



Asymptotic safety, string theory and the weak gravity conjecture

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ABSTRACT

We propose a scenario with string theory in the deep ultraviolet, an intermediate asymptotically safe scaling regime for gravity and matter, and the Standard Model in the infrared. This could provide a new perspective to tackle challenges of the two models: For instance, the gravitational Renormalization Group flow could connect a negative microscopic to a positive macroscopic cosmological constant, potentially rendering string theory on an anti-de Sitter background observationally viable. Further, the unitarity of a string-theoretic ultraviolet completion could be inherited by an asymptotically safe fixed point, despite the presence of higher-order interactions. We discuss necessary conditions on the scale of asymptotic safety and the string scale for our scenario to be viable. As a first test, we explore the weak-gravity conjecture in the context of asymptotically safe gravity.

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1. Asymptotic safety and string theory

What is the fundamental nature of the building blocks of our universe? String theory and the asymptotically safe Standard Model (ASSM) are both possible candidates. The latter relies on scale-symmetry kicking in at microscopic distance scales. If realized, it provides a predictive quantum field theory of the Standard Model plus quantum gravity, see [1–5] for RG studies and [6] for a recent review. It is based on an interacting fixed point of the Renormalization Group (RG), generalizing the concept of asymptotic freedom to a setting in which both gravity as well as Abelian gauge sectors could be included without Landau poles. Compelling indications for asymptotic safety in pure Euclidean gravity, proposed in [7], have been collected in [8–38], starting from the pioneering work [8], and matter-gravity systems have been explored, see, e.g., [1–3,39–49,4,5,50,51]. For related Monte Carlo studies see e.g. [52,53] and references therein. Potential cosmological implications are reviewed in [54] and possible consequences for black-hole physics explored, e.g., in [55–63], see [64–66] for recent reviews. See [67] for an introduction to quantum scale symmetry and [68] for a review of asymptotic safety and underlying mechanisms in various models.

The transition scale k_{tr} is defined as the energy scale at which a departure from scale-symmetry sets in, such that below k_{tr} the couplings deviate from their values in the asymptotically safe scaling regime. The additional scale-symmetry in the asymptotically safe regime can even result in enhanced predictive power, potentially fixing the values of some of the Standard-Model couplings at $k = k_{\text{tr}}$, and thereby at all scales [1–5]. For our scenario, it is key that the determination of infrared (IR) values of couplings following from scale-symmetry also carries over (at least approximately) if an asymptotically safe scaling regime is only realized over a finite, but large enough, range of scales, with new physics kicking in at very high energy scales.

On the other hand, string theory goes beyond the local quantum field theory framework, resulting in the requirement for extra dimensions as well as supersymmetry, see, e.g., [69–71] for reviews. Both the transition scale k_{tr} in asymptotic safety and the string scale M_s , are usually associated with the Planck scale. Therefore, a relation between these two candidates for a microscopic description of nature might not be immediately obvious, but could actually be possible whenever these two scales are separated, i.e., $k_{\text{tr}} < M_s$. Here, we set out to investigate a possible connection. We refer to Fig. 1 for an illustration of our proposal. Specifically, the scenario we explore assumes that string theory provides the most fundamental description of nature. Below the string scale M_s , this

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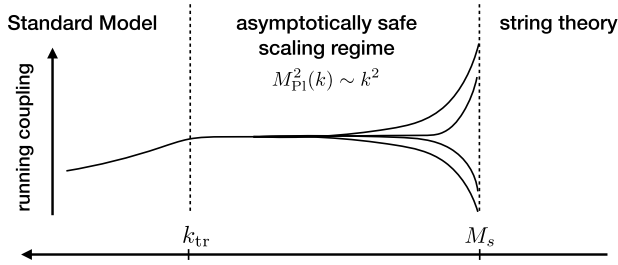


Fig. 1. We illustrate our scenario, indicating how an asymptotically safe scaling regime can generate universal predictions for couplings, coming from a range of values resulting from different choices of compactification for the string theory at the string scale M_s . The scale k_{tr} is the transition scale from an asymptotically safe scaling regime, where relevant operators kick in and drive the flow away from the scale-invariant point.

results in an effective quantum-field theoretic description¹. We assume that the values of couplings at M_s lie in the IR basin of attraction of the asymptotically safe fixed point. This assumption results in constraints on those couplings that are *relevant* at the interacting fixed point, as those are the IR-repulsive directions, cf. Fig. 2. Along the IR attractive (irrelevant) directions of the fixed point, the flow is pulled towards the fixed point. This results in an RG trajectory that spends a large amount of RG “time” close to the fixed point and then emanates from its vicinity close to the UV critical surface. In the simplest case, the compactification scale and scale of supersymmetry breaking are both close to the string scale M_s , so that the effective field theory is four-dimensional, potentially facilitating an asymptotically safe fixed point for all gauge interactions, including an Abelian one [72]. For the simplest scenario, we also assume that additional states from string theory (such as heavy moduli and superpartners) decouple at high energies (i.e., directly below the string scale), so that the effective-field theory regime contains only the Standard Model and gravity. This assumption can be relaxed to accommodate further matter fields that arise from string theory, if an asymptotically safe fixed point persists under extensions by the corresponding additional fields. We will work with general numbers of matter fields (which may include light moduli, axions, etc) in the following. In brief, our setup explores those parts of the string landscape that feature an emergent scale-symmetry.

The degree to which the asymptotically safe scaling regime determines the deep-IR physics by mapping a given range of initial conditions at the string scale to a narrow IR range of couplings, cf. Fig. 1, depends on the following two properties:

- (i) How strongly the irrelevant couplings are attracted to the asymptotically safe fixed point.
- (ii) How large the separation is between the asymptotically safe transition scale k_{tr} and the string scale M_s .

In such a setting, the physics in the deep IR is essentially determined by the ASSM. This might include the intriguing consequence that the Higgs mass [1,74,75], the top quark mass [2], the bottom quark mass [5] and the Abelian gauge coupling [76,3] could emerge as predictions of string theory. This follows since functional RG studies indicate that the respective couplings come out as *irrelevant* couplings with finite asymptotically safe fixed-point values. In turn, the relevant couplings in state-of-the-art approximations in asymptotically safe gravity are the cosmological constant, the Newton coupling, and a superposition of the 4-derivative

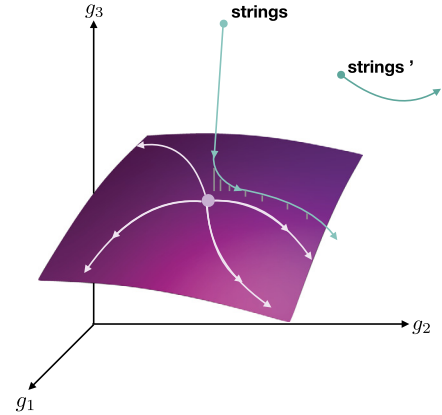


Fig. 2. We show a sketch of a three-dimensional space of couplings with an asymptotically safe fixed point (light purple) and its UV critical surface (purple). Its IR critical surface is one-dimensional, and the starting point provided by a string model (light cyan) lies within it, resulting in the effective QFT description of this string model approaching the fixed point very closely, before the RG trajectory leaves the fixed-point regime close to the UV critical surface. For an alternative string model (string’) the starting point for the QFT description (darker cyan) lies off the IR critical surface of the fixed point.

An earlier discussion explaining how models which are not fundamentally asymptotically safe can nevertheless appear effectively asymptotically safe can be found in [73].

curvature couplings, see, e.g., [16,23,30]. Thus, the constraint of reaching this fixed point with the given relevant couplings from string theory selects a highly predictive corner of the string landscape.

We also point out that the gravitational RG flow can connect a fixed-point regime at negative cosmological constant to an IR regime with a tiny, positive value of the cosmological constant, as required observationally. This could help to address a challenge in string theory, where the existence of consistent de Sitter (dS) backgrounds such as the \bar{D} (anti-D-brane) uplift of KKLT [77], see also [78–80] for other potential constructions, is under debate, see, e.g., [81–86]. Typically, in string theory, it is more natural to get anti-de Sitter (AdS) backgrounds, and in contrast to the supersymmetric AdS background of KKLT (prior to introducing \bar{D} ’s) one can even get SUSY broken AdS backgrounds [87]. In most string phenomenology discussions based on the latter, some additional input (not necessarily \bar{D} ’s, see e.g., [78–80]) is used to ‘uplift’ such an AdS minimum to dS. However, this last step is somewhat less well under control compared to the original AdS construction in [87].

The difficulty of getting stable dS vacuum configurations in string theory (see [88] for a recent discussion) has even been elevated to the level of a conjecture [89] [90], see also [91], stating that it is not possible to get a stable dS solution in a controlled approximation scheme within string theory. Be that as it may (and in fact, this conjecture is indeed controversial, see, e.g., [92]), it should be pointed out that while the effective field theory arising from string theory is expected to be defined at (or close to) the string scale, the observed positive cosmological constant is measured in the deep IR. Hence it is conceivable that a negative cosmological constant obtained from string theory is consistent with a positive cosmological constant at cosmological scales. It should be emphasized that the cosmological constant is an IR repulsive coupling of the asymptotically safe scaling regime. Hence, an RG trajectory which realizes such an AdS–dS transition is not generic but has to be set by rather specific initial conditions of the effective field theory arising from string theory. Nevertheless, it could connect a string theory with a negative microscopic cosmological constant to a positive cosmological constant in the infrared.

¹ For simplicity, we are taking the compact volume to be not that large so that the Kaluza-Klein (compactification) scale is close to the string scale.

Establishing a relation between asymptotically safe gravity and string theory is also interesting for the following reason: The presence of a fixed point of the RG flow is not sufficient to guarantee a well-defined ultraviolet completion, as the microscopic dynamics might feature kinematical instabilities, leading to a unitarity problem. Four-derivative gravity, which features an asymptotically free UV completion [93], is typically considered an example of the fact that the presence of higher-derivative terms can spoil unitarity. Note, however, that the mere existence of kinematic instabilities at the classical level or at a finite order of derivatives is far from being conclusive with regard to unitarity or its lack. The possibility of non-perturbative cures of perturbative unitarity problems has been investigated recently in e.g., [94–97].

Asymptotically safe gravity is an example of a non-perturbative setup and higher derivative terms typically come to all orders. Thus, a Taylor expansion of the inverse propagator up to finite order in momenta (which generically features additional zeros) is inadequate to answer the question if asymptotically safe gravity is unitary. For a recent discussion of this see, e.g., [98]. Of course, in turn, it makes a conclusive analysis even more intricate.

Within the scenario we explore here, the above intricacies are softened: Additional poles in the gravity propagator can be present without spoiling the consistency of the theory, as long as they lie at or beyond the string scale. Conversely, within the present scenario one can even use the scale of additional poles in order to estimate the required value of the fundamental string scale.

In Sec. 2, we present explicit conditions on the parameters of the fundamental string theory and the intermediate asymptotically safe scaling regime that are necessary to realize the required separation of scales, i.e., $k_{\text{tr}} < M_s$, cf. Fig. 1. In Sec. 3, we comment on the possibility of a transition from a negative cosmological constant at M_s transitioning to a viable positive value at macroscopic scales. In Sec. 4, we discuss first implications of the weak gravity conjecture in the presented scenario. Finally, we summarize and give an outlook in Sec. 5.

2. Conditions realizing an intermediate scaling regime

Let us now analyze the conditions on the string scale M_s and the transition scale k_{tr} that have to hold within our proposed scenario. These considerations can inform model-building efforts, both on the string-theory side as well as the asymptotically safe side. To that end, we now discuss the flow of the gravitational coupling. Define the dimensionless gravitational coupling at the momentum scale k as

$$g(k) \equiv \frac{k^2}{8\pi M_{\text{pl}}^2(k)}. \quad (1)$$

Here $M_{\text{pl}}(k)$ is the running Planck scale - the physical gravitational coupling giving the initial condition in the deep IR is $M_{\text{pl}}^2(k=0) \equiv 1/(8\pi G_{\text{Newton}})$. To lowest order in the truncation of the infinite series for the beta function of the gravitational coupling we have in a semi-perturbative approximation²

$$\beta_{\text{gravity}} = \frac{dg}{dt} = 2g - 2\frac{g^2}{g_*}. \quad (2)$$

Here $t = \ln k$ and g_* is the fixed point value of g . The fixed-point coupling g_* needs to be positive in order to have a physically

meaningful asymptotically safe theory. For a UV fixed point with $g_* < 0$ the fixed point at $g_* = 0$ shields the UV fixed point from a low-energy regime with attractive gravity, as realized in our universe. In pure gravity the UV fixed point has been found at $g_* > 0$, [8–38]. This is a consequence of gravitational fluctuations having an antiscreening effect on the Newton coupling, thereby generating an asymptotically safe fixed-point regime. Of course, matter fluctuations also drive the value of g_* , towards either larger or smaller values, as has been explored in [41–43,45–47,5,49–51]. In a first, rough, approximation we may write this dependence of g_* as

$$g_*(N_{\text{eff}}) \approx \frac{12\pi}{N_{\text{eff}}}. \quad (3)$$

Roughly speaking, N_{eff} comprises a weighted sum of the number of spin s fields with $s = 0, 1/2, 1, 3/2$ and contains the effect of metric fluctuations, $s = 2$. The higher spin modes (see [100]) are required for supersymmetric extensions of the Standard Model. The detailed fixed-point properties of fully coupled gravity-matter system -which contain higher-order as well as non-minimal interactions- is subject of current research.

We proceed with the discussion of the consequences of this setup. We first focus on the case $g_*(N_{\text{eff}}) > 0$, that is $N_{\text{eff}} > 0$, and comment on the second case below. Integrating the flow equation (2) and re-expressing in terms of the running Planck scale, cf. equation (1), we have

$$M_{\text{pl}}^2(k) = M_{\text{pl}}^2(0) + \frac{1}{8\pi g_*} k^2, \quad (4)$$

where $M_{\text{pl}}^2(0)$ is the low-energy Planck mass, i.e., we have set the low-energy reference scale $k_0 = 0$. For $k^2 \ll 8\pi M_{\text{pl}}^2(0) g_*$, the dimensionful Planck mass is essentially constant, $M_{\text{pl}}^2(k) \approx M_{\text{pl}}^2(0)$, as expected in the classical-gravity regime. In contrast, for $k^2 > 8\pi M_{\text{pl}}^2(0) g_*$, we are in the asymptotically safe scaling regime, where the Planck mass exhibits scaling, $M_{\text{pl}}(k^2) \sim k^2$.

At the transition scale $k = k_{\text{tr}}$, the scale-dependence vanishes, such that the following estimate for the transition scale holds

$$k_{\text{tr}}^2 = 8\pi M_{\text{pl}}^2(0) g_*. \quad (5)$$

If the fixed-point value is sufficiently low, fixed-point scaling can even set in well below the Planck scale. For $g_* \sim O(1)$ the quantum correction to the running (squared) Planck scale is a small ($\sim 4\%$) effect even at $k = M_{\text{pl}}(0)$. But, if $N_{\text{eff}} \gg 1$, so that $g_* \ll 1$, the quantum corrections can be significant. Such a change of the fixed-point value of the Newton coupling could follow from the impact of quantum fluctuations of matter, see e.g., [41,42,45]. Whether this is indeed realized with a suitable number of matter fields is beyond the scope of the present work.

In view of the flow equation (4), one needs to reconsider the relation between the matching scale \bar{k} , at which QFT should be replaced by string theory, and the low-energy Planck scale $M_{\text{pl}}(0)$. If an asymptotically safe scaling regime is realized, the matching relations should actually use the running Planck scale, cf. Eq. (1), at the matching scale \bar{k} , which differs from the low-energy Planck scale

$$M_{\text{pl}}^2(\bar{k}) = \frac{\bar{k}^2}{8\pi g(\bar{k})} = \frac{M_s^2 \mathcal{V}}{\sqrt{g_s}}, \quad (6)$$

where \mathcal{V} is the volume of the compact space in string units, and g_s is the string coupling. This is because the relation between the 4D Planck scale and the string scale is expected to be valid at the cutoff scale which we denote by \bar{k} . This relation can be read off from the low-energy effective action.

² In this approximation higher dependencies of the coupling g that come from the fully non-perturbative propagator are neglected. Applying this approximation to marginal couplings allows us to recover the universal 1-loop coefficients, see, e.g., [99].

The matching scale \bar{k} should be somewhat less than the Kaluza-Klein (KK) scale, which is related to the string scale by $M_{\text{KK}}^2 = M_s^2/\mathcal{V}^{1/3}$. Using Eq. (6) to solve for the string scale we arrive at

$$M_{\text{KK}}^2 = \frac{\sqrt{g_s}}{\mathcal{V}^{4/3}} \frac{\bar{k}^2}{8\pi g(\bar{k})} \gtrsim \bar{k}^2. \quad (7)$$

This gives the bound on the compact space volume

$$\frac{\mathcal{V}^{4/3}}{\sqrt{g_s}} \lesssim \frac{1}{8\pi g(\bar{k})} < \frac{1}{8\pi g_*} \frac{M_{\text{Pl}}^2(\bar{k})}{M_{\text{Pl}}^2(0)}. \quad (8)$$

The second inequality comes from the requirement that there is a scaling regime, i.e., that

$$\bar{k}^2 > k_{\text{tr}}^2. \quad (9)$$

In full theory space (i.e., the space of gravitational couplings), the existence of the scaling regime depends on the values of the gravitational couplings at \bar{k} in relation to the location of the asymptotically safe fixed point. For simplicity we now strengthen the inequality (8) by neglecting the flow of all other couplings.

Now from Eq. (4) we have

$$\frac{M_{\text{Pl}}^2(\bar{k})}{M_{\text{Pl}}^2(0)} = \left(1 - \frac{g(\bar{k})}{g_*}\right)^{-1}. \quad (10)$$

This shows that for a long scaling regime, $g(\bar{k})$ should be sufficiently close to g_* , just as one would expect.

We may rewrite the second inequality in (8) further as

$$\frac{1}{g(\bar{k})} < \frac{1}{g_* - g(\bar{k})}. \quad (11)$$

Further using that $g_* > 0$, this implies that

$$1 < \frac{g_*}{g(\bar{k})} < 2, \quad (12)$$

where the first inequality comes from the requirement of positivity of $g(\bar{k})$. The first inequality also ensures that the potential scaling regime connects the string theory to the Gaussian fixed point, i.e., to a viable IR limit. Thus, the bound on the volume in (8) can be expressed in terms of the fixed-point value of the dimensionless gravitational coupling as

$$\frac{\mathcal{V}^{4/3}}{\sqrt{g_s}} \lesssim \frac{1}{8\pi g(\bar{k})} < \frac{2}{8\pi g_*}. \quad (13)$$

The two inequalities (13) and (12) together ensure that there is a scaling regime, i.e., that $\bar{k}^2 > k_{\text{tr}}^2$, and that it connects to a viable IR limit. Accordingly these inequalities can be satisfied by either

1. a small asymptotically safe fixed-point value for g_* ,
2. or a large string coupling g_s .

Note that a third possibility i.e., $\mathcal{V} < 1$ is not realizable because of T-duality considerations. This essentially means that the string scale is a lower limit for length scales - smaller scales have to be analysed in terms of the T-dual theory. For instance a type IIB compactification with a Calabi-Yau space with some Euler character χ at below the string scale is actually a type IIA theory with Euler character $-\chi$. Thus one simply has to replace one string compactification model by another. (See for instance the discussion in [69,70] chapters 8 and 13.) Given either of these conditions the proposed scenario summarized in Fig. 1 might be realized. Let us now comment on them further.

The first option for satisfying Eq. (13) is a fixed-point value of the Newton coupling which is sufficiently small. In such a setting \bar{k} might even be as low as the infrared Planck scale, while $k_{\text{tr}}^2 < M_{\text{Pl}}^2(0)$ would need to hold. This would imply a weakly coupled asymptotically safe regime with a very small fixed-point value. It is intriguing that hints for a rather weakly-coupled (in the sense of near-Gaussian scaling behavior) asymptotically safe regime have been found in pure gravity [23,101,35], and in particular with matter [48,5,49]. The latter also might allow a near-perturbative UV completion for the Standard Model [2-4,72]. Such a scenario might be achievable under the impact of an appropriate number and type of matter degrees of freedom [41-43,45-47,5,49,50].

For the second option, the string theory would have to be strongly coupled, i.e., g_s could be sufficiently large. While this is not necessarily a regime that is computationally easy to access on the string side, it is nevertheless intriguing to observe that the strongly-coupled string regime could be related to a weakly-coupled asymptotically safe regime in our setting. However often a strongly coupled string theory is S-dual to another string theory in the weak coupling regime - for instance, type I string theory in strong/weak coupling is S-dual to heterotic SO(32) string theory, while type IIB string theory is self-dual under S-duality (in effect SL(2,Z)) transformations. Hence if one finds that a given asymptotically safe field theory is related to a strongly coupled regime of the corresponding string theory, the latter should be replaced by its S-dual weakly coupled partner and the corresponding field theory looked at for its asymptotically safe properties.³ In the case of type IIB not only the string coupling, but also the fluxes in the compactification manifold change, thus changing the phenomenology.

In the other case, $N_{\text{eff}} < 0$, there is no asymptotic safety, since $g_* < 0$, and the cutoff scale following from the running of g in Eq. (2) is

$$\bar{k}^2 \simeq \frac{96\pi^2 M_{\text{Pl}}^2(0)}{|N_{\text{eff}}|} \ll M_{\text{Pl}}^2(0), \quad (14)$$

where the inequality holds for large $|N_{\text{eff}}|$. This scale is basically the so-called species scale $k_{\text{species}} \sim M_{\text{Pl}}(0)/\sqrt{N}$ (see for instance [102], especially the argument around eqn. 5.16).

The sign of N_{eff} is crucial. If it is positive, we could have asymptotic safety and the above arguments for a potential compatibility with string theory would be valid. In this case the UV Planck mass may be much larger than the IR Planck mass and gravity is weakly coupled in the UV. On the other hand for $N_{\text{eff}} < 0$ the UV Planck mass is much smaller, so gravity becomes strongly coupled in the UV. Of course all of the above arguments are strictly valid only in the leading order truncation of the RG equations.

The above discussion would mean that the existence of an asymptotically safe fixed point would (approximately) determine the infinite set of irrelevant couplings at the string/cutoff/KK scale. In string theory terms it would mean that the bottom up physics is fixing the particular compactification, the choice of the Calabi-Yau manifold, set of fluxes etc., i.e., a particular string theory vacuum from the landscape.

3. From microscopic anti de Sitter to macroscopic de Sitter

Another important property of our scenario is the dynamical change of the cosmological constant in the UV regime. This dynamics can turn a negative cosmological constant at microscopic (UV) scales to a positive one at large (IR). This happens as the cosmological constant is not protected by symmetries in the presence

³ We wish to thank Arthur Hebecker for drawing our attention to this issue.

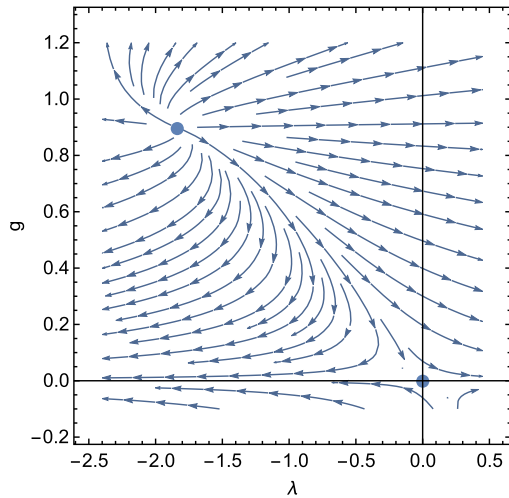


Fig. 3. We show the RG flow towards the IR in the $(g - \lambda)$ plane in the approximation of [41], which exhibits RG trajectories crossing from the fixed point at negative cosmological constant to a tiny positive cosmological constant in the infrared for $N_{\text{eff}} = 42$ and $N'_{\text{eff}} = 66$, based on Eq. (2) and Eq. (15). The chosen values for N_{eff} and N'_{eff} correspond to the matter content of the Standard Model in the approximation of [41].

of gravitational fluctuations (i.e., at $g \neq 0$). We exemplify this in the simple approximation of matter-gravity systems in [41], where the RG flow of the dimensionless cosmological constant $\lambda = \Lambda/k^2$ is given by

$$\beta_\lambda = -2\lambda + g \frac{\lambda}{6\pi} (-N_{\text{eff}} + 30) - \frac{g}{4\pi} N'_{\text{eff}}. \quad (15)$$

Here, N'_{eff} and N_{eff} depend on the number of matter fields. The last term in Eq. (15) drives the RG flow of λ across $\lambda = 0$ to positive values for $N'_{\text{eff}} > 0$. The determination of N'_{eff} and N_{eff} is subject to systematic uncertainties due to the choice of truncation, see, e.g., [41,42,45,50]. Working in the approximation of [41], we show the RG flow in the $(g - \lambda)$ plane with the desired characteristics in Fig. 3. As one can see from the flow, multiple trajectories connect the fixed-point value at negative λ_* to a positive IR-value of the cosmological constant. Since the cosmological constant is associated with a relevant direction of the fixed point, its IR value is a free parameter, allowing us to connect a negative fixed-point value with the observed value. For an example of such a concrete RG trajectory that is obtained as a solution to the system Eq. (2) and (15), see Fig. 4.

4. Weak gravity conjecture

For the above scenario to be realized, requirements from string theory for a consistent low-energy description should be satisfied by asymptotic safety. A prominent example is given by the weak gravity conjecture (WGC) [103], see also, e.g., [104–107] and, e.g., [108,109] for applications, which states that in a theory with a $U(1)$ gauge symmetry and corresponding coupling e , there should be a charged particle with charge q (we replace $eq \rightarrow e$) and mass M_{WGC} , such that

$$eM_{\text{Pl}} \geq M_{\text{WGC}}. \quad (16)$$

Here, $M_{\text{Pl}} = M_{\text{Pl}}(k)$ is the energy dependent Planck mass. In particular, it should not be confused with the low-energy value of the Planck mass $M_{\text{Pl}}(0)$ introduced earlier.

As a minimal requirement for whether asymptotically safe models could lie in the string-theoretic landscape, we will investigate whether the weak gravity conjecture holds in the ASSM.

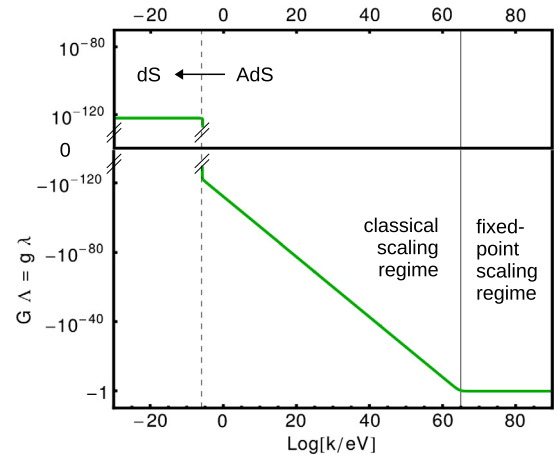


Fig. 4. We show the product $G \cdot \Lambda = g \cdot \lambda$ of the dimensionfull Newton coupling G and dimensionfull cosmological constant Λ along an RG trajectory that solves Eq. (2) and (15) for $N_{\text{eff}} = 42$ and $N'_{\text{eff}} = 66$. Along the trajectory, the dimensionfull cosmological constant, measured in units of the Planck mass, exhibits the asymptotically safe scaling regime in the UV, and a classical scaling regime in the IR, where it transitions from negative values (anti de Sitter) to positive values (de Sitter).

Within string theory, proofs of the conjecture based on various assumptions can be found, e.g., in [110–112].

A second motivation to study the WGC in the context of asymptotic safety is independent of string theory. Given the remoteness of the Planck scale from experimentally directly accessible scales, direct observational tests of quantum gravitational physics are challenging. Accordingly, deriving restrictions on particle physics that come from a consistent embedding into a more fundamental theory including quantum gravity can serve as an observational guide towards quantum gravity, highlighting the importance of understanding the interplay of quantum gravity with matter, as also emphasized, e.g., in [113]. In this spirit, asymptotic safety has been investigated in [114,41,42,44,115,46]. In string theory, this is the program of delineating the landscape with respect to the swampland [116], see [102] for a recent review. As there are more generic arguments concerning global and gauged symmetries in black-hole spacetimes [117,118,103,104] [119], the weak-gravity conjecture might be expected to hold beyond string theory. It is therefore of independent interest to determine whether asymptotically safe gravity-matter models obey the weak-gravity conjecture, irrespective of a possible embedding in string theory.

To be more specific, some comments about the inequality (16) are in order. It is introduced based on actions that describe the physics of processes at the corresponding scale. A basic lesson from quantum field theory is that all couplings depend on the energy scale used to probe any physical process. Therefore, the couplings appearing in the inequality should be interpreted as running couplings, as was already pointed out in the original paper [103]. In particular, the Planck mass, which describes the strength of gravitational interactions and the mass M_{WGC} depend on the energy, too. To describe this properly, we will focus on the ratio of M_{WGC} and Planck mass, writing the above inequality (16) as

$$e(k) \geq \frac{m_{\text{WGC}}(k)}{m_{\text{Pl}}(k)}, \quad (17)$$

where k is the energy scale of the relevant physics. m_{WGC} and m_{Pl} are the dimensionless counterparts of the two mass-scales. In particular, such dimensionless ratios of masses run, i.e., depend on the energy scale. In the scale-invariant, asymptotically safe fixed-point regime, all dimensionless counterparts of couplings are constant. This implies that in this regime

$$e(k) = e_*, \quad \frac{m_{\text{Pl}}(k)}{m_{\text{WGC}}(k)} = \frac{m_{\text{Pl},*}}{m_{\text{WGC},*}}. \quad (18)$$

Herein, e_* , $m_{\text{Pl},*}$ and $m_{\text{WGC},*}$ are the fixed-point values of these couplings. Accordingly, the fixed-point properties of asymptotically safe quantum gravity determine whether the weak-gravity conjecture holds. In the following we will only investigate a necessary condition for this, namely that (17) is satisfied at the fixed point, and will not analyze whether further constraints arise along the full RG flow.

Asymptotically safe quantum gravity is compatible with two distinct fixed-point structures in the matter sector, as discussed in [115]. The interacting nature of gravity at an asymptotically safe fixed point always percolates into the matter sector, such that it is not possible to set all matter interactions to zero, as pointed out in [114,120]. Yet, marginal interactions, such as those in the Standard Model, as well as masses, can either be finite or vanishing, depending on the respective choice of one of two possible fixed-point structures.

A first option is a maximally symmetric fixed point, at which only higher-order interactions, not relevant for our considerations, are present [114,120,121]. At this fixed point, all minimal gauge-interactions and scalar-potential-terms vanish, i.e., $m_{\text{WGC},*} = 0$ and $e_* = 0$. Accordingly, the scenario summarized in the inequality (17) does not apply and one would have to derive similar constraints for higher-order couplings.

A second fixed point, at which $m_{\text{WGC},*} \neq 0$ and $e_* = 0$ violates the WGC. Conversely, a fixed point at which $e_* \neq 0$ but $m_{\text{WGC},*} = 0$ would trivially satisfy the WGC, but such fixed-points cannot exist, since for a charged scalar, a finite fixed-point value of the mass is necessarily induced by non-vanishing gauge interactions.

The final option is a fixed point at which a finite value for the gauge coupling [76,121,3,4] as well as for the mass [122] is realized. A finite fixed-point value $e_* > 0$ could be realized in asymptotically safe gravity-matter models. In the approximations of the dynamics in [123,76,124,121,3,46,4], it arises from a balance of antiscreening quantum gravity fluctuations with screening quantum fluctuations of charged matter, encoded in the beta function as follows

$$\beta_e = -f_g e + \beta^{(1)} e^3 + \mathcal{O}(e^5), \quad (19)$$

where the second term is the standard one-loop term from charged matter. The first term arises from quantum-gravity fluctuations, and f_g depends on the gravitational couplings. Most importantly, it is proportional to the Newton coupling, i.e., to $m_{\text{Pl}}^{-1/2}$. Further, it depends on additional gravitational couplings, such as the cosmological constant. In a perturbative setting, a similar contribution has been discussed in [125–129]. In the asymptotically safe fixed-point regime, $m_{\text{Pl}} = m_{\text{Pl},*}$ such that $f_g = \text{const}$. Functional RG studies yield $f_g \geq 0$ [123,76,124,121,3,46,4]. Hence, a fixed point for the gauge coupling in the one-loop approximation arises at

$$e_* = \sqrt{\frac{f_g}{\beta^{(1)}}}. \quad (20)$$

We now distinguish between fermionic and bosonic fields as candidates for the light, charged particle in the WGC. In the Standard Model, fermions are protected from acquiring a mass at high energies by chiral symmetry, even in the presence of quantum-gravity fluctuations [114,130,44,115,131]. An explicit breaking of chiral symmetry through finite fixed-point values for Yukawa couplings is possible [132,115,2,4] in conjunction with a finite vacuum-expectation value for a scalar, leading to finite fermion masses.

Here, we assume that no spontaneous symmetry breaking occurs beyond the Planck scale, or to be more precise beyond eM_{Pl} . Therefore fermions remain massless in the UV fixed-point regime. Thus, as $e_* > 0$, the weak-gravity conjecture is trivially satisfied in this case. We conclude that asymptotically safe models in which a light charged fermion exists, which acquires its mass through spontaneous symmetry breaking below the Planck scale, appear to be compatible with the weak-gravity conjecture. Accordingly, such models could lie in the landscape of string theory.

In the following, we focus on a charged scalar field as the lightest charged particle. As a consequence of finite fixed-point values for the Planck mass and the gauge coupling, $m_{\text{WGC},*}$ must be finite, as well. Specifically, the beta function for the mass is given by

$$\begin{aligned} \beta_{m_{\text{WGC}}^2} &= k \partial_k m_{\text{WGC}}^2 \\ &= -2m_{\text{WGC}}^2 + f_m m_{\text{WGC}}^2 - \frac{3}{32\pi^2} e^2 + \dots \end{aligned} \quad (21)$$

It includes a canonical term $-2m_{\text{WGC}}^2$, a contribution from gauge-field fluctuations $\sim e^2$ and a gravitational contribution $\sim f_m$. Just as in the case of the gauge coupling, f_m depends on the gravitational couplings including the Newton coupling but also, e.g., the cosmological constant, see, e.g., [39,133–135,74] for the explicit form. For simplicity, we have omitted additional contributions due to scalar self-interactions here. At the asymptotically safe fixed point, $m_{\text{Pl}} = m_{\text{Pl},*}$ and $e = e_*$. As a consequence of $e_* \neq 0$, we cannot set $m_{\text{WGC},*} = 0$. Instead, a finite fixed-point value for the mass is generated, see also [122],

$$m_{\text{WGC},*}^2 = \frac{-3e_*^2}{32\pi^2(2 - f_m)}. \quad (22)$$

This expression requires some explanations. Depending on f_m , $m_{\text{WGC},*}^2$ can have either sign. A negative sign indicates a phase of spontaneously broken symmetry. In the following, we focus on the simpler case $f_m > 2$. The beta function Eq. (21) already shows that the quantum-gravity contribution acts like an effective change of dimensionality for the mass parameter. It is positive [39,133–135,74], and can even become larger than 2. In this case, quantum-gravity fluctuations render the Higgs mass-parameter irrelevant. This could provide a solution to the gauge-hierarchy problem, as proposed in [136]: Starting from an arbitrary value of the Higgs mass at the scale Λ_{string} , quantum fluctuations of the metric drive the mass towards zero for a sufficiently large separation between Λ_{string} and k_{tr} , such that it becomes naturally tiny at the Planck scale. This solution to the gauge-hierarchy problem also becomes available for those string models for which asymptotic safety is the effective low-energy description. We highlight that the present solution only requires new physics at the Planck scale. This is unlike most solutions to the hierarchy problem, which require new physics close to the electroweak scale. The key point about the resurgence mechanism is that the new physics – in this case quantum gravity – provides a very particular microscopic value of the Higgs mass parameter at the Planck scale, such that it is automatically much smaller than the Planck scale, even though it depends on the cutoff scale quadratically below the Planck scale. For this scenario, $f_m > 2$ must hold such that the fixed-point value for the mass is positive. Accordingly, the weak-gravity conjecture becomes a nontrivial constraint on the asymptotically safe theory, as we will show now.

Inserting the fixed-point value (22) for the mass m_{WGC} , the fixed-point value for the charge actually drops out of the inequality (17), to wit

$$g_* \leq \frac{4\pi}{3} (f_m - 2). \quad (23)$$

Herein, we have used the relation between Newton coupling and Planck mass, $g = 1/(8\pi m_{\text{pl}}^2)$. The inequality (23) actually constitutes a nontrivial constraint on the microscopic gravitational parameter space, since f_m depends on g as well as additional gravitational couplings. In the simplest approximation, this becomes a restriction on the microscopic value of the cosmological constant. Given this restriction on parameter space, one can check whether an asymptotically safe fixed point exists which lies in the string landscape.

5. Conclusions and outlook

We have found indications that the weak gravity conjecture imposes constraints on the microscopic parameter space of asymptotically safe models. This observation in itself is independent of the existence of an embedding of the ASSM into string theory.

In a scenario with string theory as the fundamental theory of quantum gravity, an intermediate asymptotically safe fixed point, see Fig. 1, is expected to be subject to the weak gravity conjecture. Moreover, such a scaling regime is a potential candidate for the low-energy effective description emerging from string theory. Our work, therefore, provides a first indication that an asymptotically safe region might exist in the landscape. We highlight that the RG flow of an asymptotically safe scaling regime could potentially connect a compactification of string theory on a background with a negative *microscopic* value of the cosmological constant to infrared physics in dS space (i.e., with a positive low-energy value of the cosmological constant). We hasten to add that further conditions beyond the weak-gravity conjecture should be satisfied. Most importantly, we have not constructed a specific choice of compactification, for which the coupling-values at \bar{k} lie in the basin of attraction of the asymptotically safe fixed point, and where $\bar{k} \gg k_{\text{tr}}$. We simply point out that such a construction could be possible. In that region of the string-theoretic landscape, the low-energy phenomenology of asymptotic safety and string theory would essentially be indistinguishable. This would, in particular, imply that first-principle calculations of Standard Model couplings, which could be possible in asymptotic safety, would also apply to string theory. On the other hand, embedding asymptotic safety in a UV completion provided by string theory places questions about unitarity in asymptotic safety [96,137] in a different light. In a string-embedding, asymptotic safety could even feature unstable propagating modes. As long as their masses are at or above the string scale, these instabilities simply constitute a signature for a more fundamental UV completion and do not pose problems for the stability of the theory. Accordingly, the class of fixed points that allows for the presented scenario might be larger than the class of fixed points that allows for fundamental asymptotic safety, where ghost modes should be absent.

There has been much discussion on the constraints on QFTs coming from the requirement of a consistent coupling to quantum gravity. Most of the discussion has been in the context of string theory - i.e., under the assumption that quantum gravity corresponds to string theory. Asymptotic safety also gives restrictions and has been explored, e.g., in terms of its implications for chiral fermions [114], a light Higgs [136], restrictions on the maximum number of matter fields [41,100,42] and the allowed interaction structures for matter [132,120,115]. It is of interest to understand to what extent such restrictions are compatible (or in conflict with), the string theory restrictions, i.e., delineate the boundaries and overlapping regions of the respective landscapes.

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