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Generating enhanced primordial GWs during inflation with intermittent violation of NEC and diminishment of GW propagating speed

Yong Cai^a and Yun-Song Piao^{b,c,d,e}

^a*School of Physics and Microelectronics, Zhengzhou University, Zhengzhou, Henan 450001, China*

^b*School of Physics, University of Chinese Academy of Sciences, Beijing 100049, China*

^c*School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China*

^d*International Center for Theoretical Physics Asia-Pacific, Beijing/Hangzhou, China*

^e*Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100190, China*

E-mail: yongcai_phy@outlook.com, yspiao@ucas.ac.cn

ABSTRACT: We investigate both the null energy condition (NEC) violating scenario and the c_T -diminishing scenario for generating enhanced power spectrum of primordial gravitational waves (GWs) during inflation, where c_T is the propagating speed of primordial GWs. Both of these two scenarios can be realized stably with theories beyond Horndeski, hence can be uniformly implemented within the framework of the effective field theory. We calculate the power spectrum of primordial GWs by assuming that the inflationary Universe undergoes three phases, where the violation of NEC or the diminishment of c_T occurs in the intermediate phase. A template of the spectrum is given for the NEC-violating scenario. We also discuss the underlying relation and discrepancy between these two scenarios with a disformal transformation.

KEYWORDS: Cosmology of Theories BSM, Classical Theories of Gravity, Spacetime Singularities

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1 Introduction

The direct detections of gravitational waves (GWs) from binary black holes [1] and a binary neutron star inspiral [2] by the LIGO and Virgo collaborations have opened a new window for exploring our Universe and the theory of gravity. Recently, evidence for a common-spectrum process, which might be interpreted as a stochastic GW background (see e.g. [3–13] for related studies, see also [14–16]), was reported by the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) collaboration in their 12.5-year dataset [17] and also reported later by the Parkes Pulsar Timing Array (PPTA) collaboration [18]. A wide multi-frequency range of observations aimed at searching for GW signals will lead us to the era of GW astronomy.

The primordial GWs [19, 20] predicted by inflation have yet to be confirmed by future observations. In the slow-roll inflation scenario, the power spectrum of primordial GWs is usually assumed as a power-law. As a result, the primordial GWs can hardly be detected by the pulsar timing arrays (PTA) or laser interferometers in the near future, since the tensor-to-scalar ratio r is severely constrained by the data of cosmic microwave background (CMB) B-mode polarization [21]. However, the primordial GW background spans a broad frequency-band ($10^{-18} - 10^{10}$ Hz), the structure of the power spectrum and the underlying physics might be far richer than expected [22–24].

Primordial null energy condition (NEC) violation (i.e., $T_{\mu\nu}k^\mu k^\nu < 0$, where $T_{\mu\nu}$ is the energy-momentum tensor and k^μ is any null vector) could occur during the inflationary

era [25–29] (see also [30–47] for the Genesis scenario) and generate GW background with rich features in the power spectrum, see e.g. [22] for a Great Wall-like spectrum. The GW power spectrum can be enhanced greatly at frequencies higher than the CMB window since the NEC violation indicates $\dot{H} > 0$ for a homogeneous and isotropic cosmological background. Fully stable NEC violation can be realized in scalar-tensor theories beyond Horndeski, as discovered in the exploration of the effective field theory (EFT) of nonsingular cosmology [48–51].

Additionally, the propagating speed of GWs (i.e., c_T) can be time-dependent in some gravity theories beyond general relativity. Although the joint detection of GWs and the gamma rays from the binary neutron star merger GW170817 has strongly supported that the propagating speed of GWs is $c_T = 1$ at present [52–56], with deviations smaller than $\mathcal{O}(10^{-15})$, it still cannot place stringent bounds on c_T in the very early Universe, see e.g., [57–60]. The nontrivial variation of c_T is able to imprint interesting features in the power spectrum of GWs [61, 62], see also [63–65]. An enhanced (or blue) power spectrum of primordial GWs may also be attributed to the diminishment of c_T [66–72], which can be implemented with scalar-tensor theories beyond Horndeski. Intriguingly, it is found that a scenario in which c_T gradually diminishes during inflation is actually a disformal dual to the superinflation [67].

In this paper, we investigate both the NEC-violating scenario and the c_T -diminishing scenario for generating enhanced power spectrum of primordial GWs during inflation within the uniform framework of EFT. In section 2, we briefly review the EFT method. In section 3, we explicitly calculate the power spectrum of primordial GWs for a scenario in which the NEC-preserving slow-roll inflation is followed by an NEC-violating expanding phase and a later NEC-preserving slow-roll inflation with a larger Hubble parameter, while $c_T = 1$ throughout. Templates of such spectra are provided. We also calculate the power spectrum of primordial GWs in section 4 for a scenario in which c_T decreases with time during an intermediate phase of the slow-roll inflation, while the NEC is preserving throughout. The underlying relation and discrepancy between these two scenarios are discussed with a disformal transformation in section 5. The speed of light is set as $c = 1$ throughout this paper.

2 The effective field theory method

It is convenient to use the 3 + 1 decomposed metric

$$ds^2 = -N^2 dt^2 + h_{ij} (dx^i + N^i dt) (dx^j + N^j dt), \quad (2.1)$$

where N is the lapse function, N^i is the shift vector, h_{ij} is the 3-dimensional metric. In the unitary gauge, the EFT of cosmological perturbations up to quadratic order can be written as

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right. \\ \left. + \frac{M_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta K \delta g^{00} - m_4^2(t) (\delta K^2 - \delta K_{\mu\nu} \delta K^{\mu\nu}) \right. \\ \left. + \frac{\tilde{m}_4^2(t)}{2} R^{(3)} \delta g^{00} \right] + S_m[g_{\mu\nu}, \psi_m], \quad (2.2)$$

where we have disregarded those higher-order spatial derivatives; $\delta g^{00} = g^{00} + 1$, $\delta K_{\mu\nu} = K_{\mu\nu} - h_{\mu\nu}H$ with H being the Hubble parameter, $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$ is the induced metric, n^μ is the unit normal vector of the constant time hypersurfaces, $K_{\mu\nu}$ is the extrinsic curvature, $R^{(3)}$ is the induced 3-dimensional Ricci scalar, S_m represents the action of matter sector, which is minimally coupled to the metric $g_{\mu\nu}$.

These functions f , Λ , c , M_2^4 , m_3^3 , m_4^2 and \tilde{m}_4^2 could be time-dependent in general so that they are able to specify different theories of gravity. Particularly, the Horndeski theory [73–75] corresponds to $m_4^2 = \tilde{m}_4^2$, the “beyond Horndeski” theory [76] requires $m_4^2 \neq \tilde{m}_4^2$, while the degenerate higher-order scalar-tensor (DHOST) theories [77] require the appearances of new operators, such as $\delta K \delta g^{00}$, $(\delta g^{00})^2$ and $(\partial_i \delta g^{00})^2$ in the action (2.2), see e.g., [78, 79].

The background evolution of the Universe is only determined by the first line of (2.2). Particularly, for $f(t) \equiv 1$, the Friedmann equations can be given as $c(t) = -M_p^2 \dot{H}$ and $\Lambda(t) = M_p^2 (\dot{H} + 3H^2)$. With a variety of designs of those coefficient functions, the EFT action (2.2) is versatile in applications to a variety of cosmological background, including inflation [80], dark energy [81–83] and nonsingular cosmology [48–50]. Furthermore, the EFT method has great advantages in the study of cosmological perturbations.

The derivation of quadratic actions for scalar and tensor perturbations of the action (2.2) can be found in [48]. In this paper, we will focus on the tensor perturbations γ_{ij} (i.e., primordial GWs), which is traceless and divergence-free, i.e., $\gamma_{ii} = 0 = \partial_i \gamma_{ij}$, where we have set

$$h_{ij} = a^2 (e^\gamma)_{ij}. \tag{2.3}$$

Generally, the quadratic actions of γ_{ij} can be formulated as

$$S_\gamma^{(2)} = \frac{M_p^2}{8} \int d^4x a^3 Q_T \left[\dot{\gamma}_{ij}^2 - c_T^2 \frac{(\partial_k \gamma_{ij})^2}{a^2} \right], \tag{2.4}$$

where $\dot{} = d/dt$, c_T is the propagating speed of primordial GWs. In order to avoid the ghost and gradient instabilities, $Q_T > 0$ and $c_T^2 > 0$ are required, respectively. From (2.2), we have

$$Q_T = f + \frac{2m_4^2}{M_p^2}, \quad c_T^2 = f/Q_T, \tag{2.5}$$

where f and m_4^2 are time-dependent coefficients defined in (2.2). When $f = 1$ and $m_4^2 = 0$, we have $Q_T = 1$ and $c_T = 1$, which are same as those in general relativity.

In the momentum space, we have

$$\gamma_{ij}(\tau, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} e^{-i\mathbf{k}\cdot\mathbf{x}} \sum_{\lambda=+, \times} \hat{\gamma}_\lambda(\tau, \mathbf{k}) \epsilon_{ij}^{(\lambda)}(\mathbf{k}), \tag{2.6}$$

where $\hat{\gamma}_\lambda(\tau, \mathbf{k}) = \gamma_\lambda(\tau, k) a_\lambda(\mathbf{k}) + \gamma_\lambda^*(\tau, -k) a_\lambda^\dagger(-\mathbf{k})$, the polarizations $\epsilon_{ij}^{(\lambda)}(\mathbf{k})$ satisfy $k_j \epsilon_{ij}^{(\lambda)}(\mathbf{k}) = 0$, $\epsilon_{ii}^{(\lambda)}(\mathbf{k}) = 0$, $\epsilon_{ij}^{(\lambda)}(\mathbf{k}) \epsilon_{ij}^{*(\lambda')}(\mathbf{k}) = \delta_{\lambda\lambda'}$ and $\epsilon_{ij}^{*(\lambda)}(\mathbf{k}) = \epsilon_{ij}^{(\lambda)}(-\mathbf{k})$; $a_\lambda(\mathbf{k})$ and $a_\lambda^\dagger(\mathbf{k}')$ satisfy $[a_\lambda(\mathbf{k}), a_{\lambda'}^\dagger(\mathbf{k}')] = \delta_{\lambda\lambda'} \delta^{(3)}(\mathbf{k} - \mathbf{k}')$. The power spectrum of primordial GWs is

$$P_T = \frac{k^3}{2\pi^2} \sum_{\lambda=+, \times} |\gamma_\lambda|^2, \tag{2.7}$$

which is evaluated after the perturbation modes exited their horizons, i.e., $aH/(c_T k) \gg 1$.

The energy density spectrum of primordial GWs (see e.g. [84], see also [85–88]) can be given by

$$\Omega_{\text{GW}}(\tau_0) = \frac{k^2}{12a_0^2 H_0^2} P_T(k) \left[\frac{3\Omega_m j_1(k\tau_0)}{k\tau_0} \sqrt{1.0 + 1.36 \frac{k}{k_{\text{eq}}} + 2.50 \left(\frac{k}{k_{\text{eq}}} \right)^2} \right]^2, \quad (2.8)$$

where k_{eq} is the comoving wavenumber of the perturbation mode that entered the horizon at the matter-radiation equality, $\Omega_m = \rho_m/\rho_c$, $\rho_c = 3M_p^2 H_0^2$ is the critical energy density. The observation of a stochastic GW background would provide us with significant information or constraints on the primordial GWs as well as the evolution history of the very early Universe.

3 Enhanced power spectrum of primordial GWs from intermittent NEC violations during inflation

Primordial NEC violation would imprint a blue-tilted power spectrum of primordial GWs. A series of intermittent NEC violations during inflation would generate a spectrum of primordial GWs with far richer structures, such as a Great Wall-like spectrum [22]. An enhanced power spectrum of primordial GWs at the frequency $\sim 1/\text{yr}$ induced by NEC violation might be compatible with the recent result of NANOGrav, provided the reported common-spectrum process could be interpreted as GWs.

In this section, we analytically calculate the power spectrum of primordial GWs in a scenario proposed by [22] and provide a template of such an enhanced GW spectrum. Follow [22], we assume that after an NEC-preserving slow-roll inflation with Hubble parameter $H \simeq H_{\text{inf}1}$, the Universe experienced an NEC-violating expansion, and then enters a subsequent NEC-preserving slow-roll inflation again but with a higher Hubble scale H ($= H_{\text{inf}2} \gg H_{\text{inf}1}$).

3.1 Set up

Instead of explicitly implementing this scenario in scalar-tensor theories, we adopt the EFT action

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \Lambda(t) - c(t) g^{00} + \frac{M_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta K \delta g^{00} + \frac{\tilde{m}_4^2(t)}{2} R^{(3)} \delta g^{00} \right], \quad (3.1)$$

where we have disregarded the action of the matter sector (i.e., S_m), which should make a negligible contribution to the evolution of the very early Universe in our scenario. The action (3.1) corresponds to setting $f(t) = 1$ and $m_4^2(t) = 0$ in action (2.2). As a result, we have

$$Q_T = 1, \quad c_T = 1 \quad (3.2)$$

in eq. (2.4). Although the covariant correspondence of action (3.1) (see [51, 89]) belongs to the “beyond Horndeski” theory, the propagation of GWs is exactly same as that in general relativity, at least at quadratic order.

The functions $c(t)$ and $\Lambda(t)$ can be determined in a reverse manner by the background evolution with relations $c(t) = -M_p^2 \dot{H}$ and $\Lambda(t) = M_p^2 (\dot{H} + 3H^2)$, while $M_2^4(t)$, $m_3^3(t)$ and $\tilde{m}_4^2(t)$ can be determined or constrained by the requirement that the scalar perturbations should be consistent with observations. Additionally, a nonzero $\tilde{m}_4^2(t)$, i.e., a contribution from the operator $R^{(3)}\delta g^{00}$, is crucial for avoiding instabilities of scalar perturbations induced by a violation of the NEC [48–50]. However, since we will focus on the primordial GWs in the following calculations, we will not be interested in the explicit formulations of these coefficient functions in the following.

The evolution history of the Universe that we will consider can be divided into three phases, which will be represented by phase 1, 2 and 3. During phase j ($j = 1, 2$ or 3), the scale factor could be parameterized with the conformal time τ as

$$a_j(\tau) = a_j(\tau_j) \left(\frac{\tau - \tau_{R,j}}{\tau_j - \tau_{R,j}} \right)^{\frac{1}{\epsilon_j - 1}} \quad (3.3)$$

for $\tau < \tau_j$, where τ_j is the conformal time at the end of phase j , $\tau_{R,j} = \tau_j - (\epsilon_j - 1)^{-1} \mathcal{H}^{-1}(\tau_j)$ is the conformal reference time, $\mathcal{H}(\tau) \equiv a^{-1} da/d\tau$, $\epsilon_j = -\dot{H}/H^2$ is constant during phase j and $dt = a d\tau$.

The continuities of $a(\tau)$ and $da/d\tau$ are required at τ_1 and τ_2 . Since phase 1 and 3 are assumed as slow-roll inflation while phase 2 is NEC-violating expansion, we will set $\epsilon_1 \approx \epsilon_3 \approx 0$ and $\epsilon_2 < 0$. A specific design of such a model can be found in [22]. The detailed variation of ϵ around the beginning or the end of the NEC-violating phase could be model-dependent. For our purpose, the simplification will not make a qualitative difference.

3.2 Power spectrum

With eqs. (2.4) and (3.2), the equation of motion for $\gamma_\lambda(\tau, k)$ can be given as

$$\frac{d^2 u_k}{d\tau^2} + \left(k^2 - \frac{d^2 a/d\tau^2}{a} \right) u_k = 0, \quad (3.4)$$

where we have defined $u_k = \gamma_\lambda(\tau, k) a M_p / 2$. The parameterization in eq. (3.3) gives

$$\frac{d^2 a_j/d\tau^2}{a_j} = \frac{\nu_j^2 - 1/4}{(\tau - \tau_{R,j})^2} \quad (3.5)$$

with $\nu_j = \frac{1}{2} + \frac{1}{1 - \epsilon_j}$. The solutions to eq. (3.4) during the three phases can be given as

$$\begin{aligned} u_{k,1}(\tau) &= \frac{\sqrt{\pi(\tau_{R,1} - \tau)}}{2} \left\{ \alpha_1 H_{\nu_1}^{(1)}[k(\tau_{R,1} - \tau)] + \beta_1 H_{\nu_1}^{(2)}[k(\tau_{R,1} - \tau)] \right\}, \quad (\tau < \tau_1), \quad (3.6) \\ u_{k,2}(\tau) &= \frac{\sqrt{\pi(\tau_{R,2} - \tau)}}{2} \left\{ \alpha_2 H_{\nu_2}^{(1)}[k(\tau_{R,2} - \tau)] + \beta_2 H_{\nu_2}^{(2)}[k(\tau_{R,2} - \tau)] \right\}, \quad (\tau_1 \leq \tau \leq \tau_2), \\ u_{k,3}(\tau) &= \frac{\sqrt{\pi(\tau_{R,3} - \tau)}}{2} \left\{ \alpha_3 H_{\nu_3}^{(1)}[k(\tau_{R,3} - \tau)] + \beta_3 H_{\nu_3}^{(2)}[k(\tau_{R,3} - \tau)] \right\}, \quad (\tau_2 < \tau \leq \tau_3), \end{aligned}$$

where α_j and β_j are k -dependent coefficients for $j = 1, 2, 3$; $H_{\nu_j}^{(1)}$ and $H_{\nu_j}^{(2)}$ are the first and second kind Hankel functions of the ν_j th order, respectively.

We set the initial state as the Bunch-Davies vacuum (see also [90–95]), i.e., $u_k = \frac{1}{\sqrt{2k}}e^{-ik\tau}$, which indicates $|\alpha_1| = 1$ and $|\beta_1| = 0$. Using the matching conditions $u_{k,j}(\tau_j) = u_{k,j+1}(\tau_j)$ and $du_{k,j}/d\tau|_{\tau=\tau_j} = du_{k,j+1}/d\tau|_{\tau=\tau_j}$, we can obtain α_2 , β_2 , α_3 and β_3 . The information of the evolution of the Universe has been encoded in α_3 and β_3 . For simplicity, we set $\epsilon_1 = \epsilon_3 = 0$, i.e., $\nu_1 = \nu_3 = 3/2$ in the following. Additionally, we have $1/2 < \nu_2 < 3/2$, since $\epsilon_2 < 0$.

The resulting power spectrum of primordial GWs is

$$P_T = \frac{4k^3}{\pi^2 M_p^2} \cdot \frac{|u_{k,3}|^2}{a^2} = P_{T,inf2} |\alpha_3 - \beta_3|^2, \quad (3.7)$$

where $P_{T,inf2} = \frac{2H_{inf2}^2}{M_p^2 \pi^2}$, $H_{inf2} = H(\tau_3)$ is the Hubble parameter at the second inflation phase (i.e., phase 3). With the matching method, we obtain that

$$\begin{aligned} \alpha_3 - \beta_3 = & \frac{\pi(1-2\nu_2)}{8\sqrt{x_1 y_1}} \left\{ H_{\nu_2}^{(1)}(y_2) \left[\left(i - \frac{1}{x_1} \right) H_{\nu_2-1}^{(2)}(x_2) + H_{\nu_2}^{(2)}(x_2) \right] \sin y_1 \right. \\ & + H_{\nu_2-1}^{(1)}(y_2) \left[\left(i - \frac{1}{x_1} \right) H_{\nu_2-1}^{(2)}(x_2) + H_{\nu_2}^{(2)}(x_2) \right] \left[\cos y_1 - \frac{1}{y_1} \sin y_1 \right] \\ & \left. - \left[\left(i - \frac{1}{x_1} \right) H_{\nu_2-1}^{(1)}(x_2) + H_{\nu_2}^{(1)}(x_2) \right] \left[H_{\nu_2}^{(2)}(y_2) \sin y_1 + H_{\nu_2-1}^{(2)}(y_2) \left(\cos y_1 - \frac{1}{y_1} \sin y_1 \right) \right] \right\}, \end{aligned} \quad (3.8)$$

where $x_1 = k/\bar{\mathcal{H}}_1$, $x_2 = (\nu_2 - 1/2)x_1$, $y_1 = k/\bar{\mathcal{H}}_2$, $y_2 = (\nu_2 - 1/2)y_1$, $\bar{\mathcal{H}}_1 = \mathcal{H}(\tau_1)$ and $\bar{\mathcal{H}}_2 = \mathcal{H}(\tau_2)$; a phase factor has been neglected. It should be emphasized again that a and $da/d\tau$ are required to be continuous at τ_1 and τ_2 , which is reasonable. In order to obtain the asymptotic behavior of $P_T/P_{T,inf2}$ with respect to the comoving wavenumber k , let us assume that the NEC-violating phase lasts a sufficient long time such that $\bar{\mathcal{H}}_1 \ll \bar{\mathcal{H}}_2$.

For perturbations that exit the horizon in the first inflationary phase, we have $k \ll \bar{\mathcal{H}}_1 \ll \bar{\mathcal{H}}_2$, i.e., $x_1, x_2, y_1, y_2 \ll 1$, provided $1/2 < \nu_2 < 3/2$. By straightforward series expansion, we find

$$\frac{P_T}{P_{T,inf2}} \Big|_{k \ll \bar{\mathcal{H}}_1} = \left(\frac{\bar{\mathcal{H}}_1}{\bar{\mathcal{H}}_2} \right)^{3-2\nu_2} + \mathcal{O} \left(\frac{k}{\bar{\mathcal{H}}_1} \right)^2. \quad (3.9)$$

As can be inferred, the power spectrum is (nearly) scale-invariant in the limit of $k \ll \bar{\mathcal{H}}_1$. Note that, according to eq. (3.3), the leading order term in the right hand side of eq. (3.9) can also be written as $(\bar{\mathcal{H}}_1/\bar{\mathcal{H}}_2)^{3-2\nu_2} = (H_1/H_2)^2$, where H_1 and H_2 are evaluated at τ_1 and τ_2 , respectively.

A similar calculation gives $\frac{P_T}{P_{T,inf2}} \Big|_{k \ll \bar{\mathcal{H}}_1} \simeq \frac{4}{9} \left(\frac{\bar{\mathcal{H}}_1}{\bar{\mathcal{H}}_2} \right)^{3-2\nu_2}$ for $\nu_2 = 1/2$, where there is a difference of a factor 4/9 at the leading order compared with (3.9). However, it should be pointed out that when $\nu_2 = 1/2$, i.e., $\epsilon_2 \rightarrow -\infty$, the parameterization given by eq. (3.3) cannot guarantee the continuity of $da/d\tau$ at τ_1 or τ_2 any more. The above derivation will no longer be applicable. Therefore, we will disregard the case of $\nu_2 = 1/2$ in this paper.

For perturbations that exit the horizon in the NEC-violating phase, we will focus on the perturbation modes which satisfy $\bar{\mathcal{H}}_1 \ll k \ll \bar{\mathcal{H}}_2$, i.e., $x_1, x_2 \gg 1$ and $y_1, y_2 \ll 1$. We obtain

$$\frac{P_T}{P_{T,inf2}} \Big|_{\bar{\mathcal{H}}_1 \ll k \ll \bar{\mathcal{H}}_2} = \frac{\Gamma^2(\nu_2)}{\pi} \left(\frac{2\nu_2 - 1}{4} \right)^{1-2\nu_2} \left(\frac{k}{\bar{\mathcal{H}}_2} \right)^{3-2\nu_2} + \mathcal{O} \left[\frac{\bar{\mathcal{H}}_1^2}{\bar{\mathcal{H}}_2^2} \left(\frac{k}{\bar{\mathcal{H}}_2} \right)^{1-2\nu_2} \right] \quad (3.10)$$

for $1/2 < \nu_2 < 3/2$. Therefore, the GWs generated during the NEC-violating phase have a blue spectrum, where the spectrum index is $n_T = 3 - 2\nu_2$.

As for the perturbation modes that exit the horizon in phase 3, namely, the second inflationary phase, we have $k \gg \bar{\mathcal{H}}_2$, which indicates $x_1, x_2 \gg 1$ and $y_1, y_2 \gg 1$. The result is trivially given as

$$\frac{P_T}{P_{T,inf2}} \Big|_{k \gg \bar{\mathcal{H}}_2} = 1 + \mathcal{O} \left(\frac{k}{\bar{\mathcal{H}}_2} \right)^{-2}, \quad (3.11)$$

where there is some oscillating corrections included in the subleading order terms.

Apparently, when $\nu_2 \rightarrow 3/2$, all of eqs. (3.9) to (3.11) reduce to $P_T/P_{T,inf2} \rightarrow 1$, which is just the result of a single inflationary phase. In a realistic inflation model which is constructed with a slowly rolling scalar field, the slow roll parameter ϵ should slightly deviate from 0. The resulting power spectrum would also deviate from scale-invariance. We assume that such a deviation could be absorbed into $P_{T,inf2}$, for simplicity.

3.3 Templates

With the asymptotic behaviors of $P_T/P_{T,inf2}$ given by eqs. (3.9) to (3.11), we can provide a template of the GW power spectrum in our scenario as

$$\frac{P_T(k)}{P_{T,inf2}} = \frac{\left(\frac{\bar{\mathcal{H}}_1}{\bar{\mathcal{H}}_2} \right)^{3-2\nu_2} + \frac{\Gamma^2(\nu_2)}{\pi} \left(\frac{2\nu_2-1}{4} \right)^{1-2\nu_2} \left(\frac{k}{\bar{\mathcal{H}}_2} \right)^{3-2\nu_2}}{1 + \frac{\Gamma^2(\nu_2)}{\pi} \left(\frac{2\nu_2-1}{4} \right)^{1-2\nu_2} \left(\frac{k}{\bar{\mathcal{H}}_2} \right)^{3-2\nu_2}}, \quad (3.12)$$

where there are only three model-dependent parameters, i.e., $\nu_2 \equiv \frac{1}{2} + \frac{1}{1-\epsilon_2}$, $\bar{\mathcal{H}}_1$ and $\bar{\mathcal{H}}_2$. Here, $\bar{\mathcal{H}}_1$ and $\bar{\mathcal{H}}_2$ can also be replaced by H_1 and H_2 , since $(\bar{\mathcal{H}}_1/\bar{\mathcal{H}}_2)^{3-2\nu_2} = (H_1/H_2)^2$. The condition $\bar{\mathcal{H}}_1 \ll \bar{\mathcal{H}}_2$ or $H_1 \ll H_2$ is required for the template to be applicable. Obviously, in the limit $k \ll \bar{\mathcal{H}}_1$, $\bar{\mathcal{H}}_1 \ll k \ll \bar{\mathcal{H}}_2$ and $k \gg \bar{\mathcal{H}}_2$, eq. (3.12) reduces to eq. (3.9), (3.10) and (3.11) up to the leading order, respectively. A comparison between the template (3.12) and the numerical result of $P_T/P_{T,inf2}$ is displayed in figure 1.

The parameterization of the template (3.12) is quite simple and straightforward. However, a shortcoming of (3.12) is the loss of the oscillating features in $P_T/P_{T,inf2}$ around $k \sim \bar{\mathcal{H}}_1$ and $k \sim \bar{\mathcal{H}}_2$. These oscillations could be model-dependent to some extent. Additionally, since $\bar{\mathcal{H}}_1 \ll \bar{\mathcal{H}}_2$ in our model, the oscillations around $k \sim \bar{\mathcal{H}}_1$ are suppressed greatly compared with those around $k \sim \bar{\mathcal{H}}_2$, as can be seen from eq. (3.8), see also figure 1. Therefore, only the oscillations around $k \sim \bar{\mathcal{H}}_2$ need to be included in the template. For this purpose, we can use the condition $k \gg \bar{\mathcal{H}}_1$ without considering the relation between k

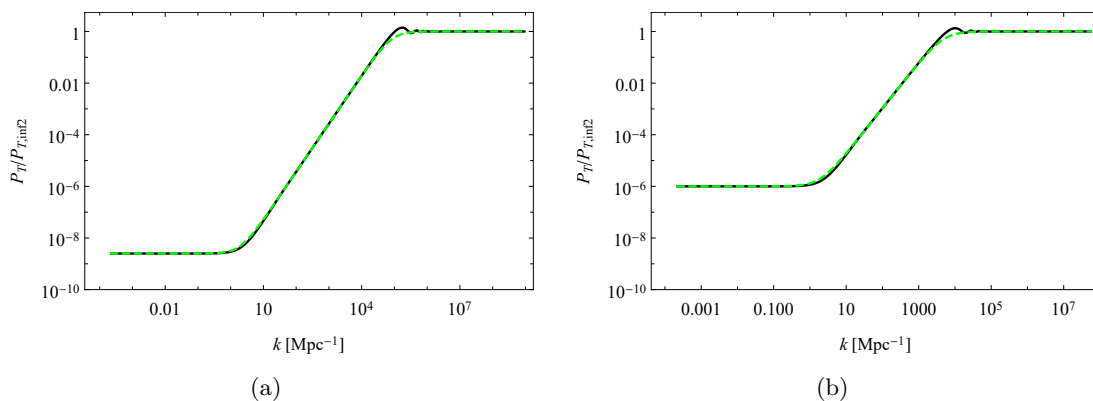


Figure 1. Comparisons between the template (3.12) (green dashed curve) and the numerical result of $P_T/P_{T,inf2}$ (black solid curve). We have set $\bar{\mathcal{H}}_1 = 10^4 a_0 H_0$, where $a_0 = 1$ and $H_0 = 67 \text{ km/s/Mpc}$. Left: $\epsilon_2 = -13.5$, $\bar{\mathcal{H}}_2 = 4.165 \times 10^4 \bar{\mathcal{H}}_1$. Right: $\epsilon_2 = -7.5$, $\bar{\mathcal{H}}_2 = 2.512 \times 10^3 \bar{\mathcal{H}}_1$.

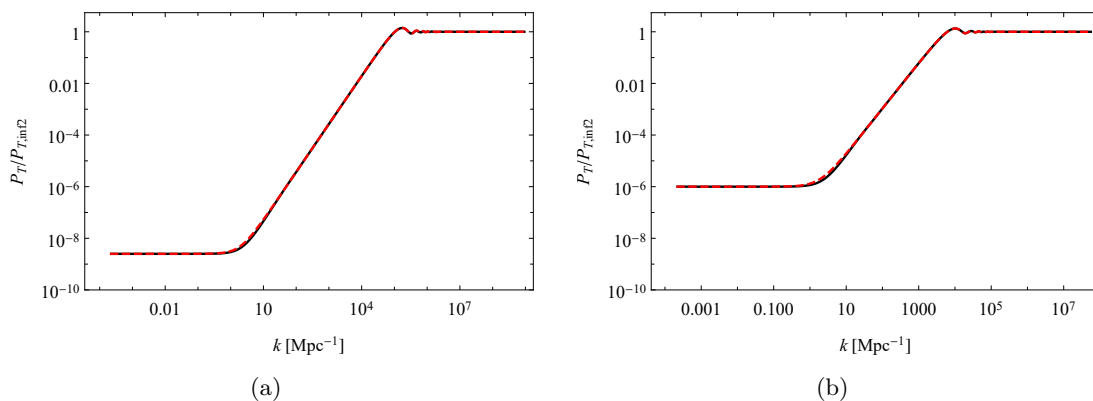


Figure 2. Comparisons between the template (3.13) (green dashed curve) and the numerical result of $P_T/P_{T,inf2}$ (black solid curve). We have set $\bar{\mathcal{H}}_1 = 10^4 a_0 H_0$, where $a_0 = 1$ and $H_0 = 67 \text{ km/s/Mpc}$. Left: $\epsilon_2 = -13.5$, $\bar{\mathcal{H}}_2 = 4.165 \times 10^4 \bar{\mathcal{H}}_1$. Right: $\epsilon_2 = -7.5$, $\bar{\mathcal{H}}_2 = 2.512 \times 10^3 \bar{\mathcal{H}}_1$.

and $\bar{\mathcal{H}}_2$. The template can be given as

$$\begin{aligned} \frac{P_T(k)}{P_{T,inf2}} = & \left(\frac{\bar{\mathcal{H}}_1}{\bar{\mathcal{H}}_2} \right)^{3-2\nu_2} + \frac{\pi(2\nu_2 - 1)}{4} \frac{k}{\bar{\mathcal{H}}_2} \text{Abs} \left\{ H_{\nu_2}^{(1)} \left[\left(\nu_2 - \frac{1}{2} \right) \frac{k}{\bar{\mathcal{H}}_2} \right] \sin \frac{k}{\bar{\mathcal{H}}_2} \right. \\ & \left. + H_{\nu_2-1}^{(1)} \left[\left(\nu_2 - \frac{1}{2} \right) \frac{k}{\bar{\mathcal{H}}_2} \right] \left[\cos \frac{k}{\bar{\mathcal{H}}_2} - \frac{\bar{\mathcal{H}}_2}{k} \sin \frac{k}{\bar{\mathcal{H}}_2} \right] \right\}^2, \end{aligned} \quad (3.13)$$

where there are still three model-dependent parameters, i.e., ν_2 , $\bar{\mathcal{H}}_1$ and $\bar{\mathcal{H}}_2$. It is obvious that eq. (3.13) reduces to (3.9) in the limit $k \ll \bar{\mathcal{H}}_1$. For $k \gtrsim \bar{\mathcal{H}}_1$, eq. (3.13) is equivalent to (3.8) at the leading order, so that the oscillating features around $k \ll \bar{\mathcal{H}}_2$ is preserved, see figure 2.

4 Enhanced power spectrum of primordial GWs from the diminishment of propagating speed of inflationary GWs

With a disformal transformation of the metric, a time-dependent c_T can be reset as unity [96]. However, the background evolution may no longer be inflationary. For example, it has been found that slow-roll inflation in which c_T decreases rapidly with time is a disformal dual to a superinflation ($\epsilon < 0$) in which $c_T \equiv 1$ [66]. The power spectrum, which is observable in principle, has to be invariant under a nonsingular disformal transformation. Therefore, a time-dependent c_T during inflation is able to imprint interesting features in the power spectra of primordial GWs [61, 62, 66, 67].¹ In this section, we calculate the enhanced power spectra of primordial GWs for a specific time-dependent c_T while assuming the background evolution as the slow-roll inflation.

4.1 Set up

Using the EFT method, we can write the action as

$$S = \int d\hat{t} d^3 \hat{x} \sqrt{-\hat{g}} \left[\frac{M_p^2}{2} \hat{R} - \hat{\Lambda}(t) - \hat{c}(t) \hat{g}^{00} - \hat{m}_4^2(t) (\delta \hat{K}^2 - \delta \hat{K}_{\mu\nu} \delta \hat{K}^{\mu\nu}) \right], \quad (4.1)$$

where ‘ $\hat{}$ ’s are used to mark the frame in which the propagating speed of GWs is time-dependent. We assume that the background evolution is the slow-roll inflation throughout, which is determined by the first line of action (4.1). Again, we have neglected S_m . However, it should be remembered that the matter sector is crucial for identifying the physical background evolution as measured relative to physical rulers, see e.g., [97]. Additionally, operators $(\delta \hat{g}^{00})^2$, $\delta \hat{K} \delta \hat{g}^{00}$ and $\hat{R}^{(3)} \delta \hat{g}^{00}$ can also appear in the action (4.1), since they will not affect the tensor perturbations at quadratic order, as can be seen from eq. (2.5). For simplicity, we will focus on the tensor perturbations and consider only the contribution of a nonzero $\hat{m}_4^2(t)$, which contributes nothing to the scalar perturbation at quadratic order.

According to eqs. (2.2) to (2.5), we can write the quadratic action of γ_{ij} as

$$S_\gamma^{(2)} = \frac{M_p^2}{8} \int d\hat{\tau} d^3 x \hat{a}^2 \hat{Q}_T \left[\left(\frac{d\gamma_{ij}}{d\hat{\tau}} \right)^2 - \hat{c}_T^2 (\partial_k \gamma_{ij})^2 \right], \quad (4.2)$$

where

$$\hat{Q}_T = \hat{c}_T^{-2}, \quad \hat{c}_T^2 = \left(1 + \frac{2\hat{m}_4^2}{M_p^2} \right)^{-1}. \quad (4.3)$$

As in the last section, we consider three phases, during which the scale factor can be parameterised as

$$\hat{a}_j(\hat{\tau}) = \hat{a}_j(\hat{\tau}_j) \left(\frac{\hat{\tau} - \hat{\tau}_{R,j}}{\hat{\tau}_j - \hat{\tau}_{R,j}} \right)^{\frac{1}{\hat{\epsilon}_j - 1}} \quad (4.4)$$

for $\hat{\tau} < \hat{\tau}_j$, where $\hat{\tau}_j$ is the conformal time at the end of phase j , $\hat{\tau}_{R,j} = \hat{\tau}_j - (\hat{\epsilon}_j - 1)^{-1} \hat{\mathcal{H}}^{-1}(\hat{\tau}_j)$, $\hat{\mathcal{H}}(\hat{\tau}) \equiv \hat{a}^{-1} d\hat{a}/d\hat{\tau}$. Since the background evolution is assumed as slow-roll inflation throughout these three phases, we will set $\hat{\epsilon}_1 = \hat{\epsilon}_2 = \hat{\epsilon}_3 = 0$ in the following, for simplicity.

¹See also [68, 69, 72] for the refractive index of the relic gravitons.

The propagating speed of primordial GWs can be parameterized as

$$\hat{c}_{T,1}(\hat{\tau}) = \bar{\hat{c}}_{T,1} := \hat{c}_{T,2}(\hat{\tau}_1), \quad (\hat{\tau} < \hat{\tau}_1), \quad (4.5)$$

$$\hat{c}_{T,2}(\hat{\tau}) = \bar{\hat{c}}_{T,2} \left(\frac{\hat{\tau} - \hat{\tau}_{R,2}}{\hat{\tau}_2 - \hat{\tau}_{R,2}} \right)^p, \quad (\hat{\tau}_1 \leq \hat{\tau} \leq \hat{\tau}_2), \quad (4.6)$$

$$\hat{c}_{T,3}(\hat{\tau}) = \bar{\hat{c}}_{T,2} := \hat{c}_{T,2}(\hat{\tau}_2), \quad (\hat{\tau}_2 < \hat{\tau} \leq \hat{\tau}_3), \quad (4.7)$$

where the parameter $p = \text{const.} > 0$, $\bar{\hat{c}}_{T,1} > \bar{\hat{c}}_{T,2}$. Namely, we assume that $\hat{c}_T(\hat{\tau})$ is constant during phase 1 and phase 3, while $\hat{c}_T(\hat{\tau})$ decreases with time during phase 2.

4.2 Power spectrum

In order to analytically calculate the power spectrum induced by a time-dependent $\hat{c}_T(\hat{\tau})$, we define

$$d\sigma = \hat{c}_T(\hat{\tau}) d\hat{\tau}, \quad (4.8)$$

so that the quadratic action (4.2) can be rewritten as

$$S_\gamma^{(2)} = \frac{M_p^2}{8} \int d\sigma d^3x \hat{a}^2 \hat{c}_T^{-1} \left[\left(\frac{d\gamma_{ij}}{d\sigma} \right)^2 - (\partial_k \gamma_{ij})^2 \right]. \quad (4.9)$$

The equation of motion for $\gamma_\lambda(\tau, k)$ is

$$\frac{d^2 v_k}{d\sigma^2} + \left(k^2 - \frac{d^2 \hat{z}_T / d\sigma^2}{\hat{z}_T} \right) v_k = 0, \quad (4.10)$$

where $v_k = \gamma_\lambda(\tau, k) / \hat{z}_T$, $\hat{z}_T = M_p \hat{a} \hat{c}_T^{-1/2} / 2$. Using eq. (2.7), the power spectrum can be given as

$$\hat{P}_T = \frac{4k^3}{\pi^2 M_p^2} \frac{\hat{c}_T}{\hat{a}^2} |v_k|^2. \quad (4.11)$$

From eq. (4.5) to (4.7), we can write the solutions to eq. (4.10) in the three phases as

$$v_{k,1}(\sigma) = \frac{\sqrt{\pi(\sigma_{R,1} - \sigma)}}{2} \left\{ \hat{\alpha}_1 H_{\hat{\nu}_1}^{(1)} [k(\sigma_{R,1} - \sigma)] + \hat{\beta}_1 H_{\hat{\nu}_1}^{(2)} [k(\sigma_{R,1} - \sigma)] \right\}, \quad (\sigma < \sigma_1), \quad (4.12)$$

$$v_{k,2}(\sigma) = \frac{\sqrt{\pi(\sigma_{R,2} - \sigma)}}{2} \left\{ \hat{\alpha}_2 H_{\hat{\nu}_2}^{(1)} [k(\sigma_{R,2} - \sigma)] + \hat{\beta}_2 H_{\hat{\nu}_2}^{(2)} [k(\sigma_{R,2} - \sigma)] \right\}, \quad (\sigma_1 \leq \sigma \leq \sigma_2),$$

$$v_{k,3}(\sigma) = \frac{\sqrt{\pi(\sigma_{R,3} - \sigma)}}{2} \left\{ \hat{\alpha}_3 H_{\hat{\nu}_3}^{(1)} [k(\sigma_{R,3} - \sigma)] + \hat{\beta}_3 H_{\hat{\nu}_3}^{(2)} [k(\sigma_{R,3} - \sigma)] \right\}, \quad (\sigma_2 < \sigma \leq \sigma_3),$$

where

$$\hat{\nu}_1 = \hat{\nu}_3 = 3/2, \quad \hat{\nu}_2 = \frac{3 + 2p}{2 + 2p}, \quad (4.13)$$

$\sigma_{R,j}$ corresponds to $\hat{\tau}_{R,j}$ according to the relation (4.8), $\hat{\alpha}_j$ and $\hat{\beta}_j$ are k -dependent coefficients. The initial condition of v_k is set as $v_k \simeq \frac{1}{\sqrt{2k}} e^{-ik\sigma}$, which indicates that $|\hat{\alpha}_1| = 1$ and $|\hat{\beta}_1| = 0$.

With eqs. (4.11) and (4.12), the power spectrum evaluated at the end of phase 3 can be given as

$$\hat{P}_T = \hat{P}_{T,inf2} |\hat{\alpha}_3 - \hat{\beta}_3|^2, \quad (4.14)$$

where $\hat{P}_{T,inf2} = \frac{2\hat{H}_3^2}{\pi^2 M_p^2 \hat{c}_{T,2}}$, \hat{H}_j is the Hubble parameter evaluated at $\hat{\tau}_j$. Since we have assumed that $\hat{\epsilon}_3 = 0$, we have $\hat{H}_3 = \hat{H}_2 = \hat{H}_{inf2}$. For a specific model of slow-roll inflation construct by scalar-tensor theories, the slow-roll parameter $\hat{\epsilon}_j$ can slightly deviate from 0. Therefore, there could be a correction in $\hat{P}_{T,inf2}$, which will be ignored in this paper for simplicity.

We can obtain $\hat{\alpha}_3$ and $\hat{\beta}_3$ by requiring the continuities of v_k and $dv_k/d\sigma$ at σ_1 and σ_2 . Note that the continuities of $\hat{\mathcal{H}}$ at $\hat{\tau}_1$ and $\hat{\tau}_2$ actually requires that $\hat{\tau}_{R,1} = \hat{\tau}_{R,2} = \hat{\tau}_{R,3}$. Hence, we also have $\sigma_{R,1} = \sigma_{R,2} = \sigma_{R,3} =: \sigma_R$. For convenience, we define $z_1 = k(\sigma_R - \sigma_1)$ and $z_2 = k(\sigma_R - \sigma_2)$. Using (4.8), it can be found that $z_1 \approx \frac{k}{(1+p)\hat{\mathcal{H}}_1/\bar{c}_{T,1}}$ and $z_2 = \frac{k}{\hat{\mathcal{H}}_2/\bar{c}_{T,2}}$. As a result, we obtain

$$\begin{aligned} \hat{\alpha}_3 - \hat{\beta}_3 = & \frac{-\pi}{16z_1^{3/2}z_2^{3/2}} \left\{ 2z_2 H_{\hat{\nu}_2-1}^{(1)}(z_2) \left[2iz_1(i+z_1)H_{\hat{\nu}_2-1}^{(2)}(z_1) \right. \right. \\ & \left. \left. + (2\hat{\nu}_2 - 3 + z_1(3i + 2z_1 - 2i\hat{\nu}_2))H_{\hat{\nu}_2}^{(2)}(z_1) \right] (z_2 \cos z_2 - \sin z_2) \right. \\ & \left. + \left[z_2(3 - 2\hat{\nu}_2) \cos z_2 + (2\hat{\nu}_2 - 3 + 2z_2^2) \sin z_2 \right] H_{\hat{\nu}_2}^{(1)}(z_2) \left[2iz_1(i+z_1)H_{\hat{\nu}_2-1}^{(2)}(z_1) \right. \right. \\ & \left. \left. + (2\hat{\nu}_2 - 3 + z_1(3i + 2z_1 - 2i\hat{\nu}_2))H_{\hat{\nu}_2}^{(2)}(z_1) \right] \right. \\ & \left. + \left[2iz_1(i+z_1)H_{\hat{\nu}_2-1}^{(1)}(z_1) + (2\hat{\nu}_2 - 3 + z_1(3i + 2z_1 - 2i\hat{\nu}_2))H_{\hat{\nu}_2}^{(1)}(z_1) \right] \right. \\ & \left. \cdot \left[2z_2 H_{\hat{\nu}_2-1}^{(2)}(z_2) (\sin z_2 - z_2 \cos z_2) \right. \right. \\ & \left. \left. + H_{\hat{\nu}_2}^{(2)}(z_2) \left[z_2(2\hat{\nu}_2 - 3) \cos z_2 + 3 \sin z_2 - 2(z_2^2 + \hat{\nu}_2) \sin z_2 \right] \right] \right\}. \end{aligned} \quad (4.15)$$

For the perturbation modes exited horizon during phase 1 (i.e., $\sigma < \sigma_1$), we have $z_1 \ll 1$ and $z_2 \ll 1$. Therefore, we find that

$$\frac{\hat{P}_T}{\hat{P}_{T,inf2}} \Big|_{k \ll \hat{\mathcal{H}}_1/\bar{c}_{T,1}} \simeq \frac{\bar{c}_{T,2}}{\bar{c}_{T,1}} + \mathcal{O}\left(\frac{k}{\hat{\mathcal{H}}_1/\bar{c}_{T,1}}\right)^2, \quad (4.16)$$

where we have used $\hat{\mathcal{H}}_2/\hat{\mathcal{H}}_1 = (\bar{c}_{T,1}/\bar{c}_{T,2})^{1/p}$, which can be obtained from the continuity of \hat{c}_T at $\hat{\tau}_1$. Note that there is actually a p -dependent factor in the leading order term of the right hand side of eq. (4.16), which appears due to the fact that $d\hat{c}_T(\hat{\tau})/d\hat{\tau}$ is not continuous at $\hat{\tau}_1$ or $\hat{\tau}_2$ according to eqs. (4.5) to (4.7). Such issues also appear in section 3 for the case where $\nu_2 = 1/2$. In fact, for $k \ll \hat{\mathcal{H}}_1/\bar{c}_{T,1}$, we can directly obtain the power spectrum by substituting $v_{k,1}$ into eq. (4.11), which gives $\hat{P}_{T,inf1} = \frac{2\hat{H}_1^2}{\pi^2 M_p^2 \bar{c}_{T,1}}$, i.e., $\hat{P}_{T,inf1}/\hat{P}_{T,inf2} = \bar{c}_{T,2}/\bar{c}_{T,1}$, provided these perturbation modes are frozen after horizon crossing.

For the perturbation modes exited horizon during phase 2 (i.e., $\sigma_1 \leq \sigma \leq \sigma_2$), we consider the situation in which $z_1 \gg 1$ and $z_2 \ll 1$. Then, we find

$$\frac{\hat{P}_T}{\hat{P}_{T,inf2}} \Big|_{\hat{\mathcal{H}}_1/\bar{c}_{T,1} \ll k \ll \hat{\mathcal{H}}_2/\bar{c}_{T,2}} \simeq \frac{2^{2\hat{\nu}_2-3}(3+2\hat{\nu}_2)^2 \Gamma^2(\hat{\nu}_2)}{9\pi} \left(\frac{k}{\hat{\mathcal{H}}_2/\bar{c}_{T,2}}\right)^{3-2\hat{\nu}_2} + \mathcal{O}\left(\frac{k}{\hat{\mathcal{H}}_1/\bar{c}_{T,1}}\right)^{2\hat{\nu}_2-5}. \quad (4.17)$$

It can be checked that $\hat{P}_T(\hat{\mathcal{H}}_1/\bar{c}_{T,1} \ll k \ll \hat{\mathcal{H}}_2/\bar{c}_{T,2}) \rightarrow \hat{P}_{T,inf2}$ in the limit $p \rightarrow 0$. Again, the discontinuity of $d\hat{c}_T(\hat{\tau})/d\hat{\tau}$ at $\hat{\tau}_1$ or $\hat{\tau}_2$ could contribute a p -dependent factor, which is not physical. What we shall be interested in is the spectrum index, i.e., $\hat{n}_{T,2} = 3 - 2\hat{\nu}_2 = p/(1+p)$, which is consistent with that obtained in [66, 67].

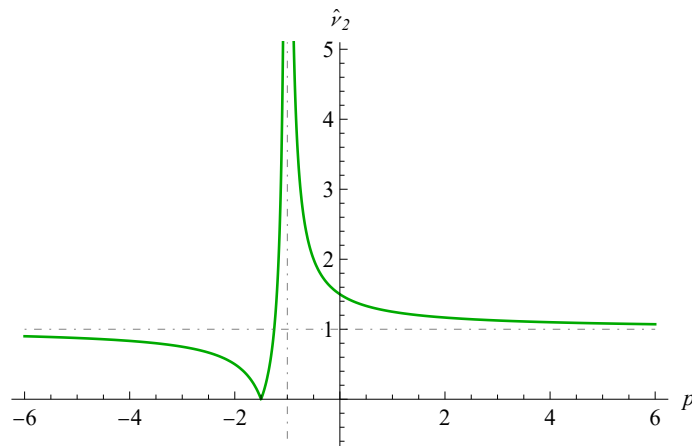


Figure 3. For a general p , we have $\hat{\nu}_2 = \left| \frac{3+2p}{2+2p} \right|$, which is depicted by the green curve.

Note that we have assumed $p > 0$ in the derivation of eq. (4.17). For a general p , we actually have $\hat{\nu}_2 = \left| \frac{3+2p}{2+2p} \right|$, which is displayed in figure 3. It can be checked that $\hat{n}_{T,2} = 3 - 2\hat{\nu}_2 = p/(1+p)$ is also valid for an increasing c_T as long as $-1 < p \leq 0$, which will predict a red-tilted (i.e., $\hat{n}_{T,2} < 0$) spectrum of primordial GWs. It is interesting to see whether a blue spectrum can be obtained by an increasing c_T with $p < -1$.

Unfortunately, the result of eq. (4.17) is no longer applicable for the situation $p < -1$. Actually, according to eqs. (4.4), (4.6) and (4.10), the effective comoving sound horizon of tensor perturbation mode, which can be defined as $\frac{1}{a|H_{\text{GW}}|} \equiv \sqrt{\left| \frac{\dot{z}_T}{d^2 \dot{z}_T / d\sigma^2} \right|} \sim \frac{\hat{c}_T}{\hat{H}} \sim |\hat{\tau} - \hat{\tau}_{R,2}|^{p+1}$, will not decrease with time when $p \leq -1$. In other words, we will have $\hat{H}_1/\bar{\hat{c}}_{T,1} \gg \hat{H}_2/\bar{\hat{c}}_{T,2}$ in eq. (4.17) for $p < -1$. Therefore, the tensor perturbation modes are not able to exit their sound horizon when $p \leq -1$. In fact, the tensor perturbation modes with comoving wavenumber $\hat{H}_1/\bar{\hat{c}}_{T,1} \gg k \gg \hat{H}_2/\bar{\hat{c}}_{T,2}$ should reenter their sound horizon during the $p < -1$ phase and eventually exit their horizon during the third phase ($\hat{\tau}_2 < \hat{\tau} \leq \hat{\tau}_3$), see figure 4 for an illustration. With this in mind, we still obtain a red-tilted spectrum, i.e., $\hat{P}_T/\hat{P}_{T,\text{inf}2} \sim k^{-(3+2\hat{\nu}_2)}$, for $\hat{H}_1/\bar{\hat{c}}_{T,1} \gg k \gg \hat{H}_2/\bar{\hat{c}}_{T,2}$. In a short summary, the case in which c_T increases with time (i.e., $p < 0$) during inflation cannot generate GWs with a blue spectrum.

For the perturbation modes exited horizon during phase 3 (i.e., $\sigma_2 < \sigma \leq \sigma_3$), we have $z_1 \gg 1$ and $z_2 \gg 1$. In this limit, the effect of the discontinuity of $d\hat{c}_T(\hat{\tau})/d\hat{\tau}$ can be negligible, as can be inferred from eq. (4.10). Therefore, we can obtain that

$$\left. \frac{\hat{P}_T}{\hat{P}_{T,\text{inf}2}} \right|_{k \gg \hat{H}_2/\bar{\hat{c}}_{T,2}} = 1 + \mathcal{O}\left(\frac{k}{\hat{H}_2/\bar{\hat{c}}_{T,2}}\right)^{-2}, \quad (4.18)$$

where the leading order term is trivial.

When $\hat{\tau} > \hat{\tau}_3$, \hat{c}_T may regain the value of the speed of light, so that the predictions will be consistent with observations of GWs at higher frequency [52]. As a consequence, the power spectrum of GWs could become red-tilted at much higher frequencies. Additionally,

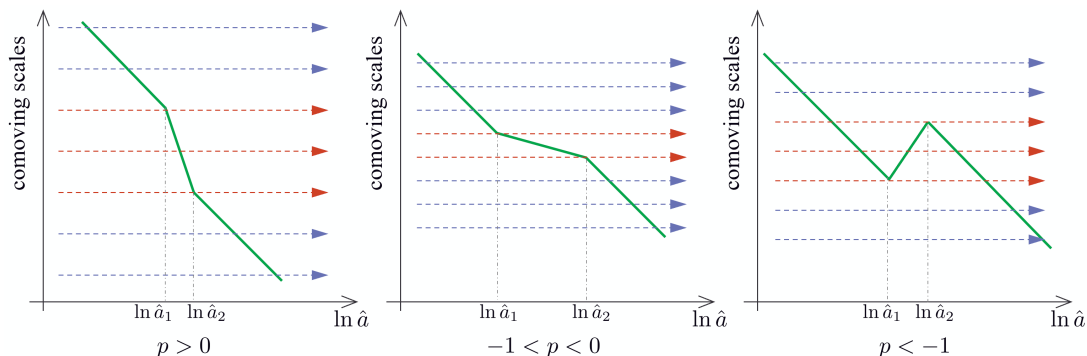


Figure 4. Horizon-crossing of primordial GWs with time-dependent propagating speed c_T parameterized by eqs. (4.5) to (4.7). The green lines represent the evolution of the effective comoving sound horizon of GWs, which is defined as $\frac{1}{a|H_{\text{GW}}|} \simeq \sqrt{\frac{\hat{z}_T}{\bar{a}^2 \hat{z}_T / d\sigma^2}} \simeq \hat{c}_T / \hat{H}$; the dashed lines with arrows represent the GW modes, in which the modes with comoving wavenumber $\hat{H}_1 / \bar{c}_{T,1} < k < \hat{H}_2 / \bar{c}_{T,2}$ for $p > -1$ (and $\hat{H}_1 / \bar{c}_{T,1} > k > \hat{H}_2 / \bar{c}_{T,2}$ for $p < -1$) are colored red. During the stage where c_T increases with time while $p < -1$, the GW modes cannot exit their sound horizon.

the discontinuity of $d\hat{c}_T(\hat{\tau})/d\hat{\tau}$ at $\hat{\tau}_1$ or $\hat{\tau}_2$ is a defect of the parameterizations (4.5) to (4.7). For a generic smooth $\hat{c}_T(\hat{\tau})$, the power spectrum can be solved numerically from the action (4.2), see the next section for two examples.

5 Discussion of relation and discrepancy between two scenarios

Both actions (3.1) and (4.1) belong to the category of the “beyond Horndeski” theories, which correspond to $m_4^2 \neq \tilde{m}_4^2$ in action (2.2). The quadratic action (4.3) can be related to (2.4) when $c_T = 1$ and $Q_T = 1$ by a disformal transformation $\hat{g}_{\mu\nu} \rightarrow \hat{c}_T^{-1}[\hat{g}_{\mu\nu} + (1 - \hat{c}_T^2)\hat{n}_\mu\hat{n}_\nu]$, which indicates that $\hat{\tau}$ and \hat{a} satisfy

$$d\tau = \hat{c}_T d\hat{\tau}, \quad a = \hat{c}_T^{-1/2} \hat{a}, \quad (5.1)$$

see [96] for details. Therefore, we find

$$\mathcal{H}(\tau) = \frac{1}{\hat{c}_T(\hat{\tau})} \left[\hat{\mathcal{H}}(\hat{\tau}) - \frac{1}{2} \frac{d\hat{c}_T/d\hat{\tau}}{\hat{c}_T(\hat{\tau})} \right]. \quad (5.2)$$

In section 4.1, we have assumed that $\hat{c}_T(\hat{\tau})$ is constant during phase 1 and phase 3. As a result, we can obtain $(\bar{\mathcal{H}}_1/\bar{\mathcal{H}}_2)^{3-2\nu_2} = \bar{c}_{T,2}/\bar{c}_{T,1}$ and $H_{inf2} = \hat{H}_{inf2} \bar{c}_{T,2}^{-1/2}$. It is easy to identify that

$$\hat{P}_{T,inf2} = P_{T,inf2}, \quad (5.3)$$

see eq. (3.7) and (4.14) for the definitions of $P_{T,inf2}$ and $\hat{P}_{T,inf2}$, respectively. Therefore, the power spectra of the two scenarios coincide with each other at the leading order for the perturbation modes exited their horizon in phase 1 and phase 3.

Regarding the NEC-violating phase and the \hat{c}_T -diminishing phase, we can find $p = -\frac{2\epsilon_2}{1+\epsilon_2}$ by using (5.1), which indicates that $\hat{\nu}_2 = \nu_2$, provided there exists such a disformal

transformation. Therefore, for perturbation modes that exited their horizon during the intermediate phase, their spectra indexes are same in these two scenarios at the leading order. However, there is a discrepancy in the coefficients of the leading order terms in eq. (3.10) and (4.17), which should be attributed to the discontinuity of $d\hat{c}_T(\hat{\tau})/d\hat{\tau}$. As can be seen from eq. (5.2), even if $d\hat{c}_T(\hat{\tau})/d\hat{\tau}$ is continuous everywhere, the oversimplified parameterizations of a (or \mathcal{H}) and \hat{c}_T cannot result in the equivalence of these two scenarios around the matching time τ_1 (or $\hat{\tau}_1$) and τ_2 (or $\hat{\tau}_2$), unless $d\hat{c}_T/d\hat{\tau}/\hat{c}_T$ is negligible. For this reason, we should focus on the comparison of spectra indices of these two scenarios, which are evaluated for $\hat{\mathcal{H}}_1/\hat{c}_{T,1} \ll k \ll \hat{\mathcal{H}}_2/\hat{c}_{T,2}$ so that they can hardly be affected by the details around the matching time.

In the NEC-violating scenario, $\nu_2 = 1/2 + 1/(1 - \epsilon_2)$, which indicates that $1/2 < \nu_2 < 3/2$ and

$$0 < n_T = 3 - 2\nu_2 < 2. \tag{5.4}$$

However, in the \hat{c}_T -diminishing scenario, $\hat{\nu}_2 = (3 + 2p)/(2 + 2p)$ with $p > 0$, which results in $1 < \hat{\nu}_2 < 3/2$ and

$$0 < \hat{n}_T = 3 - 2\hat{\nu}_2 < 1, \tag{5.5}$$

where the background evolution has been assumed as inflation. During inflation, a blue-tilted power spectrum of primordial GWs with $0 < n_T < 1$ could be generated by either the NEC violation or the diminishment of the propagating speed of primordial GWs, while $n_T > 1$ can only be attributed to the violation of NEC. In fact, in the limit $p \rightarrow \infty$, the relation (5.2) will be invalid due to the divergence of $d\hat{c}_T/d\hat{\tau}/\hat{c}_T$, namely, the correspondence of the two scenarios will be broken. This is also due to the fact that we have required an inflationary background in the $\hat{c}_T \neq 1$ case.

The NEC-violating case with $\epsilon_2 < -1$, which gives $1 < n_T < 2$, cannot be related to a varying c_T with $p > -1$ on an inflationary background by a disformal transformation. To see this, we can assume that $\hat{a}_2(\hat{\tau}) \sim (\hat{\tau}_{R,j} - \hat{\tau})^{\frac{1}{\epsilon_j - 1}} \sim (\hat{\tau}_{R,j} - \hat{\tau})^{\frac{2}{1 + 3\hat{w}_2}}$ and $\hat{c}_{T,2}(\hat{\tau}) \sim (\hat{\tau}_{R,j} - \hat{\tau})^p$, where $\hat{w}_2 = \hat{p}_2/\hat{\rho}_2$ is the equation of state parameter of phase 2. It can be checked that $p > -1$ is still required so that the tensor perturbation modes are able to exit their effective sound horizon during phase 2 ($\hat{\tau}_2 < \hat{\tau} \leq \hat{\tau}_3$). Under the conditions $\epsilon_2 < -1$ and $p > -1$, the disformal transformation (or the relation given by eq. (5.1)) actually gives $\hat{w}_2 > -1/3$ or $\hat{w}_2 < -5/3$, which indicates that the background evolution in the $\hat{c}_T \neq 1$ frame should be either a contraction² or an NEC-violating expansion, namely non-inflationary, see figure 5 for an illustration. In other words, even if we give up the requirement of having an inflationary background in the $\hat{c}_T \neq 1$ frame, we would conclude that an observation of $n_T > 1$ would support the existence of a primordial NEC violation.

Since we have focused on the \hat{c}_T -diminishing scenario on the inflationary background in section 4, the resulted range of \hat{n}_T given by (5.5) cannot cover that of the NEC-violating scenario (i.e., (5.4)). Therefore, in the sense of the prediction of blue-tilted primordial GWs, these two scenarios may be identified by observations of the spectra index when $n_T > 1$, provided the background is assumed as inflation in the $\hat{c}_T \neq 1$ frame. Note that physics

²A contracting Universe has to go through an NEC-violating bounce later to enter the Big Bang expansion.

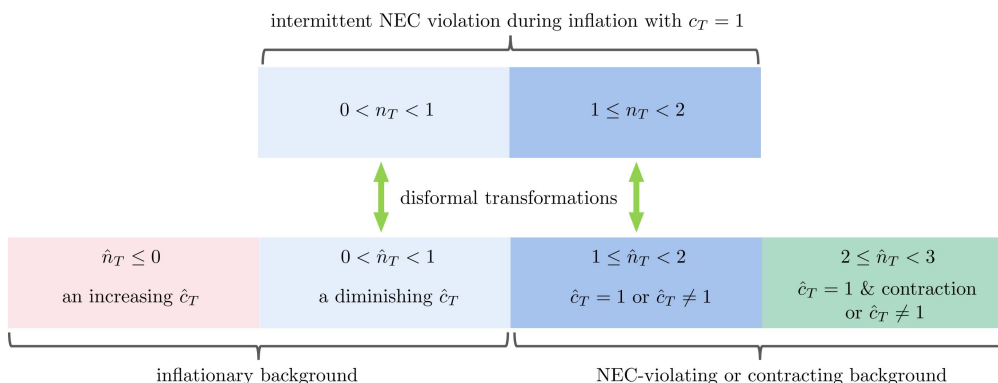


Figure 5. Relation and discrepancy between the NEC-violating scenario and the c_T -diminishing scenario. In the sense of the prediction of blue-tilted spectrum of primordial GWs with $0 < n_T < 1$, these two scenarios cannot be distinguished by observations of n_T when there exists a disformal dual. However, the NEC-violating scenario with $c_T = 1$ which predicts $1 \leq n_T < 2$ cannot be related to the $c_T \neq 1$ case by a disformal transformation, unless the background evolution is contracting or NEC-violating (namely, non-inflationary).

should not depend on the chosen frame. Our result does not mean the breakdown of the equivalence between the two scenarios which give $1 \leq n_T < 2$ in figure 5, but highlighted the significance of primordial NEC violation in generating GWs with $n_T > 1$.

In the context of $0 < n_T < 1$, we can numerically demonstrate the correspondence of the NEC-violating scenario and \hat{c}_T -diminishing scenario. For simplicity, we consider two examples of the step-like \hat{c}_T and obtain the corresponding H with eqs. (5.1) and (5.2), which are displayed in figure 6. Obviously, when $d\hat{c}_T/d\hat{\tau}/\hat{c}_T$ is significant, the corresponding H can no longer be mimicked by the parameterization (3.3), see figure 6(a). The resulted power spectra in two scenarios exactly coincide with each other for smooth \hat{c}_T regardless of the significance of $d\hat{c}_T/d\hat{\tau}/\hat{c}_T$, as displayed in figure 7, see also [66, 67]. As a contrast, we also plotted the template (3.12) in figure 7(b). Note that the numerical method used here follows that in [66].

In order to be consistent with the current observations of LIGO, the spectrum may decrease with respect to the frequency at much higher frequencies. We can assume that \hat{c}_T will regain the value of the speed of light or H will go down to a smaller reasonable value before the end of inflation.

6 Conclusion

The history of the inflationary era could be much more complicated than expected [98–100]. The primordial GWs generated during inflation span a frequency band from 10^{-18} Hz to 10^{10} Hz, hence could encode rich information about the history of the inflationary Universe. Inflationary GWs with enhanced power spectra may be promising to explain the common-spectrum process reported recently by NANOGrav and PPTA.

Within the framework of EFT, we investigated an intermittent NEC violation as well as a diminishment of propagating speed of primordial GWs for generating an enhanced

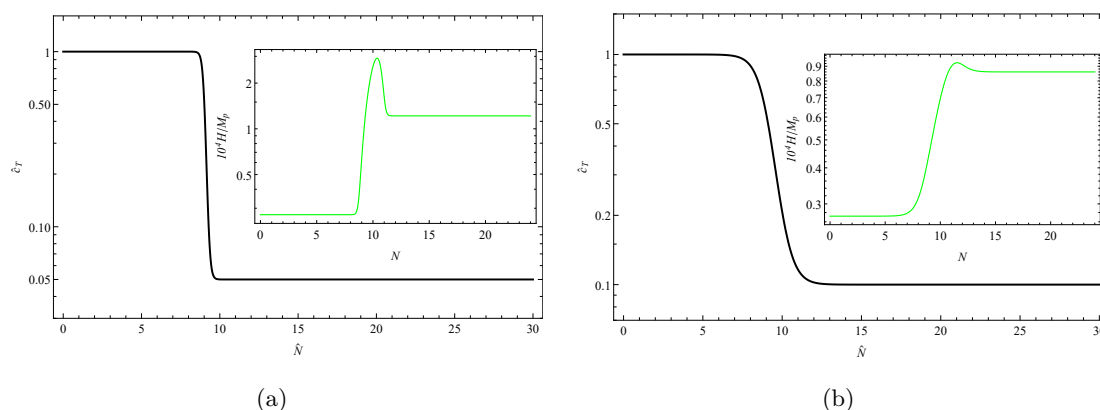


Figure 6. The diminishing step-like \hat{c}_T and the corresponding H , where the parameters N and \hat{N} satisfy $\Delta N = \Delta \ln a$ and $\Delta \hat{N} = \Delta \ln \hat{a}$, respectively. See ref. [66] for more details about the numerical method.

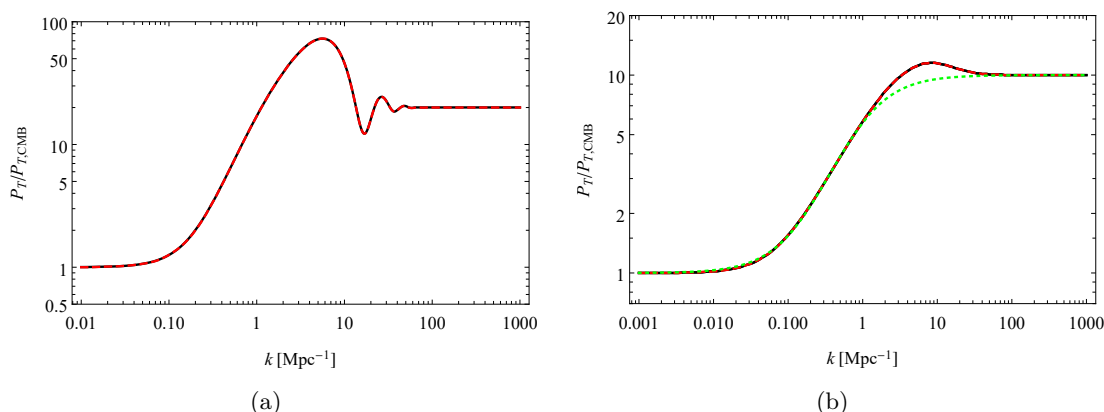


Figure 7. Comparisons between the power spectra of primordial GWs in the NEC-violating scenario (red dashed curves) and the \hat{c}_T -diminishing scenario (black solid curves), where $P_{T,\text{CMB}}$ is assumed to be the power spectrum of primordial GWs at the CMB frequency band. The green dotted curve is plotted by multiplying a factor 10 with the right hand side of eq. (3.12), where $\nu_2 = 0.88$, $\mathcal{H}_1 = 0.18 \text{ Mpc}^{-1}$ and $\mathcal{H}_2 = 1.15 \text{ Mpc}^{-1}$. Left: the power spectra induced by \hat{c}_T and H displayed in figure 6(a). Right: the power spectra induced by \hat{c}_T and H displayed in figure 6(b). See ref. [66] for more details about the numerical method.

power spectrum of GW background during inflation. Both of the NEC violation and the time-dependent \hat{c}_T can be realized stably in the “beyond Horndeski” theory. We explicitly calculated the power spectra of primordial GWs for both the NEC-violating scenario and the c_T -diminishing scenario. Interestingly, the range of the spectra index in the NEC-violating scenario is $0 < n_T < 2$, while it is $0 < n_T < 1$ in the c_T -diminishing scenario. Therefore, these two scenarios can be identified by the spectra index when $n_T > 1$, while they could be dual to each other by a disformal transformation of the metric in the sense of generating primordial GWs for the spectra index $n_T < 1$.

Compared with previous works [22, 96], our work discussed explicitly the relation and discrepancy between two scenarios for different ranges of the spectra index n_T of the primordial GW spectrum. We pointed out that the GW modes are not able to exit their effective sound horizon during the phase in which the propagating speed of GWs increases with time too fast (i.e., $p < -1$ in eq. (4.6)). As a result, a blue-tilted spectrum of GWs with $n_T > 1$ cannot be attributed to a diminishing or increasing c_T during inflation, unless the background evolution is replaced by non-inflationary ones, i.e., a contraction or NEC-violating expansion. Therefore, our work highlighted the significance of primordial NEC violation in generating a blue spectrum of GWs with $1 < n_T < 2$.

Our work reinforces the possibility that the primordial GWs could be non-negligible and essential for surveys of a stochastic GW background. Future observational surveys of GWs at the frequency band of the CMB, PTA and interferometers could be combined to put constraints on the NEC-violating scenario as well as the c_T -varying scenario. The result of this paper can be generalized to the case where there are multiple violations of the NEC or nontrivial variations of \hat{c}_T . Additionally, taking account of the enhancement of chirality of primordial GWs during inflation would also be interesting from both theoretical and observational standpoints [101–109].

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