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Alcubierre warp drive in spherical coordinates with some matter configurations

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Abstract In this work we introduce the Alcubierre warp metric using spherical symmetry. In this way we write the Einstein equations for a perfect fluid and for an anisotropic fluid with cosmological constant. Analysing the energy conditions for both cases, we find that these cases are flexible enough to allow them to be satisfied. We also find that in the time-independent case of the warp bubble, the metric admits a timelike Killing vector and all the energy conditions are satisfied except for the strong energy condition. Moreover, in the time-independent case a barotropic equation of state known from cosmological models naturally arises.

1 Introduction

It is well known that, locally, particles cannot exceed the speed of light. However in general relativity particles can travel globally at superluminal velocities [1–4]. This idea was explored by Alcubierre [5] to propose a way to propel material at velocities higher than the speed of light. The mechanism proposed by Alcubierre creates a distortion of space-time, called a warp bubble, resulting in spacetime contracting in front of the bubble and expanding behind of the bubble as the bubble moves through a geodesic. The line element proposed by Alcubierre was

$$ds^2 = -dt^2 + (dx - f(r_s)v_s dt)^2 + dy^2 + dz^2, \tag{1}$$

with $r_s = \sqrt{(x - x_s)^2 + y^2 + z^2}$ and $v_s = \frac{dx_s}{dt}$. This corresponds to an ADM-like decomposition of the line element [6–8]. Studying this metric, Alcubierre concluded that the proposed warp implied the violation of energy conditions, since it seemed that a negative energy density would be necessary for the creation of the bubble.

Since Alcubierre's original work, there have been numerous papers that contribute to a better understanding of the physics behind the warp metric. One interesting aspect has been to understand the properties of the metric that would allow superluminal velocities to be achieved [3,4]. Another relevant contribution to the warp drive theory has been to study modifications of the original metric that allow for a significant decrease in the energy involved in the creation of the warp bubble [9]. In [10] Natario proposes a new warp drive with zero expansion. Lobo and Visser [11] discuss the characteristics of the matter inside the warp bubble and determine that it must be massless at the centre. However, it is found that in these studies the amount of energy needed is significantly reduced. One aspect that has received much attention from the very beginning has been the study of energy conditions as a means of validating the physical feasibility of the warp drive [12–24]. The occurrence of horizons [25–27] and closed time curves has also been studied [28]. This has been the subject of extensive debate in the community and although much progress has been made, there is still no definitive consensus.

In a series of papers [29–33], Santos et al. propose to study the warp problem from the point of view of matter distribution. The question they ask is how the elements of the warp metric should be constrained by some kind of matter, for example dust or a perfect fluid. In this way they were able to obtain some relations for the deformation function given by the Einstein equations. This form serves as an alternative to the way the problem has been treated in early works.

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In this paper we approach the warp drive problem from the point of view of Santos' work. In Sect. 2, we propose a new version of the warp metric in spherical coordinates. The idea is to exploit the symmetries of these coordinates and to obtain a set of Einstein equations that are simpler to interpret. We then propose an energy–momentum tensor corresponding to a perfect fluid, and calculate the Einstein equations. With this configuration we proceed to the study of the energy conditions. We then investigate in Sect. 3 a more general parametrization of the energy–momentum tensor that takes into account the incorporation of anisotropy and heat. With this setup we calculate the Einstein equations and verify the energy conditions. Finally we make some concluding remarks and further considerations.

2 The spherical warp drive

The original Natario's warp drive metric is a globally hyperbolic spacetime, which is given in Cartesian coordinates by the line element [10],

$$ds^2 = -dt^2 + \sum_{i=1}^{3}\left(dx^i - X^i dt\right)^2, \tag{2}$$

where functions X^i corresponds to the X, Y, and Z components of a vector field defined in Euclidean 3-space. This can be expanded to obtain,

$$ds^2 = -dt^2(1-(X^i)^2) + (dx^i)^2 - 2X^i dx^i dt. \tag{3}$$

We can identify the mixed component $dx^i dt$ and the spacial flat part of the metric dx^i.

Based on the above expression, we write the following line element

$$\begin{aligned} ds^2 &= -dt^2 + (dr - Rdt)^2 + r^2 d\Omega^2 \\ &= -dt^2\left(1-R^2\right) + dr^2 + r^2 d\Omega^2 - 2Rdrdt, \end{aligned} \tag{4}$$

where we can easily identify the flat part of the metric $dr^2 + r^2 d\Omega^2$ with $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ and the mixed component $2Rdtdr$. The main idea is that the flat part of the warp is described in spherical coordinates.

There is no direct mapping between metrics (3) and (4), albeit the physical mechanism in both follows the same reasoning. Alcubierre assumed the motion of the warp bubble occurred in a defined direction, for instance x. In this case the line element is just,

$$ds^2 = -dt^2(1-X^2) + dx^2 - 2Xdxdt \tag{5}$$

He proposed $X = vf(r_s)$, where $v = \frac{dx}{dt}$ and $f(r_s)$ is a bounded function that defines the shape of the bubble in this Cartesian type of spacetime. Note that $r_s^2 = (x - x_s(t))^2 + y^2 + z^2$ corresponds to the radius of the bubble and its origin is in general different from the origin of the spatial part or the metric. On the other hand, the metric (2) propagates in an arbitrary direction described by the coordinates x, y and z. The metric (4) propagates in an arbitrary radial direction.

In [34] Bobrick et al., a scheme to construct warp drive metrics based on their very definite properties is proposed. They consider formally warp drive spacetimes that admit a global Killing vector field, ξ, which is aligned with the four-velocity of the boundary of the region of the warp. This field establishes a global frame of rest with respect to the warped region. Any physical motion of an observer relative to the warp drive, in local terms, is the motion relative to the global reference frame defined by this Killing field.

A stationary warp drive spacetime may be associated with a coordinate comoving system, such that the Killing vector field ξ defines the rest frame of the bubble. Also, a global coordinate system can be assumed asymptotically at infinity, which approaches the coordinate system of the resting observer outside of the bubble. Since the two charts cover the whole spacetime and overlap, it is, in principle, possible to introduce a mapping from one to another. Following this principle, they report a procedure for constructing an axisymmetric warp drive spacetime from this subclass which relies basically on three very general steps,

- Choosing a map between the resting outside observer and the bubble comoving observer x_{co}.
- Propose a spacetime with the following structure,

$$\begin{aligned} ds^2 = &-[dt(1-f_t) + f_t dt_{co}]^2 \\ &+ \sum_{i=1}^{3}\left[dx^i(1-f_{x^i}) + f_{x^i}dx^i_{co}\right]^2. \end{aligned} \tag{6}$$

- Choose a set of shape functions of the bubble that approaches to 1 inside the bubble and 0 outside.

These conditions are not dependent on any coordinate system, hence we can generalize the third item above, such that x^i denotes any general suitable coordinate system. We can rewrite this metric,

$$\begin{aligned} ds^2 = &-[dt + f_t(dt_{co} - dt)]^2 \\ &+ \sum_{i=1}^{3}\left[dx^i + f_{x^i}(dx^i_{co} - dx^i)\right]^2, \end{aligned} \tag{7}$$

with coordinates $dx^1 = dr$, $dx^2 = rd\theta$ and $dx^3 = r\sin\theta d\phi$. We choose a one-to-one mapping,

$$dt_{co} = dt\,, \tag{8}$$

$$dr_{co} = dr - v_s dt\,, \tag{9}$$

$$d\theta_{co} = d\theta\,, \tag{10}$$

$$d\phi_{co} = d\phi\,. \tag{11}$$

The metric (7) can be written as,

$$ds^2 = -dt^2 + [dr + f_r(dr_{co} - dr)]^2 + r^2 d\Omega^2$$
$$= -dt^2 + (dr - f_r v_s dt)^2 + r^2 d\Omega^2, \quad (12)$$

following that, $f_r = f(r_s)$, with $r_s = \|r - R_s\|$ and $v_s = v_s(t) \equiv \frac{dR_s}{dt}$.

With these considerations we propose the following line element

$$ds^2 = -(1-\beta^2)dt^2 - 2\beta dt dr + dr^2 + r^2 d\Omega^2, \quad (13)$$

with $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ and $\beta = f_s v_s$. This is analogous to Alcubierre's metric with a warp bubble on radial direction and flat space components described by spherical coordinates. Following the previous results, we found that the metric (13) has the same form of the line element given by equation (4), where the function R corresponds with β. At this point, it is worth mentioning that in order for this metric to admit a timelike Killing vector in the form $\xi_i = (-1, 0, 0, 0)$, spacetime needs to be stationary, meaning the warp bubble function β is time-independent.

In the following we will discuss the consequences of this metric associated with different matter configurations.

2.1 Einstein tensor components

We are interested in exploring the main features of the spherical symmetric warp proposed in the last section. To achieve this, we study the properties of Einstein's equations,

$$G_{\mu\nu} = 8\pi T_{\mu\nu}, \quad (14)$$

with $G_{\mu\nu}$ the Einstein tensor and $T_{\mu\nu}$ the energy–momentum tensor. Note that we are working in units where $c = G = 1$. In this way, using the line element (13), the components of the Einstein tensor are

$$G_{00} = \frac{\beta}{r^2}\left[\left(1-\beta^2\right)\left(\beta + 2r\frac{\partial\beta}{\partial r}\right) - 2r\beta\frac{\partial\beta}{\partial t}\right], \quad (15)$$

$$G_{01} = \frac{\beta}{r^2}\left(\beta^2 + 2r\beta\frac{\partial\beta}{\partial r} + 2r\frac{\partial\beta}{\partial t}\right), \quad (16)$$

$$G_{11} = -\frac{1}{r^2}\left(\beta^2 + 2r\beta\frac{\partial\beta}{\partial r} + 2r\frac{\partial\beta}{\partial t}\right), \quad (17)$$

$$G_{22} = -r\left\{\beta\left(2\frac{\partial\beta}{\partial r} + r\frac{\partial^2\beta}{\partial r^2}\right) + \frac{\partial\beta}{\partial t} + r\left[\left(\frac{\partial\beta}{\partial r}\right)^2 + \frac{\partial^2\beta}{\partial t\partial r}\right]\right\}, \quad (18)$$

$$G_{33} = -r\sin^2\theta\left\{\beta\left(2\frac{\partial\beta}{\partial r} + r\frac{\partial^2\beta}{\partial r^2}\right) + \frac{\partial\beta}{\partial t} + r\left[\left(\frac{\partial\beta}{\partial r}\right)^2 + \frac{\partial^2\beta}{\partial t\partial r}\right]\right\}. \quad (19)$$

Here, note that we have considered the most general case for $\beta = \beta(t, r)$ as a function of both coordinates t and r, which can be seem from the definition $\beta = f(r_s)v(t)$ and noticing that $r_s = \|r - R_s\|$, $v(t) = \frac{dR_s}{dt}$.

Now we want to include matter content. To this end we consider the timelike and future-directed unit vectors normal to the slicing hypersurfaces taken as the 4-velocity of the so-called Eulerian observers characterized by,

$$u_\mu = \{-1, 0, 0, 0\}, \quad (20)$$

which corresponds with the temporal Killing vector mentioned above. With this parametrization, we proceed to study the energy–momentum tensor.

2.2 Perfect fluid energy–momentum tensor

In order to write Einstein's equations we first consider a perfect fluid system. It is defined

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}. \quad (21)$$

Using the parametrization (20), this could be written in matrix form as

$$T_{\mu\nu} = \begin{bmatrix} \rho + \beta^2 p & -\beta p & 0 & 0 \\ -\beta p & p & 0 & 0 \\ 0 & 0 & r^2 p & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta\ p \end{bmatrix}, \quad (22)$$

with ρ and p the matter–energy density and pressure respectively. It is worth mentioning that from Einstein's equations and knowing that $u^\mu = \{1, \beta, 0, 0\}$ we obtain the relation

$$T_{\mu\nu}u^\mu u^\nu = \frac{1}{8\pi}G_{\mu\nu}u^\mu u^\nu$$
$$= \frac{1}{8\pi}\left(G_{00} + 2\beta G_{01} + \beta^2 G_{11}\right)$$
$$= \frac{\beta^2}{8\pi r^2}\left(\beta + 2r\frac{\partial\beta}{\partial t}\right). \quad (23)$$

But we also have the relation $T_{\mu\nu}u^\mu u^\nu = \rho$, thus we could write the relation

$$\frac{\beta^2}{8\pi r^2}\left(\beta + 2r\frac{\partial\beta}{\partial t}\right) = \rho. \quad (24)$$

This expression relates matter–energy density with the metric coefficients.

Let's write Einstein's equations. Using the Einstein tensor components (15)–(19) and (24) we obtain

$$\begin{aligned} &\beta\left(\beta+2r\frac{\partial\beta}{\partial r}\right)=8\pi r^2\rho,\\ &\beta\left(\beta+2r\frac{\partial\beta}{\partial r}\right)+2r\frac{\partial\beta}{\partial t}=-8\pi r^2 p,\\ &\beta^2+r\frac{\partial\beta}{\partial t}-r^2\left[\frac{\partial}{\partial r}\left(\beta\frac{\partial\beta}{\partial r}\right)\right]=0. \end{aligned} \tag{25}$$

These expressions correspond to time-dependent isotropic warp. If we constraint to $\beta=\beta(r)$ we have a time-independent isotropic warp given by

$$\begin{aligned} &\beta\left(\beta+2r\frac{d\beta}{dr}\right)=8\pi r^2\rho,\\ &\beta\left(\beta+2r\frac{d\beta}{dr}\right)=-8\pi r^2 p,\\ &\beta^2-r^2\left[\frac{d}{dr}\left(\beta\frac{d\beta}{dr}\right)\right]=0. \end{aligned} \tag{26}$$

It is important to notice that for time-independent warp, we necessarily have the equation of state $p=-\rho$, as is easily seen from the first two equations. This equation characterizes the matter content in the context of a Friedmann–Robertson–Walker (FRW) universe with cosmological constant, which is consistent with observations [35].

Another important aspect is that, although there are five non-zero components of the Einstein tensor, there are only three independent equations and three unknowns ρ, p and β.

2.3 Energy conditions

Energy conditions are constraints imposed on the energy–momentum tensor so that one can control non-physical aspects of the system [7,35–39].

2.3.1 Weak energy condition

The weak energy condition (WEC) requires that for every future-pointing like vector u^μ, $T_{\mu\nu}u^\mu u^\nu\geq 0$, so we find that

$$\text{If}\quad \rho\geq 0,\qquad T_{\mu\nu}u^\mu u^\nu=\rho\geq 0. \tag{27}$$

So weak energy condition is satisfied if $\rho\geq 0$.

2.3.2 Dominant energy condition

The dominant energy condition (DEC) is equivalent to the WEC, with the additional requirement that $T^\mu_\nu u^\nu$ is a future-pointing causal vector. Thus, the weak energy condition and $F^\mu F_\mu\leq 0$, with $F^\mu=T^{\mu\nu}u_\nu$, must be satisfied. So, we already prove that given $\rho\geq 0$, WEC is satisfied and just remains to assess $F^\mu F_\mu$. After some straightforward calculations, we find that

$$F^\mu=\{-\rho,-p\rho,0,0\},\quad F_\mu=\{\rho,0,0,0\}, \tag{28}$$

then, $F^\mu F_\mu=-\rho^2\leq 0$. Satisfying both conditions essentially show that with respect to an observer, the local energy density is non-negative and the local energy flow must be non-spacelike. If the conservation of the $T_{\mu\nu}$ is also considered, these conditions guarantee the causal structure in local matter configurations.

2.3.3 Strong energy condition

The strong energy condition (SEC) imposes a bound on a more complicated expression in 4 dimensions

$$\left(T_{\mu\nu}-\frac{1}{2}Tg_{\mu\nu}\right)u^\mu u^\nu\geq 0\,. \tag{29}$$

The left side is

$$\left(T_{\mu\nu}-\frac{1}{2}Tg_{\mu\nu}\right)u^\mu u^\nu=\frac{1}{2}(\rho+3p)\,, \tag{30}$$

so SEC implies that

$$\rho+3p\geq 0\quad\Rightarrow\quad p\geq-\frac{1}{3}\rho. \tag{31}$$

Now, for the particular case of time-independent warps we found that, $p=-\rho$, then we can check that this condition is always fulfilled

$$p\geq-\frac{1}{3}\rho=\frac{1}{3}p\quad\Rightarrow\quad 1\geq\frac{1}{3}. \tag{32}$$

We find that at least for the time-independent warps, SEC holds for pressure. However, if we write this condition for density in the time-independent regime we obtain

$$\frac{1}{2}(\rho+3p)=\frac{1}{2}(\rho-3\rho)=-\rho\geq 0\quad\Rightarrow\quad\rho\leq 0. \tag{33}$$

It requires, then, positive pressure and negative energy density. This energy density problem has been studied since the original Alcubierre paper, and much work has been done in order to understand its causes and how to circumvent it. This is consistent with results in Santos's paper [33].

2.3.4 Null energy condition

The null energy condition (NEC) is analogous to WEC, with the timelike vector replaced by a null vector k^μ. That is

$$T_{\mu\nu}k^\mu k^\nu\geq 0\,. \tag{34}$$

Assuming the following vector, $k^\mu=\{a,b,0,0\}$, which is a light vector $k^\mu k_\mu=0$. This imposes a constraint over the components with two possible solutions for a

$$a_{\pm} = \frac{b}{\beta \pm 1}, \tag{35}$$

With this result, the left hand side of Eq. (34) is

$$T_{\mu\nu}k^{\mu}k^{\nu} = \left(\frac{b}{\beta \pm 1}\right)^2 (\rho + p) \tag{36}$$

So we find that, $T_{\mu\nu}k^{\mu}k^{\nu} \geq 0$ imposes the following constraint $\Rightarrow p + \rho \geq 0$. Moreover, we find that given a time-independent warp, which implies $p = -\rho$, we find that $p + \rho = 0$ and NEC always holds.

3 Anisotropic warp with cosmological constant

In this section we explore a system with a more general energy–momentum tensor that includes cosmological constant. In this case, the Einstein equation is

$$G_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} \,. \tag{37}$$

The Einstein tensor followed the metric (4) is given by

$$G_{00} = (1-\beta^2)\Lambda + \frac{\beta}{r^2}\left[\left(1-\beta^2\right)\left(\beta + 2r\frac{\partial\beta}{\partial r}\right) - 2r\beta\frac{\partial\beta}{\partial t}\right], \tag{38}$$

$$G_{01} = \beta\Lambda + \frac{\beta}{r^2}\left(\beta^2 + 2r\beta\frac{\partial\beta}{\partial r} + 2r\frac{\partial\beta}{\partial t}\right), \tag{39}$$

$$G_{11} = -\Lambda - \frac{1}{r^2}\left(\beta^2 + 2r\beta\frac{\partial\beta}{\partial r} + 2r\frac{\partial\beta}{\partial t}\right), \tag{40}$$

$$G_{22} = -r^2\Lambda - r\left\{\beta\left(2\frac{\partial\beta}{\partial r} + r\frac{\partial^2\beta}{\partial r^2}\right) + \frac{\partial\beta}{\partial t} + r\left[\left(\frac{\partial\beta}{\partial r}\right)^2 + \frac{\partial^2\beta}{\partial t\partial r}\right]\right\}, \tag{41}$$

$$G_{33} = -r^2\sin^2\theta\Lambda - r\sin^2\theta\left\{\beta\left(2\frac{\partial\beta}{\partial r} + r\frac{\partial^2\beta}{\partial r^2}\right) + \frac{\partial\beta}{\partial t} + r\left[\left(\frac{\partial\beta}{\partial r}\right)^2 + \frac{\partial^2\beta}{\partial t\partial r}\right]\right\}. \tag{42}$$

In order to study the matter content, we consider an Eulerian observer as it was done in the previous section.

3.1 Generalized fluid energy–momentum tensor

The perfect fluid energy–momentum tensor motivates a generalized expression which could include contributions from pressure anisotropy and heat transfer. We propose the ansatz.

$$T_{\mu\nu} = \begin{bmatrix} \rho + \beta^2 p_r & -\beta D & 0 & 0 \\ -\beta D & A & 0 & 0 \\ 0 & 0 & r^2 B & 0 \\ 0 & 0 & 0 & r^2\sin(\theta)^2 C \end{bmatrix}. \tag{43}$$

Here, p_r is the radial pressure and A, B, C, D are functions to be determined. From these expressions and Einstein's equations, it is straightforward to verify that $A = D$ and $B = C$. Moreover, consistency with Einstein's equations and also the perfect fluid case requires that the more general energy–momentum tensor consistent with the metric is given by

$$A = D = p_r\,, \quad B = C = p_{\perp}. \tag{44}$$

This corresponds to an anisotropic fluid, which reduces to isotropic case when $p_{\perp} = p_r$. Note that heat fluxes are not allowed for the metric proposed.

Now we write Einstein's equations in a convenient form. For this, we use the relation $T_{\mu\nu}u^{\mu}u^{\nu} = \rho$. This implies

$$8\pi T_{\mu\nu} = G_{\mu\nu} \quad \Rightarrow \quad \rho = T_{\mu\nu}u^{\mu}u^{\nu} = \frac{1}{8\pi}G_{\mu\nu}u^{\mu}u^{\nu}\,. \tag{45}$$

Using this relation it is possible to write Einstein's equations as

$$\Lambda + \frac{\beta}{r^2}\left(\beta + 2r\frac{\partial\beta}{\partial r}\right) = 8\pi\rho, \tag{46}$$

$$\Lambda + \frac{\beta}{r^2}\left(\beta + 2r\frac{\partial\beta}{\partial r}\right) + \frac{2}{r}\frac{\partial\beta}{\partial t} = -8\pi p_r, \tag{47}$$

$$\beta^2 + r\frac{\partial\beta}{\partial t} - r^2\left[\frac{\partial}{\partial r}\left(\beta\frac{\partial\beta}{\partial r}\right) + \frac{\partial^2\beta}{\partial t\partial r}\right] = 8\pi r^2\Delta, \tag{48}$$

where $\Delta = p_{\perp} - p_r$ is the anisotropy factor. These equations correspond with a time-dependent anisotropic warp. For time-independent anisotropic warp, we have the following expressions

$$\Lambda + \frac{\beta}{r^2}\left(\beta + 2r\frac{d\beta}{dr}\right) = 8\pi\rho, \tag{49}$$

$$\Lambda + \frac{\beta}{r^2}\left(\beta + 2r\frac{d\beta}{dr}\right) = -8\pi p_r, \tag{50}$$

$$\beta^2 - r^2\left[\frac{d}{dr}\left(\beta\frac{d\beta}{dr}\right)\right] = 8\pi r^2\Delta. \tag{51}$$

As in the isotropic case, time-independent warp implies the equation of state $p_r = -\rho$. This more general model allows us to adjust several parameters and obtain other interesting models, e.g. $\{\Lambda = 0, \Delta \neq 0\}$ or $\{\Lambda \neq 0, \Delta = 0\}$.

In the next sections we examine energy conditions for Eulerian observer using the anisotropic energy–momentum tensor.

3.2 Energy conditions

Now we explore the energy conditions for the generalized fluid energy–momentum tensor.

3.2.1 Weak and dominant energy conditions

For Weak and Dominant energy conditions we do not have any modifications, and the analysis is exactly the same as in the perfect fluid system, so given $\rho \geq 0$, both conditions are fulfilled.

3.2.2 Strong energy condition

The strong energy condition is given by

$$\left(T_{\mu\nu} - \frac{1}{2}T g_{\mu\nu}\right) u^{\mu}u^{\nu} = \rho + p_r + 2p_{\perp} \geq 0. \tag{52}$$

Because for time-independent warp we have $p_r = -\rho$, we obtain the following condition for tangential pressure

$$p_{\perp} \geq 0. \tag{53}$$

Tangential pressure has to be positive for the condition to be satisfied with time-independent warp.

3.2.3 Null energy condition

The null energy condition implies

$$T_{\mu\nu}k^{\mu}k^{\nu} \geq 0 \quad \rightarrow \quad \rho + p_r \geq 0. \tag{54}$$

Again, if time independent warp is considered, we have $p_r = -\rho$ and the NEC holds.

So, at least for time-independent warp, we have found that energy conditions are fulfilled for the proposed metric.

4 Final remarks

In this work we have analyzed a spherically symmetric line element that allows warp motion, in the sense that it introduces an inertial moving shell material, which encloses a region or bubble characterized by a function β. We started with a general time and space dependent warp, with a mapping between the outer and inner observers similar to Alcubierre's, and obtained a set of field equations for the bubble shape functions and the matter content. Further restrictions on the metric were imposed, for instance, a stationary metric is considered, but allowing a space dependence on the function β. As matter sources we have used the isotropic perfect fluid and the anisotropic fluid [33]. The latter reduces to the case of isotropic perfect fluid as we have already seen.

By writing the metric using spherical symmetry, it allows us to find a cleaner system of equations from which it is possible to draw conclusions with greater clarity, thereby eliminating the spurious relations from dependent equations. This is particularly useful when dealing with problems involving material distributions, where one can clearly read the links with matter and the energy conditions involved.

An interesting aspect is that by imposing a time-independent warp, there naturally appears an equation of state of the form $p_r = -\rho$ which is consistent with the system having a temporal Killing vector. This equation of state is well known from cosmological Λ-Cold Dark Matter (ΛCDM) models. We believe that this aspect deserves further study.

We also calculated the weak, dominant, strong and null energy conditions, from which we have been able to extract constraints for the components of the energy–momentum tensor and thus relations for the matter that sustains the warp condition. We found that, at least for time independent warps, all energy conditions are satisfied with the exception of the SEC. It is worth mentioning that there are signs that WEC and NEC may be violated at the quantum-microscopic scale [40–49], although they seem to be satisfied at the macroscopic scales. However, SEC seems to be violated at the largest cosmological scales [50–54]. We believe that this fact is very interesting and promising. Also, it is worth noting that the spherically symmetric warp metric (13) does not support heat dissipation, as we have shown.

Summarizing, it can be said that the study of non-trivial matter configurations is fundamental for the understanding of warp drives as well as their possible physical feasibility and limitations. These matter configurations will necessarily influence the shape of the warp bubble and its possibilities as a form of propulsion. The configurations studied in this work open new possibilities for obtaining a physical viable warp drive.

Based on the results obtained, we believe that it is essential to continue the research on different and more complex matter configurations. Different metric realizations that could include dissipation, heat flux and electromagnetic fields can be studied on the basis of our proposed metric. Understanding these aspects may be the key to achieving the stabilisation of the warp drive geometry and thus to develop a viable warp drive system.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: All the required theoretical data and the figures were already provided by the authors.]

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References

1. M.S. Morris, K.S. Thorne, Wormholes in space-time and their use for interstellar travel: a tool for teaching general relativity. Am. J. Phys. **56**, 395–412 (1988). https://doi.org/10.1119/1.15620
2. M. Visser, Traversable wormholes: some simple examples. Phys. Rev. D **39**, 3182–3184 (1989). https://doi.org/10.1103/PhysRevD.39.3182. arXiv:0809.0907 [gr-qc]
3. S.V. Krasnikov, Hyperfast travel in general relativity. Phys. Rev. D **57**, 4760–4766 (1998). https://doi.org/10.1103/PhysRevD.57.4760. arXiv:gr-qc/9511068
4. A.E. Everett, T.A. Roman, A superluminal subway: the Krasnikov tube. Phys. Rev. D **56**, 2100–2108 (1997). https://doi.org/10.1103/PhysRevD.56.2100. arXiv:gr-qc/9702049
5. M. Alcubierre, The Warp drive: Hyperfast travel within general relativity. Class. Quantum Gravity **11**, 73–77 (1994). https://doi.org/10.1088/0264-9381/11/5/001. arXiv:gr-qc/0009013
6. R.L. Arnowitt, S. Deser, C.W. Misner, The dynamics of general relativity. Gen. Relativ. Gravit. **40**, 1997–2027 (2008). https://doi.org/10.1007/s10714-008-0661-1. arXiv:gr-qc/0405109
7. R.M. Wald, General Relativity (1984)
8. M. Alcubierre, Introduction to 3+1 Numerical Relativity (2008)
9. C. Van Den Broeck, A 'Warp drive' with reasonable total energy requirements. Class. Quantum Gravity **16**, 3973–3979 (1999). https://doi.org/10.1088/0264-9381/16/12/314. arXiv:gr-qc/9905084
10. J. Natário, Warp drive with zero expansion. Class. Quantum Gravity **19**, 1157–1166 (2002). https://doi.org/10.1088/0264-9381/19/6/308. arXiv:gr-qc/0110086
11. F.S.N. Lobo, M. Visser, Linearized warp drive and the energy conditions. In: 27th Spanish Relativity Meeting: Beyond General Relativity (ERE 2004) (2004)
12. K.D. Olum, Superluminal travel requires negative energies. Phys. Rev. Lett. **81**, 3567–3570 (1998). https://doi.org/10.1103/PhysRevLett.81.3567. arXiv:gr-qc/9805003
13. M.J. Pfenning, Quantum inequality restrictions on negative energy densities in curved space-times. Other thesis (1998)
14. R.J. Low, Speed limits in general relativity. Class. Quantum Gravity **16**, 543–549 (1999). https://doi.org/10.1088/0264-9381/16/2/016. arXiv:gr-qc/9812067
15. C. Van Den Broeck, On the (im)possibility of warp bubbles (1999). arXiv:gr-qc/9906050
16. C. Barcelo, M. Visser, Scalar fields, energy conditions, and traversable wormholes. Class. Quantum Gravity **17**, 3843–3864 (2000). https://doi.org/10.1088/0264-9381/17/18/318. arXiv:gr-qc/0003025
17. F. Lobo, P. Crawford, Weak energy condition violation and superluminal travel. Lect. Notes Phys. **617**, 277–291 (2003). arXiv:gr-qc/0204038
18. C. Barcelo, M. Visser, Twilight for the energy conditions? Int. J. Mod. Phys. D **11**, 1553–1560 (2002). https://doi.org/10.1142/S0218271802002888. arXiv:gr-qc/0205066
19. F.S.N. Lobo, M. Visser, Fundamental limitations on 'warp drive' spacetimes. Class. Quantum Gravity **21**, 5871–5892 (2004). https://doi.org/10.1088/0264-9381/21/24/011. arXiv:gr-qc/0406083
20. B. McMonigal, G.F. Lewis, P. O'Byrne, The Alcubierre warp drive: on the matter of matter. Phys. Rev. D **85**, 064024 (2012). https://doi.org/10.1103/PhysRevD.85.064024. arXiv:1202.5708 [gr-qc]
21. M. Alcubierre, F.S.N. Lobo, Warp drive basics. Fundam. Theor. Phys. **189**, 257–279 (2017). https://doi.org/10.1007/978-3-319-55182-1_11
22. M. Alcubierre, F.S.N. Lobo, *Wormholes, Warp Drives and Energy Conditions*, vol. 189 (Springer, Berlin, 2017). https://doi.org/10.1007/978-3-319-55182-1
23. E.W. Lentz, Breaking the warp barrier: hyper-fast solitons in Einstein–Maxwell-plasma theory. Class. Quantum Gravity **38**(7), 075015 (2021). https://doi.org/10.1088/1361-6382/abe692. arXiv:2006.07125 [gr-qc]
24. J. Santiago, S. Schuster, M. Visser, Generic warp drives violate the null energy condition. Phys. Rev. D **105**(6), 064038 (2022). https://doi.org/10.1103/PhysRevD.105.064038. arXiv:2105.03079 [gr-qc]
25. W.A. Hiscock, Quantum effects in the Alcubierre warp drive spacetime. Class. Quantum Gravity **14**, 183–188 (1997). https://doi.org/10.1088/0264-9381/14/11/002. arXiv:gr-qc/9707024
26. C. Clark, W.A. Hiscock, S.L. Larson, Null geodesics in the Alcubierre warp drive space-time: the view from the bridge. Class. Quantum Gravity **16**, 3965–3972 (1999). https://doi.org/10.1088/0264-9381/16/12/313. arXiv:gr-qc/9907019
27. P.F. Gonzalez-Diaz, On the warp drive space-time. Phys. Rev. D **62**, 044005 (2000). https://doi.org/10.1103/PhysRevD.62.044005. arXiv:gr-qc/9907026
28. A.E. Everett, Warp drive and causality. Phys. Rev. D **53**, 7365–7368 (1996). https://doi.org/10.1103/PhysRevD.53.7365
29. O.L. Santos-Pereira, E.M.C. Abreu, M.B. Ribeiro, Dust content solutions for the Alcubierre warp drive spacetime. Eur. Phys. J. C **80**(8), 786 (2020). https://doi.org/10.1140/epjc/s10052-020-8355-2. arXiv:2008.06560 [gr-qc]
30. O.L. Santos-Pereira, E.M.C. Abreu, M.B. Ribeiro, Perfect fluid warp drive solutions with the cosmological constant. Eur. Phys. J. Plus **136**(9), 902 (2021). https://doi.org/10.1140/epjp/s13360-021-01899-7. arXiv:2108.10960 [gr-qc]
31. O.L. Santos-Pereira, E.M.C. Abreu, M.B. Ribeiro, Warp drive dynamic solutions considering different fluid sources. In: *16th Marcel Grossmann Meeting on Recent Developments in Theoretical and Experimental General Relativity, Astrophysics and Relativistic Field Theories* (2021)
32. O.L. Santos-Pereira, E.M.C. Abreu, M.B. Ribeiro, Charged dust solutions for the warp drive spacetime. Gen. Relativ. Gravit. **53**(2), 23 (2021). https://doi.org/10.1007/s10714-021-02799-y. arXiv:2102.05119 [gr-qc]
33. O.L. Santos-Pereira, E.M.C. Abreu, M.B. Ribeiro, Fluid dynamics in the warp drive spacetime geometry. Eur. Phys. J. C **81**(2), 133 (2021). https://doi.org/10.1140/epjc/s10052-021-08921-3. arXiv:2101.11467 [gr-qc]
34. A. Bobrick, G. Martire, Introducing physical warp drives. Class. Quantum Gravity **38**(10), 105009 (2021). https://doi.org/10.1088/1361-6382/abdf6e. arXiv:2102.06824 [gr-qc]
35. S.M. Carroll, Spacetime and Geometry. An Introduction to General Relativity (2004)
36. M. Visser, Lorentzian Wormholes. From Einstein to Hawking (1995)
37. E. Poisson, A Relativist's Toolkit: The Mathematics of Black-hole Mechanics (2004)
38. E. Curiel, A primer on energy conditions. Einstein Stud. **13**, 43–104 (2017). https://doi.org/10.1007/978-1-4939-3210-8_3. arXiv:1405.0403 [physics.hist-ph]
39. E.-A. Kontou, K. Sanders, Energy conditions in general relativity and quantum field theory. Class. Quantum Gravity **37**(19), 193001 (2020). https://doi.org/10.1088/1361-6382/ab8fcf. arXiv:2003.01815 [gr-qc]

40. L.H. Ford, T.A. Roman, Averaged energy conditions and quantum inequalities. Phys. Rev. D **51**, 4277–4286 (1995). https://doi.org/10.1103/PhysRevD.51.4277. arXiv:gr-qc/9410043
41. M. Visser, Gravitational vacuum polarization. 1: energy conditions in the Hartle–Hawking vacuum. Phys. Rev. D **54**, 5103–5115 (1996). https://doi.org/10.1103/PhysRevD.54.5103. arXiv:gr-qc/9604007
42. M. Visser, Gravitational vacuum polarization. 2: energy conditions in the Boulware vacuum. Phys. Rev. D **54**, 5116–5122 (1996). https://doi.org/10.1103/PhysRevD.54.5116. arXiv:gr-qc/9604008
43. M. Visser, Gravitational vacuum polarization. 3: energy conditions in the (1+1) Schwarzschild space-time. Phys. Rev. D **54**, 5123–5128 (1996). https://doi.org/10.1103/PhysRevD.54.5123. arXiv:gr-qc/9604009
44. M. Visser, Gravitational vacuum polarization. 4: energy conditions in the Unruh vacuum. Phys. Rev. D **56**, 936–952 (1997). https://doi.org/10.1103/PhysRevD.56.936. arXiv:gr-qc/9703001
45. C.J. Fewster, T.A. Roman, Null energy conditions in quantum field theory. Phys. Rev. D **67**, 044003 (2003) [Erratum: Phys. Rev. D 80, 069903 (2009)]. https://doi.org/10.1103/PhysRevD.67.044003. arXiv:gr-qc/0209036
46. L.H. Ford, Spacetime in semiclassical gravity. 100 years of relativity: space-time structure: Einstein and beyond, pp. 293–310 (2005). https://doi.org/10.1142/9789812700988_0011. arXiv:gr-qc/0504096
47. C.J. Fewster, G.J. Galloway, Singularity theorems from weakened energy conditions. Class. Quantum Gravity **28**, 125009 (2011). https://doi.org/10.1088/0264-9381/28/12/125009. arXiv:1012.6038 [gr-qc]
48. P. Martín-Moruno, M. Visser, Classical and quantum flux energy conditions for quantum vacuum states. Phys. Rev. D **88**(6), 061701 (2013). https://doi.org/10.1103/PhysRevD.88.061701. arXiv:1305.1993 [gr-qc]
49. P. Martin-Moruno, M. Visser, Semiclassical energy conditions for quantum vacuum states. JHEP **09**, 050 (2013). https://doi.org/10.1007/JHEP09(2013)050. arXiv:1306.2076 [gr-qc]
50. M. Visser, General relativistic energy conditions: the Hubble expansion in the epoch of galaxy formation. Phys. Rev. D **56**, 7578–7587 (1997). https://doi.org/10.1103/PhysRevD.56.7578. arXiv:gr-qc/9705070
51. M. Visser, Energy conditions in the epoch of galaxy formation. Science **276**, 88–90 (1997). https://doi.org/10.1126/science.276.5309.88. arXiv:1501.01619 [gr-qc]
52. M. Visser, C. Barcelo, Energy conditions and their cosmological implications. In: *3rd International Conference on Particle Physics and the Early Universe*, pp. 98–112 (2000). https://doi.org/10.1142/9789812792129_0014
53. C. Cattoen, M. Visser, Cosmodynamics: energy conditions, Hubble bounds, density bounds, time and distance bounds. Class. Quantum Gravity **25**, 165013 (2008). https://doi.org/10.1088/0264-9381/25/16/165013. arXiv:0712.1619 [gr-qc]
54. E.-A. Kontou, K.D. Olum, Energy conditions allow eternal inflation. JCAP **03**, 097 (2021). https://doi.org/10.1088/1475-7516/2021/03/097. arXiv:2008.01878 [gr-qc]