# Placing the newly observed state $B_{J}(5840)$ in bottom spectra along with states $B_{1}(5721), B_{2}^{*}(5747), B_{s 1}(5830), B_{2 s}^{*}(5840)$, and $B_{J}(5970)$ 

Pallavi Gupta and A. Upadhyay<br>School of Physics and Materials Science, Thapar Institute of Engineering and Technology, Patiala 147004, Punjab, India

(Received 30 September 2018; published 31 May 2019)


#### Abstract

We apply the formalism of P. Colangelo et al. [Phys. Rev. D 86, 054024 (2012)] to discuss the quantum number assignments for the recently observed $B_{J}(5840)$ state by R. Aaij et al. (LHCb Collaboration) [J. High Energy Phys. 04 (2015) 024], and we classify the six possible $J^{P}$,s for this state on the basis of the theoretically available masses. By analyzing the strong decay widths and the branching ratios for all six of these cases of $B_{J}(5840)$, we justify one of them to be the most favorable assignment. We also examine the recently observed bottom state $B_{J}(5970)$ as $2 \mathrm{S1}^{-}$and states $B_{J}(5721)$ and $B_{2}^{*}(5747)$ with their strange partners $B_{s 1}(5830)$ and $B_{2 s}^{*}(5840)$ for their $J^{P}$,s as $1 P_{3 / 2} 1^{+}$and $1 P_{3 / 2} 2^{+}$, respectively. The predicted coupling constants $g_{X H}, \tilde{g}_{H H}$, and $g_{T H}$ help in redeeming the strong decay width of experimentally missing bottom states $B\left(2^{1} S_{0}\right), B_{s}\left(2^{3} S_{1}\right), B_{s}\left(2^{1} S_{0}\right), B\left(1^{1} D_{2}\right), B_{s}\left(1^{3} D_{1}\right)$, and $B_{s}\left(1^{1} D_{2}\right)$. These predictions provide crucial information for upcoming experimental studies.


DOI: 10.1103/PhysRevD.99.094043

## I. INTRODUCTION

In recent decades, significant experimental progress has been achieved in studying the heavy-light meson spectroscopy. Heavy-light mesons composed of one heavy quark Q and a light quark $\bar{q}$ are useful in understanding the strong interactions in the nonperturbative regime. Recently, many new charm states like $D_{2}^{*}(3000), D_{J}(3000), D_{J}^{*}(3000)$, $D_{3}^{*}(2760), D_{1}^{*}(2680), D_{2}^{*}(2460), D_{J}^{*}(2650)^{0}, D_{J}^{*}(2760)^{0}$, etc., announced by LHCb [1,2] and BABAR [3] have successfully stimulated charm meson spectroscopy. In the bottom sector, however, only ground states $B^{0}(5279), \quad B^{ \pm}(5279), \quad B^{*}(5324), \quad B_{s}(5366), \quad B_{s}^{*}(5415)$, and a few of the low-lying excited bottom mesons $B_{1}(5721), B_{2}^{*}(5747)$ are experimentally well known [49]; they are listed in a paper by the PDG [10]. But the information for other excited bottom mesons is rather limited compared to the charm mesons. However, the recent measurement of newly observed bottom mesons by LHCb have opened the gate to extending our understanding of these higher excited bottom states. In 2015, LHCb reported the observation of $B_{J}(5721)^{0,+}$ and $B_{2}^{*}(5747)^{0,+}$ states, along with the observation of two new resonances, $B_{J}(5840)^{0,+}$ and $B_{J}(5960)^{0,+}$, in the

[^0]$p p$ collision data, at center-of-mass energies of 7 and 8 TeV [11]. Also, in 2013, the CDF Collaboration analyzed a new state $B_{J}(5960)$ in both the $B^{0} \pi^{+}$and $B^{+} \pi^{-}$mass distribution from the $p \bar{p}$ collision data at $\sqrt{s}=$ 1.96 TeV [12].

And in the strange sector of the bottom mesons, $B_{1 s}(5830)$ and $B_{2 s}^{*}(5840)$ states are well observed by the CDF [7,12], D0 [13], and LHCb [14] collaborations and are assigned the $J^{P}$ states $1 P_{3 / 2} 1^{+}$and $1 P_{3 / 2} 2^{+}$, respectively. The masses and the widths of the recently measured experimental bottom states $B_{J}(5721), \quad B_{2}^{*}(5747)$, $B_{J}(5840), B_{J}(5960), B_{s J}(5830)$, and $B_{2 s}^{*}(5840)$ are listed in Table I. Assigning a place in the mass spectra for such newly observed experimental states is very important, as the $J^{P}$ helps in redeeming many crucial strong interaction properties of the states. To assign a $J^{P}$, many theoretical models are available, such as the quark model [15-18], heavy quark effective theory (HQET) [19], the ${ }^{3} P_{0}$ model [18,20], and many more [21]. Many theoretical predictions have been made for assigning a particular $J^{P}$ to these newly observed states. Different theoretical approaches use different theoretical parameters, and therefore the predictions are not completely consistent with each other; hence, a particular $J^{P}$ is not confirmed to these experimentally observed bottom states.

The first two bottom states in Table I, $B_{J}(5721)$ and $B_{2}^{*}(5747)$, have been analyzed theoretically with various models [18,22-25], and their analyses have interpreted the $B_{2}^{*}(5747)$ state to belong to $J^{P} 2^{+}$. For the $B_{J}(5721)$ state, some of the theoretical works [19,24] favor it to be the spin

TABLE I. Values of the masses and the decay widths of bottom mesons observed by various collaborations.

| State | $J^{P}$ | Mass (MeV) | Width (MeV) | Experiment | Observed decay mode |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $B_{J}(5721)$ | $1^{+}$ | $5727.7 \pm 0.7$ | $30.1 \pm 1.5$ | LHCb [11] | $B^{*} \pi$ |
|  |  | $5720.6 \pm 2.4$ | ... | D0 [8] | $B^{*} \pi$ |
|  |  | $5725.3 \pm 1.6$ | $\ldots$ | CDF [9] | $B^{*} \pi$ |
| $B_{2}^{*}(5747)$ | $2^{+}$ | $5739.44 \pm 0.37$ | $24.5 \pm 1.0$ | LHCb [11] | $B^{*} \pi, B \pi$ |
|  |  | $5746.8 \pm 2.4$ |  | D0 [8] | $B^{*} \pi, B \pi$ |
|  |  | $5740.2 \pm 1.7$ | $22.7 \pm 3.2$ | CDF [9] | $B^{*} \pi, B \pi$ |
| $B_{J}(5840)$ | $\ldots$ | $5862.9 \pm 5.0$ | $127.4 \pm 16.7$ | LHCb [11] | $B \pi$ |
| $B_{J}(5960)$ | $\ldots$ | $5978 \pm 5$ | . 1 | CDF [9] | $B \pi$ |
|  |  | $5969.2 \pm 2.9$ | $82.3 \pm 7.7$ | LHCb [11] | $B \pi$ |
| $B_{s 1}(5830)$ | $1^{+}$ | $5828.40 \pm 0.04$ | . ${ }^{\text {. }}$ | LHCb [14] | $B^{*} K$ |
|  |  | $5828.3 \pm 0.1$ | $0.5 \pm 0.3$ | CDF [12] | $B^{*} K$ |
|  |  | $5829.4 \pm 0.7$ | ... | CDF [7] | $B^{*} K$ |
| $B_{s 2}^{*}(5840)$ | $2^{+}$ | $5839.6 \pm 1.1$ | $\ldots$ | D0 [13] | $B^{*} K, \mathrm{BK}$ |
|  |  | $5839.70 \pm 0.7$ |  | CDF [7] | $B^{*} K$, BK |
|  |  | $5839.70 \pm 0.1$ | $1.40 \pm 0.4$ | CDF [12] | $B^{*} K$, BK |
|  |  | $5839.99 \pm 0.05$ | $1.56 \pm 0.13$ | LHCb [14] | $B^{*} K, \mathrm{BK}$ |

partner of the $B_{2}^{*}(5747)$ state, and hence $J^{P}$ as $1^{+}$for $j_{l}=$ $3 / 2 P$-wave bottom meson. And other papers [22,23,25] suggest $B_{J}(5721)$ to be the mixture of the $1 P_{1 / 2}$ and $1 P_{3 / 2}$ state. The other two bottom states, $B_{1 s}(5830)$ and $B_{2 s}^{*}(5840)$, being the strange partners of $B_{J}(5721)$ and $B_{2}^{*}(5747)$ states, also belong to $1 P_{s 3 / 2} 1^{+}$and $1 P_{s 3 / 2} 2^{+}$ $J^{P}$, s, respectively.

For the bottom state $B_{J}(5960)$, Godfrey et al. [26] claimed that the properties of the $B(5970)$ seen by the CDF Collaboration [12] are consistent with the properties of the $B_{J}(5960)$ measured by LHCb [11], so they may be the same state. The theoretical analysis made by studying decay widths for $B_{J}(5960)$ using the quark pair creation model [25] and HQET [27] favors it belonging to the $2 \mathrm{~S} 1^{-}$ state. This prediction is also supported by work in Ref. [28], where Liu and Lu have used the relativistic quark model. In our previous work [29], where masses were predicted using the QCD and $1 / m_{Q}$ corrections to the flavor independent parameters $\Delta_{F}$ and $\lambda_{F}, B_{J}(5960)$ is again favored to be the $2 \mathrm{S1}^{-}$state. But Lu et al., in Ref. [23], studied masses and strong decays of $B_{J}(5960)$ states with a different spinparity hypothesis and identified that the $B_{J}(5960)$ belongs to the $1 \mathrm{D} 3^{-}$state. A review of the open charm and open bottom systems by Chen in Ref. [30] undergoes various theoretical analyses with the conclusion that $B_{J}(5960)$ belongs to the $2 \mathrm{S1}^{-}$state.

As most of the analyses favored the $2 S 1^{-}$spin parity, $B_{J}(5960)$ is considered to be the radial excited $2 S 1^{-}$state. And lastly, for the $B_{J}(5840)$ bottom state, two spinparity proposals have been put forward. The first one is given by Lu et al. [23], who suggested that it belongs to the $2 \mathrm{SO}^{-}$state. This interpretation matches with the LHCb Collaboration analysis [11]. A second possible $J^{P}$ is given in Ref. [28], where Liu and Lu suggested that the $B_{J}(5840)$
state is a member of a $1 \mathrm{P} 1^{+}$doublet with $j_{l}=1 / 2$. As $B_{J}(5840)$ has been studied in only a few papers, in this scenario, $B_{J}(5840)$ needs to be properly placed in the bottom meson spectra. In Ref. [23], the $J^{P}$ for bottom state $B_{J}(5840)$ has been analyzed by predicting the masses and decay widths using the nonrelativistic quark model and the ${ }^{3} P_{0}$ model, respectively. Conversely, the $J^{P}$ in Ref. [28] has been decided just on the basis of theoretically predicted bottom meson masses. In both of the references, the models have some unknown parameters, which are fitted by using experimental data like the decay width of bottom state $B_{2}^{*}(5747)$. So the accuracy of these predictions cannot be completely justified.

We apply HQET to discuss the assignments of the quantum numbers $J^{P}$ of the open bottom states recently reported by LHCb [11]. In past decades, HQET has successfully explained the properties of heavy-light hadrons. The effectiveness of this theory lies in the fact that a heavy quark is treated as a dynamical degree of freedom (d.o.f.). As a result, the number of unknown parameters is greatly reduced using heavy quark spin and flavor symmetry. Another peculiar property of HQET is that each effective quark field is written in terms of four-vector $v_{\mu}$ of the heavy quark, which remains conserved in the strong interactions in the infinitely heavy quark mass limit. In our paper, we predict $J^{P}$ using the branching ratio $\frac{B \pi}{B^{*} \pi}$, which is free from any theoretical parameter; hence, the prediction made by HQET is supposed to be more accurate and logical. HQET was originally proposed to study the interactions among heavy-light charm and bottom mesons through the emission of light pseudoscalar mesons ( $\pi, \eta, K$ ) [31-40]. The paper is arranged as follows: Sec. II gives the description about the model "heavy quark effective theory." Section III represents the numerical analysis where we
investigated the $B_{J}(5840)$ state by considering all of the possible quark model assignments based on its mass and spin parity and analyze the branching ratio $\frac{B \pi}{B^{*} \pi}$ for all of these possible $J^{P}$ states and confirm one of them to be the suitable $J^{P}$ for this state. Along with this, we also analyze the bottom states $B_{1}(5721), B_{2}^{*}(5747), B_{1 s}(5830), B_{2 s}^{*}(5840)$, and $B_{J}(5960)$ for their respective $J^{P}$, s. In addition to this, we also study the strong decays for the experimentally unobserved but theoretically predicted states $B\left(2^{1} S_{0}\right), B_{s}\left(2^{3} S_{1}\right)$, $B_{s}\left(2^{1} S_{0}\right), B\left(1^{1} D_{2}\right), B_{s}\left(1^{3} D_{1}\right)$, and $B_{s}\left(1^{1} D_{2}\right)$ and discuss their strong coupling constants involved. Section IV presents the summary of our work.

## II. FRAMEWORK

We use heavy quark effective theory for the study of heavy-light bottom mesons. As in our analysis, we use the decay width expressions calculated in Ref. [19] by means of HQET; it is useful to remind the reader about the theoretical framework of this theory. In the heavy quark limit $m_{Q} \rightarrow \infty \quad(Q=c, b)$, spin of the heavy quark $s_{Q}$ decouples from the light d.o.f. which includes the light antiquark and the gluons. Thus the spin of the heavy quark $s_{Q}$ and the total angular momentum of light d.o.f. $s_{l}$ are separately conserved. The total angular momentum of light d.o.f. $s_{l}$ is given by $\left(s_{l}=s_{\bar{q}}+l\right)$, where $s_{\bar{q}}$ is the spin of the light quark and 1 is the orbital angular momentum of the light d.o.f.; therefore the resultant angular momentum J for each heavy-light meson is $J=s_{l}+s_{Q}$. Thus for each state, there is a degenerate doublet of meson state with $J^{P}=$ $s_{l}^{P} \pm 1 / 2$ which for $S$-wave $(l=0)$ gives the doublet which is represented by $\left(P, P^{*}\right)$ with $J_{s_{l}}^{P}=\left(0^{-}, 1^{-}\right)_{\frac{1}{2}}$. For the $P$-wave $(l=1)$, we get two doublets which are represented by $\left(P_{0}^{*}, P_{1}^{\prime}\right)$ and $\left(P_{1}, P_{2}^{*}\right)$ with $J_{s_{l}}^{P}=\left(0^{+}, 1^{+}\right)_{\frac{1}{2}}$ and $\left(1^{+}, 2^{+}\right)_{\frac{3}{2}}$, respectively. Similarly, two doublets for the D-wave $(l=2)$ are represented by $\left(P_{1}^{*}, P_{2}\right)$ and $\left(P_{2}^{\prime}, P_{3}^{*}\right)$, belonging to $J_{s_{l}}^{P}=$ $\left(1^{-}, 2^{-}\right)_{\frac{3}{2}}$ and $\left(2^{-}, 3^{-}\right)_{\frac{5}{2}}$, respectively. The above-mentioned doublets for each wave are expressed by the effective superfield $H_{a}, T_{a}, X_{a}$ and $Y_{a}[40,41]$ :

$$
\begin{gather*}
H_{a}=\frac{1+\not p}{2}\left\{P_{a \mu}^{*} \gamma^{\mu}-P_{a} \gamma_{5}\right\}  \tag{1}\\
T_{a}^{\mu}=\frac{1+\not p}{2}\left\{P_{2 a}^{* \mu \nu} \gamma_{\nu}-P_{1 a \nu} \sqrt{\frac{3}{2}} \gamma_{5}\left[g^{\mu \nu}-\frac{\gamma^{\nu}\left(\gamma^{\mu}-v^{\mu}\right)}{3}\right]\right\}  \tag{2}\\
X_{a}^{\mu}=\frac{1+\not p}{2}\left\{P_{2 a}^{\mu \nu} \gamma_{5} \gamma_{\nu}-P_{1 a \nu}^{*} \sqrt{\frac{3}{2}}\left[g^{\mu \nu}-\frac{\gamma_{\nu}\left(\gamma^{\mu}+v^{\mu}\right)}{3}\right]\right\}  \tag{3}\\
Y_{a}^{\mu \nu}= \\
\frac{1+\not p}{2}\left\{P_{3 a}^{* \mu \nu \sigma} \gamma_{\sigma}-P_{2 a}^{\alpha \beta} \sqrt{\frac{5}{3}} \gamma_{5}\left[g_{\alpha}^{\mu} g_{\beta}^{\nu}\right.\right.  \tag{4}\\
\left.\left.-\frac{g_{\beta}^{\nu} \gamma_{\alpha}\left(\gamma^{\mu}-v^{\mu}\right)}{5}-\frac{g_{\alpha}^{\mu} \gamma_{\beta}\left(\gamma^{\nu}-v^{\nu}\right)}{5}\right]\right\}
\end{gather*}
$$

where the field $H_{a}$ describes the doublet of S -wave, and field $T_{a}$ represents the P-wave doublet $\left(1^{+}, 2^{+}\right)_{\frac{3}{2}}$. D-wave doublets are represented by $X_{a}$ and $Y_{a}$ fields for the $\left(1^{-}, 2^{-}\right)_{\frac{3}{2}}$ and $\left(2^{-}, 3^{-}\right)_{\frac{5}{2}} J^{P}$,s, respectively. Here indices a and b in the subsequent fields are $S U(3)$ flavor indices ( $\mathrm{u}, \mathrm{d}$, and s). The heavy-meson field $P^{(*)}$ contains a factor $\sqrt{m_{Q}}$ with mass dimension of $\frac{1}{2}$. For the radially excited states for radial quantum number $\mathrm{n}=2$, these states are replaced by notation with " $\sim$ ", e.g., $\tilde{P}, \tilde{P}^{*}$, and so on. The strong interaction for these heavy-light mesons involves their decay with the emission of light pseudoscalar mesons $(\pi, \eta, K)$, which can be studied with the help of chiral perturbation theory.

The light pseudoscalar mesons are described by the fields $\xi=\exp ^{\frac{i \mathcal{M}}{f_{\pi}}}$, where $\mathcal{M}$ is defined as

$$
\mathcal{M}=\left(\begin{array}{ccc}
\frac{1}{\sqrt{2}} \pi^{0}+\frac{1}{\sqrt{6}} \eta & \pi^{+} & K^{+}  \tag{5}\\
\pi^{-} & -\frac{1}{\sqrt{2}} \pi^{0}+\frac{1}{\sqrt{6}} \eta & K^{0} \\
K^{-} & \bar{K}^{0} & -\sqrt{\frac{2}{3}} \eta
\end{array}\right) .
$$

The pion octet is introduced by the vector and axial vector combinations $V^{\mu}=\frac{1}{2}\left(\xi \partial^{\mu} \xi^{\dagger}+\xi^{\dagger} \partial^{\mu} \xi\right)$ and $A^{\mu}=\frac{1}{2}\left(\xi \partial^{\mu} \xi^{\dagger}-\xi^{\dagger} \partial^{\mu} \xi\right)$. We choose $f_{\pi}=130 \mathrm{MeV}$. The Dirac structure of the chiral Lagrangian is given by the velocity vector $\mathrm{v} / \mathrm{c}$. The interaction terms between the ground state doublet $\left(H_{a}\right)$ and the excited states $\left(T_{a}, X_{a}, Y_{a}\right)$ through light pseudoscalar mesons are written as

$$
\begin{gather*}
L_{H H}=g_{H H} \operatorname{Tr}\left\{\bar{H}_{a} H_{b} \gamma_{\mu} \gamma_{5} A_{b a}^{\mu}\right\}  \tag{6}\\
L_{T H}=\frac{g_{T H}}{\Lambda} \operatorname{Tr}\left\{\bar{H}_{a} T_{b}^{\mu}\left(i D_{\mu} A+i \not D A_{\mu}\right)_{b a} \gamma_{5}\right\}+\text { H.c. }  \tag{7}\\
L_{X H}=\frac{g_{X H}}{\Lambda} \operatorname{Tr}\left\{\bar{H}_{a} X_{b}^{\mu}\left(i D_{\mu} \mathbb{A}+i \not D A_{\mu}\right)_{b a} \gamma_{5}\right\}+\text { H.c. }  \tag{8}\\
L_{Y H}=\frac{1}{\Lambda^{2}} \operatorname{Tr}\left\{\overline { H } _ { a } Y _ { b } ^ { \mu \nu } \left[k_{1}^{Y}\left\{D_{\mu}, D_{\nu}\right\} A_{\lambda}+k_{2}^{Y}\left(D_{\mu} D_{\lambda} A_{\nu}\right.\right.\right. \\
\left.\left.\left.+D_{\nu} D_{\lambda} A_{\mu}\right)\right]_{b a} \gamma^{\lambda} \gamma_{5}\right\}+ \text { H.c. } \tag{9}
\end{gather*}
$$

In these equations,

$$
\begin{align*}
D_{\mu}= & \partial_{\mu}+V_{\mu}, \quad\left\{D_{\mu}, D_{\nu}\right\}=D_{\mu} D_{\nu}+D_{\nu} D_{\mu} \\
\left\{D_{\mu}, D_{\nu} D_{\rho}\right\}= & D_{\mu} D_{\nu} D_{\rho}+D_{\mu} D_{\rho} D_{\nu}+D_{\nu} D_{\mu} D_{\rho} \\
& +D_{\nu} D_{\rho} D_{\mu}+D_{\rho} D_{\mu} D_{\nu}+D_{\rho} D_{\nu} D_{\mu} \tag{10}
\end{align*}
$$

Here $g_{H H}, g_{T H}, g_{X H}$, and $g_{Y H}=k_{1}^{Y}+k_{2}^{Y}$ are the strong coupling constants, $\Lambda$ is the chiral symmetry breaking scale which is taken as 1 GeV . Using the Lagrangian $L_{H H}, L_{T H}$, $L_{X H}$, and $L_{Y H}$, the two body strong decay widths of $Q \bar{q}$ heavy-light bottom mesons are calculated in Ref. [19] as

TABLE II. Numerical value of the meson masses used in this work [10].

| States | $B^{0}$ | $B^{ \pm}$ | $B^{*}$ | $B_{s}$ | $B_{s}^{*}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Masses $(\mathrm{MeV})$ | 5279.58 | 5279.25 | 5325.20 | 5366.77 | 5415.40 |
| States | $\pi^{ \pm}$ | $\pi^{0}$ | $\eta$ | $K^{+}$ | $K^{0}$ |
| Masses $(\mathrm{MeV})$ | 139.57 | 134.97 | 547.85 | 493.67 | 497.61 |

$$
\begin{equation*}
\Gamma=\frac{1}{(2 J+1)} \sum \frac{p_{M}}{8 \pi M_{i}^{2}}|A|^{2} \tag{11}
\end{equation*}
$$

where $A$ is the scattering amplitude, and $p_{M}$ and $m_{M}$ are the final momentum and mass of the light pseudoscalar meson with $\quad p_{M}=\sqrt{\lambda\left(M_{i}^{2}, m_{M}^{2}, M_{f}^{2}\right)} / 2 M_{i}$, where $\lambda(a, b, c)=$ $a^{2}+b^{2}+c^{2}-2 a b-2 a c-2 b c$ is the Källen function. $M_{i}$ and $M_{f}$ stand for initial and final heavy-meson mass:
$\left(0^{-}, 1^{-}\right) \rightarrow\left(0^{-}, 1^{-}\right)+M$

$$
\begin{align*}
& \Gamma\left(1^{-} \rightarrow 1^{-}\right)=C_{M} \frac{g_{H H}^{2} M_{f} p_{M}^{3}}{3 \pi f_{\pi}^{2} M_{i}}  \tag{12}\\
& \Gamma\left(1^{-} \rightarrow 0^{-}\right)=C_{M} \frac{g_{H H}^{2} M_{f} p_{M}^{3}}{6 \pi f_{\pi}^{2} M_{i}}  \tag{13}\\
& \Gamma\left(0^{-} \rightarrow 1^{-}\right)=C_{M} \frac{g_{H H}^{2} M_{f} p_{M}^{3}}{2 \pi f_{\pi}^{2} M_{i}} \tag{14}
\end{align*}
$$

$$
\left(1^{+}, 2^{+}\right) \rightarrow\left(0^{-}, 1^{-}\right)+M
$$

$$
\begin{equation*}
\Gamma\left(2^{+} \rightarrow 1^{-}\right)=C_{M} \frac{2 g_{T H}^{2} M_{f} p_{M}^{5}}{5 \pi f_{\pi}^{2} \Lambda^{2} M_{i}} \tag{15}
\end{equation*}
$$

$$
\begin{equation*}
\Gamma\left(2^{+} \rightarrow 0^{-}\right)=C_{M} \frac{4 g_{T H}^{2} M_{f} p_{M}^{5}}{15 \pi f_{\pi}^{2} \Lambda^{2} M_{i}} \tag{16}
\end{equation*}
$$

$$
\begin{equation*}
\Gamma\left(1^{+} \rightarrow 1^{-}\right)=C_{M} \frac{2 g_{T H}^{2} M_{f} p_{M}^{5}}{3 \pi f_{\pi}^{2} \Lambda^{2} M_{i}} \tag{17}
\end{equation*}
$$

$$
\left(1^{-}, 2^{-}\right) \rightarrow\left(0^{-}, 1^{-}\right)+M
$$

$$
\begin{equation*}
\Gamma\left(1^{-} \rightarrow 0^{-}\right)=C_{M} \frac{4 g_{X H}^{2}}{9 \pi f_{\pi}^{2} \Lambda^{2}} \frac{M_{f}}{M_{i}}\left[p_{M}^{3}\left(m_{M}^{2}+p_{M}^{2}\right)\right] \tag{18}
\end{equation*}
$$

$$
\begin{equation*}
\Gamma\left(1^{-} \rightarrow 1^{-}\right)=C_{M} \frac{2 g_{X H}^{2}}{9 \pi f_{\pi}^{2} \Lambda^{2}} \frac{M_{f}}{M_{i}}\left[p_{M}^{3}\left(m_{M}^{2}+p_{M}^{2}\right)\right] \tag{19}
\end{equation*}
$$

$$
\begin{equation*}
\Gamma\left(2^{-} \rightarrow 1^{-}\right)=C_{M} \frac{2 g_{X H}^{2}}{3 \pi f_{\pi}^{2} \Lambda^{2}} \frac{M_{f}}{M_{i}}\left[p_{M}^{3}\left(m_{M}^{2}+p_{M}^{2}\right)\right] \tag{20}
\end{equation*}
$$

$$
\left(2^{-}, 3^{-}\right) \rightarrow\left(0^{-}, 1^{-}\right)+M
$$

$$
\begin{equation*}
\Gamma\left(2^{-} \rightarrow 1^{-}\right)=C_{M} \frac{4 g_{Y H}^{2}}{15 \pi f_{\pi}^{2} \Lambda^{4}} \frac{M_{f}}{M_{i}}\left[p_{M}^{7}\right] \tag{21}
\end{equation*}
$$

$$
\begin{gather*}
\Gamma\left(3^{-} \rightarrow 0^{-}\right)=C_{M} \frac{4 g_{Y H}^{2}}{35 \pi f_{\pi}^{2} \Lambda^{4}} \frac{M_{f}}{M_{i}}\left[p_{M}^{7}\right]  \tag{22}\\
\Gamma\left(3^{-} \rightarrow 1^{-}\right)=C_{M} \frac{16 g_{Y H}^{2}}{105 \pi f_{\pi}^{2} \Lambda^{4}} \frac{M_{f}}{M_{i}}\left[p_{M}^{7}\right] . \tag{23}
\end{gather*}
$$

In these equations, the coefficients $C_{\pi^{ \pm}}, C_{K^{ \pm}}, C_{K^{0}}$, $C_{\bar{K}^{0}}=1, C_{\pi^{0}}=\frac{1}{2}$, and $C_{\eta}=\frac{2}{3}$ or $\frac{1}{6}$ as from Ref. [19]. Different values of $C_{\eta}$ corresponds to the initial state being $b \bar{u}, b \bar{d}$, or $b \bar{s}$, respectively. For the decay within $\mathrm{n}=1$, the hadronic coupling constants are notated as $g_{H H}, g_{T H}$, etc., and for the decay from $\mathrm{n}=2$ to $\mathrm{n}=1$, these couplings are notated as $\tilde{g}_{H H}, \tilde{g}_{T H}$, respectively. Higher order corrections for spin and flavor violation of order $\frac{1}{m_{Q}}$ are excluded to avoid new unknown coupling constants. The coupling constants involved in these widths either can be theoretically predicted or can be determined indirectly from the known experimental values of the decay widths. The numerical masses of various mesons used in the calculation are listed in Table II.

## III. NUMERICAL ANALYSIS

To assign a particular $J^{P}$ to the experimental available states is very important, as the $J^{P}$ helps in redeeming many crucial strong interaction properties of the states like their decay widths, masses, branching ratios, hadronic coupling constants, etc. The recently observed state $B_{J}(5840)$ has gone through various theoretical analyses $[23,28]$ for its strong decay, but a unique $J^{P}$ is not yet confirmed for it.

In this paper, we confirm a particular $J^{P}$ to the bottom state $B_{J}(5840)$ recently observed by LHCb. On the basis of the theoretically predicted masses Refs. [23,28,42-44], $B_{J}(5840)$ can be a member of the doublets for radially excited S-wave $2 \mathrm{~S}\left(0^{-}, 1^{-}\right)$, or for orbitally excited D -wave doublet $1 \mathrm{D}\left(1^{-}, 2^{-}\right)$or $1 \mathrm{D}\left(2^{-}, 3^{-}\right)$. These six possible $J^{P}$ states are tabulated in Table III with their allowed strong decays to the ground state bottom mesons $1 \mathrm{~S}\left(0^{-}, 1^{-}\right)$.

To choose the best possible $J^{P}$ among these, we study the branching ratio

$$
\begin{equation*}
B R=R_{1}=\frac{\Gamma\left(B_{J}(5840) \rightarrow B \pi\right)}{\Gamma\left(B_{J}(5840) \rightarrow B^{*} \pi\right)} \tag{24}
\end{equation*}
$$

for all of these suggested $J^{P}$,s and their masses. This ratio $R_{1}$ is effective in distinguishing these six possible assignments, as this ratio $R_{1}$ gives a result independent of the

TABLE III. Strong decay channels for all of the six possible spin-parity $J^{P}$ values for the $B_{J}(5840)$ state.

| Decay mode | $2 \mathrm{~S} 0^{-}$ | $2 \mathrm{~S} 1^{-}$ | $1 \mathrm{D} 1^{-}$ | $1 \mathrm{D} 2_{3 / 2}^{-}$ | $1 \mathrm{D} 2_{5 / 2}^{-}$ | $1 \mathrm{D} 3^{-}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $B^{0} \pi^{0}$ | $\ldots$ | $220.64 \tilde{g}_{H H}^{2}$ | $182.58 g_{X H}^{2}$ | $\ldots$ | $\ldots$ | $12.84 g_{Y H}^{2}$ |
| $B^{+} \pi^{-}$ | $\ldots$ | $439.40 \tilde{g}_{H H}^{2}$ | $364.14 g_{X H}^{2}$ | $\ldots$ | $\ldots$ | $25.44 g_{Y H}^{2}$ |
| $B^{0} \eta$ | $\ldots$ | $12.47 \tilde{g}_{H H}^{2}$ | $11.60 g_{X H}^{2}$ | $\ldots$ | $\ldots$ | $0.01 g_{Y H}^{2}$ |
| $B_{s} K$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $B^{*} \pi^{0}$ | $523.71 \tilde{g}_{H H}^{2}$ | $347.46 \tilde{g}_{H H}^{2}$ | $61.63 g_{X H}^{2}$ | $184.91 g_{X H}^{2}$ | $16.98 g_{Y H}^{2}$ | $9.70 g_{Y H}^{2}$ |
| $B^{*} \pi^{+}$ | $1040.10 \tilde{g}_{H H}^{2}$ | $690.08 \tilde{g}_{H H}^{2}$ | $122.46 g_{X H}^{2}$ | $367.39 g_{X H}^{2}$ | $33.41 g_{Y H}^{2}$ | $19.09 g_{Y H}^{2}$ |
| $B^{*} \eta$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $B_{s}^{*} K$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| Total | $1563.82 \tilde{g}_{H H}^{2}$ | $1710.09 \tilde{g}_{H H}^{2}$ | $742.43 g_{X H}^{2}$ | $552.30 g_{X H}^{2}$ | $50.40 g_{Y H}^{2}$ | $67.09 g_{Y H}^{2}$ |
| Ratio $R_{1}$ | 0 | 0.63 | 2.96 | 0 | 0 | 1.32 |

coupling constants $\tilde{g}_{H H}, g_{X H}$, and $g_{Y H}$, thus making the predictions model independent. This ratio gives different values for all these six states, thus allowing us to notate the proper $J^{P}$ for the bottom state $B_{J}(5840)$.

We have also plotted the graphs for the $R_{1}$ with the masses for these $J^{P}$ states which are shown in Fig. 1. It is worth noticing that, as Fig. 1(a) shows, the $R_{1}$ remains 0 for the entire mass range, which indicates that the $B \pi$ decay mode is either suppressed or not allowed for $J^{P}$, $2 \mathrm{SO}^{-}$, $1 D_{3 / 2} 2^{-}$, and $1 D_{5 / 2} 2^{-}$. Figures $1(\mathrm{~b})-1(\mathrm{~d})$ show the variation of $R_{1}$ with the masses and give the values of $R_{1}$ as 0.63 ,
2.96, and 1.32 for the $J^{P}$ states $2 S 1^{-}, 1 D 1^{-}$, and $1 D 3^{-}$, respectively, corresponding to $\mathrm{M}(5840)=5862.90 \mathrm{MeV}$. The values 2.96 for $1 D 1^{-}$and 1.32 for $1 D 3^{-}$point towards the dominancy of the $B \pi$ mode, whereas the value 0.63 for the $2 S 1^{-}$indicates the dominance of the $B^{*} \pi$ decay mode. The calculation of the total decay widths for all six of these classifications of $B_{J}(5840)$ requires the values of the coupling constants $\tilde{g}_{H H}, g_{X H}$, and $g_{Y H}$, which are experimentally unknown. Nevertheless, on the basis of the theoretically available values of these couplings, the following results can be seen:


FIG. 1. Branching ratio $\Gamma\left(B_{J}(5840)\right) \rightarrow B \pi / B^{*} \pi$ for all six possible $J^{P}$, s for the $B_{J}(5840)$ state, where three possible $J^{P}$, s $2 S(0-), 1 D(2-) 3 / 2,1 D(2-) 5 / 2$ are shown in (a) and $J^{P}$,s $2 S(1-), 1 D(1-)$ and $1 D(3-)$ are shown in (b), (c) and (d) respectively.

TABLE IV. Strong decay width of newly observed bottom meson $B_{J}^{*}(5840)$ and its spin and strange partners $B\left(1^{1} D_{2}\right), B_{s}\left(1^{1} D_{2}\right)$, and $B_{s}^{*}\left(1^{3} D_{1}\right)$. The ratio in the fifth column represents $\hat{\boldsymbol{\Gamma}}=\frac{\Gamma}{\Gamma\left(B_{J}^{(*)} \rightarrow B^{*+} \pi^{-}\right)}$for the nonstrange mesons, and $\hat{\boldsymbol{\Gamma}}=\frac{\Gamma}{\Gamma\left(B_{s,}^{*} \rightarrow B^{* 0} K^{+}\right)}$for the strange mesons. Branching fraction (B.F.) gives the percentage of the partial decay width with respect to the total decay width.

| State | $n L s_{l} J^{P}$ | Decay channel | Decay width (MeV) | Ratio | B.F. \% | Experimental/theoretical value ( MeV ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $B_{J}(5840)$ | $1 D_{3 / 2} 1^{-}$ | $B^{*} \pi^{+}$ | $122.46 g_{X H}^{2}$ | 1 | 16.49 |  |
|  |  | $B^{*} \pi^{0}$ | $61.63 g_{X H}^{2}$ | 0.50 | 8.30 |  |
|  |  | $B^{*} \eta$ | 61.63 | ... | ... |  |
|  |  | $B_{s}^{*} K$ | ... | $\ldots$ | $\ldots$ |  |
|  |  | $B^{0} \pi^{0}$ | $182.58 g_{X H}^{2}$ | 1.49 | 24.59 |  |
|  |  | $B^{+} \pi^{-}$ | $364.14 g_{X H}^{2}$ | 2.97 | 49.04 |  |
|  |  | $B^{0} \eta$ | 11.60 | 0.09 | 1.56 |  |
|  |  | $B_{s} K$ | ... | $\ldots$ | $\ldots$ |  |
|  |  | Total | $742.43 g_{X H}^{2}$ |  |  | 127.40 [11], 127 [18] |
| $B_{J}(5967.20)$ | $1 D_{3 / 2} 2^{-}$ | $B^{*} \pi^{+}$ | $866.42 g_{X H}^{2}$ | 1 | 60.34 |  |
|  |  | $B^{*} \pi^{0}$ | $435.16 g_{X H}^{2}$ | 0.50 | 30.31 |  |
|  |  | $B^{*} \eta$ | $94.61 g_{X H}^{2}$ | 0.10 | 6.58 |  |
|  |  | $B_{s}^{*} K$ | $39.42 g_{X H}^{2}$ | 0.04 | 2.74 |  |
|  |  | Total | $1435.69 g_{X H}^{2}$ |  |  | 250.69 [23], 98 [18] |
| $B_{s J}(6083.00)$ | $1 D_{s(3 / 2)} 1^{-}$ | $B_{s}^{*} \pi^{0}$ | $175.31 g_{X H}^{2}$ | 0.57 | 5.64 |  |
|  |  | $B_{s}^{*} \eta$ | $12.39 g_{X H}^{2}$ | 0.04 | 0.39 |  |
|  |  | $B^{* 0} K^{0}$ | $300.48 g_{X H}^{2}$ | 0.98 | $9.67$ |  |
|  |  | $B^{*-} K^{+}$ | $305.62 g_{X H}^{2}$ | 1 | 9.84 |  |
|  |  | $B_{s}^{0} \pi^{0}$ | $488.80 g_{X H}^{2}$ | 1.59 | 15.73 |  |
|  |  | $B_{s} \eta$ | $49.01 g_{X H}^{2}$ | 0.16 | 1.57 |  |
|  |  | $B^{+} K^{-}$ | $894.37 g_{X H}^{2}$ | 2.92 | 28.79 |  |
|  |  | $B^{0} K^{0}$ | $879.81 g_{X H}^{2}$ | 2.87 | 28.32 |  |
|  |  | Total | $3105.83 g_{X H}^{2}$ |  |  | 213.38 [23], 137 [18] |
| $B_{s J}(6057.50)$ | $1 D_{s(3 / 2)} 2^{-}$ | $B_{s}^{*} \pi^{0}$ | $436.79 g_{X H}^{2}$ | 0.60 | 23.04 |  |
|  |  | $B_{s}^{*} \eta$ | 23.62 ${ }_{\text {XH }}^{2}$ | 0.03 | 1.24 |  |
|  |  | $B^{*-} K^{+}$ | $724.42 g_{X H}^{2}$ | 1 | 38.21 |  |
|  |  | $B^{* 0} K^{0}$ | $710.60 g_{X H}^{2}$ | 0.98 | 37.48 |  |
|  |  | Total | $1895.45 g_{X H}^{2}$ |  |  | 198.64 [23], 89 [18] |

(a) If $B_{J}(5840)$ is classified as that of the member of the doublet $2 \mathrm{~S}\left(0^{-}, 1^{-}\right)$, then the total decay width for these states comes out to be 150.28 and 165.13 MeV , respectively, for $2 \mathrm{SO}^{-}$and $2 \mathrm{S1}^{-}$. This prediction is made using the theoretical data $\tilde{g}_{H H}=0.31$ [45]. Both of these decay widths match very well with the experimentally observed broad decay width of 127 MeV for $B_{J}(5840)$. The $2 S 1^{-}$state is known to be filled by the experimentally seen bottom state $B_{J}(5970)$, and the experimentally observed decay mode $B^{0} \pi^{+}$is not possible for $2 \mathrm{SO}^{-}$. So the possibility of both of these $J^{P}$,s, $2 S 0^{-}$and $2 S 1^{-}$, are excluded.
(b) If $B_{J}(5840)$ is the member of the doublet $1 \mathrm{D}\left(1^{-}, 2^{-}\right)$ with $s_{l}^{P}=3 / 2^{-}$, then the total strong decay width comes out to be 42.76 and 31.81 MeV for $J^{P}$, $1^{-}$and $2^{-}$, respectively. For this, $g_{X H}$ is taken as 0.24 , which is derived using the charm state $D_{1}^{*}(2760)$ information
observed by LHCb in 2016 [46]. The $0 R_{1}$ value and the narrow decay width for state 1D2 ${ }^{-}$also rules out this option for $B_{J}(5840)$.
(c) The last possibility for $B_{J}(5840)$ can be the member of the doublets $1 \mathrm{D}\left(2^{-}, 3^{-}\right)$. Using the available data for coupling constant $g_{Y H}=0.61$ [45], the total decay widths for $J^{P}$ states $2^{-}$and $3^{-}$come out to be 18.75 and 24.96 MeV , respectively. Even for such a high value of $g_{Y H}$, the decay widths are very narrow. So the classification of $B_{J}(5840)$ as a member of $1 \mathrm{D}\left(2^{-}, 3^{-}\right)$ is completely ruled out.
Thus the left out possibility of spin parity for $B_{J}(5840)$ is $1 D\left(1^{-}\right)_{3 / 2}$. It is interesting to note that the ratio $R_{1}$ of $B_{J}(5840)$ for $J^{P}$ state $1 D\left(1^{-}\right)_{3 / 2}$ also comes out to be maximum with a value of 2.96 , thus favoring $1 D\left(1^{-}\right)_{3 / 2}$ as the most favorable $J^{P}$ for $B_{J}(5840)$. However, if we consider the fact that the photon from $B^{*} \rightarrow B \gamma$ is too
low in energy to be detected, and $B^{*}$ mesons are partially reconstructed as B mesons. Then the $J^{P}$ option for $B_{J}(5840)$ belonging to $2 S 0^{-}$cannot be fully ignored because of its large decay width. The identification of $B_{J}(5840)$ as $2 S 0^{-}$is also supported by the work in Refs. [11,23]. So in the future, one may expect experimental information about the decay modes for $B_{J}(5840)$ to broaden up, to clearly identify the exact $J^{P}$ for this state. In this paper, because of the only $B^{0} \pi^{+}$experimentally observed decay mode, we expect $B_{J}(5840)$ to belong to the $1 D\left(1^{-}\right)_{3 / 2} \mathrm{~J}$ value.

## A. Prediction of spin and strange partners for $\boldsymbol{B}_{\boldsymbol{J}}(\mathbf{5 8 4 0})$

On the basis of the spin-parity assignment of $B_{J}(5840)$, it is interesting to look for some features of its spin and strange partners. As discussed, $B_{J}(5840)$ is assigned the orbitally excited D-wave state with $J^{P}$ as $1 \mathrm{D} 1^{-}$. The complete picture of the partial decay widths for $B\left(1^{1} D_{2}\right)$, $B_{s}\left(1^{3} D_{1}\right)$, and $B_{s}\left(1^{1} D_{2}\right)$ being the spin and strange partners of the $B_{J}(5840)$ state is listed in Table IV. Along with the partial decay widths, the table also shows the branching ratios $\hat{\boldsymbol{\Gamma}}=\frac{\Gamma}{\Gamma\left(B_{J}(5840) \rightarrow B^{*+} \pi^{-}\right)}, \hat{\boldsymbol{\Gamma}}=\frac{\Gamma}{\Gamma\left(B_{s, J}^{*} \rightarrow B^{* 0} K^{+}\right)}$and branching fractions for all of the mentioned decay modes. Apart from the decay channels mentioned in this table, $B_{J}(5840)$ being $1 D\left(1^{-}\right)$also decays to $1 P\left(1^{+}\right), 1 P^{\prime}\left(1^{+}\right)$, and $1 P\left(2^{+}\right)$states along with pseudoscalar mesons $(\pi, \eta, K)$. Since these decays occur via D-wave, their contribution is relatively suppressed. Here we mentioned only the dominant decay modes with which total decay width of $B_{J}(5840)$ comes out to be $742.43 g_{X H}^{2}$.

The information in the Table IV reveals that, for the $B_{J}(5840)$ state, $B^{+} \pi^{-}$and $B^{0} \pi^{0}$ are the main decay modes compared to the $B^{*+} \pi^{-}$and $B^{* 0} \pi^{0}$ modes. The decay width obtained in this work is finally compared with the experimental data, and hence the coupling constant $g_{X H}$ is obtained to be

$$
\begin{equation*}
g_{X H}=0.41 \pm 0.02 \tag{25}
\end{equation*}
$$

This information can be beneficial in finding the total and partial decay widths of unobserved highly excited bottom meson states. Theoretically, these coupling values are also obtained as 0.45 [47], 0.53 [48], and 0.19 [24] from the charm states $D_{S J}(2860), D_{J}(2600)$ and bottom state $B_{J}(5960)$ assuming them to be in $1 D 1^{-}$state. As the $D_{J}(2600)$ and $B_{J}(5960)$ belong to $2 S 1^{-}$, the last two values of the coupling $g_{X H}=0.53$ and 0.19 predicted from $D_{J}(2600)$ and $B_{J}(5960)$ are not useful for our study.

Theoretically, mass of the spin partner of $B_{J}(5840)$, i.e., $B\left(1^{1} D_{2}\right)$ is predicted to be $5967.20 \pm 30 \mathrm{MeV}$ in Ref. [23,28,42-44]. Column 5 of the table gives the ratio of the partial decay widths for $B\left(1^{1} D_{2}\right)$ with respect to its partial decay width $B^{*-} \pi^{+}$. Apart from the decay channels listed in this table, $B\left(1^{1} D_{2}\right)$ also decays to P-wave bottom
meson states $1 P\left(0^{+}\right), 1 P\left(1^{+}\right), 1 P^{\prime}\left(1^{+}\right)$, and $1 P\left(2^{+}\right)$, which occur via D -wave, and thus due to the small phase space, these decay modes are suppressed when compared to decays to ground state S -wave mesons and hence are not shown in Table IV. From the listed decay channels, $B^{*-} \pi^{+}$comes out to be the dominant decay mode for $B\left(1^{1} D_{2}\right)$ with branching fraction $60.34 \%$. Decay width calculated in Ref. [18], also shows $B^{*} \pi$ to be the dominant decay mode. Hence, the decay mode $B^{*-} \pi^{+}$can be a motivation for the experimental search for the missing bottom state $B\left(1^{1} D_{2}\right)$ in future. Using the value of the coupling constant $g_{X H}$ obtained from Eq. (25), the total decay width of the bottom state $B\left(1^{1} D_{2}\right)$ is obtained as 241.33 MeV . This decay width value is in the same range as given in Ref. [23] with $3.87 \%$ deviation.

Masses for the strange partners of these bottom states are taken as 6083.06 and 6057.50 MeV from the theoretical work [23,28,42-44]. Referring to the branching fractions in Table IV, $B^{+} K^{0}$ and $B^{*-} K^{+}$seem to be the dominant decay modes with contribution $28.79 \%$ and $38.21 \%$ for the bottom strange states $B_{s 1}^{*}$ and $B_{s 2}$, respectively, which are comparable to the dominant modes seen in Ref. [18]. These strange states also allow decays to $P$-wave bottom meson states but are relatively suppressed. Hence, the total decay width for these strange state comes out to be

$$
\begin{align*}
& \Gamma\left(B_{s 1}^{*}\right)=522.09 \mathrm{MeV}  \tag{26}\\
& \Gamma\left(B_{s 2}\right)=318.62 \mathrm{MeV} \tag{27}
\end{align*}
$$

The results conclude that $B_{s 1}^{*}$ is a broader state than its spin partner $B_{s 2}$. Moreover, if we use the coupling $g_{X H}=$ 0.45 obtained in Ref. [47], the decay widths for states $B\left(1^{1} D_{2}\right), B_{s}\left(1^{3} D_{1}\right)$, and $B_{s}\left(1^{1} D_{2}\right)$ deviate from our results by $16 \%$.

## B. Analysis for bottom states $\boldsymbol{B}_{1}(5721), B_{2}^{*}(5747)$, $B_{1 s}(5830)$, and $B_{2 s}^{*}(5840)$

We have also analyzed the bottom states $B_{1}(5721)$, $B_{2}^{*}(5747), B_{1 s}(5830)$, and $B_{2 s}^{*}(5840)$ for their $J^{P}$ 's. On the basis of their available theoretical and experimental information, the states $B_{1}(5721), B_{2}^{*}(5747), B_{1 s}(5830)$, and $B_{2 s}^{*}(5840)$ are identified as the P -wave bottom mesons with $j_{l}=3 / 2$ :

$$
\begin{equation*}
\left(B_{1}(5721), B_{2}^{*}(5747)\right)=\left(1^{+}, 2^{+}\right)_{3 / 2} \text { with } n=1, L=1 \text {, } \tag{28}
\end{equation*}
$$

$\left(B_{1 s}(5830), B_{2 s}^{*}(5840)\right)=\left(1^{+}, 2^{+}\right)_{3 / 2}$ with $n=1, L=1$.

We study their strong decay widths using Eqs. (14)-(16) and calculate the various branching ratios involved. The numerical value of the partial decay widths for the bottom

TABLE V. Strong decay width of newly observed bottom mesons $B_{1}(5721)$ and $B_{2}^{*}(5747)$ and their strange partners $B_{1 s}(5830)$ and $B_{2 s}^{*}(5840)$. The ratio in the fifth column represents the $\hat{\boldsymbol{\Gamma}}=\frac{\Gamma}{\Gamma\left(B_{J}^{(*)} \rightarrow B^{*+} \pi^{-}\right)}$for the nonstrange mesons, and $\hat{\Gamma}=\frac{\Gamma}{\Gamma\left(B_{s J}^{*} \rightarrow B^{* 0} K^{+}\right)}$for the strange mesons. The branching fraction (B.F.) gives the percentage of the partial decay width with respect to the total decay width.

| State | $n L s_{l} J^{P}$ | Decay channel | Decay width (MeV) | Ratio | B.F. \% | Experimental/theoretical value (MeV) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $B_{1}(5721)$ | $1 P_{3 / 2} 1^{+}$ | $B^{*} \pi^{+}$ | $74.83 g_{T H}^{2}$ | 1 | 66.16 |  |
|  |  | $B^{*} \pi^{0}$ | $38.25 g_{\text {TH }}^{2}$ | 0.51 | 33.82 |  |
|  |  | Total | $113.09 g_{T H}^{2}$ |  |  | 30.1 [11], 55 [18] |
| $B_{2}^{*}(5747)$ | $1 P_{3 / 2} 2^{+}$ | $B^{*} \pi^{+}$ | $52.47 g_{T H}^{2}$ | 1 | 30.65 |  |
|  |  | $B^{*} \pi^{0}$ | $26.78 g_{T H}^{2}$ | 0.51 | 15.64 |  |
|  |  | $B^{0} \pi^{0}$ | $30.91 g_{T H}^{2}$ | 0.58 | 18.05 |  |
|  |  | $B^{+} \pi^{-}$ | $61.02 g_{T H}^{2}$ | 1.16 | 35.64 |  |
|  |  | Total | $171.18 g_{T H}^{2}$ |  |  | 24.5[11], 7 [18] |
| $B_{1 s}(5830)$ | $1 P_{s 3 / 2} 1^{+}$ | $B_{s}^{*} \pi^{0}$ | $44.45 g_{T H}^{2}$ | $\ldots$ | 100 |  |
|  |  | $B^{*+} K^{0}$ | $\sim \mathcal{O}\left(10^{-10}\right) g_{T H}^{2}$ | ... | ... |  |
|  |  | $B^{*-} K^{+}$ | $\sim \mathcal{O}\left(10^{-10}\right) g_{T H}^{2}$ | $\ldots$ | $\ldots$ |  |
|  |  | Total | $44.45 g_{T H}^{2}$ |  |  | 0.5 [12], 30 [18] |
| $B_{2 s}^{*}(5840)$ | $1 P_{s 3 / 2} 2^{+}$ | $B_{s}^{*} \pi^{0}$ | $30.41 g_{\text {TH }}^{2}$ | 87.38 | 40.27 |  |
|  |  | $B^{*+} K^{0}$ | $0.20 g_{T H}^{2}$ | 0.59 | 0.27 |  |
|  |  | $B^{*-} K^{+}$ | $0.34 g_{T H}^{2}$ | 1 | 0.46 |  |
|  |  | $B^{+} K^{-}$ | $3.97 g_{T H}^{2}$ | 11.40 | 5.25 |  |
|  |  | $B^{0} K^{0}$ | $4.64 g_{T H}^{2}$ | 13.33 | 6.14 |  |
|  |  | $B_{s} \pi^{0}$ | $35.92 g_{\text {TH }}^{2}$ | 103.21 | 47.53 |  |
|  |  | Total | $75.57 g_{T H}^{2}$ |  |  | 1.40[12], 1 [18] |

states $B_{1}(5721), B_{2}^{*}(5747), B_{1 s}(5830)$, and $B_{2 s}^{*}(5840)$ are given in Table V. The obtained decay widths are then compared to the experimental data to obtain the strong coupling constant $g_{T H}$. Since the strange states $B_{1 s}(5830)$ and $B_{2 s}^{*}(5840)$ are very narrow, we exclude them to calculate the coupling constant $g_{T H} \cdot g_{T H}$ comes out to be $0.50 \pm 0.01$ and $0.37 \pm 0.01$ for bottom states $B_{1}(5721)$ and $B_{2}^{*}(5747)$, respectively. This is consistent with the theoretical values of $g_{T H}$ in Refs. [24,45,49] obtained from charm mesons. Here the consistency in the hadronic coupling constant $g_{T H}$ beautifully describes the heavy quark symmetry between the charm and bottom mesons. We also obtained the ratios $R_{2}, R_{3}$, and $R_{4}$ as

$$
\begin{gather*}
R_{2}=\frac{\Gamma\left(B_{1}(5721)\right)}{\Gamma\left(B_{1}(5721)\right)+\Gamma\left(B_{2}^{*}(5747)\right)}=0.60,  \tag{30}\\
R_{3}=\frac{\Gamma\left(B_{2}^{*} \rightarrow B^{*} \pi\right)}{\left(\Gamma\left(B_{2}^{*} \rightarrow B^{*} \pi\right)+\Gamma\left(B_{2}^{*} \rightarrow B \pi\right)\right)}=0.46,  \tag{31}\\
R_{4}=B_{2}^{*}(5747) \rightarrow \frac{B^{*+} \pi^{-}}{B^{+} \pi^{-}}=0.85, \tag{32}
\end{gather*}
$$

which are consistent with their experimental values $R_{2}=$ $0.47 \pm 0.06$ and $R_{3}=0.47 \pm 0.09$ observed by the D0 Collaboration [8], and $R_{4}=0.71 \pm 0.14$ measured by LHCb [11]. Table V also shows the decay widths of the
strange bottom states $B_{1 s}(5830)$ and $B_{2 s}^{*}(5840)$. The negligible values of the decay widths (of order of $10^{-10} \mathrm{MeV}$ ) for the $B_{1 s}(5830)$ state decaying to $B^{*+} K^{-}$ and $B^{*-} K^{0}$ are consistent with its very small decay width 0.5 MeV measured by the CDF Collaboration [12] in 2014. Table V reveals that $B^{*+} \pi^{-}$and $B^{+} \pi^{-}$are the main decay modes for $B_{1}(5721)$ and $B_{2}^{*}(5747)$ with branching fractions $66.16 \%$ and $35.64 \%$, respectively. Similarly, $B_{s}^{*} \pi^{0}$ and $B_{s} \pi^{0}$ are observed to be the dominating decay modes for their strange partners $B_{1 s}(5830)$ and $B_{2 s}^{*}(5840)$, respectively.

## C. Prediction of spin and strange partners for $\boldsymbol{B}_{\boldsymbol{J}}(\mathbf{5 9 7 0})$

Now, we will proceed in a similar manner to study the spin and strange partners for bottom state $B_{J}(5970)$. As we have discussed, $B_{J}(5970)$ is fitted to be the radially excited state with $J^{P} 1^{-}$. Table VI shows the partial decay widths for $B_{J}(5970)$ along with its spin and strange partners $B\left(2^{1} S_{0}\right), B_{s}\left(2^{3} S_{1}\right)$, and $B_{s}\left(2^{1} S_{0}\right)$. Along with the partial decay widths, the table also shows the branching ratio $\hat{\boldsymbol{\Gamma}}=$ $\frac{\Gamma}{\Gamma\left(B_{J}^{(*)} \rightarrow B^{*+} \pi^{-}\right)}$and $\hat{\boldsymbol{\Gamma}}=\frac{\Gamma}{\Gamma\left(B_{s J}^{*} \rightarrow B^{* 0} K^{+}\right)}$for the nonstrange and strange states $B\left(2^{1} S_{0}\right), B\left(2^{3} S_{1}\right), B_{s}\left(2^{3} S_{1}\right)$, and $B_{S}\left(2^{1} S_{0}\right)$, respectively.

From the experimental decay widths of $B_{J}(5970)$, we obtain the strong coupling constant $\tilde{g}_{H H}$ as

TABLE VI. Strong decay width of bottom meson $B_{J}(5970)$ with its spin and strange partners $B\left(2^{1} S_{0}\right), B_{s}\left(2^{1} S_{0}\right)$, and $B_{s}\left(2^{3} S_{1}\right)$. The ratio in the fifth column represents the $\hat{\boldsymbol{\Gamma}}=\frac{\Gamma}{\Gamma\left(B_{J}^{(*)} \rightarrow B^{*+} \pi^{-}\right)}$for the nonstrange mesons, and $\hat{\boldsymbol{\Gamma}}=\frac{\Gamma}{\Gamma\left(B_{s J}^{*} \rightarrow B^{* 0} K^{+}\right)}$for the strange mesons. The branching fraction (B.F.) gives the percentage of the partial decay width with respect to the total decay width.

| State | $n L s_{l} J^{P}$ | Decay channel | Decay width (MeV) | Ratio | B.F. \% | Experimental/theoretical value (MeV) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $B_{0}(5881)$ | $2^{1} S_{0} 0^{-}$ | $B^{*} \pi^{+}$ | $1148.08 \tilde{g}_{H H}^{2}$ | 1 | 66.48 |  |
|  |  | $B^{*} \pi^{0}$ | $577.80 \tilde{g}_{H H}^{2}$ | 0.50 | 33.27 |  |
|  |  | $B^{*} \eta$ | $1.00 \tilde{g}_{H H}^{2}$ | 0.00 | 0.05 |  |
|  |  | $B_{s}^{*} K$ |  | ... | ... |  |
|  |  | Total | $1726.89 \tilde{g}_{H H}^{2}$ |  |  | 91 [18] |
| $B_{J}(5970)$ | $2^{3} S_{1} 1^{-}$ | $B^{*} \pi^{+}$ | $1178.23 \tilde{g}_{H H}^{2}$ | 1 | 36.30 |  |
|  |  | $B^{*} \pi^{0}$ | $591.95 \tilde{g}_{H H}^{2}$ | 0.50 | 18.23 |  |
|  |  | $B^{*} \eta$ | $122.22 \tilde{g}_{H H}^{2}$ | 0.10 | 3.76 |  |
|  |  | $B_{s}^{*} K$ | $69.94 \tilde{g}_{H H}^{2}$ | 0.05 | 2.15 |  |
|  |  | $B^{0} \pi^{0}$ | $359.11 \tilde{g}_{H H}^{2}$ | 0.30 | 11.06 |  |
|  |  | $B^{+} \pi^{-}$ | $716.21 \tilde{g}_{H H}^{2}$ | 0.60 | 22.06 |  |
|  |  | $B^{0} \eta$ | $113.37 \tilde{g}_{H H}^{2}$ | 0.09 | 3.49 |  |
|  |  | $B_{s} K$ | $94.43 \tilde{g}_{H H}^{2}$ | 0.08 | 2.90 |  |
|  |  | Total | $3245.49 \tilde{g}_{H H}^{2}$ |  |  | 82.30 [11], 107 [18] |
| $B_{s 0}(5976.0)$ | $\left(2^{1} S_{0}\right) 0^{-}$ | $B^{* 0} K^{0}$ | $521.96 \tilde{g}_{H H}^{2}$ | 0.96 | 31.38 |  |
|  |  | $B^{*+} K^{-}$ | $539.41 \tilde{g}_{H H}^{2}$ | 1 | 32.43 |  |
|  |  | $B_{s}^{*} \pi^{0}$ | $593.72 \tilde{g}_{H H}^{2}$ | 1.10 | 35.70 |  |
|  |  | $B_{s}^{*} \eta$ | $7.85 \tilde{g}_{H H}^{2}$ | 0.01 | 0.47 |  |
|  |  | Total | $1662.97 \tilde{g}_{H H}^{2}$ |  |  | 75.80 [26], 106 [18] |
| $B_{s}^{*}(6007.8)$ | $2^{3} S_{1} 1^{-}$ | $B^{0} K^{0}$ | $342.71 \tilde{g}_{H H}^{2}$ | 0.97 | 15.87 |  |
|  |  | $B^{+} K^{-}$ | $350.66 \tilde{g}_{H H}^{2}$ | 1 | 16.24 |  |
|  |  | $B_{s} \pi^{0}$ | $292.67 \tilde{g}_{H H}^{2}$ | 0.83 | 13.55 |  |
|  |  | $B_{s} \eta$ | $58.29 \tilde{g}_{H H}^{2}$ | 0.16 | 2.70 |  |
|  |  | $B^{* 0} K^{0}$ | $474.96 \tilde{g}_{H H}^{2}$ | 1.35 | 22.00 |  |
|  |  | $B^{*+} K^{-}$ | $486.47 \tilde{g}_{H H}^{2}$ | 1.38 | 22.53 |  |
|  |  | $B_{s}^{*} \pi^{0}$ | $466.53 \tilde{g}_{H H}^{2}$ | 1.33 | 21.61 |  |
|  |  | $B_{s}^{*} \eta$ | $36.98 \tilde{g}_{H H}^{2}$ | 0.10 | 1.71 |  |
|  |  | Total | $2158.65 \tilde{g}_{H H}^{2}$ |  |  | 114.0 [26], 127 [18] |

$$
\begin{equation*}
\tilde{g}_{H H}=0.15 \pm 0.01 \tag{33}
\end{equation*}
$$

The error in the value of coupling comes from the statistical error in experimental mass and decay width values of these bottom states. Using HQET, this coupling constant $\tilde{g}_{H H}$ is also predicted as 0.14 [24], 0.31 [45], 0.28 [19], and 0.40 [48]. The first value is obtained from bottom state $B_{J}(5960)$, and the other three values are obtained from the charm state sector by assuming that the charm states to be in the $2 S 0^{-}$state.

From the listed decay channels mentioned in Table VI, $B^{*-} \pi^{+}$comes out as the dominant decay mode for $B_{J}(5970)$ and its spin partner $B\left(2^{1} S_{0}\right)$ with branching fractions $36.30 \%$ and $66.48 \%$, respectively.

Apart from the decay channels listed in Table VI, we also find its partial decays to $1 P\left(0^{+}\right), 1 P\left(2^{+}\right), 1 D\left(1^{-}\right)$, and $1 D\left(3^{-}\right)$states, but due to the small phase space, these decay modes are suppressed and are not considered in this work.

And for their strange partners $B_{s 0}$ and $B_{s 1}^{*}$, we observe $B^{*+} K^{-}$and $B_{s}^{*} \pi^{0}$ as the dominant decay modes for the $B_{s 1}^{*}$ and $B_{s 0}$ bottom states, respectively. Thus these decay modes are suitable for the experimental search for these missing radially excited strange bottom mesons $B_{s 1}^{*}$ and $B_{s 0}$. Using the result in Eq. (33), their total decay widths corresponding to the mass $M\left(B_{0}\right)=5881.00 \mathrm{MeV}$, $M\left(B_{s 1}^{*}\right)=6007.80 \mathrm{MeV}$, and $M\left(B_{s 0}\right)=5976.00 \mathrm{MeV}$ [23,28,42-44] are obtained as

$$
\begin{align*}
& \Gamma\left(B_{0}\right)=38.85 \mathrm{MeV}  \tag{34}\\
& \Gamma\left(B_{s 1}^{*}\right)=37.41 \mathrm{MeV}  \tag{35}\\
& \Gamma\left(B_{s 0}\right)=48.56 \mathrm{MeV} \tag{36}
\end{align*}
$$

This shows that the strange partners follow the same pattern as the nonstrange bottom states, i.e., $B_{s 1}^{*}$ state is seen to be
broader than its spin partner $B_{s 0}$. From the other available theoretical coupling values of $\tilde{g}_{H H}$, the uppermost theoretical value predicted from the charm states is 0.40 [48]. The results for the decay widths obtained using this higher value are very large, $(\simeq 254 \pm 35) \mathrm{MeV}$, from the values obtained in our result. However, if we use the coupling value $\tilde{g}_{H H}=0.14$ [24] obtained from the bottom sector, it gives decay widths of $33.84,32.59$, and 42.30 MeV for the states $B\left(2^{1} S_{0}\right), B_{s}\left(2^{1} S_{0}\right)$, and $B_{s}\left(2^{3} S_{1}\right)$, which deviates by $11 \%$ from our results. The results for dominating decay modes for all of these four states are the same as observed in Ref. [18]. If we look at the leading order terms of the coupling constants, it will remain the same for both the charm and bottom sectors. It may vary if we go for corrections up to $1 / m_{Q}$ order. So using the coupling constant values obtained from experimental charm states to theoretically predict the information for bottom states may change the actual results. Moreover, one can also extend the work by studying the decays decaying to ground state through vector mesons with $J^{P}=1^{-}$[50].

## IV. CONCLUSION

In this article, we have used the heavy quark effective theory of Ref. [19] to investigate the recently observed bottom mesons, $B_{J}(5840), B_{2}^{*}(5747), B_{J}(5840), B_{J}^{*}(5960)$, $B_{s 1}^{*}(5830)$, and $B_{s 2}^{*}(5840)$ by calculating the $B / B^{*}$-light
pseudoscalar meson decay widths. We also calculate the strong decay widths for the experimentally unobserved but theoretically predicted states $B\left(2^{1} S_{0}\right), B_{s}\left(2^{3} S_{1}\right), B_{s}\left(2^{1} S_{0}\right)$, $B\left(1^{1} D_{2}\right), B_{s}\left(1^{3} D_{1}\right)$, and $B_{s}\left(1^{1} D_{2}\right)$. In particular, we have identified the six possible spin-parity assignments for the $B_{J}(5840)$ state, observed by the LHCb in 2015 [11]. We have analyzed the total decay widths and branching ratio $\left(R_{1}\right) \frac{B \pi}{B^{*} \pi}$ for all six of the assignments in Table III and concluded that the only favorable J value for $B_{J}(5840)$ state is $1 D 1^{-}$. This ratio has very different values for $B_{J}(5840)$ belonging to these two $J^{P}$, s, so experimental measurement of such a branching ratio in the future will be very helpful in clearly identifying one of them to be the most favorable $J^{P}$, s for $B_{J}(5840)$.

We have also obtained coupling constant $g_{X H}, \tilde{g}_{H H}$, and $g_{T H}$ governing the strong decays of bottom states to the light pseudoscalar mesons. These obtained couplings allowed us to compute the strong decay widths of the above-mentioned experimentally missing bottom states. Along with this, we examine the recently observed bottom states $B_{J}(5721)$ and $B_{2}^{*}(5747)$ and their strange partners $B_{s J}(5830)$ and $B_{2 s}^{*}(5840)$ for their $J^{P}$ 's as $1 P_{3 / 2} 1^{+}$and $1 P_{3 / 2} 2^{+}$, respectively. Thus these predictions have opened a window to investigate the higher excitations of bottom mesons at LHCb, D0, and CDF.
[1] R. Aaij et al. (LHCb Collaboration), Phys. Rev. D 94, 072001 (2016).
[2] R. Aaij et al. (LHCb Collaboration), J. High Energy Phys. 09 (2013) 145.
[3] P. del Amo Sanchez et al. (BABAR Collaboration), Phys. Rev. D 82, 111101 (2010).
[4] S. Behrends et al. (CLEO Collaboration), Phys. Rev. Lett. 50, 881 (1983).
[5] K. Han et al., Phys. Rev. Lett. 55, 36 (1985).
[6] J. Lee-Franzini, U. Heintz, D. M. J. Lovelock, M. Narain, R. D. Schamberger, J. Willins, C. Yanagisawa, P. Franzini, and P. M. Tuts, Phys. Rev. Lett. 65, 2947 (1990).
[7] T. Aaltonen et al. (CDF Collaboration), Phys. Rev. Lett. 100, 082001 (2008).
[8] V. M. Abazov et al., Phys. Rev. Lett. 99, 172001 (2007).
[9] T. Aaltonen et al. (CDF Collaboration), Phys. Rev. Lett. 102, 102003 (2009).
[10] C. Patrignani et al. (Particle Data Group), Chin. Phys. C 40, 100001 (2016).
[11] R. Aaij et al. (LHCb Collaboration), J. High Energy Phys. 04 (2015) 024.
[12] T. A. Aaltonen et al. (CDF Collaboration), Phys. Rev. D 90, 012013 (2014).
[13] V. M. Abazov et al. (D0 Collaboration), Phys. Rev. Lett. 100, 082002 (2008).
[14] R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 110, 151803 (2013).
[15] S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985).
[16] D. Ebert, R. N. Faustov, and V. O. Galkin, Eur. Phys. J. C 66, 197 (2010).
[17] L. Y. Xiao and X. H. Zhong, Phys. Rev. D 90, 074029 (2014).
[18] J. Ferretti, and E. Santopinto, Phys. Rev. D 97, 114020 (2018).
[19] P. Colangelo, F. De Fazio, F. Giannuzzi, and S. Nicotri, Phys. Rev. D 86, 054024 (2012).
[20] Z. G. Luo, X. L. Chen, and X. Liu, Phys. Rev. D 79, 074020 (2009).
[21] P. Gelhausen, A. Khodjamirian, A. A. Pivovarov, and D. Rosenthal, Eur. Phys. J. C 74, 2979 (2014).
[22] X.-H. Zhong and Q. Zhao, Phys. Rev. D 78, 014029 (2008).
[23] Q.-F. Lu, T.-T. Pan, Y.-Y. Wang, E. Wang, and D.-M. Li, Phys. Rev. D 94, 074012 (2016).
[24] Z.-G. Wang, Eur. Phys. J. Plus 129, 186 (2014).
[25] Y. Sun, Q.-T. Song, D.-Y. Chen, X. Liu, and S.-L. Zhu, Phys. Rev. D 89, 054026 (2014).
[26] S. Godfrey, K. Moats, and E. S. Swanson, Phys. Rev. D 94, 054025 (2016).
[27] H. Xu, X. Liu, and T. Matsuki, Phys. Rev. D 89, 097502 (2014).
[28] J.-B. Liu and C.-D. Lu, Eur. Phys. J. C 77, 312 (2017).
[29] P. Gupta and A. Upadhyay, Eur. Phys. J. A 54, 160 (2018).
[30] H.-X. Chen, W. Chen, X. Liu, Y.-R. Liu, and S.-L. Zhu, Rep. Prog. Phys. 80, 076201 (2017).
[31] M. B. Wise, Phys. Rev. D 45, R2188 (1992).
[32] T.-M. Yan, H.-Y. Cheng, C.-Y. Cheung, G.-L. Lin, Y. C. Lin, and H.-L. Yu, Phys. Rev. D 46, 1148 (1992).
[33] G. Burdman and J. F. Donoghue, Phys. Lett. B 280, 287 (1992); 68, 2887(E) (1992).
[34] R.Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Lett. B 292, 371 (1992).
[35] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, F. Feruglio, R. Gatto, and G. Nardulli, Phys. Lett. B 294, 106 (1992).
[36] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, F. Feruglio, R. Gatto, and G. Nardulli, Phys. Lett. B 299, 139 (1993).
[37] J. Schechter and A. Subbaraman, Phys. Rev. D 48, 332 (1993).
[38] U. Kilian, J. C. Korner, and D. Pirjol, Phys. Lett. B 288, 360 (1992).
[39] D. Ebert, T. Feldmann, R. Friedrich, and H. Reinhardt, Nucl. Phys. B434, 619 (1995).
[40] A. F. Falk, and M. Luke, Phys. Lett. B 292, 119 (1992).
[41] Iain W. Stewart, Nucl. Phys. B529, 62 (1998).
[42] J. Zeng, J. W. Van Orden, and W. Roberts, Phys. Rev. D 52, 5229 (1995).
[43] M. Di Pierro and E. Eichten, Phys. Rev. D 64, 114004 (2001).
[44] T. A. Lahde, C. J. Nyfalt, and D. O. Riska, Nucl. Phys. A674, 141 (2000).
[45] P. Gupta and A. Upadhyay, Phys. Rev. D 97, 014015 (2018).
[46] R. Aaij et al. (LHCb Collaboration), Phys. Rev. D 91, 092002 (2015); 93, 119901(E) (2016).
[47] P. Colangelo, F. De Fazio, and S. Nicotri, Phys. Lett. B 642, 48 (2006).
[48] M. Batra and A. Upadhayay, Eur. Phys. J. C 75, 319 (2015).
[49] Z. G. Wang, Phys. Rev. D 88, 114003 (2013).
[50] S. Campanella, P. Colangelo, and F. De Fazio, Phys. Rev. D 98, 114028 (2018).


[^0]:    Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP ${ }^{3}$.

