tag{15}$$

One can also read off $\langle V_{C^{-1}} V_{\phi^0} V_{A^{-1}}\rangle$ accordingly as follows:

$$\begin{aligned} &2^{1/2}\xi_{1i}\xi_{2a}\int_{-\infty}^{\infty} dx\left(1+x^2\right)^{2t-1}(2x)^{-2t}\Big(k_{1b}\mathrm{Tr}\left(P_- H_{(n)} M_p \Gamma^{bai}\right) \\ &- p^i \mathrm{Tr}\left(P_- H_{(n)} M_p \gamma^a\right)\Big). \end{aligned} \tag{16}$$

Using momentum conservation along the world volume of brane $(k_1+k_2+p)^a=0$, due to symmetry structures and to get consistent result for both above symmetric amplitudes, one gets to derive the following restricted Bianchi identity for RR's field strength as below:

$$p_b \epsilon^{a_0\cdots a_{p-2}ba} H^i_{a_0\cdots a_{p-2}} + p^i \epsilon^{a_0\cdots a_{p-1}a} H_{a_0\cdots a_{p-1}} = 0. \quad (17)$$

In the next section, we would like to deal with a more complicated analysis.

3. S-Matrix of $\langle V_A(x_1)V_C(z_1,\bar z_1)V_C(z_2,\bar z_2)\rangle$ of Type IIB

The complete form of the S-matrix element of two closed string RR field $\langle V_C^{(-1)}(x_1,x_2)V_C^{(-1)}(x_4,x_5)\rangle$ has been carried out in [43] which has the following form:

$$\begin{aligned}&\int dx_1dx_2dx_4dx_5 \left(P_- H_{(1n)}M_p\right)^{\alpha\beta}\\&\cdot\left(P_-H_{(2n)}M_p\right)^{\gamma\delta}(x_{12}x_{14}x_{15}x_{24}x_{25}x_{45})^{-1}\\&\times\frac{1}{2}\left[(\gamma^\mu C)_{\alpha\beta}(\gamma^\mu C)_{\gamma\delta}x_{15}x_{24}-(\gamma^\mu C)_{\gamma\beta}(\gamma^\mu C)_{\alpha\delta}x_{12}x_{45}\right]\\&\times|x_{12}x_{45}|^{-s/2}|x_{14}x_{25}|^{-t/2}|x_{15}x_{24}|^{(s+t)/2}.\end{aligned} \quad (18)$$

It is shown that by choosing the gauge fixing as $(x_1,x_2,x_4,x_5)=(iy,-iy,i,-i)$ for the moduli space, one maps the moduli space to unit disk, and the final amplitude of two closed string RRs in IIB was read off as follows:

$$\begin{aligned}\mathcal{A}^{CC}_{IIB}=&\frac{i\mu_{1p}\mu_{2p}}{p!p!}\left(\frac{2}{s}-t\sum_{n,m=0}^{\infty}h_{n,m}(ts)^n(t+s)^m\right)\\&\times\epsilon_1^{a_0\cdots a_{p-1}a}H_{1a_0\cdots a_{p-1}}\epsilon_2^{a_0\cdots a_{p-1}a}H_{2a_0\cdots a_{p-1}}.\end{aligned} \quad (19)$$

It was also shown that the only massless pole can be reconstructed by using the following subamplitude in an EFT:

$$\mathcal{A}=V^a_\alpha(C_{1p-1},A)G^{ab}_{\alpha\beta}(A)V^b_\beta(C_{2p-1},A), \quad (20)$$

where the vertex $V^a_\alpha(C_{1p-1},A)$ is derived from $i(2\pi\alpha')\mu_{1p}\int_{\Sigma_{p+1}}C_{1p-1}\wedge F$ and the following vertices were needed:

$$\begin{aligned}V^a_\alpha(C_{1p-1},A)&=i\left(2\pi\alpha'\right)\frac{\mu_{1p}}{p!}\epsilon_1^{a_0\cdots a_{p-1}a}H_{1a_0\cdots a_{p-1}}\mathrm{Tr}(\lambda_\alpha),\\G^{ab}_{\alpha\beta}(A)&=\frac{-1}{\left(2\pi\alpha'\right)^2}\frac{\delta^{ab}\delta_{\alpha\beta}}{k^2},\\V^b_\beta(C_{2p-1},A)&=i\left(2\pi\alpha'\right)\frac{\mu_{2p}}{p!}\epsilon_2^{a_0\cdots a_{p-1}b}H_{2a_0\cdots a_{p-1}}\mathrm{Tr}(\lambda_\beta).\end{aligned} \quad (21)$$

$k^2=(p_1+D.p_1)^2=-s$ is taken inside the propagator while $t=-2p_1.p_2$. Due to having the entire and closed form of S-matrix of two RRs in (20), one can apply properly higher derivative corrections on C_1 and C_2 so that $(ts)^n$ can be produced by the following all-order α' corrections to two closed string RRs of IIB:

$$\begin{aligned}&\sum_{n,m=0}^{\infty}h_{n,m}\left(\alpha'\right)^{2n+1}\left(D_{a_1}\cdots D_{a_{n+1}}(D_bD_b)^nC_{1a_0\cdots a_{p-2}}D^{a_1}\cdots D^{a_{n+1}}C_{2a_0\cdots a_{p-2}}\right)\\&\times\frac{\mu_{1p}\mu_{2p}}{(p-1)!(p-1)!}\epsilon_1^{a_0\cdots a_{p-2}a}\epsilon_2^{a_0\cdots a_{p-2}a}.\end{aligned} \quad (22)$$

$(t+s)^m$ can also be produced by the following all-order α' corrections of IIB:

$$\left(\alpha'/2\right)^m\left((D_aD^a)C_{1a_0\cdots a_{p-2}}C_{2a_0\cdots a_{p-2}}+\alpha' D^dC_{1a_0\cdots a_{p-2}}D_dC_{2a_0\cdots a_{p-2}}\right)^m. \quad (23)$$

Now in this section, we would like to carry out direct CFT methods to derive the entire S-matrix elements of two closed string RRs and a gauge field on the world volume of BPS branes in IIB. The aim was to explore singularity structures as well as α' corrections and also to see if the above prescription holds or not. Hence, the five point function $\langle V_A^{(0)}(x_1)V_C^{(-1)}(z_1,\bar z_1)V_C^{(-1)}(z_2,\bar z_2)\rangle$ in the type IIB string theory ($z_1=x_2+ix_3$, $z_2=x_4+ix_5$; $x_{ij}=x_i-x_j$) is given by the following correlation functions:

$$\begin{aligned}I_{CCA}=&\int_{\mathbb{R}}dx_1\int_{\mathcal{H}^+}dx_2dx_3\int_{\mathcal{H}^+}dx_4dx_5\left(P_-H_{(1n)}M_p\right)^{\alpha\beta}\\&\cdot\left(P_-H_{(2n)}M_p\right)^{\gamma\delta}\times(x_{23}x_{24}x_{25}x_{34}x_{35}x_{45})^{-\frac14}\mathcal{J}\xi_{1a}\\&\cdot\big(\langle :S_\alpha(x_2): \ :S_\beta(x_3): \ :S_\gamma(x_4): \ :S_\delta(x_5):\rangle\mathcal{J}^a\\&+2ik_{1c}\langle :S_\alpha(x_2): \ :S_\beta(x_3): \ :S_\gamma(x_4): \ :S_\delta(x_5): \ :\psi^c\psi^a(x_1):\rangle\big),\end{aligned} \quad (24)$$

where $\mathcal{J}$, and $\mathcal{J}^a$ take the form

$$\begin{aligned}\mathcal{J}=&|x_{12}|^{2k_1\cdot p_1}|x_{13}|^{2k_1\cdot p_1}|x_{14}|^{2k_1\cdot p_2}|x_{15}|^{2k_1\cdot p_2}|x_{23}|^{p_1\cdot D\cdot p_1}\\&\cdot|x_{24}|^{p_1\cdot p_2}|x_{25}|^{p_1\cdot D\cdot p_2}\times|x_{34}|^{p_2\cdot D\cdot p_1}|x_{35}|^{p_1\cdot p_2}|x_{45}|^{p_2\cdot D\cdot p_2},\\\mathcal{J}^a=&ip_1^a\frac{x_{52}}{x_{12}x_{15}}+ip_2^a\frac{x_{34}}{x_{14}x_{13}}.\end{aligned} \quad (25)$$

with the definition of the Mandelstam variables as

$$\begin{aligned}s&=-(p_1+k_1)^2=-2k_1\cdot p_1,\\v&=-(p_2+D\cdot p_2)^2=-2p_2\cdot D\cdot p_2,\\w&=-(p_1+p_2)^2=-2p_1\cdot p_2.\end{aligned} \quad (26)$$

Substituting the definition of the Mandelstam variables into $\mathcal{J}$, we obtain it as

$$\mathcal{J} = |x_{12}|^{-s}|x_{13}|^{-s}|x_{14}|^{s}|x_{15}|^{s}|x_{23}|^{2s-\frac{v}{2}}|x_{24}|^{-\frac{w}{2}} \cdot |x_{25}|^{-s+\frac{w+v}{2}}|x_{34}|^{-s+\frac{w+v}{2}}|x_{35}|^{-\frac{w}{2}}|x_{45}|^{-\frac{v}{2}}. \quad (27)$$

The correlation function of four spin operators in IIB is read by

$$\begin{aligned}\mathcal{S}_{\alpha\beta\gamma\delta}(x_2,x_3,x_4,x_5) &\coloneqq \left\langle :S_\alpha(x_2): \ :S_\beta(x_3): \ :S_\gamma(x_4): \ :S_\delta(x_5): \right\rangle \\ &= \left[(\gamma^\mu C)_{\alpha\beta}\left(\gamma_\mu C\right)_{\gamma\delta}x_{25}x_{34} - (\gamma^\mu C)_{\gamma\beta}\left(\gamma_\mu C\right)_{\alpha\delta}x_{23}x_{45}\right] \\ &\quad \cdot \frac{1}{2(x_{23}x_{24}x_{25}x_{34}x_{35}x_{45})^{3/4}},\end{aligned} \quad (28)$$

The correlation function of four spin operators and one current has been obtained in [48] to be

$$\begin{aligned}\widehat{\mathcal{S}}^{ca}_{\alpha\beta\gamma\delta}(x_1,x_2,x_3,x_4,x_5) &\coloneqq \left\langle :S_\alpha(x_2): \ :S_\beta(x_3): \ :S_\gamma(x_4): \ :S_\delta(x_5): \ :\psi^c\psi^a(x_1): \right\rangle \\ &= \frac{(x_{23}x_{24}x_{25}x_{34}x_{35}x_{45})^{-3/4}}{4(x_{12}x_{13}x_{14}x_{15})}\Big[\Big((\gamma^c C)_{\gamma\beta}(\gamma^a C)_{\alpha\delta} \\ &\quad - (\gamma^c C)_{\alpha\delta}(\gamma^a C)_{\gamma\beta}\Big)(x_{12}x_{14}x_{35} - x_{15}x_{13}x_{24}) \times x_{23}x_{45} \\ &\quad - \Big((\gamma^c C)_{\alpha\beta}(\gamma^a C)_{\gamma\delta} + (\gamma^c C)_{\gamma\delta}(\gamma^a C)_{\alpha\beta}\Big)x_{25}x_{34}(x_{15}x_{13}x_{24} \\ &\quad + x_{14}x_{12}x_{35}) + \left(\Gamma^{ca\lambda}C\right)_{\alpha\beta}(\gamma_\lambda C)_{\gamma\delta}x_{23}x_{25}x_{34}(x_{14}x_{15}) \\ &\quad + \left(\Gamma^{ca\lambda}C\right)_{\gamma\delta}(\gamma_\lambda C)_{\alpha\beta}x_{25}x_{34}x_{45}(x_{12}x_{13}) \\ &\quad - \left(\Gamma^{ca\lambda}C\right)_{\alpha\delta}(\gamma_\lambda C)_{\gamma\beta}x_{23}x_{25}x_{45}(x_{14}x_{13}) \\ &\quad + \left(\Gamma^{ca\lambda}C\right)_{\gamma\beta}(\gamma_\lambda C)_{\alpha\delta}x_{23}x_{34}x_{45}(x_{12}x_{15})\Big].\end{aligned} \quad (29)$$

Gauge fixing of $SL(2,\mathbb{R})$ invariance for $\langle V_A^{(0)}(x_1)V_C^{(-1)}(z_1,\bar{z}_1)V_C^{(-1)}(z_2,\bar{z}_2)\rangle$ can be chosen as follows:

$$(z_1,\bar{z}_1,z_2,\bar{z}_2,x_1) = (i,-i,z,\bar{z},\infty). \quad (30)$$

The Jacobian for this transformation will be $Jac = -2ix_1^2$. After gauge fixing, the expressions for the amplitude in (24) were simplified to

$$\begin{aligned}\mathcal{A}^{A^0C^{-1}C^{-1}} &= (-2i)\int_{\mathcal{H}^+} dz d\bar{z}\left(P\_H_{(1n)}M_p\right)^{\alpha\beta} \\ &\quad \cdot \left(P\_H_{(2n)}M_p\right)^{\gamma\delta}\left[(2i)|z+i|^2|z-i|^2(z-\bar{z})\right]^{-\frac{1}{4}} \\ &\quad \times \mathcal{J}^{(g.f.)}\xi_{1a}\left(\mathcal{S}^{(g.f.)}_{\alpha\beta\gamma\delta}(z,\bar{z})\mathcal{J}_A^{(g.f.),a} + 2ik_{1c}\widehat{\mathcal{S}}^{(g.f.),ca}_{\alpha\beta\gamma\delta}(z,\bar{z})\right).\end{aligned} \quad (31)$$

The various gauge fixed quantities appearing in the above amplitudes are summarised (the momentum conservation along the brane is $(2k_1 + p_1 + D.p_1 + p_2 + D.p_2)^a = 0$, also note that $(k_1+p_1)^2 = p_{2a}p_{2a}$, $(k_1+p_2)^2 = p_{1a}p_{1a}$, and as expected, the expansion energy is low, $s = -p_{2a}p_{2a}, p_{1a}p_{1a}, v, w \longrightarrow 0$) in the following formulae ($z = x + iy$; $y \geq 0$):

$$\begin{aligned}\mathcal{J}^{(g.f.)} &= (2i)^{2s-\frac{v}{2}}|z-i|^{-w}|z+i|^{-2s+w+v}(z-\bar{z})^{-\frac{v}{2}}, \\ \mathcal{J}_A^{(g.f.),a} &= ip_1^a(\bar{z}-i) + ip_2^a(-i-z).\end{aligned} \quad (32)$$

Now, if we apply on-shell condition for the gauge field $k_1.\xi_1 = 0$, then one gets to derive

$$\begin{aligned}\mathcal{J}_A^{(g.f.),a} &= \frac{i}{2}(z+\bar{z})(p_1-p_2)^a, \\ \mathcal{S}^{(g.f.)}_{\alpha\beta\gamma\delta}(z,\bar{z}) &= \frac{1}{2\left[(2i)(z-\bar{z})|z+i|^2|z-i|^2\right]^{3/4}} \\ &\quad \cdot\left[|z+i|^2(\gamma^\mu C)_{\alpha\beta}\left(\gamma_\mu C\right)_{\gamma\delta} - 2i(z-\bar{z})(\gamma^\mu C)_{\gamma\beta}\left(\gamma_\mu C\right)_{\alpha\delta}\right], \\ \widehat{\mathcal{S}}^{(g.f.),ca}_{\alpha\beta\gamma\delta}(z,\bar{z}) &= \frac{1}{4\left[(2i)(z-\bar{z})|z+i|^2|z-i|^2\right]^{3/4}} \\ &\quad \cdot\Big\{2i(\bar{z}-z)(2i+(\bar{z}-z))\Big[(\gamma^c C)_{\gamma\beta}(\gamma^a C)_{\alpha\delta} \\ &\quad - (\gamma^c C)_{\alpha\delta}(\gamma^a C)_{\gamma\beta}\Big] + (z+\bar{z})|z+i|^2 \\ &\quad \cdot\Big[(\gamma^c C)_{\alpha\beta}(\gamma^a C)_{\gamma\delta} + (\gamma^c C)_{\gamma\delta}(\gamma^a C)_{\alpha\beta}\Big] \\ &\quad + 2i|z+i|^2\left(\Gamma^{ca\lambda}C\right)_{\alpha\beta}(\gamma_\lambda C)_{\gamma\delta} + (z-\bar{z})|z+i|^2 \\ &\quad \cdot\left(\Gamma^{ca\lambda}C\right)_{\gamma\delta}(\gamma_\lambda C)_{\alpha\beta} + 2i(\bar{z}-i)(z-\bar{z})\left(\Gamma^{ca\lambda}C\right)_{\alpha\delta} \\ &\quad \cdot(\gamma_\lambda C)_{\gamma\beta} - 2i(z+i)(z-\bar{z})\left(\Gamma^{ca\lambda}C\right)_{\gamma\beta}(\gamma_\lambda C)_{\alpha\delta}\Big\}.\end{aligned} \quad (33)$$

Let us define the following integral:

$$\mathbf{A}[a,b,c,d] = \int_{\mathcal{H}^+} dz d\bar{z}|z+i|^a|z-i|^b(z-\bar{z})^c(z+\bar{z})^d, \quad (34)$$

where a, b, c are written down in terms of the Mandelstam variables and $d = 0, 1$ for this amplitude; hence, the final result for the amplitude $\langle V_A^{(0)}(x_1)V_C^{(-1)}(z_1,\bar{z}_1)V_C^{(-1)}(z_2,\bar{z}_2)\rangle$ in IIB can be expressed as

$$\begin{aligned}\mathscr{A}^{A^0C^{-1}C^{-1}} &= -(2i)^{2s-v/2}\left(P_{-}H_{(1n)}M_p\right)^{\alpha\beta}\left(P_{-}H_{(2n)}M_p\right)^{\gamma\delta} \\ &\cdot\left\{\frac{i}{4}\xi_{1a}(p_1-p_2)^a(\gamma^\mu C)_{\alpha\beta}\left(\gamma_\mu C\right)_{\gamma\delta}\mathbf{A}_5 \right.\\ &+\frac{1}{2}\xi_{1a}(p_1-p_2)^a(\gamma^\mu C)_{\gamma\beta}\left(\gamma_\mu C\right)_{\alpha\delta}\mathbf{A}_6 \\ &-k_{1c}\xi_{1a}\left[\left[(\gamma^c C)_{\gamma\beta}(\gamma^a C)_{\alpha\delta}-(\gamma^c C)_{\alpha\delta}(\gamma^a C)_{\gamma\beta}\right]\right.\\ &\times(-2i\mathbf{A}_2+\mathbf{A}_4)-\frac{i}{2}\left[(\gamma^c C)_{\alpha\beta}(\gamma^a C)_{\gamma\delta}+(\gamma^c C)_{\gamma\delta}(\gamma^a C)_{\alpha\beta}\right]\mathbf{A}_5 \\ &+\left(\Gamma^{ca\lambda}C\right)_{\alpha\beta}(\gamma_\lambda C)_{\gamma\delta}\mathbf{A}_1-\frac{i}{2}\left(\Gamma^{ca\lambda}C\right)_{\gamma\delta}(\gamma_\lambda C)_{\alpha\beta}\mathbf{A}_3 \\ &+\frac{1}{2}\left(\Gamma^{ca\lambda}C\right)_{\alpha\delta}(\gamma_\lambda C)_{\gamma\beta}(\mathbf{A}_6-\mathbf{A}_4-2i\mathbf{A}_2) \\ &\left.\left.-\frac{1}{2}\left(\Gamma^{ca\lambda}C\right)_{\gamma\beta}(\gamma_\lambda C)_{\alpha\delta}(\mathbf{A}_6+\mathbf{A}_4+2i\mathbf{A}_2)\right]\right\},\end{aligned} \tag{35}$$

where $A_1, A_2, A_3, A_4, A_5, A_6$ are given by

$$\begin{aligned}\mathbf{A}_1 &= \mathbf{A}\left[-2s+w+v,-w-2,-\frac{v}{2}-1,0\right],\\ \mathbf{A}_2 &= \mathbf{A}\left[-2(s+1)+w+v,-w-2,-\frac{v}{2},0\right],\\ \mathbf{A}_3 &= \mathbf{A}\left[-2s+w+v,-w-2,-\frac{v}{2},0\right],\\ \mathbf{A}_4 &= \mathbf{A}\left[-2(s+1)+w+v,-w-2,-\frac{v}{2}+1,0\right],\\ \mathbf{A}_5 &= \mathbf{A}\left[-2s+w+v,-w-2,-\frac{v}{2}-1,1\right],\\ \mathbf{A}_6 &= \mathbf{A}\left[-2(s+1)+w+v,-w-2,-\frac{v}{2},1\right].\end{aligned} \tag{36}$$

One can show that A_5 and A_6 have no contribution to our S-matrix, due to the fact that their integrations are zero on the upper half plane.

4. World Volume Singularity Structures of IIB

The low-energy expansions of all the functions can be found by using the package of HypExp [49, 50], and we just point out to some of the expansions. For instance for $n = 0$ (see the Appendix) and at the first order of the expansion, one gets the following values:

$$\begin{aligned}\mathbf{A}_1 &= \frac{-i\pi(2s-v-w)}{2(2s-v)w},\\ \mathbf{A}_2 &= \frac{-\pi(-2s+v)}{4(-2s+v+w)w},\\ \mathbf{A}_3 &= \frac{\pi(-2s+v+w)}{(2s-v)w},\\ \mathbf{A}_4 &= \frac{-i\pi(2s-v-2w)}{2(-2s+v+w)w}.\end{aligned} \tag{37}$$

Table 1: $A1\varepsilon$series$[1,1]$.

ε order	Coefficient
-1	0
0	0
1	$1/4\pi^2(2s-v-w)$

Table 2: $A2\varepsilon$series$[1,1]$.

ε order	Coefficient
-1	0
0	0
1	$1/8i\pi^2 v$

Table 3: $A3\varepsilon$series$[1,2]$.

ε order	Coefficient
-1	0
0	0
1	0
2	$((i+1)/4)\pi^2 v(-2s+v+w)$

Let us deal with the singularities of the S-matrix. The amplitude makes sense for $C_{p-3}, C_{p-1}, C_{p+1}$ cases. One can summarise the expansions of the functions in terms of $A\varepsilon$ series$[n,\varepsilon\text{order}]$ accordingly. In particular, for the other cases, we find the following expansions with the explicit coefficients as written in Tables 1–4.

One can show that the expansions of $A3\varepsilon$series$[1,2]$, $A3$ εseries$[1,3]$, and $A3\varepsilon$series$[1,4]$ have no poles and made out of just contact interactions of the sort of the following form:

$$(-2s+v+w)\sum_{m,p,q=0}^{\infty}h_{m,p,q}w^m v^p s^q. \tag{38}$$

Given the above structure, for $p = n$ case, now one can apply α' higher derivative corrections to explore corrections in type IIB as follows:

$$\begin{aligned}&\sum_{m,p,q=0}^{\infty}h_{m,p,q}\left(\alpha'\right)^{m+q}\left(D^{a_1}\cdots D^{a_m}(D_aD^a)^pC_{2a_0\cdots a_{p-2}}D^{a_1}\cdots D^{a_q}F_{a_{p-1}a_p}\right.\\ &\left.\times D_{a_1}\cdots D_{a_m}D_{a_1}\cdots D_{a_q}C_{1a_0\cdots a_p}\right)\frac{\mu_{1p}\mu_{2p}}{(p-1)!(p+1)!}\varepsilon_1^{a_0\cdots a_p}\varepsilon_2^{a_0\cdots a_p},\end{aligned} \tag{39}$$

where $(-2s+v+w)$ is an overall factor and $p_2.D.p_2$ can be constructed out by applying the sum of momenta as $1/2(D^a D_a)C_2\wedge F$. Note that for all the other functions starting from $n=3$, $\varepsilon = 0,1,2$, the only nonzero values for the expansion would be at the first order expansion and have the following nonzero values $(\pi^2/12)(2s-v-w)$, $(i\pi^2/24)v$, and $(i\pi^2/12)v$ for $A1[1,3]$, $A2[1,3]$, and $A4[1,3]$ accordingly.

Table 4: $A4\varepsilon$series[1, 1].

ε order	Coefficient
-1	0
0	$i\pi^2/2$
1	$(-i\pi^2/4)(v + \text{EulerGamma}v + 3i\pi v + 4s\log[2] + v\text{PolyGamma}[0, 1/2])$

Now given the low-energy expansion and in order to produce the world volume singularity structures, we try to extract the traces and carry out algebraic simplifications.

Indeed, if λ in the S-matrix takes the world volume index $\lambda = d$, then the $p_1.D.p_1$ channel pole in IIB can be produced by the following EFT subamplitude:

$$\mathscr{A}^{CCA}_{IIB} = \frac{i\pi\mu_{1p}\mu_{2_{p-2}}}{p!(p-2)!}\frac{1}{2p_1.D.p_1}\varepsilon_1^{a_0\cdots a_{p-1}d}H_{1a_0\cdots a_{p-1}}\varepsilon_2^{a_0\cdots a_{p-3}cad}H_{2a_0\cdots a_{p-3}}\xi_{1a}k_{1c}, \tag{40}$$

where $(\mu_{1p}, \mu_{2_{p-2}})$ are RR charges and it is renormalised by $1/2^6$. Hence, if λ picks up the world volume index, then all the traces have nonzero contributions for $C_{1_{p-1}}$ as well as for $C_{2_{p-3}}$ cases. Note that this obviously confirms that we do have a gauge field singularity structure and also all various α' higher derivative corrections to two closed string RRs in type IIB that can be constructed out later on.

The gauge singularity structure for this particular case is regenerated by the following EFT subamplitude:

$$\mathscr{A} = V^a_\alpha(C_{1p-1}, A)G^{ab}_{\alpha\beta}(A)V^b_\beta(C_{2p-3}, A, A_1), \tag{41}$$

where the Chern-Simons coupling $i(2\pi\alpha')\mu_{1p}\int_{\Sigma_{p+1}} C_{1p-1}\wedge F$ is needed; also note that it is shown that this coupling does not receive any corrections either.

One can reveal all the simple propagators by employing the kinetic terms that appeared in DBI action as $(2\pi\alpha')^2 F_{ab} F^{ab}$ and $((2\pi\alpha')^2/2)\mathrm{Tr}(D_a\phi^i D^a\phi_i)$ where the kinetic terms of gauge fields and scalars will receive no correction either, because they are fixed in the low-energy DBI action as well. One readily gets to derive the EFT vertex operators for the above amplitude as follows:

$$\begin{aligned} V^a_\alpha(C_{1p-1}, A) &= i(2\pi\alpha')\frac{\mu_{1p}}{p!}\varepsilon_1^{a_0\cdots a_{p-1}a}H_{1a_0\cdots a_{p-1}}\mathrm{Tr}(\lambda_\alpha), \\ G^{ab}_{\alpha\beta}(A) &= \frac{-1}{(2\pi\alpha')^2}\frac{\delta^{ab}\delta_{\alpha\beta}}{k^2}, \\ V^b_\beta(C_{2p-3}, A, A_1) &= i(2\pi\alpha')^2\frac{\mu_{2_{p-2}}}{(p-2)!}\varepsilon_2^{a_0\cdots a_{p-1}b} \\ &\quad \cdot H_{2a_0\cdots a_{p-3}}\xi_{1a_{p-2}}k_{1a_{p-1}}\mathrm{Tr}(\lambda_1\lambda_\beta), \end{aligned} \tag{42}$$

where the following interaction for the 2nd Chern-Simons coupling has been taken into account:

$$i(2\pi\alpha')^2\mu_{2_{p-2}}\int_{\Sigma_{p+1}} C_{2_{p-3}}\wedge F\wedge F. \tag{43}$$

Notice that in the propagator, one considers $k^2 = -(p_1 + D.p_1)^2$. Now, by replacing (42) into (41), we would be able to precisely reconstruct in the EFT of IIB the following simple pole:

$$\frac{i\pi\mu_{1p}\mu_{2_{p-2}}}{p!(p-2)!}\frac{1}{2p_1.D.p_1}\varepsilon_1^{a_0\cdots a_{p-1}d}H_{1a_0\cdots a_{p-1}}\varepsilon_2^{a_0\cdots a_{p-1}d}H_{2a_0\cdots a_{p-3}}\xi_{1a_{p-2}}k_{1a_{p-1}}. \tag{44}$$

This is the same gauge field singularity structure of $C CA$ that appeared in the IIB string theory. (Notice that due to symmetries, likewise, the $p_2.D.p_2$ pole can also be generated. Given the fact that after all $\mathrm{Tr}(\lambda_1)$ is zero for $SU(N)$, we come to know that there is no s channel pole. $4k_1.p_2 = -4k_1.p_1 = 2s$). It would be nice to explore two RR couplings, with a scalar field, and their bulk singularity structures to actually find out their corrections in type IIA as well.

One can show that indeed if λ in the S-matrix takes the transverse index $\lambda = j$, then the scalar field pole in IIB is produced by the following EFT subamplitude:

$$\frac{i\pi\mu_{1_{p+2}}\mu_{2_p}}{(p+2)!(p)!}\frac{1}{2p_1.p_2}\varepsilon_1^{a_0\cdots a_p}H^j_{1a_0\cdots a_p}\varepsilon_2^{a_0\cdots a_{p-2}ca}H^j_{2a_0\cdots a_{p-2}}\xi_{1a}k_{1c}. \tag{45}$$

Hence, if λ picks up the transverse index, then the amplitude and the traces have nonzero contributions for $C_{1_{p+1}}$ as well as for $C_{2_{p-1}}$ cases, and we do have a scalar field singularity structure that can be shown to be matched in an EFT.

The singularity structure for this case is also produced by the following EFT counterpart:

$$V^i_\alpha(C_{1p+1}, \phi)G^{ij}_{\alpha\beta}(\phi)V^j_\beta(C_{2_{p-1}}, \phi, A_1), \tag{46}$$

where $V^i_\alpha(C_{1_{p+1}}, \phi)$ was obtained from $i(2\pi\alpha')\mu_{1p}\int_{\Sigma_{p+1}}\partial^i C_{1_{p+1}}\phi_i$ which is indeed the Taylor expansion in EFT. We clarify the EFT vertex operators for the above amplitude as follows:

$$V^i_\alpha\left(C_{1_{p+1}}, \phi\right) = i\left(2\pi\alpha'\right)\frac{\mu_{1_{p+2}}}{(p+2)!}\varepsilon_1^{a_0\cdots a_p} H^i_{1a_0\cdots a_p}\mathrm{Tr}(\lambda_\alpha),$$
$$G^{ij}_{\alpha\beta}(\phi) = \frac{-1}{\left(2\pi\alpha'\right)^2}\frac{\delta^{ij}\delta_{\alpha\beta}}{k^2},$$
$$V^j_\beta\left(C_{2_{p-1}}, \phi, A_1\right) = i\left(2\pi\alpha'\right)^2\frac{\mu_{2_p}}{(p)!}\varepsilon_2^{a_0\cdots a_p} \cdot H^j_{2a_0\cdots a_{p-2}}\xi_{1a_{p-1}}k_{1a_p}\mathrm{Tr}\left(\lambda_1\lambda_\beta\right), \tag{47}$$

where the mixed WZ and CS interaction $i(2\pi\alpha')^2\mu_{2_p}\int_{\Sigma_{p+1}}\partial_i C_{2_{p-1}}\wedge F\phi^i$ for the second coupling is taken. Having set that (we have also used the fact that $(k_1+p_2)^a = -p_1^a$ and also $2k_1{\cdot}p_2 = -2p_1{\cdot}p_2$), one regenerates the $p_1{\cdot}p_2$ singularity structure in EFT which is the same pole that appeared in (45) of IIB. Finally, one can show that the other singularity of the amplitude can be reconstructed in an EFT. Indeed, if one makes use of the same EFT rule and applies the mixed pull-back of the brane and Chern-Simons coupling of the derived form as below

$$\frac{i\left(2\pi\alpha'\right)^2\mu_{1p}}{(p-1)!}\int_{\Sigma_{p+1}} C^i_{1_{p-2}}\wedge F\wedge D\phi^i, \tag{48}$$

then one would be able to regenerate the singularity in an EFT side as well.

5. Conclusion

In this paper, first, we have computed both asymmetric and symmetric S-matrices of a four-point amplitude of a gauge, a scalar field, and a closed string RR for different nonvanishing traces of all different BPS branes. We then figured out all world volume, the Taylor and pull-back couplings, and their α' corrections. Thanks to symmetry structures, various restricted BPS Bianchi identities are also revealed. A prescription for the corrections of two closed string RR couplings in type IIB was found out. We have also gained the closed form of all the correlators of two closed string RRs and a gauge field in the type IIB string theory.

The algebraic forms of the integrals for the most general integrations on the upper half plane that are of the sort of $\int d^2z|z-i|^a|z+i|^b(z-\bar{z})^c(z+\bar{z})^d$ are explored where the outcome is written down in terms of Pochhammer and some analytic functions. Lastly, various singularity structures in both the EFT and IIB string theory are reconstructed. We have also worked out with different contact term analysis and reproduced different contact interactions of the S-matrices as well as their α' higher derivative corrections, and eventually, some concrete points are clarified in detail. Various world volume couplings just in IIB as well as their explicit corrections are discovered as well.

Now, let us just address in detail the technical issues related to solving integrals in the Appendix.

Appendix

A. Solving the Integrals of Two RRs and an NS Field

We did gauge fixing as $(x_1\longrightarrow\infty, z_1 = i, \bar{z}_1 = -i, z_2 = z, \bar{z}_2 = \bar{z})$ and eventually one needs to take integrations on the location of the second closed string on the upper half plane as follows:

$$I = \int_{H^+} d^2z|z-i|^a|z+i|^b(z-\bar{z})^c(z+\bar{z})^d, \tag{A.1}$$

where $d = 0, 1$ and a, b, c are written down in terms of three independent Mandelstam variables. We first use the following transformations:

$$|z+i|^b = \frac{1}{\Gamma(-(b/2))}\int_0^\infty dt t^{-\frac{b}{2}-1}e^{-t|i+z|^2},$$
$$|z-i|^a = \frac{1}{\Gamma(-(a/2))}\int_0^\infty du u^{-\frac{a}{2}-1}e^{-u|z-i|^2}, \tag{A.2}$$

where $z = x+iy$ and the integration on x is readily done as $\int_{-\infty}^{\infty}dxe^{-(t+u)x^2} = \sqrt{\pi}/(t+u)^{1/2}$,.

Let us first solve it for $d = 0$ so that the integration on y becomes

$$I_y = (2i)^c\int_0^\infty dy\left(y+\frac{u-t}{u+t}\right)^c e^{-(t+u)(y)^2-\frac{4tu}{t+u}}. \tag{A.3}$$

Using a simple algebraic analysis and change of variables, one writes down the integration on y as follows:

$$I_y = (-2i)^c\left(\frac{t-u}{u+t}\right)^{c+1}\int_0^\infty dy(1-y)^c e^{-\frac{(t-u)^2}{t+u}y^2}e^{-\frac{4tu}{t+u}}. \tag{A.4}$$

Now, one can make use of the Pochhammer definition as follows:

$${}_1F_0(a|z) = (1-z)^{-a} = \sum_{n=0}^{\infty}\frac{(a)_n z^n}{n!}, \int_0^\infty dy\, y^c e^{-(s+u)y^2} = \frac{\Gamma((1+c)/2)}{2(s+u)^{(1+c)/2}}. \tag{A.5}$$

If we consider (A.5), then one gets to derive the whole y integration. Now, one can collect all the results of x and y integration inside I to get to

$$I = \sum_{n=0}^{\infty} \frac{(-c)_n \Gamma((1+n)/2)}{n!} \cdot \frac{\sqrt{\pi}(-2i)^c}{2\Gamma(-a/2)\Gamma(-b/2)} \int_0^{\infty}\int_0^{\infty} \cdot \frac{dtdu}{(t+u)^{c+1-n/2}} (t-u)^{c-n} u^{-a/2-1} t^{-b/2-1} e^{-\frac{4ut}{t+u}}. \tag{A.6}$$

If one uses the following change of variables

$$u = \frac{x}{s}, \quad t = \frac{x}{1-s}, \quad dtdu = Jdxds = \frac{xdxds}{(s(1-s))^2}. \tag{A.7}$$

By substituting the above variables and making use of the Jacobian, one eventually gets to gain the final integral as

$$I = \int_0^{\infty} dxe^{-4x} x^{\frac{-4-(a+b+n)}{2}} \int_0^1 dss^{(n+a)/2}(1-s)^{(n+b)/2}(1-2s)^{c-n} \times \sum_{n=0}^{\infty} \frac{(-c)_n(-1)^{c-n}\Gamma(1+n/2)}{n!} \frac{\sqrt{\pi}(-2i)^c}{2\Gamma(-a/2)\Gamma(-b/2)}. \tag{A.8}$$

If we use $\Gamma(z) = (z-1)!$, then one reads off the final answer to be

$$I = \int_0^1 dss^{(n+a)/2}(1-s)^{(n+b)/2}(1-2s)^{c-n} \times \sum_{n=0}^{\infty} \frac{(-c)_n(-1)^{2c-n}\Gamma((1+n)/2)}{n!} \cdot \frac{\sqrt{\pi}(2i)^c}{\Gamma(-a/2)\Gamma(-b/2)} \Gamma\left(-1-\frac{a+b+n}{2}\right) 2^{a+b+n+1}, \tag{A.9}$$

where the integration $\int_0^1 dss^{(n+a)/2}(1-s)^{(n+b)/2}(1-2s)^{c-n}$ can also be computed in terms of the hypergeometric function as follows:

$$\frac{(-2)^{c-n}\Gamma((1/2)(b+n+2))\Gamma((a/2)+c-(n/2)+1)\,{}_2F_1((1/2)(-a-b-2c-2), n-c; (1/2)(-a-2c+n); 1/2)}{\Gamma((1/2)(a+b+2c+4))} + \frac{\left((-1)^c e^{(1/2)i\pi(a+n+2)} \csc((1/2)\pi(a+2c-n)) + (-1)^n \cot((1/2)\pi(a+n)) + i(-1)^n\right)}{\Gamma(-a/2-n/2)\Gamma((a/2)+c-(n/2)+2)} \times \pi(-1)^{-n} 2^{-\frac{a}{2}-\frac{n}{2}-1} e^{-\frac{1}{2}i\pi(a+n+2)} \Gamma(c-n+1)\,{}_2F_1\left(\frac{1}{2}(-b-n), \frac{1}{2}(a+n+2); \frac{1}{2}(a+2c-n+4); \frac{1}{2}\right). \tag{A.10}$$

Data Availability

The data used to support the findings of this study have been made available at Arxiv as follows https://arxiv.org/abs/1805.09455.

Conflicts of Interest

The author declares that they have no conflicts of interest.

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