Systematic studies of charmonium-, bottomonium-, and B_c-like tetraquark states

Jing Wu[†]
School of Science, Shandong Jianzhu University, Jinan 250101, China

Xiang Liu[‡]
School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China and Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China

Yan-Rui Liu[*]
School of Physics, Shandong University, Jinan 250100, China

Shi-Lin Zhu[§]
School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China and Collaborative Innovation Center of Quantum Matter, Beijing 100871, China and Center of High Energy Physics, Peking University, Beijing 100871, China

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We study the mass splittings of $Q_1 q_2 \bar{Q}_3 \bar{q}_4$ ($Q = c$, b, $q = u$, d, s) tetraquark states with chromomagnetic interactions between their quark components. Assuming that $X(4140)$ is the lowest $J^{PC} = 1^{++}$ $cs\bar{c}\bar{s}$ tetraquark, we estimate the masses of the other tetraquark states. From the obtained masses and defined measure reflecting effective quark interactions, we find the following assignments for several exotic states: (1) both $X(3860)$ and the newly observed $Z_c(4100)$ seem to be 0^{++} $cn\bar{c}\bar{n}$ tetraquarks; (2) $Z_c(4200)$ is probably a 1^{+-} $cn\bar{c}\bar{n}$ tetraquark; (3) $Z_c(3900)$, $X(3940)$, and $X(4160)$ are unlikely compact tetraquarks; (4) $Z_c(4020)$ is unlikely a compact tetraquark, but seems the hidden-charm correspondence of $Z_b(10650)$ with $J^{PC} = 1^{+-}$; and (5) $Z_c(4250)$ can be a tetraquark candidate but the quantum numbers cannot be assigned at present. We hope further studies may check the predictions and assignments given here.

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I. INTRODUCTION

A hot topic in hadron physics study is to identify multiquark states from the observed exotic structures. Through explorations on their masses, productions, and decay properties, we may understand the problem how the strong interaction forces nonobservable quarks and gluons to form observable hadrons. Before 2003, the situation in understanding hadron structures was simple because the quark model gave a successful and satisfactory description for hadron spectra [1], although there exist a few hadrons difficult to understand. In 2003, experimentalists opened the Pandora's box for exotic states through the observation of $X(3872)$ [2]. Since then, more and more unexpected XYZ states were observed and the situation for hadron physics study became complicated [3–12]. To understand a little more of the above mentioned problem, the discussions in this work aim at basic features of ground charmonium-like, bottomonium-like, and B_c-like tetraquark states with even P-parities.

As the first exotic charmonium-like state above the $D\bar{D}$ threshold, the $X(3872)$ motivated heated discussions on its nature [5,10]. Its J^{PC} are determined to be 1^{++} but the mass is tens of MeV lower than the quark model prediction if it is a charmonium. Since the meson is extremely close to the $D^0\bar{D}^{*0}$ threshold, it is widely regarded as a loosely bound $D\bar{D}^*$ molecule. Discussions in the tetraquark picture and

[*]Corresponding author.
yrliu@sdu.edu.cn
[†]wujing18@sdjzu.edu.cn
[‡]xiangliu@lzu.edu.cn
[§]zhusl@pku.edu.cn

hybrid picture are also performed. However, without a $c\bar{c}$ core, it is difficult to understand the measured ratios $\mathcal{B}(X(3872) \to \Psi(2S)\gamma):\mathcal{B}(X(3872) \to J/\Psi\gamma) = 2.46 \pm 0.64 \pm 0.29$ by LHCb [13]. The $X(3872)$ seems to be a charmonium affected significantly by the $D\bar{D}^*$ threshold [14,15]. Anyway, one cannot identify this exotic meson as a pure tetraquark state.

To identify multiquark states, we may look for structures according to several ideas. The easiest approach is to study structures with explicitly exotic quantum numbers, e.g., charged charmonium-like or bottomonium-like states. The quark content of the charged hidden-charm or hidden-bottom mesons should be at least four if their nonresonance interpretations are excluded. Up to now, experiments have observed several charged structures, $Z_c(4430)$ [16–18], $Z_c(4050)$ [19], $Z_c(4250)$ [19], $Z_c(3900)$ [20–24], $Z_c(3885)$ [25–27], $Z_c(4020)$ [28,29], $Z_c(4025)$ [30,31], and so on. Very recently, LHCb found the evidence for a charged charmonium-like resonance $Z_c^-(4100)$ in the decay $B^0 \to Z_c^-(4100)K^+ \to \eta_c\pi^-K^+$ [32]. The measured mass and width are $4096 \pm 20^{+18}_{-22}$ MeV and $152 \pm 58^{+60}_{-35}$ MeV, respectively. Its possible quantum numbers are $J^P = 0^+$ or 1^-. They are certainly four-quark state candidates. However, it is not easy to justify whether they are compact tetraquarks or meson-meson molecules. In this paper, we will try to understand whether parts of these charged states are compact tetraquarks or just molecules.

It is also possible to identify a multiquark state from its high mass that a conventional hadron cannot have. The observed $P_c(4380)$ and $P_c(4450)$ by the LHC Collaboration [33] are two such states. They look like excited nucleons but can be identified as pentaquark states because an orbital or radial excitation energy larger than 3 GeV for light quarks is an unnatural interpretation for the high masses while the creation of a $c\bar{c}$ pair can naturally explain. Ref. [34] predicted the existence of hidden-charm pentaquarks with this idea. Similarly, one may identify other high mass states looking like conventional hadrons as multiquark states if experiments could observe them. However, one still cannot easily distinguish compact tetraquarks from molecules except the $QQ\bar{Q}\bar{q}$ case [35–37] in this possibility.

If experiments could observe an exotic structure that the molecule picture is not applicable, it is possible to identify it as a compact tetraquark. In Refs. [38,39], the D0 Collaboration claimed an exotic $B_s^0\pi^\pm$ state and named it $X(5568)$. This meson contains four different flavors. From its low mass (~200 MeV lower than the $B\bar{K}$ threshold), the $X(5568)$ is unlikely a molecule. If it really exists, it might be a compact tetraquark. Unfortunately, the LHCb [40], CMS [41], CDF [42], and ATLAS [43] Collaborations did not confirm this state. The identification of compact tetraquarks along this idea has not been achieved yet.

We have one more possibility to identify compact multiquarks through number of states. The exotic structure $X(4140)$ was first observed by the CDF Collaboration [44] in the invariant mass distribution of $J/\psi\phi$. In the latter measurements with the same channel by various collaborations [45–49], LHCb confirmed the $X(4140)$, determined its quantum numbers to be $J^{PC} = 1^{++}$, established another 1^{++} state $X(4274)$, and observed two more 0^{++} structures $X(4500)$ and $X(4700)$. The existence of two 1^{++} states does not support the molecule interpretations for them [49]. On the other hand, the $cs\bar{c}\bar{s}$ tetraquark configuration can account for such an observation [50]. This picture also favors the assignment for the Belle $X(4350)$ [51] as their 0^{++} tetraquark partner [52]. In this paper, we identify the $X(4140)$ as the lowest 1^{++} $cs\bar{c}\bar{s}$ tetraquark state and use its mass as an input to estimate the masses of other charmonium-, bottomonium-, and B_c-like tetraquark states.

This paper is organized as follows. After the introductory Sec. I, we present the theoretical formalism in Sec. II by showing necessary wave functions and Hamiltonian matrices. In Sec. III, we determine model parameters, present strategy for the estimation of tetraquark masses, list numerical results, analyze possible assignments for the observed exotic mesons, and predict possible tetraquarks. The last section is for discussions and summary.

II. FORMALISM

In this article, we use the notation $Q_1q_2\bar{Q}_3\bar{q}_4$ ($Q = c, b$; $q = n, s$; $n = u, d$) to generally denote the considered system. If the system is truly neutral, $Q_1 = Q_3 = Q$, $q_2 = q_4 = q$ and the notation becomes $Qq\bar{Q}\bar{q}$. From the $SU(3)_f$ symmetry, the tetraquarks belong to 8_f and 1_f representations. Since the flavor symmetry is broken, the isoscalar states would mix with some angle. In principle, the resulting flavor wave functions of the physical $I = 0$ states contain both $Q_1n\bar{Q}_3\bar{n}$ and $Q_1s\bar{Q}_3\bar{s}$ parts. At present, we just consider the ideal mixing case, i.e., $Q_1n\bar{Q}_3\bar{n}$ and $Q_1s\bar{Q}_3\bar{s}$ do not mix.

The effective Hamiltonian in the adopted chromomagnetic interaction (CMI) model reads,

$$H = \sum_i m_i + H_{\mathrm{CM}} = \sum_i m_i - \sum_{i<j} C_{ij}\tilde{\lambda}_i \cdot \tilde{\lambda}_j \sigma_i \cdot \sigma_j, \qquad (1)$$

where $\tilde{\lambda}_i = \lambda_i$ $(-\lambda_i^*)$ for quarks (antiquarks). The involved parameters are only effective coupling constants C_{ij} and effective masses m_i containing various effects. This Hamiltonian is reduced from a realistic model, which can be found in Refs. [53,54]. Then the formula for the mass estimation is

$$M = \sum_{i=1} m_i + \langle H_{\mathrm{CM}} \rangle. \qquad (2)$$

In calculating the last term, we use the diquark-antidiquark bases to express the wave functions for the S-wave

$Q_1q_2\bar{Q}_3\bar{q}_4$ systems whose P-parities are always positive. Here, the notation "diquark" just means two quarks and does not mean a compact substructure. If one uses the meson-meson bases, the same eigenvalues after diagonalization will be obtained. In the present case, the Pauli principle has no restriction on the total wave functions, but one should notice the possible C-parity once a state is truly neutral. The involved color (spin) wave functions $\phi_{1,2}$ ($\chi_{1,2,\ldots,6}$) are

$$\begin{aligned}
&\phi_1 = |\bar{3}_c, 3_c, 1_c\rangle, \qquad \phi_2 = |6_c, \bar{6}_c, 1_c\rangle,\\
&\chi_1 = |1_S, 1_S, 2_S\rangle, \qquad \chi_2 = |1_S, 1_S, 1_S\rangle, \qquad \chi_3 = |1_S, 1_S, 0_S\rangle,\\
&\chi_4 = |1_S, 0_S, 1_S\rangle, \qquad \chi_5 = |0_S, 1_S, 1_S\rangle, \qquad \chi_6 = |0_S, 0_S, 0_S\rangle,
\end{aligned} \tag{3}$$

where the color representations (spins) in order in ϕ_i (χ_j) are for diquark, antidiquark, and system, respectively. We define the total wave function as

$$\phi_i\chi_j \equiv (Q_1q_2\bar{Q}_3\bar{q}_4) \otimes \phi_i \otimes \chi_j. \tag{4}$$

Compared with the $cs\bar{c}\bar{s}$ case where a C-parity can be given, the CMI matrices in the present cases are the generalized ones in Ref. [52]. Now we have

$$\langle H_{\rm CM}\rangle_{J^P=2^+} = \begin{pmatrix} \frac{4}{3}(2\tau+\alpha) & 2\sqrt{2}\nu \\ & \frac{2}{3}(5\alpha-2\tau) \end{pmatrix}, \tag{5}$$

$$\langle H_{\rm CM}\rangle_{J^P=0^+} = \begin{pmatrix} \frac{8}{3}(\tau-\alpha) & 4\sqrt{2}\nu & -\frac{4}{\sqrt{3}}\nu & 2\sqrt{6}\alpha \\ & -\frac{4}{3}(\tau+5\alpha) & 2\sqrt{6}\alpha & -\frac{10}{\sqrt{3}}\nu \\ & & -8\tau & 0 \\ & & & 4\tau \end{pmatrix}, \tag{6}$$

and

$$\langle H_{\rm CM}\rangle_{J^P=1^+} = \begin{pmatrix} \frac{4}{3}(2\tau-\alpha) & \frac{4\sqrt{2}}{3}\beta & -\frac{4\sqrt{2}}{3}\mu & 2\sqrt{2}\nu & -4\mu & 4\beta \\ & \frac{8}{3}(2\theta-\tau) & \frac{4}{3}\nu & -4\mu & 0 & -2\sqrt{2}\alpha \\ & & -\frac{8}{3}(\tau+2\theta) & 4\beta & -2\sqrt{2}\alpha & 0 \\ & & & -\frac{2}{3}(2\tau+5\alpha) & \frac{10\sqrt{2}}{3}\beta & -\frac{10\sqrt{2}}{3}\mu \\ & & & & \frac{4}{3}(\tau-2\theta) & \frac{10}{3}\nu \\ & & & & & \frac{4}{3}(\tau+2\theta) \end{pmatrix}, \tag{7}$$

where the defined variables are

$$\begin{aligned}
&\tau = C_{12}+C_{34}, \qquad \theta = C_{12}-C_{34},\\
&\alpha = C_{13}+C_{24}+C_{14}+C_{23},\\
&\beta = C_{13}-C_{24}-C_{14}+C_{23},\\
&\mu = C_{13}-C_{24}+C_{14}-C_{23},\\
&\nu = C_{13}+C_{24}-C_{14}-C_{23}
\end{aligned} \tag{8}$$

and the corresponding base vectors for the matrices are $(\phi_1\chi_1, \phi_2\chi_2)^T$, $(\phi_1\chi_3, \phi_2\chi_3, \phi_1\chi_6, \phi_2\chi_6)^T$, and $(\phi_1\chi_2, \phi_1\chi_4, \phi_1\chi_5, \phi_2\chi_2, \phi_2\chi_4, \phi_2\chi_5)^T$, respectively. When the considered state is truly neutral, the matrices for the cases $J^{PC}=2^{++}$ and 0^{++} are the same as above, but that for the case $J^{PC}=1^{++}$ is

$$\langle H_{\rm CM}\rangle = \begin{pmatrix} -\frac{4}{3}(4C_{Qq}-C_{Q\bar{Q}}-C_{q\bar{q}}+2C_{Q\bar{q}}) & -2\sqrt{2}(C_{Q\bar{Q}}+C_{q\bar{q}}+2C_{Q\bar{q}}) \\ & \frac{2}{3}(4C_{Qq}+5C_{Q\bar{Q}}+5C_{q\bar{q}}-10C_{Q\bar{q}}) \end{pmatrix} \tag{9}$$

and that for the case $J^{PC}=1^{+-}$ is

$$\langle H_{\mathrm{CM}}\rangle = \begin{pmatrix} \frac{4}{3}\begin{pmatrix} 4C_{Qq}-2C_{Q\bar{q}} \\ -C_{Q\bar{Q}}-C_{q\bar{q}} \end{pmatrix} & 2\sqrt{2}\begin{pmatrix} C_{Q\bar{Q}}+C_{q\bar{q}} \\ -2C_{Q\bar{q}} \end{pmatrix} & \frac{8}{3}(C_{Q\bar{Q}}-C_{q\bar{q}}) & -4\sqrt{2}(C_{Q\bar{Q}}-C_{q\bar{q}}) \\ & -\frac{2}{3}\begin{pmatrix} 4C_{Qq}+10C_{Q\bar{q}} \\ +5C_{Q\bar{Q}}+5C_{q\bar{q}} \end{pmatrix} & -4\sqrt{2}(C_{Q\bar{Q}}-C_{q\bar{q}}) & \frac{20}{3}(C_{Q\bar{Q}}-C_{q\bar{q}}) \\ & & -\frac{4}{3}\begin{pmatrix} 4C_{Qq}-2C_{Q\bar{q}} \\ +C_{Q\bar{Q}}+C_{q\bar{q}} \end{pmatrix} & 2\sqrt{2}\begin{pmatrix} C_{Q\bar{Q}}+C_{q\bar{q}} \\ +2C_{Q\bar{q}} \end{pmatrix} \\ & & & \frac{2}{3}\begin{pmatrix} 4C_{Qq}+10C_{Q\bar{q}} \\ -5C_{Q\bar{Q}}-5C_{q\bar{q}} \end{pmatrix} \end{pmatrix}. \tag{10}$$

Their corresponding base vectors are $(\phi_1\chi_e, \phi_2\chi_e)^T$ and $(\phi_1\chi_2, \phi_2\chi_2, \phi_1\chi_o, \phi_2\chi_o)^T$, respectively. Here $\phi_i\chi_e$ ($\phi_i\chi_o$) represents C-even (C-odd) wave function. The forms of such wave functions are similar to those obtained in Ref. [52]. Since we also consider the color structure $|6_c, \bar{6}_c, 1_c\rangle$ for the tetraquarks, the above Eqs. (5)–(10) can be actually thought of as generalizations of those for $|\bar{3}_c, 3_c, 1_c\rangle$ tetraquarks given in Ref. [55].

III. NUMERICAL ANALYSIS

A. Parameters and estimation strategy

The parameters in the CMI model are effective masses of the quarks and coupling strengths between quark components. We need 14 coupling strengths in the present study: C_{cn}, C_{cs}, C_{bn}, C_{bs}, $C_{c\bar{n}}$, $C_{c\bar{s}}$, $C_{b\bar{n}}$, $C_{b\bar{s}}$, $C_{c\bar{c}}$, $C_{b\bar{b}}$, $C_{c\bar{b}}$, $C_{n\bar{n}}$, $C_{s\bar{s}}$, and $C_{n\bar{s}}$. Most of them can be extracted from the measured masses [56] of the low-lying conventional hadrons (see Table I), but the determination of $C_{s\bar{s}}$ and $C_{c\bar{b}}$ needs approximations. We here assume $C_{s\bar{s}} = C_{ss}C_{n\bar{n}}/C_{nn} = 10.5$ MeV and adopt $C_{c\bar{b}} = 3.3$ MeV extracted from $M_{B_c^*} - M_{B_c} = 70$ MeV [1]. Parts of spectroscopic coupling parameters have been derived in Ref. [55]. The values of our coupling parameters are consistent with those in that paper, see discussions in Refs. [52,57]. The effective quark masses we extracted are $m_n = 361.7$ MeV, $m_s = 540.3$ MeV, $m_c = 1724.6$ MeV, and $m_b = 5052.8$ MeV, which are close to those obtained in Ref. [58].

When one substitutes these parameters into the mass formula (2), the tetraquark masses may be estimated. However, if we check the numerical values for the masses of the conventional hadrons with this formula and the above parameters, deviations from experimental results are found

TABLE I. Chromomagnetic interactions for various hadrons and obtained effective coupling constants in units of MeV.

Hadron	$\langle H_{\mathrm{CM}}\rangle$	Hadron	$\langle H_{\mathrm{CM}}\rangle$	C_{ij}
N	$-8C_{nn}$	Δ	$8C_{nn}$	$C_{nn} = 18.4$
Σ	$\frac{8}{3}C_{nn} - \frac{32}{3}C_{ns}$	Σ^*	$\frac{8}{3}C_{nn} + \frac{16}{3}C_{ns}$	$C_{ns} = 12.4$
Ξ^0	$\frac{8}{3}(C_{ss} - 4C_{ns})$	Ξ^{*0}	$\frac{8}{3}(C_{ss} + C_{ns})$	
Ω	$8C_{ss}$			$C_{ss} = 6.5$
Λ	$-8C_{nn}$			
π	$-16C_{n\bar{n}}$	ρ	$\frac{16}{3}C_{n\bar{n}}$	$C_{n\bar{n}} = 29.8$
K	$-16C_{n\bar{s}}$	K^*	$\frac{16}{3}C_{n\bar{s}}$	$C_{n\bar{s}} = 18.7$
D	$-16C_{c\bar{n}}$	D^*	$\frac{16}{3}C_{c\bar{n}}$	$C_{c\bar{n}} = 6.7$
D_s	$-16C_{c\bar{s}}$	D_s^*	$\frac{16}{3}C_{c\bar{s}}$	$C_{c\bar{s}} = 6.7$
B	$-16C_{b\bar{n}}$	B^*	$\frac{16}{3}C_{b\bar{n}}$	$C_{b\bar{n}} = 2.1$
B_s	$-16C_{b\bar{s}}$	B^*	$\frac{16}{3}C_{b\bar{s}}$	$C_{b\bar{s}} = 2.3$
η_c	$-16C_{c\bar{c}}$	J/ψ	$\frac{16}{3}C_{c\bar{c}}$	$C_{c\bar{c}} = 5.3$
η_b	$-16C_{b\bar{b}}$	Υ	$\frac{16}{3}C_{b\bar{b}}$	$C_{b\bar{b}} = 2.9$
Σ_c	$\frac{8}{3}C_{nn} - \frac{32}{3}C_{cn}$	Σ_c^*	$\frac{8}{3}C_{nn} + \frac{16}{3}C_{cn}$	$C_{cn} = 4.0$
Ξ_c'	$\frac{8}{3}C_{ns} - \frac{16}{3}C_{cn} - \frac{16}{3}C_{cs}$	Ξ_c^*	$\frac{8}{3}C_{ns} + \frac{8}{3}C_{cn} + \frac{8}{3}C_{cs}$	$C_{cs} = 4.5$
Σ_b	$\frac{8}{3}C_{nn} - \frac{32}{3}C_{bn}$	Σ_b^*	$\frac{8}{3}C_{nn} + \frac{16}{3}C_{bn}$	$C_{bn} = 1.3$
Ξ_b'	$\frac{8}{3}C_{ns} - \frac{16}{3}C_{bn} - \frac{16}{3}C_{bs}$	Ξ_b^*	$\frac{8}{3}C_{ns} + \frac{8}{3}C_{bn} + \frac{8}{3}C_{bs}$	$C_{bs} = 1.2$

(see Table IV of Ref. [54]). Usually, the obtained masses are larger than the measured values, which indicates that the attractions between quark components are not sufficiently considered in the simple model. The application of this formula to multiquark states should also lead to higher masses than those they should be. On the theoretical side, such values can be treated as upper limits of the tetraquark masses.

The reason for the overestimated masses is because of the adopted assumption that the above extracted parameters are applicable to every system. In principle, each system has its own values of parameters. From the reduction procedure for the model Hamiltonian and the fact that the spacial wave functions are not the same for different systems, this assumption certainly induces uncertainties. The uncertainties in coupling strengths affect the mass splittings between the considered tetraquark states and the effects should not be large. On the other hand, the uncertainties in the effective quark masses affect the mass shifts of the states, which may be significant. To reduce the uncertainties in mass estimation, we adopt another method by introducing a reference system and modifying the mass formula to be

$$M = (M_{\mathrm{ref}} - \langle H_{\mathrm{CM}}\rangle_{\mathrm{ref}}) + \langle H_{\mathrm{CM}}\rangle. \tag{11}$$

Here, M_{ref} and $\langle H_{\mathrm{CM}}\rangle_{\mathrm{ref}}$ are the physical mass of the reference system and the corresponding CMI eigenvalue, respectively. For M_{ref}, one may use the mass of a reference multiquark state or use the threshold of a reference hadron-hadron system whose quark content is the same as the considered multiquark states. With this method, the problem of using extracted quark masses from conventional hadrons in multiquark systems [50] is evaded and part of missed attractions between quark components is phenomenologically compensated. In previous studies [36,52,54,57,59–62], we mainly adopted hadron-hadron thresholds. One finds that the estimated multiquark masses with this method are always lower than those with Eq. (2). Since the number of thresholds may be more than 1, there is a question which threshold leads to more reasonable masses. As a multibody system, the size of a tetraquark state should be larger than that of a conventional hadron and the distance between two quark components in tetraquarks may be larger than that in a conventional meson. The resulting effect is that the attraction between quark components should be weaker. Thus, although we cannot give a definite answer, probably the meson-meson threshold leading to higher masses gives more reasonable tetraquark masses. In the present study, besides the possible hadron-hadron thresholds, we may additionally turn to $X(4140)$ by assuming it as the ground $cs\bar{c}\bar{s}$ tetraquark state with $J^{PC} = 1^{++}$. It seems that using $X(4140)$ as an input is a better approach than the adoption of meson-meson thresholds. In Ref. [52], we have performed the exploration for the $cs\bar{c}\bar{s}$ states with this input and gotten higher masses than with the $D_s\bar{D}_s$ threshold. This observation probably indicates that the highest masses estimated with various hadron-hadron thresholds are still lower than the tetraquark masses. The discrepancy may be understood with the additional kinetic energy [63]. From the comparison for results in the current model [59] and in a dynamical study [64], the calculated masses of heavy-full tetraquark states are truly higher than the highest masses estimated with meson-meson thresholds but lower than the theoretical upper limits. In the following discussions, we use this feature as a criterion for reasonable tetraquark masses. The reasonability of the results may be tested in future studies.

B. Effective interactions and supplemental results for the $cs\bar{c}\bar{s}$ system

In Ref. [59], we have discussed the effects on the tetraquark masses due to change of coupling parameters and argued the stability of $QQ\bar{Q}\bar{Q}$ states by using the effective color-spin interactions in the case that the mixing of different color-spin structures is considered. In Ref. [57], we further introduced a dimensionless measure to reflect the effective color-spin interaction between the ith quark component and the jth quark component,

$$K_{ij} = \frac{\Delta M}{\Delta C_{ij}} \to \frac{\partial M}{\partial C_{ij}}. \tag{12}$$

With such measures, one may rewrite the multiquark masses as

$$M = M_0 + \sum_{i<j} K_{ij} C_{ij}. \tag{13}$$

When K_{ij} is a negative (positive) number, the effective interaction between the ith and jth quark components is attractive (repulsive). If K_{12} and K_{34} are negative but K_{13}, K_{14}, K_{23}, and K_{24} are positive, the tetraquark state $Q_1 q_2 \bar{Q}_3 \bar{q}_4$ is probably more stable than other cases. If only K_{12} or K_{34} is negative, the state is probably less stable than the mentioned case but more stable than other cases. In the following parts, we qualitatively discuss the stability of tetraquarks with such effective interactions.

In ours previous work [52], we considered the spectrum of $cs\bar{c}\bar{s}$ states. Here, we do not repeat the results given there, but present the supplemental results about effective interactions. The obtained K_{ij}'s of Eq. (13) are listed in Table II. The order of states for each case of J^{PC} is the same as the order of masses from high to low. From the results, the highest 2^{++}, the highest 1^{++}, and the second highest 0^{++} states are probably more stable than other states. Although the $X(4274)$ as another 1^{++} $cs\bar{c}\bar{s}$ state is higher than the $X(4140)$, its width can be narrower than that of $X(4140)$. This feature is not contradicted with the recent LHCb measurement [49].

TABLE II. K_{ij}'s for $cs\bar{c}\bar{s}$ states. The order of states for each case of J^{PC} is the same as that of masses from high to low.

	$cs\bar{c}\bar{s}$ system					$cn\bar{c}\bar{n}$ system			
J^{PC}	K_{cs}	$K_{c\bar{c}}$	$K_{c\bar{s}}$	$K_{s\bar{s}}$	J^{PC}	K_{cn}	$K_{c\bar{c}}$	$K_{c\bar{n}}$	$K_{n\bar{n}}$
2^{++}	$\begin{bmatrix} -2.1 \\ 4.7 \end{bmatrix}$	$\begin{bmatrix} 4.7 \\ 0.0 \end{bmatrix}$	$\begin{bmatrix} 3.4 \\ 5.9 \end{bmatrix}$	$\begin{bmatrix} 4.7 \\ 0.0 \end{bmatrix}$	2^{++}	$\begin{bmatrix} -0.5 \\ 3.2 \end{bmatrix}$	$\begin{bmatrix} 5.3 \\ -0.6 \end{bmatrix}$	$\begin{bmatrix} 0.6 \\ 8.7 \end{bmatrix}$	$\begin{bmatrix} 5.3 \\ -0.6 \end{bmatrix}$
1^{++}	$\begin{bmatrix} -0.4 \\ -2.3 \end{bmatrix}$	$\begin{bmatrix} 5.3 \\ -0.7 \end{bmatrix}$	$\begin{bmatrix} 0.3 \\ -9.7 \end{bmatrix}$	$\begin{bmatrix} 5.3 \\ -0.7 \end{bmatrix}$	1^{++}	$\begin{bmatrix} -0.3 \\ -2.4 \end{bmatrix}$	$\begin{bmatrix} 5.3 \\ -0.7 \end{bmatrix}$	$\begin{bmatrix} 0.3 \\ -9.6 \end{bmatrix}$	$\begin{bmatrix} 5.3 \\ -0.7 \end{bmatrix}$
1^{+-}	$\begin{bmatrix} -0.1 \\ 3.8 \\ -1.7 \\ -1.9 \end{bmatrix}$	$\begin{bmatrix} -1.0 \\ -5.2 \\ -8.1 \\ 5.0 \end{bmatrix}$	$\begin{bmatrix} 10.4 \\ -1.5 \\ -6.3 \\ -2.6 \end{bmatrix}$	$\begin{bmatrix} 1.1 \\ 1.2 \\ 2.9 \\ -14.6 \end{bmatrix}$	1^{+-}	$\begin{bmatrix} 2.4 \\ -3.8 \\ 2.1 \\ -0.7 \end{bmatrix}$	$\begin{bmatrix} -7.1 \\ -8.1 \\ 0.6 \\ 5.3 \end{bmatrix}$	$\begin{bmatrix} 7.3 \\ 1.8 \\ -8.4 \\ -0.7 \end{bmatrix}$	$\begin{bmatrix} 3.2 \\ 3.1 \\ 0.2 \\ -15.9 \end{bmatrix}$
0^{++}	$\begin{bmatrix} 7.0 \\ -12.1 \\ 6.0 \\ -6.2 \end{bmatrix}$	$\begin{bmatrix} 3.9 \\ 2.6 \\ -4.6 \\ -11.2 \end{bmatrix}$	$\begin{bmatrix} 7.2 \\ 4.9 \\ -16.5 \\ -14.2 \end{bmatrix}$	$\begin{bmatrix} 3.9 \\ 2.6 \\ -4.6 \\ -11.2 \end{bmatrix}$	0^{++}	$\begin{bmatrix} 5.7 \\ -10.3 \\ 1.6 \\ -2.4 \end{bmatrix}$	$\begin{bmatrix} 4.6 \\ 2.3 \\ -0.7 \\ -15.5 \end{bmatrix}$	$\begin{bmatrix} 5.2 \\ 4.9 \\ -25.8 \\ -3.0 \end{bmatrix}$	$\begin{bmatrix} 4.6 \\ 2.3 \\ -0.7 \\ -15.5 \end{bmatrix}$

C. The $cn\bar{c}\bar{n}$, $bn\bar{b}\bar{n}$, and $bs\bar{b}\bar{s}$ systems

These three systems have similar structures to the $cs\bar{c}\bar{s}$ case but involve different values of parameters. They are related to most of the charmonium-like or bottomonium-like XYZ states observed in various processes. The quantum numbers of the tetraquark states may be $J^{PC}=2^{++}$, 1^{++}, 1^{+-}, or 0^{++}. In the literature, there are lots of studies relevant with such tetraquarks, see e.g., Refs. [35,65–72]. Here, one gets the eigenvalues of the chromomagnetic interactions by substituting $\tau=2C_{Qq}$, $\theta=0$, $\alpha=C_{Q\bar{Q}}+C_{q\bar{q}}+2C_{Q\bar{q}}$, $\beta=\mu=C_{Q\bar{Q}}-C_{q\bar{q}}$, and $\nu=C_{Q\bar{Q}}+C_{q\bar{q}}-2C_{Q\bar{q}}$ into Eqs. (5) and (6) and by diagonalizing matrices in Eqs. (5), (6), (9), and (10). For the $Qn\bar{Q}\bar{n}$ systems, the isospin $=1$ and isospin $=0$ states are both allowed. In the current model, the obtained isovector and isoscalar $Qn\bar{Q}\bar{n}$ states are degenerate.

First, we consider the $cn\bar{c}\bar{n}$ system. When Eq. (2) is adopted, one gets the theoretically highest tetraquark masses listed in the fourth column of Table III. When the reference system is chosen as $\eta_c\pi$, the lowest masses in our strategy are obtained and listed in the fifth column of Table III. When the reference mass is chosen as the $D\bar{D}$ threshold, one gets more reasonable masses shown in the sixth column of Table III, but they are probably still lower than the realistic values. If there were one meson that we may identify as a compact $cn\bar{c}\bar{n}$ tetraquark, the relatively reliable masses of its partner states may be estimated with the CMI eigenvalues. However, as argued in Sec. I, it is hard for us to identify such a tetraquark state. To perform a better mass estimation, an alternative method we may try is to relate the $cn\bar{c}\bar{n}$ masses to that of the $X(4140)$. To do that, we rewrite the mass formula Eq. (2) as

TABLE III. Calculated CMI eigenvalues and estimated tetraquark masses for the $cn\bar{c}\bar{n}$ systems in units of MeV. The masses in the fourth column (Upper limits) are obtained with Eq. (2) and those in the last three columns with various reference states.

	$cn\bar{c}\bar{n}$ system					
J^{PC}	$\langle H_{\rm CM}\rangle$	Eigenvalues	Upper limits	$\eta_c\pi$	$D\bar{D}$	X(4140)
2^{++}	$\begin{pmatrix} 86.0 & -61.4 \\ -61.4 & 151.0 \end{pmatrix}$	$\begin{pmatrix} 188.0 \\ 49.0 \end{pmatrix}$	$\begin{pmatrix} 4361 \\ 4222 \end{pmatrix}$	$\begin{pmatrix} 3873 \\ 3734 \end{pmatrix}$	$\begin{pmatrix} 4132 \\ 3993 \end{pmatrix}$	$\begin{pmatrix} 4237 \\ 4098 \end{pmatrix}$
1^{++}	$\begin{pmatrix} 7.6 & -137.2 \\ -137.2 & 83.0 \end{pmatrix}$	$\begin{pmatrix} 187.6 \\ -97.0 \end{pmatrix}$	$\begin{pmatrix} 4360 \\ 4076 \end{pmatrix}$	$\begin{pmatrix} 3873 \\ 3588 \end{pmatrix}$	$\begin{pmatrix} 4132 \\ 3847 \end{pmatrix}$	$\begin{pmatrix} 4236 \\ 3952 \end{pmatrix}$
1^{+-}	$\begin{pmatrix} -43.3 & 61.4 & -65.3 & 138.6 \\ 61.4 & -172.3 & 138.6 & -163.3 \\ -65.3 & 138.6 & -50.3 & 137.2 \\ 138.6 & -163.3 & 137.2 & -61.7 \end{pmatrix}$	$\begin{pmatrix} 116.4 \\ 47.0 \\ -38.8 \\ -452.3 \end{pmatrix}$	$\begin{pmatrix} 4289 \\ 4220 \\ 4134 \\ 3720 \end{pmatrix}$	$\begin{pmatrix} 3802 \\ 3732 \\ 3646 \\ 3233 \end{pmatrix}$	$\begin{pmatrix} 4060 \\ 3991 \\ 3905 \\ 3492 \end{pmatrix}$	$\begin{pmatrix} 4165 \\ 4096 \\ 4010 \\ 3597 \end{pmatrix}$
0^{++}	$\begin{pmatrix} -108.0 & 122.8 & -50.1 & 237.6 \\ 122.8 & -334.0 & 237.6 & -125.3 \\ -50.1 & 237.6 & -64.0 & 0.0 \\ 237.6 & -125.3 & 0.0 & 32.0 \end{pmatrix}$	$\begin{pmatrix} 221.0 \\ 71.8 \\ -192.7 \\ -574.1 \end{pmatrix}$	$\begin{pmatrix} 4394 \\ 4244 \\ 3980 \\ 3598 \end{pmatrix}$	$\begin{pmatrix} 3906 \\ 3757 \\ 3492 \\ 3111 \end{pmatrix}$	$\begin{pmatrix} 4165 \\ 4016 \\ 3751 \\ 3370 \end{pmatrix}$	$\begin{pmatrix} 4270 \\ 4121 \\ 3856 \\ 3475 \end{pmatrix}$

$$\begin{aligned} M_{(cn\bar{c}\bar{n})} &= 2m_c + 2m_n + \langle H_{\rm CM}\rangle_{(cn\bar{c}\bar{n})} \\ &= (2m_c + 2m_s) - 2(m_s - m_n) + \langle H_{\rm CM}\rangle_{(cn\bar{c}\bar{n})}. \end{aligned} \tag{14}$$

For the $(2m_c + 2m_s)$ term, we replace it by $M_{X(4140)} - \langle H_{\rm CM}\rangle_{X(4140)}$. Then the additional attraction that m_c ($= 1724.6$ MeV) and m_s ($= 540.3$ MeV) should incorporate is partly compensated. For the $(m_s - m_n)$ ($= 178.6$ MeV) term, we also need a modification. Now the problem of mass estimation becomes the problem to determine mass gap between different quarks. The extracted $(m_s - m_n)$ from the conventional hadrons varies from 90.8 MeV to 187.1 MeV (see Table IV). If one replaces the larger value 178.6 MeV by the smaller value $m_s - m_n = 90.8$ MeV, i.e., the mass formula in estimating the $cn\bar{c}\bar{n}$ masses is

$$\begin{aligned} M_{(cn\bar{c}\bar{n})} &= M_{X(4140)} - 2m_{B_s} + 2m_B + 2\langle H_{\rm CM}\rangle_{B_s} \\ &\quad - 2\langle H_{\rm CM}\rangle_B - \langle H_{\rm CM}\rangle_{X(4140)} + \langle H_{\rm CM}\rangle_{(cn\bar{c}\bar{n})}, \end{aligned} \tag{15}$$

higher masses than those with the $D\bar{D}$ threshold are obtained (see Table III). If one uses the larger value $m_s - m_n = 187.1$ MeV, the obtained tetraquark masses are $2 \times (187.1 - 90.8) = 192.6$ MeV lower than those in the last column. Then the masses are not reasonable according to the above criterion. Considering the quark environment, probably the quark mass difference between m_s in the $cs\bar{c}\bar{s}$ system and m_n in the $cn\bar{c}\bar{n}$ system is close to that between D_s and D. If this is the case, the $cn\bar{c}\bar{n}$ masses are just 25.4 MeV lower than those in the last column. In the following, we assume that the masses in the last column of Table III are reasonable values. Of course, further studies are required to test this method of mass estimation.

One should note that the $cn\bar{c}\bar{n}$ masses in Table III are both for isovector and for isoscalar tetraquark states.

TABLE IV. Quark mass differences (units: MeV) determined with various hadrons. The values from the extracted effective quark masses are $m_s - m_n = 178.6$ MeV and $m_b - m_c = 3328.2$ MeV.

Hadron	Hadron	$(m_s - m_n)$	Hadron	Hadron	$(m_b - m_c)$
D_s	D	103.5	B	D	3340.9
B_s	B	90.8	B_s	D_s	3328.2
Σ	N	187.1	η_b	η_c	3188.4
Λ	N	177.4	Λ_b	Λ_c	3333.1
Ω_c	Σ_c	158.8	Σ_b	Σ_c	3328.5
Ω_b	Σ_b	147.9	Ξ_b	Ξ_c	3326.2
Ξ_c	Λ_c	133.4	Ω_b	Ω_c	3315.7
Ξ_c	Σ_c	119.5			
Ξ_b	Λ_b	126.9			
Ξ_b	Σ_b	117.6			

The important mixing effects for all the quantum numbers are not small. In Fig. 1, we show the relative positions for the $cn\bar{c}\bar{n}$ tetraquark states, predicted QM charmonia [1], relevant observed states, and various meson-meson thresholds. For the meson-mesons channels, we label their S-wave J^{PC} in the subscripts of their symbols. It is convenient to judge whether a state can decay into a meson-meson channel from the J, P, and C conservations or not. In Table II, we also show the obtained K_{ij}'s of Eq. (13) for the $cn\bar{c}\bar{n}$ states from which one may guess relatively stable tetraquarks.

With the help of the relative positions in Fig. 1, one may discuss possible assignments for the exotic charmonium-like mesons shown in the figure. For convenience, we summarize the mesons we will discuss, their quantum numbers, masses, widths, and finding channels in Table V. In the particle data book [56], the $Z_c(3900)$ and $Z_c(3885)$ are assumed as the same state and $Z_c(4020)$ and $Z_c(4025)$ are treated as the same state. Here, we also adopt such assignments.

We start the discussions with the newly observed charged $Z_c(4100)^-$. Its quark content should be $cd\bar{c}\bar{u}$. From Fig. 1, this state ($J^P = 0^+$ or 1^-) is ~80 MeV above the threshold of D^*D^* and ~190 MeV below the threshold of $D\bar{D}_1$. It is unlikely an S- or P-wave meson-meson state, but definite conclusion needs detailed investigations. Our results indicate that the second highest $J^{PC} = 0^{++}$ $cnc\bar{n}$ tetraquark (the C-parity of the neutral partner is $+$) has a mass close to that of $Z_c(4100)$. One may interpret the

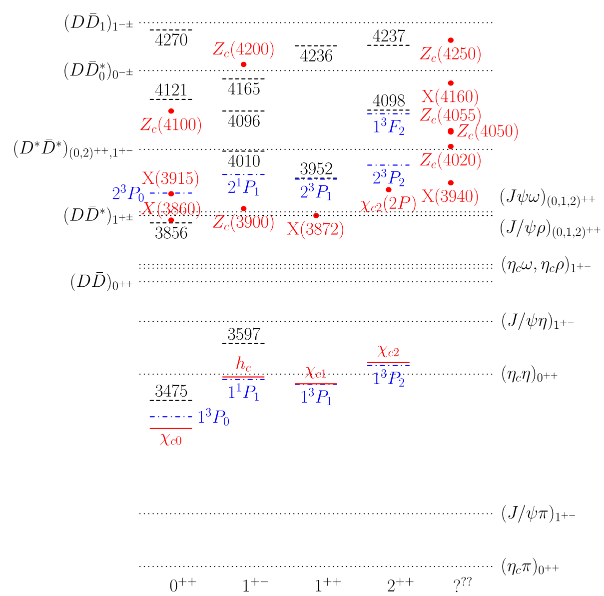

FIG. 1. Relative positions for $cn\bar{c}\bar{n}$ tetraquarks (black dashed lines), predicted charmonia (blue dash-dotted lines), observed charmonia (red solid lines), states with exotic properties (red solid dots), and various meson-meson thresholds (black dotted lines). The masses are given in units of MeV. The subscripts of threshold symbols are J^{PC} in the S-wave case.

TABLE V. Properties of mesons related with ground $cn\bar{c}\,\bar{n}$ tetraquark states [56].

States	$I^G(J^{PC})$	Mass (MeV)	Width (MeV)	Finding channels
$X(3860)$	$0^+(0^{++})$	3862^{+26+40}_{-32-13}	$201^{+154+88}_{-67-82}$	Belle: $D\bar{D}$ [73]
$X(3872)$	$0^+(1^{++})$	3871.69 ± 0.17	< 1.2	Belle: $J\psi\pi\pi$ [2]
$X(3915)$	$0^+(0/2^{++})$	3918.4 ± 1.9	20 ± 5	Belle: $J/\psi\omega$ [74]
$X(3940)$	$?^?(?^{??})$	$3942^{+7}_{-6} \pm 6$	$37^{+26}_{-15} \pm 8$	Belle: $D\bar{D}^*$ [75]
$X(4160)$	$?^?(?^{??})$	$4156^{+25}_{-20} \pm 15$	$139^{+111}_{-61} \pm 21$	Belle: $D^*\bar{D}^*$ [75]
$Z_c(3900)$	$1^+(1^{+-})$	3886.6 ± 2.4	28.2 ± 2.6	BESIII: $J/\psi\pi$ [20],$D\bar{D}^*$ [25]
$Z_c(4020)$	$1^+(?^{?-})$	4024.1 ± 1.9	13 ± 5	BESIII: $h_c\pi$ [28], $D^*\bar{D}^*$ [30]
$Z_c(4050)$	$1^-(?^{?+})$	$4051 \pm 14^{+20}_{-41}$	82^{+21+47}_{-17-22}	Belle: $\chi_{c1}\pi$ [19]
$Z_c(4055)$	$1^+(?^{?-})$	$4054 \pm 3 \pm 1$	$45 \pm 11 \pm 6$	Belle: $\psi(2S)\pi$ [76]
$Z_c(4100)$	$1^?(0^{+?}/1^{-?})$	$4096 \pm 20^{+18}_{-22}$	$152 \pm 58^{+60}_{-35}$	LHCb: $\eta_c\pi$ [32]
$Z_c(4200)$	$1^+(1^{+-})$	4196^{+31+17}_{-29-13}	$370 \pm 70^{+70}_{-132}$	Belle: $J/\psi\pi$ [77]
$Z_c(4250)$	$1^-(?^{?+})$	$4248^{+44+180}_{-29-35}$	$177^{+54+316}_{-39-61}$	Belle: $\chi_{c1}\pi$ [19]
$Z_c(4430)$	$1^+(1^{+-})$	4478^{+15}_{-18}	181 ± 31	Belle: $\psi(2S)\pi$ [16]

$Z_c(4100)$ as a scalar tetraquark state. By comparing relative positions for the state and thresholds in Fig. 1, the $Z_c(4100)$ can decay into $D^*\bar{D}^*$, $D\bar{D}$, $J/\psi\rho$, and $\eta_c\pi$ through S-wave interactions if it is really a state. Its width should be very broad and the state may be even unobservable. If we check Table II, from $K_{cn} = -10.3$, the color-spin interaction between the charm quark and the light quark is effectively attractive. Although the color-spin interactions between other quark components are effectively repulsive, the coefficient for the cn interaction is larger than those for others. This means that the state has a relatively stable tetraquark structure. The observed $\Gamma = 152$ MeV for this high mass state $Z_c(4100)$ is also qualitatively consistent with the argument that it is a scalar tetraquark. In fact, in a study with the QCD sum rule (QSR) [70], the calculation also indicates that a 0^{++} tetraquark around 4.1 GeV is possible. If the J^P of $Z_c(4100)$ are 1^-, the J^{PC} of its neutral partner will be 1^{-+}. In the QSR calculation, such a tetraquark has a mass around 4.6 GeV [78], which means that the $Z_c(4100)$ is more like a scalar state. In addition, Ref. [79] fails to reproduce the mass of the $Z_c(4100)$ with an interpolating current for vector tetraquarks. All these results favor the 0^{++} tetraquark assignment for the $Z_c(4100)$. If this is a correct interpretation, the state may also be observed in the $J/\psi\rho$, $D\bar{D}$, and $D^*\bar{D}^*$ channels. Because of the degeneracy of isovector and isoscalar tetraquarks in the present model, an isoscalar state around 4.1 GeV is also possible. Experimentally, it can be searched for in the $J/\psi\omega$ and $\eta_c\eta$ channels.

If the above assignment is correct, probably the $X(3860)$ is another 0^{++} tetraquark. This state was observed in the $D\bar{D}$ channel at Belle [73] and the $J^{PC} = 0^{++}$ assignment is more favored than 2^{++}. From our estimation, the second lowest 0^{++} tetraquark is close to it, which is a signal that the $X(3860)$ is probably a $cn\bar{c}\bar{n}$ tetraquark state. The QSR calculation also gives an isoscalar scalar tetraquark around 3.81 GeV [70] which is consistent with the $X(3860)$. Both the mass and width of a scalar tetraquark consistent with the $X(3860)$ are obtained in another QSR investigation [80]. If we check the amplitudes for the effective color-spin interactions in Table II, one finds that the $c\bar{n}$ interaction for the tetraquark is stronger and this state should not be stable like the second highest tetraquark. The resulting width may be comparable to that of the $Z_c(4100)$ although it is below the $Z_c(4100)$. The observed width ∼201 MeV for the $X(3860)$ is qualitatively consistent with this feature. In Ref. [73], the $X(3860)$ was interpreted as the $\chi_{c0}(2P)$ because the $\chi_{c0}(2P)$ should decay dominantly into $D\bar{D}$ while the $X(3915)$ which was once identified as $\chi_{c0}(2P)$ does not satisfy this requirement. However, the analysis in Ref. [81] indicates that the $\chi_{c0}(2P)$ has a narrow width, which is inconsistent with Belle's result. The $X(3860)$ is unlikely to be a charmonium from its broad width. In order to understand its nature, $\eta_c\eta$ and $\eta_c\pi$ channels are proposed to search for such a state and its isospin partner state, respectively.

In the 0^{++} case, four additional tetraquark states, two around 3470 MeV ($I = 1$ and $I = 0$) and two around 4270 MeV ($I = 1$ and $I = 0$), are also possible. The low mass $cn\bar{c}\bar{n}$ tetraquark states are around the predicted χ_{c0}. The only S-wave rearrangement decay mode for the isovector state is $\eta_c\pi$ and no rearrangement decay modes exist for the isoscalar one. Their widths may not be broad if they do exist. We wait for experimental measurements to test this judgement. The high mass $cn\bar{c}\bar{n}$ states around 4.2 GeV should be broader than the $Z_c(4100)$. Experimentally, there is a charged $Z_c(4250)$ in this mass region, which is observed in the $\pi\chi_{c1}$ channel and has a width around 177 MeV. It is unlikely a $D_1\bar{D}$ or $D_0\bar{D}^*$ molecule [82]. At present, assigning the $Z_c(4250)$ as a ground tetraquark with highest mass can not be excluded. If this assignment is correct, an additional isoscalar tetraquark around 4.2 GeV should also be measurable. However, since the quantum numbers of $Z_c(4250)$ can also be 1^{++}, 2^{++}, or others and the highest tetraquarks in the 1^{++} and 2^{++} cases are also in this mass region, there are still other possibilities for its assignment.

The χ_{c0} charmonium has been established, but its radially excited $\chi_{c0}(2P)$ not. This state should be around 3920 MeV. Experimentally, three narrow states in this mass region, $X(3915)$ in the $J/\psi\omega$ channel [74,83–85], $\chi_{c2}(2P)$ in the $D\bar{D}$ channel [86,87], and $X(3940)$ in the $D\bar{D}^*$ channel [75], were observed. The angular momentum of the state observed in the $D\bar{D}$ channel has been determined to be 2 and this state is identified as the predicted 2^3P_2 charmonium. For the other two states, the assignment problem is still unsettled. From Refs. [81,88], the $X(3915)$ and the state in $D\bar{D}$ are probably the same 2^{++} state while the real $\chi_{c0}(2P)$ is probably around 3860 MeV with a narrow width. For the $X(3940)$ state, there is no appropriate position if it is a P-even charmonium ($X(3872)$ should be the $\chi_{c1}(2P)$ state). From a study of the decay width in Ref. [89], this exotic state seems to be a good candidate of $\eta_c(3S)$. If the above assignments for the $Z_c(4100)$ and $X(3860)$ are correct, the widths of the $cn\bar{c}\bar{n}$ tetraquark states should not be small. Since $\Gamma_{X(3940)} \sim 37$ MeV, the consistency of decay widths does not support its tetraquark interpretation. Based on our results, it seems that no tetraquark assignments are favored for states around 3940 MeV. These three or two states around 3920 MeV are probably conventional charmonia or molecules.

Now let us move on to the $J^{PC} = 2^{++}$ states. In the charmonium sector, the lowest χ_{c2} has been established and the $\chi_{c2}(2P)$ is also identified. No evidence for other charmonia has been reported. In the tetraquark sector, we have two states around 4.1 GeV ($I = 1$ and $I = 0$) and two states around 4.2 GeV ($I = 1$ and $I = 0$). Their dominant decay channels should be $D^*\bar{D}^*$. For the isovector (isoscalar) states, the decay mode $J/\psi\rho$ ($J/\psi\omega$) is also allowed. Whether such tetraquark states exist or not needs to be answered by future measurements. As mentioned above, the $Z_c(4250)$ can also be a candidate of the high mass 2^{++} tetraquark. From Table II, the cn effective color-spin interaction in this state is weakly attractive, which probably narrows its width.

In the $J^{PC} = 1^{++}$ case, the most intriguing state is $X(3872)$ which is probably the predicted $\chi_{c1}(2P)$ charmonium but affected strongly by channels coupling to it. The lowest 1^{++} tetraquark we obtain is also around the predicted $\chi_{c1}(2P)$ charmonium which is tens of MeV above the physical $X(3872)$. This indicates that the $X(3872)$ should not be a pure tetraquark, which is consistent with the results in Refs. [68,69]. If the coupling between the predicted charmonium and the isoscalar $cn\bar{c}\bar{n}$ tetraquark state is considered, it is possible to obtain the physical mass of the $X(3872)$. On the other hand, the isovector $cn\bar{c}\bar{n}$, in principle, does not couple to conventional charmonium states. Its dominant decay modes are $D\bar{D}^*$ and $J/\psi\rho$. If experiments observed an isovector state around 3950 MeV (with probably broad width), it will be a good tetraquark candidate. Around 4.2 GeV, we have two higher tetraquarks ($I = 1$ and $I = 0$). The dominant decay modes for the isovector state are still $D\bar{D}^*$ and $J/\psi\rho$. Those for the isoscalar are $D\bar{D}^*$ and $J/\psi\omega$. The large phase spaces for decay indicate that both of them should be broad if they exist. Note that probably the $Z_c(4250)$ can also be the high mass 1^{++} tetraquark whose effective cn interaction is weakly attractive.

The remaining quantum numbers that the ground tetraquarks involve are $J^{PC} = 1^{+-}$. The exotic 1^{+-} $Z_c(3900)$ states have been observed around the $D\bar{D}^*$ threshold [20–22]. From our estimated masses, one cannot interpret the $Z_c(3900)$ states as ground compact tetraquarks. They should be isovector $D\bar{D}^*$ bound or resonant states, or nonresonant effects, as explored in the literature [5,90–95]. Another state consistent with $J^{PC} = 1^{+-}$ is $Z_c(4200)$ observed in the $J/\psi\pi$ channel. Its width is about 370 MeV, which implies that this state is probably a tetraquark. From our results of estimation, one may assign it as the highest ground $cn\bar{c}\bar{n}$ tetraquark. In QSR analyses [96,97], the tetraquark assignment for the $Z_c(4200)$ is also favored. In this mass region, there is an exotic $X(4160)$ which was observed in the $D^*\bar{D}^*$ channel by Belle [75]. Although its width (~139 MeV) is broad, assigning it as a 1^{+-} tetraquark seems to be problematic because (1) its mass is larger than the $Z_c(4100)$, (2) it has more decay channels than the $Z_c(4100)$, and (3) the cn diquark is not effectively attractive, but the width is not larger than the $Z_c(4100)$. Possible assignments may be $\eta_{c2}(2D)$ [98], $\eta_c(4S)$ [89], or $D_s^*\bar{D}_s^*$ molecule with $I^G(J^{PC}) = 0^+(2^{++})$ [99]. The $Z_c(4430)$ observed in the $\pi\psi(2S)$ channel [17] also has the quantum numbers $J^{PC} = 1^{+-}$. It is much higher than the $Z_c(4200)$ and should be an excited state. We do not discuss its nature here. Three charged tetraquark states, one around 4.1 GeV, the other around 4.0 GeV, and the third around 3.6 GeV, are also possible. The lowest one has only one rearrangement decay mode $J/\psi\pi$ and is probably not a broad tetraquark. The other two should be broad. For the isoscalar 1^{+-} tetraquark states, there is still no candidate we can assign. The $J/\psi\eta$ may be an ideal channel to identify them because the decay of the conventional 1^{+-} charmonia into J/ψ involves spin-flip and is suppressed. The lowest $I = 0$ tetraquark (around 3.6 GeV) should be very narrow if it really exists.

In the above discussions, we do not mention the three charged states in the mass range 4.0–4.1 GeV, $Z_c(4055)$, $Z_c(4050)$, and $Z_c(4020)$, although we need experimental candidates to assign as tetraquarks. These three states do not have broad enough widths that consistent assignments for ground tetraquark states require. Their nature should be accounted for by other interpretations. For example, the $Z_c(4020)$ can be explained as coupled channel cusp effect [100] or 1^{+-} molecule-like state [101–104] and the $Z_c(4050)$ as 3^{++} molecule-like state [103]. They can also be radially or orbitally excited four-quark states, which has not been widely studied in the literature [5].

From the symmetry consideration, its tetraquark partners should also exist if the $Z_c(4100)$ is really a 0^{++} $cn\bar{c}\bar{n}$ tetraquark state. Quite a few broad isovector and isoscalr exotic states can be searched for with the help of Fig. 1. Four low mass (and probably narrow) $cn\bar{c}\bar{n}$ tetraquarks are also possible. If such additional states could be observed, we will be sure that more compact tetraquark states in other systems exist. The corresponding state of $Z_c(4100)$ in the $cs\bar{c}\bar{s}$ case (mass ~ 4.2 GeV) should also be relatively stable and can be observed since the cs color-spin interaction is also effectively attractive. Further experimental measurements are definitely needed.

Second, we consider the $bn\bar{b}\bar{n}$ system. The obtained CMI eigenvalues, theoretical upper limits for the tetraquark masses, and estimated values with the $\eta_b\pi$ and $B\bar{B}$ thresholds are listed in Table VI. Similar to the estimation procedure for the $cn\bar{c}\bar{n}$ states, when one relates the masses to that of the $X(4140)$, we rewrite the mass formula to be

$$\begin{aligned} M_{(bn\bar{b}\bar{n})} &= 2m_b + 2m_n + \langle H_{\mathrm{CM}}\rangle_{(bn\bar{b}\bar{n})} \\ &= (2m_c + 2m_s) + 2(m_b - m_c) - 2(m_s - m_n) \\ &\quad + \langle H_{\mathrm{CM}}\rangle_{(bn\bar{b}\bar{n})}. \end{aligned} \tag{16}$$

Then the $(2m_c + 2m_s)$ term is replaced by $M_{X(4140)} - \langle H_{\mathrm{CM}}\rangle_{X(4140)}$ and the $(m_s - m_n)$ term is replaced by 90.8 MeV. For the mass difference $(m_b - m_c)$, there are some uncertainties with typical values of tens of MeV (see Table IV). By using 3340.9 MeV extracted from B and D mesons, one may get higher masses than those determined with the $B\bar{B}$ threshold. If the value from the η_b and η_c is

TABLE VI. Calculated CMI eigenvalues and estimated tetraquark masses for the $bn\bar{b}\,\bar{n}$ and $bs\bar{b}\,\bar{s}$ systems in units of MeV. The masses in the forth column (Upper limits) are obtained with Eq. (2) and those in the last three columns with various reference states.

$bn\bar{b}\bar{n}$ system						
J^{PC}	$\langle H_{\mathrm{CM}}\rangle$	Eigenvalues	Upper limits	$\eta_b\pi$	$B\bar{B}$	X(4140)
2^{++}	$\begin{pmatrix} 56.1 & -80.6 \\ -80.6 & 119.5 \end{pmatrix}$	$\begin{pmatrix} 174.5 \\ 1.2 \end{pmatrix}$	$\begin{pmatrix} 11003 \\ 10830 \end{pmatrix}$	$\begin{pmatrix} 10236 \\ 10063 \end{pmatrix}$	$\begin{pmatrix} 10800 \\ 10627 \end{pmatrix}$	$\begin{pmatrix} 10905 \\ 10732 \end{pmatrix}$
1^{++}	$\begin{pmatrix} 31.1 & -104.4 \\ -104.4 & 98.5 \end{pmatrix}$	$\begin{pmatrix} 174.4 \\ -44.9 \end{pmatrix}$	$\begin{pmatrix} 11003 \\ 10784 \end{pmatrix}$	$\begin{pmatrix} 10236 \\ 10017 \end{pmatrix}$	$\begin{pmatrix} 10800 \\ 10581 \end{pmatrix}$	$\begin{pmatrix} 10905 \\ 10686 \end{pmatrix}$
1^{+-}	$\begin{pmatrix} -42.3 & 80.6 & -71.7 & 152.2 \\ 80.6 & -126.5 & 152.2 & -179.3 \\ -71.7 & 152.2 & -44.9 & 104.4 \\ 152.2 & -179.3 & 104.4 & -91.5 \end{pmatrix}$	$\begin{pmatrix} 115.5 \\ 58.3 \\ -17.2 \\ -461.7 \end{pmatrix}$	$\begin{pmatrix} 10944 \\ 10887 \\ 10812 \\ 10367 \end{pmatrix}$	$\begin{pmatrix} 10177 \\ 10120 \\ 10045 \\ 9600 \end{pmatrix}$	$\begin{pmatrix} 10741 \\ 10684 \\ 10609 \\ 10164 \end{pmatrix}$	$\begin{pmatrix} 10846 \\ 10789 \\ 10713 \\ 10269 \end{pmatrix}$
0^{++}	$\begin{pmatrix} -91.5 & 161.2 & -65.8 & 180.8 \\ 161.2 & -249.5 & 180.8 & -164.5 \\ -65.8 & 180.8 & -20.8 & 0.0 \\ 180.8 & -164.5 & 0.0 & 10.4 \end{pmatrix}$	$\begin{pmatrix} 178.8 \\ 67.1 \\ -72.9 \\ -524.3 \end{pmatrix}$	$\begin{pmatrix} 11008 \\ 10896 \\ 10756 \\ 10305 \end{pmatrix}$	$\begin{pmatrix} 10241 \\ 10129 \\ 9989 \\ 9538 \end{pmatrix}$	$\begin{pmatrix} 10805 \\ 10693 \\ 10553 \\ 10102 \end{pmatrix}$	$\begin{pmatrix} 10909 \\ 10798 \\ 10658 \\ 10206 \end{pmatrix}$
$bs\bar{b}\,\bar{s}$ system						
J^{PC}	$\langle H_{\mathrm{CM}}\rangle$	Eigenvalues	Upper limits	$\eta_b\varphi$	$B_s\bar{B}_s$	X(4140)
2^{++}	$\begin{pmatrix} 30.4 & -25.0 \\ -25.0 & 56.9 \end{pmatrix}$	$\begin{pmatrix} 71.9 \\ 15.4 \end{pmatrix}$	$\begin{pmatrix} 11258 \\ 11202 \end{pmatrix}$	$\begin{pmatrix} 10481 \\ 10424 \end{pmatrix}$	$\begin{pmatrix} 10879 \\ 10823 \end{pmatrix}$	$\begin{pmatrix} 10984 \\ 10928 \end{pmatrix}$
1^{++}	$\begin{pmatrix} 5.4 & -51.0 \\ -51.0 & 32.6 \end{pmatrix}$	$\begin{pmatrix} 71.8 \\ -33.8 \end{pmatrix}$	$\begin{pmatrix} 11258 \\ 11152 \end{pmatrix}$	$\begin{pmatrix} 10481 \\ 10375 \end{pmatrix}$	$\begin{pmatrix} 10879 \\ 10774 \end{pmatrix}$	$\begin{pmatrix} 10984 \\ 10879 \end{pmatrix}$
1^{+-}	$\begin{pmatrix} -17.6 & 25.0 & -20.3 & 43.1 \\ 25.0 & -63.3 & 43.1 & -50.8 \\ -20.3 & 43.1 & -18.2 & 51.0 \\ 43.1 & -50.8 & 51.0 & -26.2 \end{pmatrix}$	$\begin{pmatrix} 35.3 \\ 6.8 \\ -13.4 \\ -154.1 \end{pmatrix}$	$\begin{pmatrix} 11222 \\ 11193 \\ 11173 \\ 11032 \end{pmatrix}$	$\begin{pmatrix} 10444 \\ 10416 \\ 10395 \\ 10255 \end{pmatrix}$	$\begin{pmatrix} 10843 \\ 10814 \\ 10794 \\ 10653 \end{pmatrix}$	$\begin{pmatrix} 10948 \\ 10919 \\ 10899 \\ 10758 \end{pmatrix}$
0^{++}	$\begin{pmatrix} -41.7 & 49.9 & -20.4 & 88.3 \\ 49.9 & -123.4 & 88.3 & -51.0 \\ -20.4 & 88.3 & -19.2 & 0.0 \\ 88.3 & -51.0 & 0.0 & 9.6 \end{pmatrix}$	$\begin{pmatrix} 81.4 \\ 29.4 \\ -67.3 \\ -218.2 \end{pmatrix}$	$\begin{pmatrix} 11268 \\ 11216 \\ 11119 \\ 10968 \end{pmatrix}$	$\begin{pmatrix} 10490 \\ 10438 \\ 10341 \\ 10191 \end{pmatrix}$	$\begin{pmatrix} 10889 \\ 10837 \\ 10740 \\ 10589 \end{pmatrix}$	$\begin{pmatrix} 10994 \\ 10942 \\ 10845 \\ 10694 \end{pmatrix}$

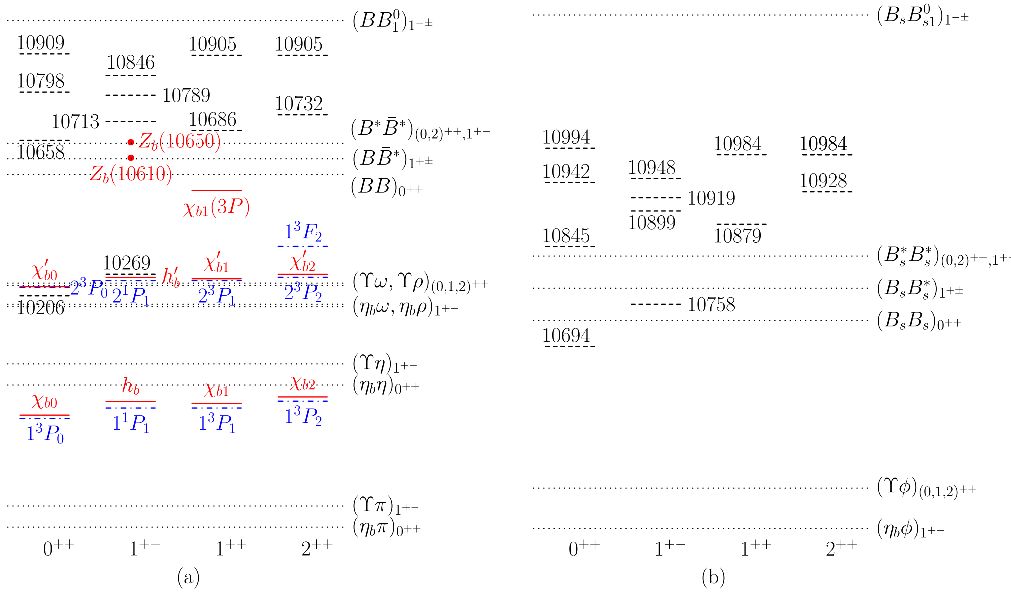

FIG. 2. Relative positions for (a) $bn\bar{b}\,\bar{n}$ and (b) $bs\bar{b}\,\bar{s}$ tetraquarks (black dashed lines), predicted bottomonia (blue dash-dotted lines), observed bottomonia (red solid lines), states with exotic properties (red solid dots), and various meson-meson thresholds (black dotted lines). The masses are given in units of MeV. The subscripts of threshold symbols are J^{PC} in the S-wave case.

adopted, tetraquark masses lower than those determined with the $B\bar{B}$ threshold are obtained. They should not be reasonable values according to our criterion. In the following discussions, we assume the masses listed in the seventh column are closer to the realistic values. With such masses, we plot in Fig. 2 relative positions for the $bn\bar{b}\bar{n}$ tetraquarks, predicted QM bottomonia, relevant observed states, and various meson-meson thresholds. Unlike the $cn\bar{c}\bar{n}$ case, only two narrow exotic bottomonium-like states, $Z_b(10650)$ and $Z_b(10610)$ [105], were observed in the present case. To understand effective color-spin interactions in the tetraquark states, in Table VII, we give the involved K_{ij}'s for the $bn\bar{b}\bar{n}$ states.

From comparison for Figs. 1 and 2(a), the mass distribution for the $bn\bar{b}\bar{n}$ system is similar to that for $cn\bar{c}\bar{n}$. Figure 2 tells us that most $bn\bar{b}\bar{n}$ tetraquarks have open-bottom decay channels and should be broad. The lowest 0^{++} tetraquark with $I=1$ ($I=0$) mainly decays into $\eta_b\pi$ ($\eta_b\eta$) through S-wave interactions. The lowest 1^{+-} tetraquark with $I=1$ ($I=0$) mainly decays into $\eta_b\rho$ and $\Upsilon\pi$ ($\eta_b\omega$ and $\Upsilon\eta$). Maybe they are not broad states. From Table VII, the bn interaction in the second highest $I^G(J^{PC})=1^-(0^{++})$ tetraquark, the corresponding state of $Z_c(4100)$ in the hidden-bottom case, is also effectively attractive. It should also be a measurable broad state. So does its degenerate $I=0$ partner state (mass around

TABLE VII. K_{ij}'s for $bn\bar{b}\,\bar{n}$ and $bs\bar{b}\,\bar{s}$ states. The order of states is the same as that in Table VI.

	$bn\bar{b}\,\bar{n}$ system					$bs\bar{b}\,\bar{s}$ system			
J^{PC}	K_{bn}	$K_{b\bar{b}}$	$K_{b\bar{n}}$	$K_{n\bar{n}}$	J^{PC}	K_{bs}	$K_{b\bar{b}}$	$K_{b\bar{s}}$	$K_{s\bar{s}}$
2^{++}	$\begin{bmatrix}-0.1\\2.8\end{bmatrix}$	$\begin{bmatrix}5.3\\-0.7\end{bmatrix}$	$\begin{bmatrix}0.1\\9.2\end{bmatrix}$	$\begin{bmatrix}5.3\\-0.7\end{bmatrix}$	2^{++}	$\begin{bmatrix}-0.5\\3.2\end{bmatrix}$	$\begin{bmatrix}5.3\\-0.6\end{bmatrix}$	$\begin{bmatrix}0.6\\8.7\end{bmatrix}$	$\begin{bmatrix}5.3\\-0.6\end{bmatrix}$
1^{++}	$\begin{bmatrix}-0.1\\-2.6\end{bmatrix}$	$\begin{bmatrix}5.3\\-0.7\end{bmatrix}$	$\begin{bmatrix}0.1\\-9.4\end{bmatrix}$	$\begin{bmatrix}5.3\\-0.7\end{bmatrix}$	1^{++}	$\begin{bmatrix}-0.3\\-2.4\end{bmatrix}$	$\begin{bmatrix}5.3\\-0.7\end{bmatrix}$	$\begin{bmatrix}0.3\\-9.6\end{bmatrix}$	$\begin{bmatrix}5.3\\-0.7\end{bmatrix}$
1^{+-}	$\begin{bmatrix}1.6\\-2.7\\1.3\\-0.2\end{bmatrix}$	$\begin{bmatrix}-15.2\\-1.3\\1.9\\5.3\end{bmatrix}$	$\begin{bmatrix}1.8\\2.2\\-3.7\\-0.2\end{bmatrix}$	$\begin{bmatrix}5.2\\2.0\\-0.6\\-16.0\end{bmatrix}$	1^{+-}	$\begin{bmatrix}0.8\\-1.5\\1.3\\-0.7\end{bmatrix}$	$\begin{bmatrix}-2.9\\-11.1\\-0.6\\5.3\end{bmatrix}$	$\begin{bmatrix}8.9\\0.7\\-8.9\\-0.7\end{bmatrix}$	$\begin{bmatrix}2.1\\3.7\\0.7\\-15.9\end{bmatrix}$
0^{++}	$\begin{bmatrix}2.4\\-5.0\\-2.1\\-0.6\end{bmatrix}$	$\begin{bmatrix}5.2\\2.0\\-0.6\\-16.0\end{bmatrix}$	$\begin{bmatrix}2.5\\3.9\\-24.4\\-0.7\end{bmatrix}$	$\begin{bmatrix}5.2\\2.0\\-0.6\\-16.0\end{bmatrix}$	0^{++}	$\begin{bmatrix}5.2\\-9.7\\1.1\\-2.0\end{bmatrix}$	$\begin{bmatrix}4.8\\2.2\\-0.6\\-15.7\end{bmatrix}$	$\begin{bmatrix}4.8\\5.2\\-26.3\\-2.4\end{bmatrix}$	$\begin{bmatrix}4.8\\2.2\\-0.6\\-15.7\end{bmatrix}$

10.8 GeV). Similarly, the highest 2^{++} and 1^{++} $bn\bar{b}\bar{n}$ tetraquarks are probably measurable since their effective bn interactions are weakly attractive while the effective interactions for other quark components in them are repulsive.

From the discussions in the $cn\bar{c}\bar{n}$ case, it seems that tetraquark states generally have broad widths. By changing the (anti)charm quark to the (anti)bottom quark, this basic feature of tetraquark states probably does not change. The observed two Z_b states have narrow widths, $\Gamma_{Z_b(10610)} = 18.4$ MeV and $\Gamma_{Z_b(10650)} = 11.5$ MeV [105], and they should not be compact tetraquarks. Since they are near-threshold states, the natural explanation is that they are hadronic molecules [106–108]. The $Z_c(3900)$ near the $D\bar{D}^*$ threshold has a width around 28 MeV and is basically thought as a 1^{+-} $D\bar{D}^*$ molecule. As its hidden-bottom partner, the $Z_b(10610)$ is a 1^{+-} $B\bar{B}^*$ molecule. If the $Z_b(10650)$ is a 1^{+-} $B^*\bar{B}^*$ molecule, the $Z_c(4020)$ with $\Gamma \sim 13$ MeV looks like its hidden-charm partner and the J^{PC} of $Z_c(4020)$ should be 1^{+-}, too. The assignment for the J^{PC} from this simple comparison may be tested with future measurements.

Searching for more exotic states in the hidden-bottom realm is an intriguing task since the bottom and charm quarks have different properties. The observation of them will be crucial for us to understand the quark interactions in conventional hadrons and in multiquark states, no matter the observed width is broad or narrow. We hope the results in the present work may provide useful information for further studies.

Finally, we consider the $bs\bar{b}\bar{s}$ system. We present relevant masses in Table VI and various K_{ij}'s in Table VII. When relating the masses to that of the $X(4140)$, we modify terms in

$$M_{(bs\bar{b}\bar{s})} = 2m_b + 2m_s + \langle H_{\rm CM}\rangle_{(bs\bar{b}\bar{s})}$$
$$= (2m_c + 2m_s) + 2(m_b - m_c) + \langle H_{\rm CM}\rangle_{(bs\bar{b}\bar{s})}. \quad (17)$$

By replacing $(2m_c + 2m_s)$ with $M_{X(4140)} - \langle H_{CN}\rangle_{X(4140)}$ and $(m_b - m_c)$ with 3340.9 MeV, we get $bs\bar{b}\bar{s}$ masses larger than those determined with the $B_s\bar{B}_s$ threshold. They are shown in the last column of Table VI. We treat them as the realistic masses and plot relative tetraquark positions in Fig. 2(b). Relevant rearrangement decay channels and their thresholds are also shown.

From the figure, these $bs\bar{b}\bar{s}$ tetraquarks can be searched for either in $\eta_b\phi$ or $\Upsilon\phi$ channel. All of them seem to have open-bottom decay channels, which is a feature different from the $cs\bar{c}\bar{s}$ case [52]. However, it is unclear whether they are broad or narrow states because the $X(4140)$ as a tetraquark has a narrow width around 22 MeV [56]. We hope future investigations may answer this puzzle. From Table VII, the highest 2^{++}, 1^{++}, and the second highest 0^{++} states seem to be more stable than other states since the effective cs color-spin interactions are attractive.

D. The $cn\bar{c}\bar{s}$, $bn\bar{b}\bar{s}$, $cn\bar{b}\bar{n}$, and $cs\bar{b}\bar{s}$ systems

The existence of isovector charmonium-like and bottomonium-like tetraquark states also implies that of more exotic tetraquarks. One may find some predictions about the $cn\bar{c}\bar{s}$, $bn\bar{b}\bar{s}$, $cn\bar{b}\bar{n}$, and $cs\bar{b}\bar{s}$ states in Refs. [35,67,109–111]. The $cn\bar{c}\bar{s}$ and $bn\bar{b}\bar{s}$ states look like excited kaon mesons from the quantum numbers but the masses are much higher. If such a high-mass kaon were observed, one may identify its tetraquark nature since the orbital or radial excitation energy larger than 3 GeV for light quarks in a conventional kaon is unlikely. The creation of a heavy quark-antiquark pair can naturally explain its high mass. The $cn\bar{b}\bar{n}$ and $cs\bar{b}\bar{s}$ states look like excited B_c mesons, but probably they are not easy to be isolated from the conventional B_c mesons. All such tetraquark states do not have C-parities. To get numerical results in the present model, the matrices in Eqs. (5)–(7) need to be diagonalized after appropriate parameters are used.

We consider temporarily the kaon-like heavy tetraquark states. With Eqs. (5)–(7), the numerical results for the chromomagnetic interactions can be easily gotten. We list them in Table VIII. The theoretical upper limits for the tetraquark masses, the masses estimated with the $\eta_c K$ ($\eta_b K$) threshold, and those with the $\bar{D}D_s$ ($B\bar{B}_s$) threshold are given in the forth, fifth, and sixth columns, respectively. The masses with $\bar{D}D_s$ ($B\bar{B}_s$) are higher than those with $\eta_c K$ ($\eta_b K$). To estimate the masses with the help of $X(4140)$, we adopt modified mass formulas of

$$M_{(cn\bar{c}\bar{s})} = 2m_c + m_n + m_s + \langle H_{\rm CM}\rangle_{(cn\bar{c}\bar{s})}$$
$$= (2m_c + 2m_s) - (m_s - m_n) + \langle H_{\rm CM}\rangle_{(cn\bar{c}\bar{s})}, \quad (18)$$

and

$$M_{(bn\bar{b}\bar{s})} = 2m_b + m_n + m_s + \langle H_{\rm CM}\rangle_{(bn\bar{b}\bar{s})}$$
$$= (2m_c + 2m_s) + 2(m_b - m_c)$$
$$- (m_s - m_n) + \langle H_{\rm CM}\rangle_{(bn\bar{b}\bar{s})}. \quad (19)$$

By making the replacements $(2m_c + 2m_s) \to M_{X(4140)} - \langle H_{\rm CM}\rangle_{X(4140)}$, $(m_b - m_c) \to 3340.9$ MeV, and $(m_s - m_n) \to 90.8$ MeV, we obtain much higher masses in the last column of Table VIII. We treat them as more reasonable values in the following discussions. The relative positions for the kaon-like heavy tetraquark states and relevant meson-meson channels and thresholds are illustrated in Fig. 3. Contributions of effective quark interactions for each pair of quark components are easy to recover with Eq. (13) and the coefficients K_{ij}'s in Table IX.

TABLE VIII. Calculated CMI eigenvalues and estimated tetraquark masses for the $cn\bar{c}\bar{s}$ and $bn\bar{b}\bar{s}$ systems in units of MeV. The masses in the forth column (Upper limits) are obtained with Eq. (2) and those in the last three columns with various reference states.

$cn\bar{c}\bar{s}$ system						
J^P	$\langle H_{\mathrm{CM}}\rangle$	Eigenvalues	Upper limits	$\eta_c K$	$\bar{D}D_s$	X(4140)
2^+	$\begin{pmatrix} 72.5 & -30.0 \\ -30.0 & 113.3 \end{pmatrix}$	$\begin{pmatrix} 129.2 \\ 56.7 \end{pmatrix}$	$\begin{pmatrix} 4480 \\ 4408 \end{pmatrix}$	$\begin{pmatrix} 3991 \\ 3918 \end{pmatrix}$	$\begin{pmatrix} 4177 \\ 4104 \end{pmatrix}$	$\begin{pmatrix} 4269 \\ 4196 \end{pmatrix}$
1^+	$\begin{pmatrix} -27.2 & -25.3 & 25.3 & 30.0 & 53.6 & -53.6 \\ -25.3 & -25.3 & 14.1 & 53.6 & 0.0 & -105.8 \\ 25.3 & 14.1 & -20.0 & -53.6 & -105.8 & 0.0 \\ 30.0 & 53.6 & -53.6 & -136.0 & -63.2 & 63.2 \\ 53.6 & 0.0 & -105.8 & -63.2 & 12.7 & 35.3 \\ -53.6 & -105.8 & 0.0 & 63.2 & 35.3 & 10.0 \end{pmatrix}$	$\begin{pmatrix} 128.5 \\ 87.9 \\ 10.9 \\ -45.5 \\ -90.3 \\ -277.3 \end{pmatrix}$	$\begin{pmatrix} 4480 \\ 4439 \\ 4362 \\ 4306 \\ 4261 \\ 4074 \end{pmatrix}$	$\begin{pmatrix} 3990 \\ 3950 \\ 3872 \\ 3816 \\ 3771 \\ 3584 \end{pmatrix}$	$\begin{pmatrix} 4176 \\ 4135 \\ 4058 \\ 4002 \\ 3957 \\ 3770 \end{pmatrix}$	$\begin{pmatrix} 4268 \\ 4228 \\ 4151 \\ 4094 \\ 4049 \\ 3862 \end{pmatrix}$
0^+	$\begin{pmatrix} -77.1 & 60.0 & -24.5 & 183.2 \\ 60.0 & -260.7 & 183.2 & -61.2 \\ -24.5 & 183.2 & -68.0 & 0.0 \\ 183.2 & -61.2 & 0.0 & 34.0 \end{pmatrix}$	$\begin{pmatrix} 172.9 \\ 42.7 \\ -180.5 \\ -406.9 \end{pmatrix}$	$\begin{pmatrix} 4524 \\ 4394 \\ 4171 \\ 3944 \end{pmatrix}$	$\begin{pmatrix} 4035 \\ 3904 \\ 3681 \\ 3455 \end{pmatrix}$	$\begin{pmatrix} 4220 \\ 4090 \\ 3867 \\ 3641 \end{pmatrix}$	$\begin{pmatrix} 4313 \\ 4182 \\ 3959 \\ 3733 \end{pmatrix}$
$bn\bar{b}\bar{s}$ system						
J^P	$\langle H_{\mathrm{CM}}\rangle$	Eigenvalues	Upper limits	$\eta_b K$	$B\bar{B}_s$	X(4140)
2^+	$\begin{pmatrix} 41.3 & -48.6 \\ -48.6 & 83.3 \end{pmatrix}$	$\begin{pmatrix} 115.3 \\ 9.3 \end{pmatrix}$	$\begin{pmatrix} 11123 \\ 11017 \end{pmatrix}$	$\begin{pmatrix} 10354 \\ 10248 \end{pmatrix}$	$\begin{pmatrix} 10832 \\ 10726 \end{pmatrix}$	$\begin{pmatrix} 10937 \\ 10831 \end{pmatrix}$
1^+	$\begin{pmatrix} -28.0 & -30.2 & 29.4 & 48.6 & 62.4 & -64.0 \\ -30.2 & -6.1 & 22.9 & 62.4 & 0.0 & -73.5 \\ 29.4 & 22.9 & -7.2 & -64.0 & -73.5 & 0.0 \\ 48.6 & 62.4 & -64.0 & -90.0 & -75.4 & 73.5 \\ 62.4 & 0.0 & -73.5 & -75.4 & 3.1 & 57.3 \\ -64.0 & -73.5 & 0.0 & 73.5 & 57.3 & 3.6 \end{pmatrix}$	$\begin{pmatrix} 115.3 \\ 61.2 \\ 34.1 \\ -12.4 \\ -38.5 \\ -284.3 \end{pmatrix}$	$\begin{pmatrix} 11123 \\ 11069 \\ 11042 \\ 10995 \\ 10969 \\ 10723 \end{pmatrix}$	$\begin{pmatrix} 10354 \\ 10300 \\ 10272 \\ 10226 \\ 10200 \\ 9954 \end{pmatrix}$	$\begin{pmatrix} 10832 \\ 10778 \\ 10751 \\ 10704 \\ 10678 \\ 10432 \end{pmatrix}$	$\begin{pmatrix} 10937 \\ 10883 \\ 10856 \\ 10809 \\ 10783 \\ 10537 \end{pmatrix}$
0^+	$\begin{pmatrix} -62.7 & 97.3 & -39.7 & 127.4 \\ 97.3 & -176.7 & 127.4 & -99.3 \\ -39.7 & 127.4 & -20.0 & 0.0 \\ 127.4 & -99.3 & 0.0 & 10.0 \end{pmatrix}$	$\begin{pmatrix} 121.7 \\ 45.5 \\ -69.2 \\ -347.4 \end{pmatrix}$	$\begin{pmatrix} 11129 \\ 11053 \\ 10938 \\ 10660 \end{pmatrix}$	$\begin{pmatrix} 10360 \\ 10284 \\ 10169 \\ 9891 \end{pmatrix}$	$\begin{pmatrix} 10838 \\ 10762 \\ 10647 \\ 10369 \end{pmatrix}$	$\begin{pmatrix} 10943 \\ 10867 \\ 10752 \\ 10474 \end{pmatrix}$

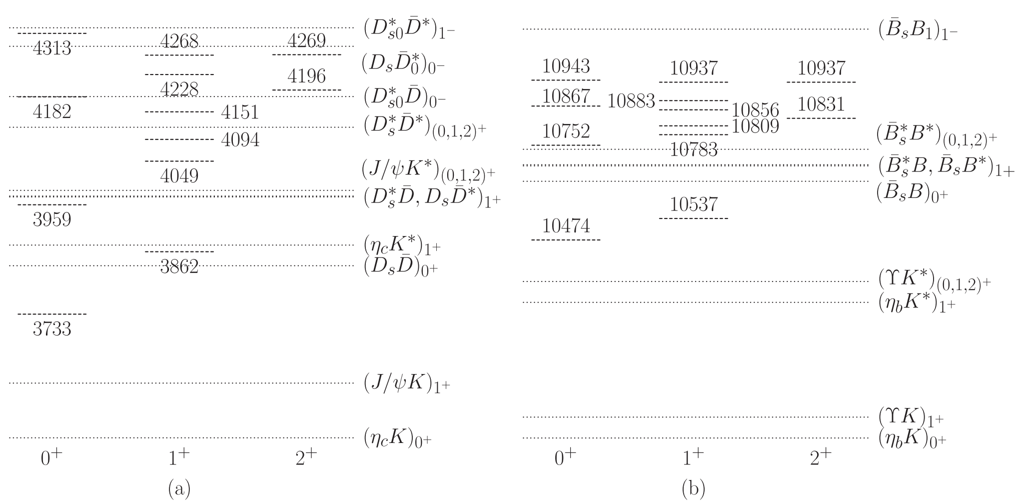

FIG. 3. Relative positions for kaon-like heavy tetraquarks (black dashed lines): (a) $cn\bar{c}\bar{s}$ and (b) $bn\bar{b}\bar{s}$ and various meson-meson thresholds (black dotted lines). The masses are given in units of MeV. The subscripts of threshold symbols are J^{PC} in the S-wave case.

TABLE IX. K_{ij}'s for $cn\bar{c}\,\bar{s}$ and $bn\bar{b}\,\bar{s}$ states. The order of states is the same as that in Table VIII.

$cn\bar{c}\,\bar{s}$ system							$bn\bar{b}\,\bar{s}$ system						
J^P	K_{cn}	$K_{c\bar{c}}$	$K_{c\bar{s}}$	$K_{c\bar{n}}$	$K_{s\bar{n}}$	K_{cs}	J^P	K_{bn}	$K_{b\bar{b}}$	$K_{b\bar{s}}$	$K_{b\bar{n}}$	$K_{s\bar{n}}$	K_{bs}
2^+	$\begin{bmatrix}-0.5\\1.8\end{bmatrix}$	$\begin{bmatrix}5.2\\-0.6\end{bmatrix}$	$\begin{bmatrix}0.6\\4.1\end{bmatrix}$	$\begin{bmatrix}0.6\\4.1\end{bmatrix}$	$\begin{bmatrix}5.2\\-0.6\end{bmatrix}$	$\begin{bmatrix}-0.5\\1.8\end{bmatrix}$	2^+	$\begin{bmatrix}-0.1\\1.5\end{bmatrix}$	$\begin{bmatrix}5.3\\-0.7\end{bmatrix}$	$\begin{bmatrix}0.1\\4.5\end{bmatrix}$	$\begin{bmatrix}0.1\\4.5\end{bmatrix}$	$\begin{bmatrix}5.3\\-0.7\end{bmatrix}$	$\begin{bmatrix}-0.1\\1.5\end{bmatrix}$
1^+	$\begin{bmatrix}-0.5\\0.7\\-0.2\\0.3\\-1.1\\-0.5\end{bmatrix}$	$\begin{bmatrix}5.3\\-3.0\\-9.2\\-2.4\\-0.7\\5.3\end{bmatrix}$	$\begin{bmatrix}0.1\\4.7\\0.3\\-4.2\\-5.1\\-0.5\end{bmatrix}$	$\begin{bmatrix}0.2\\4.6\\0.5\\-4.8\\-4.5\\-0.7\end{bmatrix}$	$\begin{bmatrix}5.3\\2.0\\3.0\\1.3\\-0.7\\-15.6\end{bmatrix}$	$\begin{bmatrix}0.1\\0.1\\-0.1\\0.4\\-1.3\\-0.6\end{bmatrix}$	1^+	$\begin{bmatrix}-0.0\\1.4\\-2.3\\1.0\\-1.3\\-0.2\end{bmatrix}$	$\begin{bmatrix}5.3\\-11.6\\-4.6\\1.6\\-0.7\\5.3\end{bmatrix}$	$\begin{bmatrix}0.1\\2.3\\1.0\\-1.8\\-6.0\\-0.2\end{bmatrix}$	$\begin{bmatrix}0.0\\2.3\\0.8\\-4.2\\-3.5\\-0.2\end{bmatrix}$	$\begin{bmatrix}5.3\\4.3\\2.6\\-0.3\\-0.7\\-16.0\end{bmatrix}$	$\begin{bmatrix}-0.1\\1.5\\-2.1\\0.8\\-1.2\\-0.2\end{bmatrix}$
0^+	$\begin{bmatrix}3.3\\-5.8\\1.8\\-2.0\end{bmatrix}$	$\begin{bmatrix}4.3\\2.5\\-1.6\\-14.5\end{bmatrix}$	$\begin{bmatrix}3.1\\2.4\\-11.8\\-3.1\end{bmatrix}$	$\begin{bmatrix}3.1\\2.4\\-11.8\\-3.1\end{bmatrix}$	$\begin{bmatrix}4.3\\2.5\\-1.6\\-14.5\end{bmatrix}$	$\begin{bmatrix}3.3\\-5.8\\1.8\\-2.0\end{bmatrix}$	0^+	$\begin{bmatrix}1.8\\-3.5\\-0.4\\-0.5\end{bmatrix}$	$\begin{bmatrix}5.1\\2.0\\-0.5\\-15.9\end{bmatrix}$	$\begin{bmatrix}1.7\\2.4\\-12.9\\-0.6\end{bmatrix}$	$\begin{bmatrix}1.7\\2.4\\-12.9\\-0.6\end{bmatrix}$	$\begin{bmatrix}5.1\\2.0\\-0.5\\-15.9\end{bmatrix}$	$\begin{bmatrix}1.8\\-3.5\\-0.4\\-0.5\end{bmatrix}$

For the $cn\bar{c}\bar{s}$ system, from Fig. 3(a), all the tetraquark states have rearrangement decay channels. Most of the states have open-charm decay modes while the lowest 0^+ and 1^+ not. Unlike the conventional mesons where the OZI rule works, at present, we do not know whether the tetraquarks are broad or not even if the state has only hidden-charm decay channels. From Table IX, the highest 2^+ and the second highest 0^+ states should be relatively stable since the diquarks have effectively attractive color-spin interactions while quark-antiquark interactions are effectively repulsive. Probably the highest 1^+ is also not very broad because of the weakly attractive cn interaction. Further studies on decay widths can help to understand the properties.

For the $bn\bar{b}\bar{s}$ system, from Fig. 3(b), one sees that the lowest 0^+ and 1^+ states do not have open-bottom rearrangement decay modes while others have, a feature similar to $cn\bar{c}\bar{s}$. From Table IX, in these tetraquarks, possible relatively stable states are the highest 2^+, highest 1^+, and the second highest 0^+.

Now we move on to the B_c-like tetraquark states. With appropriate substitutions of coupling parameters, one can obtain the eigenvalues of the CMI matrices in Eqs. (5)–(7). Further, the tetraquark masses can be estimated in various approaches mentioned above. We list these numerical results in Table X. The values in the last column are determined with the help of the $X(4140)$ which is treated as a $cs\bar{c}\bar{s}$ tetraquark. In this case, the mass formulas we modify are

$$M_{(cn\bar{b}\bar{n})} = m_b + m_c + 2m_n + \langle H_{\rm CM}\rangle_{(cn\bar{b}\bar{n})} = (2m_c + 2m_s) + (m_b - m_c) - 2(m_s - m_n) + \langle H_{\rm CM}\rangle_{(cn\bar{b}\bar{n})}, \tag{20}$$

and

$$M_{(cs\bar{b}\bar{s})} = m_b + m_c + 2m_s + \langle H_{\rm CM}\rangle_{(cs\bar{b}\bar{s})} = (2m_c + 2m_s) + (m_b - m_c) + \langle H_{\rm CM}\rangle_{(cs\bar{b}\bar{s})}. \tag{21}$$

After the replacements we have used in previous systems are made, one gets higher masses than those estimated with the DB/D_sB_s threshold. We perform discussions with such masses. In Figs. 4(a) and 4(b), we display the mass spectra with dashed lines for $cn\bar{b}\bar{n}$ and $cs\bar{b}\bar{s}$ tetraquarks, respectively. Relevant meson-meson thresholds are also shown with dotted lines. The calculated coefficients of effective color-spin interactions between quark components, K_{ij} in Eq. (13), are given in Table XI.

Again one should note that the masses for $cn\bar{b}\bar{n}$ states in the present model correspond to both the $I = 1$ case and the $I = 0$ case. From Fig. 4, the lowest isoscalar 0^+ state seems to be narrow if it exists while its isovector partner may be broader. If the mass difference $m_{B_c^*} - m_{B_c} = 70$ MeV [1] is used, the lowest 1^+ tetraquark may decay into $B_c^*\pi$ or $B_c^*\eta$. The isovector state seems to have a broader width than its isoscalar partner. From Table XI, the highest 1^+ states and the second highest 0^+ states probably have relatively stable structures (for both $I = 1$ and $I = 0$ cases).

For the $cs\bar{b}\bar{s}$ states, the lowest 0^+ is around the threshold of D_sB_s and probably not broad, while the lowest 1^+ may decay into $D_sB_s^*$, $B_c\phi$, and $B_c^*\phi$. Other tetraquarks should have broader widths. However, from Table XI, the highest 2^+ and the second highest 0^+ probably have relatively stable structures.

In the hidden-charm (hidden-bottom) case, the minimal excitation energy for the creation of a light quark-antiquark pair is around 370 (740) MeV while the orbital excitation energy is around 300 (400) MeV. In the present B_c case, the excitation energy for a light quark-antiquark pair is

TABLE X. Calculated CMI eigenvalues and estimated tetraquark masses for the $cn\bar{b}\,\bar{n}$ and $cs\bar{b}\,\bar{s}$ systems in units of MeV. The masses in the forth column (Upper limits) are obtained with Eq. (2) and those in the last three columns with various reference states.

$cn\bar{b}\,\bar{n}$ system						
J^P	$\langle H_{\mathrm{CM}}\rangle$	Eigenvalues	Upper limits	$B_c\pi$	DB	X(4140)
2^+	$\begin{pmatrix} 70.0 & -68.7 \\ -68.7 & 132.6 \end{pmatrix}$	$\begin{pmatrix} 176.8 \\ 25.8 \end{pmatrix}$	$\begin{pmatrix} 7678 \\ 7527 \end{pmatrix}$	$\begin{pmatrix} 7121 \\ 6970 \end{pmatrix}$	$\begin{pmatrix} 7462 \\ 7311 \end{pmatrix}$	$\begin{pmatrix} 7567 \\ 7416 \end{pmatrix}$
1^+	$\begin{pmatrix} -41.7 & -58.6 & 41.3 & 68.7 & 87.6 & -124.4 \\ -58.6 & 0.3 & 32.4 & 87.6 & 0.0 & -118.5 \\ 41.3 & 32.4 & -28.5 & -124.4 & -118.5 & 0.0 \\ 68.7 & 87.6 & -124.4 & -146.7 & -146.6 & 103.2 \\ 87.6 & 0.0 & -118.5 & -146.6 & -0.1 & 81.0 \\ -124.4 & -118.5 & 0.0 & 103.2 & 81.0 & 14.3 \end{pmatrix}$	$\begin{pmatrix} 179.9 \\ 118.4 \\ 58.1 \\ -1.5 \\ -95.5 \\ -462.0 \end{pmatrix}$	$\begin{pmatrix} 7681 \\ 7619 \\ 7559 \\ 7499 \\ 7405 \\ 7039 \end{pmatrix}$	$\begin{pmatrix} 7124 \\ 7063 \\ 7002 \\ 6943 \\ 6849 \\ 6482 \end{pmatrix}$	$\begin{pmatrix} 7465 \\ 7403 \\ 7343 \\ 7283 \\ 7189 \\ 6823 \end{pmatrix}$	$\begin{pmatrix} 7570 \\ 7508 \\ 7448 \\ 7388 \\ 7294 \\ 6928 \end{pmatrix}$
0^+	$\begin{pmatrix} -97.6 & 137.5 & -56.1 & 205.3 \\ 137.5 & -286.4 & 205.3 & -140.3 \\ -56.1 & 205.3 & -42.4 & 0.0 \\ 205.3 & -140.3 & 0.0 & 21.2 \end{pmatrix}$	$\begin{pmatrix} 193.4 \\ 69.0 \\ -132.8 \\ -534.8 \end{pmatrix}$	$\begin{pmatrix} 7694 \\ 7570 \\ 7368 \\ 6966 \end{pmatrix}$	$\begin{pmatrix} 7138 \\ 7013 \\ 6811 \\ 6409 \end{pmatrix}$	$\begin{pmatrix} 7478 \\ 7354 \\ 7152 \\ 6750 \end{pmatrix}$	$\begin{pmatrix} 7583 \\ 7459 \\ 7257 \\ 6855 \end{pmatrix}$
$cs\bar{b}\,\bar{s}$ system						
J^P	$\langle H_{\mathrm{CM}}\rangle$	Eigenvalues	Upper limits	$B_c\varphi$	D_sB_s	X(4140)
2^+	$\begin{pmatrix} 45.6 & -13.7 \\ -13.7 & 68.5 \end{pmatrix}$	$\begin{pmatrix} 74.9 \\ 39.3 \end{pmatrix}$	$\begin{pmatrix} 7933 \\ 7897 \end{pmatrix}$	$\begin{pmatrix} 7366 \\ 7330 \end{pmatrix}$	$\begin{pmatrix} 7554 \\ 7518 \end{pmatrix}$	$\begin{pmatrix} 7646 \\ 7611 \end{pmatrix}$
1^+	$\begin{pmatrix} -15.2 & -21.9 & 5.3 & 13.7 & 11.3 & -46.5 \\ -21.9 & 2.4 & 6.4 & 11.3 & 0.0 & -64.6 \\ 5.3 & 6.4 & -32.8 & -46.5 & -64.6 & 0.0 \\ 13.7 & 11.3 & -46.5 & -83.7 & -54.8 & 13.3 \\ 11.3 & 0.0 & -64.6 & -54.8 & -1.2 & 16.1 \\ -46.5 & -64.6 & 0.0 & 13.3 & 16.1 & 16.4 \end{pmatrix}$	$\begin{pmatrix} 83.2 \\ 48.7 \\ 11.4 \\ -14.3 \\ -79.0 \\ -164.1 \end{pmatrix}$	$\begin{pmatrix} 7941 \\ 7907 \\ 7869 \\ 7844 \\ 7779 \\ 7694 \end{pmatrix}$	$\begin{pmatrix} 7374 \\ 7340 \\ 7302 \\ 7277 \\ 7212 \\ 7127 \end{pmatrix}$	$\begin{pmatrix} 7562 \\ 7528 \\ 7491 \\ 7465 \\ 7400 \\ 7315 \end{pmatrix}$	$\begin{pmatrix} 7655 \\ 7620 \\ 7583 \\ 7557 \\ 7492 \\ 7407 \end{pmatrix}$
0^+	$\begin{pmatrix} -45.7 & 27.3 & -11.1 & 111.8 \\ 27.3 & -159.8 & 111.8 & -27.9 \\ -11.1 & 111.8 & -45.6 & 0.0 \\ 111.8 & -27.9 & 0.0 & 22.8 \end{pmatrix}$	$\begin{pmatrix} 106.5 \\ 23.0 \\ -116.9 \\ -240.9 \end{pmatrix}$	$\begin{pmatrix} 7964 \\ 7881 \\ 7741 \\ 7617 \end{pmatrix}$	$\begin{pmatrix} 7398 \\ 7314 \\ 7174 \\ 7050 \end{pmatrix}$	$\begin{pmatrix} 7586 \\ 7502 \\ 7362 \\ 7238 \end{pmatrix}$	$\begin{pmatrix} 7678 \\ 7594 \\ 7455 \\ 7331 \end{pmatrix}$

around 570 MeV, a value between the hidden-charm and hidden-bottom cases. That for orbital excitation should be less than 400 MeV, e.g., 370 MeV. Then the mass of $B_{c0}(1P)$ is probably around 6.7 GeV and the mass of $B_{c2}(1P)$ is likely to be less than 6.8 GeV. From the QM calculations [1], we may also guess that the mass for the $B_{c2}(1F)$ meson is probably in the range 7.2–7.3 GeV. From these numbers, it seems that only radially excited B_c states with $J^P = 0^+$, 1^+, and 2^+ can fall into the mass region for the B_c-like tetraquarks.

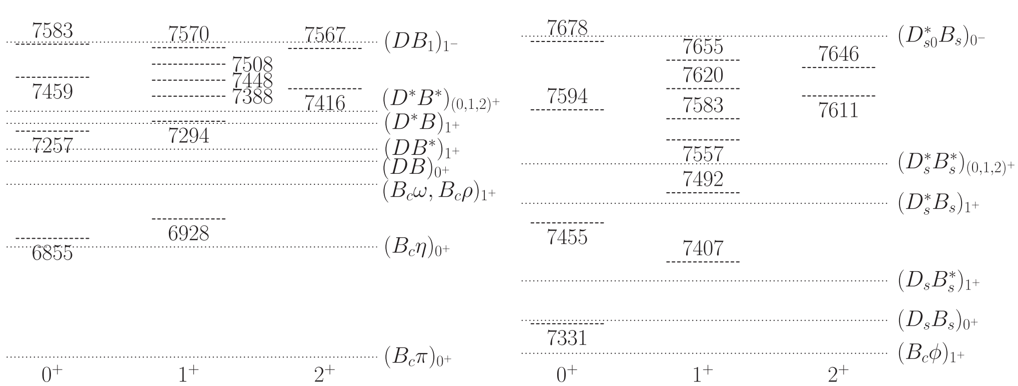

FIG. 4. Relative positions for B_c-like heavy tetraquarks (black dashed lines): (a) $cn\bar{b}\,\bar{n}$ and (b) $cs\bar{b}\,\bar{s}$ and various meson-meson thresholds (black dotted lines). The masses are given in units of MeV. The subscripts of threshold symbols are J^{PC} in the S-wave case.

TABLE XI. K_{ij}'s for $cn\bar{b}\bar{n}$ and $cs\bar{b}\bar{s}$ states. The order of states is the same as that in Table X.

$cn\bar{b}\bar{n}$ system							$cs\bar{b}\bar{s}$ system						
J^P	K_{cn}	$K_{b\bar{c}}$	$K_{c\bar{n}}$	$K_{b\bar{n}}$	$K_{n\bar{n}}$	K_{bn}	J^P	K_{cs}	$K_{b\bar{c}}$	$K_{c\bar{s}}$	$K_{b\bar{s}}$	$K_{s\bar{s}}$	K_{bs}
2^+	$\begin{bmatrix}-0.2\\1.5\end{bmatrix}$	$\begin{bmatrix}5.3\\-0.7\end{bmatrix}$	$\begin{bmatrix}0.2\\4.5\end{bmatrix}$	$\begin{bmatrix}0.2\\4.5\end{bmatrix}$	$\begin{bmatrix}5.3\\-0.7\end{bmatrix}$	$\begin{bmatrix}-0.2\\1.5\end{bmatrix}$	2^+	$\begin{bmatrix}-0.6\\2.0\end{bmatrix}$	$\begin{bmatrix}5.1\\-0.5\end{bmatrix}$	$\begin{bmatrix}0.8\\3.9\end{bmatrix}$	$\begin{bmatrix}0.8\\3.9\end{bmatrix}$	$\begin{bmatrix}5.1\\-0.5\end{bmatrix}$	$\begin{bmatrix}-0.6\\2.0\end{bmatrix}$
1^+	$\begin{bmatrix}0.9\\1.0\\-3.4\\1.1\\-0.4\\-0.6\end{bmatrix}$	$\begin{bmatrix}5.1\\-12.8\\-3.2\\1.1\\-0.1\\5.3\end{bmatrix}$	$\begin{bmatrix}0.9\\1.6\\1.9\\4.0\\-12.5\\-0.6\end{bmatrix}$	$\begin{bmatrix}-0.7\\2.1\\0.9\\-10.5\\3.5\\0.1\end{bmatrix}$	$\begin{bmatrix}5.2\\4.7\\2.3\\-0.5\\-0.6\\-15.9\end{bmatrix}$	$\begin{bmatrix}-0.9\\1.6\\-0.9\\-0.7\\-0.5\\0.1\end{bmatrix}$	1^+	$\begin{bmatrix}2.8\\-2.5\\2.6\\-4.0\\2.2\\-2.4\end{bmatrix}$	$\begin{bmatrix}3.7\\-1.0\\-4.0\\-8.3\\0.2\\4.6\end{bmatrix}$	$\begin{bmatrix}2.1\\3.7\\1.7\\2.6\\-10.7\\-4.0\end{bmatrix}$	$\begin{bmatrix}-0.4\\4.2\\-4.3\\-8.5\\3.0\\1.2\end{bmatrix}$	$\begin{bmatrix}4.6\\2.4\\1.2\\3.0\\-2.1\\-13.8\end{bmatrix}$	$\begin{bmatrix}-2.7\\2.9\\-1.2\\0.8\\-1.9\\0.6\end{bmatrix}$
0^+	$\begin{bmatrix}2.3\\-4.3\\0.0\\-0.7\end{bmatrix}$	$\begin{bmatrix}4.9\\2.1\\-0.6\\-15.8\end{bmatrix}$	$\begin{bmatrix}2.1\\2.4\\-13.0\\-0.9\end{bmatrix}$	$\begin{bmatrix}2.1\\2.4\\-13.0\\-0.9\end{bmatrix}$	$\begin{bmatrix}4.9\\2.1\\-0.6\\-15.8\end{bmatrix}$	$\begin{bmatrix}2.3\\-4.3\\0.0\\-0.7\end{bmatrix}$	0^+	$\begin{bmatrix}3.4\\-5.9\\2.3\\-2.4\end{bmatrix}$	$\begin{bmatrix}4.1\\2.5\\-2.3\\-13.7\end{bmatrix}$	$\begin{bmatrix}3.3\\2.4\\-10.9\\-4.2\end{bmatrix}$	$\begin{bmatrix}3.3\\2.4\\-10.9\\-4.2\end{bmatrix}$	$\begin{bmatrix}4.1\\2.5\\-2.3\\-13.7\end{bmatrix}$	$\begin{bmatrix}3.4\\-5.9\\2.3\\-2.4\end{bmatrix}$

TABLE XII. Calculated CMI eigenvalues and estimated tetraquark masses for the $cn\bar{b}\bar{s}$ and $cs\bar{b}\bar{n}$ systems in units of MeV. The masses in the fourth column (Upper limits) are obtained with Eq. (2) and those in the last three columns with various reference states.

$cn\bar{b}\bar{s}$ system						
J^P	$\langle H_{\rm CM}\rangle$	Eigenvalues	Upper limits	B_cK	D_sB	X(4140)
2^+	$\begin{pmatrix}54.9 & -37.3\\-37.3 & 95.7\end{pmatrix}$	$\begin{pmatrix}117.9\\32.8\end{pmatrix}$	$\begin{pmatrix}7797\\7712\end{pmatrix}$	$\begin{pmatrix}7238\\7153\end{pmatrix}$	$\begin{pmatrix}7506\\7421\end{pmatrix}$	$\begin{pmatrix}7598\\7513\end{pmatrix}$
1^+	$\begin{pmatrix}-27.2 & -37.7 & 20.4 & 37.3 & 43.2 & -80.0\\-37.7 & 1.1 & 17.6 & 43.2 & 0.0 & -87.1\\20.4 & 17.6 & -28.8 & -80.0 & -87.1 & 0.0\\37.3 & 43.2 & -80.0 & -109.6 & -94.3 & 50.9\\43.2 & 0.0 & -87.1 & -94.3 & -0.5 & 44.0\\-80.0 & -87.1 & 0.0 & 50.9 & 44.0 & 14.4\end{pmatrix}$	$\begin{pmatrix}122.5\\72.0\\29.2\\1.4\\-89.0\\-286.6\end{pmatrix}$	$\begin{pmatrix}7802\\7751\\7709\\7681\\7590\\7393\end{pmatrix}$	$\begin{pmatrix}7243\\7193\\7150\\7122\\7032\\6834\end{pmatrix}$	$\begin{pmatrix}7511\\7460\\7418\\7390\\7299\\7102\end{pmatrix}$	$\begin{pmatrix}7603\\7553\\7510\\7482\\7392\\7194\end{pmatrix}$
0^+	$\begin{pmatrix}-68.3 & 74.7 & -30.5 & 150.9\\74.7 & -212.3 & 150.9 & -76.2\\-30.5 & 150.9 & -41.6 & 0.0\\150.9 & -76.2 & 0.0 & 20.8\end{pmatrix}$	$\begin{pmatrix}140.1\\45.0\\-125.8\\-360.7\end{pmatrix}$	$\begin{pmatrix}7819\\7724\\7554\\7319\end{pmatrix}$	$\begin{pmatrix}7261\\7166\\6995\\6760\end{pmatrix}$	$\begin{pmatrix}7528\\7433\\7263\\7028\end{pmatrix}$	$\begin{pmatrix}7621\\7526\\7355\\7120\end{pmatrix}$
$cs\bar{b}\bar{n}$ system						
J^P	$\langle H_{\rm CM}\rangle$	Eigenvalues	Upper limits	$B_c\bar{K}$	DB_s	X(4140)
2^+	$\begin{pmatrix}56.8 & -36.8\\-36.8 & 95.6\end{pmatrix}$	$\begin{pmatrix}117.8\\34.6\end{pmatrix}$	$\begin{pmatrix}7797\\7714\end{pmatrix}$	$\begin{pmatrix}7238\\7155\end{pmatrix}$	$\begin{pmatrix}7493\\7410\end{pmatrix}$	$\begin{pmatrix}7598\\7515\end{pmatrix}$
1^+	$\begin{pmatrix}-25.9 & -37.3 & 20.7 & 36.8 & 44.0 & -79.2\\-37.3 & 1.6 & 17.3 & 44.0 & 0.0 & -87.7\\20.7 & 17.3 & -32.5 & -79.2 & -87.7 & 0.0\\36.8 & 44.0 & -79.2 & -111.1 & -93.3 & 51.9\\44.0 & 0.0 & -87.7 & -93.3 & -0.8 & 43.3\\-79.2 & -87.7 & 0.0 & 51.9 & 43.3 & 16.3\end{pmatrix}$	$\begin{pmatrix}122.9\\72.9\\28.3\\-1.3\\-88.2\\-287.1\end{pmatrix}$	$\begin{pmatrix}7802\\7752\\7708\\7678\\7591\\7392\end{pmatrix}$	$\begin{pmatrix}7244\\7194\\7149\\7119\\7032\\6834\end{pmatrix}$	$\begin{pmatrix}7499\\7449\\7404\\7374\\7288\\7089\end{pmatrix}$	$\begin{pmatrix}7604\\7554\\7509\\7479\\7392\\7194\end{pmatrix}$
0^+	$\begin{pmatrix}-67.2 & 73.5 & -30.0 & 151.9\\73.5 & -214.4 & 151.9 & -75.1\\-30.0 & 151.9 & -46.4 & 0.0\\151.9 & -75.1 & 0.0 & 23.2\end{pmatrix}$	$\begin{pmatrix}142.4\\42.4\\-127.8\\-361.8\end{pmatrix}$	$\begin{pmatrix}7822\\7722\\7552\\7318\end{pmatrix}$	$\begin{pmatrix}7263\\7163\\6993\\6759\end{pmatrix}$	$\begin{pmatrix}7518\\7418\\7248\\7014\end{pmatrix}$	$\begin{pmatrix}7623\\7523\\7353\\7119\end{pmatrix}$

TABLE XIII. K_{ij}'s for $cn\bar{b}\bar{s}$ and $cs\bar{b}\bar{n}$ states. The order of states is the same as that in Table XII.

	$cn\bar{b}\bar{s}$ system							$cs\bar{b}\bar{n}$ system					
J^P	K_{cn}	$K_{b\bar{c}}$	$K_{c\bar{s}}$	$K_{b\bar{n}}$	$K_{s\bar{n}}$	K_{bs}	J^P	K_{cs}	$K_{b\bar{c}}$	$K_{c\bar{n}}$	$K_{b\bar{s}}$	$K_{s\bar{n}}$	K_{bn}
2^+	$\begin{bmatrix}-0.3\\1.6\end{bmatrix}$	$\begin{bmatrix}5.3\\-0.6\end{bmatrix}$	$\begin{bmatrix}0.3\\4.3\end{bmatrix}$	$\begin{bmatrix}0.3\\4.3\end{bmatrix}$	$\begin{bmatrix}5.3\\-0.6\end{bmatrix}$	$\begin{bmatrix}-0.3\\1.6\end{bmatrix}$	2^+	$\begin{bmatrix}-0.3\\1.6\end{bmatrix}$	$\begin{bmatrix}5.3\\-0.6\end{bmatrix}$	$\begin{bmatrix}0.3\\4.4\end{bmatrix}$	$\begin{bmatrix}0.3\\4.4\end{bmatrix}$	$\begin{bmatrix}5.3\\-0.6\end{bmatrix}$	$\begin{bmatrix}-0.3\\1.6\end{bmatrix}$
1^+	$\begin{bmatrix}1.5\\-0.1\\-2.1\\-0.2\\0.5\\-1.0\end{bmatrix}$	$\begin{bmatrix}4.8\\-6.9\\-7.5\\-0.1\\-0.1\\5.2\end{bmatrix}$	$\begin{bmatrix}1.3\\3.0\\1.1\\3.9\\-12.7\\-1.2\end{bmatrix}$	$\begin{bmatrix}-0.9\\3.8\\1.2\\-12.7\\3.7\\0.3\end{bmatrix}$	$\begin{bmatrix}5.1\\3.5\\2.9\\0.1\\-0.6\\-15.6\end{bmatrix}$	$\begin{bmatrix}-1.5\\2.3\\-1.6\\0.1\\-0.9\\0.2\end{bmatrix}$	1^+	$\begin{bmatrix}1.7\\-0.1\\-1.8\\-0.5\\0.4\\-1.0\end{bmatrix}$	$\begin{bmatrix}4.7\\-7.0\\-7.0\\-0.5\\-0.1\\5.2\end{bmatrix}$	$\begin{bmatrix}1.4\\2.8\\1.2\\3.7\\-12.5\\-1.3\end{bmatrix}$	$\begin{bmatrix}-0.8\\3.7\\1.3\\-12.7\\3.5\\0.3\end{bmatrix}$	$\begin{bmatrix}5.1\\3.5\\2.7\\0.3\\-0.7\\-15.6\end{bmatrix}$	$\begin{bmatrix}-1.6\\2.4\\-1.7\\0.3\\-0.9\\0.2\end{bmatrix}$
0^+	$\begin{bmatrix}2.9\\-5.2\\0.9\\-1.3\end{bmatrix}$	$\begin{bmatrix}4.6\\2.3\\-0.8\\-15.5\end{bmatrix}$	$\begin{bmatrix}2.7\\2.5\\-12.9\\-1.6\end{bmatrix}$	$\begin{bmatrix}2.7\\2.5\\-12.9\\-1.6\end{bmatrix}$	$\begin{bmatrix}4.6\\2.3\\-0.8\\-15.5\end{bmatrix}$	$\begin{bmatrix}2.9\\-5.2\\0.9\\-1.3\end{bmatrix}$	0^+	$\begin{bmatrix}3.0\\-5.3\\0.9\\-1.3\end{bmatrix}$	$\begin{bmatrix}4.6\\2.3\\-0.9\\-15.4\end{bmatrix}$	$\begin{bmatrix}2.7\\2.4\\-12.7\\-1.8\end{bmatrix}$	$\begin{bmatrix}2.7\\2.4\\-12.7\\-1.8\end{bmatrix}$	$\begin{bmatrix}4.6\\2.3\\-0.9\\-15.4\end{bmatrix}$	$\begin{bmatrix}3.0\\-5.3\\0.9\\-1.3\end{bmatrix}$

E. The $cn\bar{b}\bar{s}$ and $cs\bar{b}\bar{n}$ systems

These states are composed of four different flavors, a similar feature to the $X(5568)$. Some results can be found in Ref. [35]. In fact, the isovector B_c-like systems also contain quarks with four different flavors.

According to the expressions for $\langle H_{\mathrm{CM}}\rangle$ in Eqs. (5)–(7) and the values of C_{cn}, $C_{b\bar{c}}$, $C_{c\bar{s}}$, $C_{b\bar{n}}$, $C_{n\bar{s}}$, C_{bs}, C_{cs}, $C_{c\bar{n}}$, $C_{b\bar{s}}$, and $C_{b\bar{n}}$ in Table I, we obtain numerical values and eigenvalues of the CMI matrices for $cn\bar{b}\bar{s}$ and $cs\bar{b}\bar{n}$ systems. These data together with the estimated tetraquark masses in various approaches are shown in Table XII. We have obtained masses with the help of the $X(4140)$ in the last column by modifying

$$M_{(cn\bar{b}\bar{s})} = m_b + m_c + m_n + ms + \langle H_{\mathrm{CM}}\rangle_{(cn\bar{b}\bar{s})} = (2m_c + 2m_s) + (m_b - m_c) - (m_s - m_n) + \langle H_{\mathrm{CM}}\rangle_{(cn\bar{b}\bar{s})} \tag{22}$$

and

$$M_{(cs\bar{b}\bar{n})} = m_b + m_c + m_n + m_s + \langle H_{\mathrm{CM}}\rangle_{(cs\bar{b}\bar{n})} = (2m_c + 2m_s) + (m_b - m_c) - (m_s - m_n) + \langle H_{\mathrm{CM}}\rangle_{(cs\bar{b}\bar{n})}. \tag{23}$$

That is, the relevant formula is (to get more reasonable masses)

$$M = M_{X(4140)} - m_{B_s} - m_D + 2m_B + \langle H_{\mathrm{CM}}\rangle_{B_s} + \langle H_{\mathrm{CM}}\rangle_D - 2\langle H_{\mathrm{CM}}\rangle_B - \langle H_{\mathrm{CM}}\rangle_{X(4140)} + \langle H_{\mathrm{CM}}\rangle. \tag{24}$$

The mass differences between the $cn\bar{b}\bar{s}$ and $cs\bar{b}\bar{n}$ states mainly come from the chromomagnetic interactions. From Table XII, their differences are very small. If we check the variables defined in Eq. (8), the differences in expressions are related to $(C_{cn} - C_{cs}) \pm (C_{bn} - C_{bs})$, $(C_{c\bar{n}} - C_{c\bar{s}}) \pm (C_{b\bar{n}} - C_{b\bar{s}})$, and $-(C_{c\bar{n}} - C_{c\bar{s}}) \pm (C_{b\bar{n}} - C_{b\bar{s}})$, i.e., with the $SU(3)_f$ symmetry breaking when heavy quark is involved. Numerically, Table I tells us that their absolute values are all less than 1 MeV and Table XII lets us know that the resulting mass difference is at most 3 MeV. Because of the existent heavy quark, the mass of a $c\bar{b}n\bar{s}$ is not exactly the same as that of $c\bar{b}s\bar{n}$. We display the relative positions for the tetraquark states and relevant thresholds in Fig. 5. One notes that mass differences (10–20 MeV) between D_sB and DB_s, D_sB^* and DB_s^*, and so on also exist, but the properties of these two systems are very similar. In Table XIII, we present values of K_{ij}'s for the present systems. The data for the two systems are also close to each other. One may concentrate only on one system, e.g., $cn\bar{b}\bar{s}$.

If the mass difference $m_{B_c^*} - m_{B_c}$ is around 70 MeV [1], from Fig. 5, all these tetraquarks have rearrangement decay modes and probably are not narrow states. From Table XIII, the highest 2^+ and the second highest 0^+ states have relatively stable structures. The width of the lowest 0^+ state is probably not very large since it has only one rearrangement decay channel B_cK. To understand whether such states exist or not and whether the adopted method is reasonable or not, searching for them in possible decay channels is a worthwhile work.

Now we move on to the problem about the nature of a state below the B_cK threshold. From Table XII, the lowest tetraquark (6760 MeV) we can obtain is only ~10 MeV lower than the B_cK threshold. If a B_cK molecule exists, the binding energy should be small since the B_cK interaction is weak (the small scattering length $a_{\eta_c\pi}$ in Ref. [112] as a reference). If experiments could observe a state below the threshold with a large energy gap, a similar situation to the

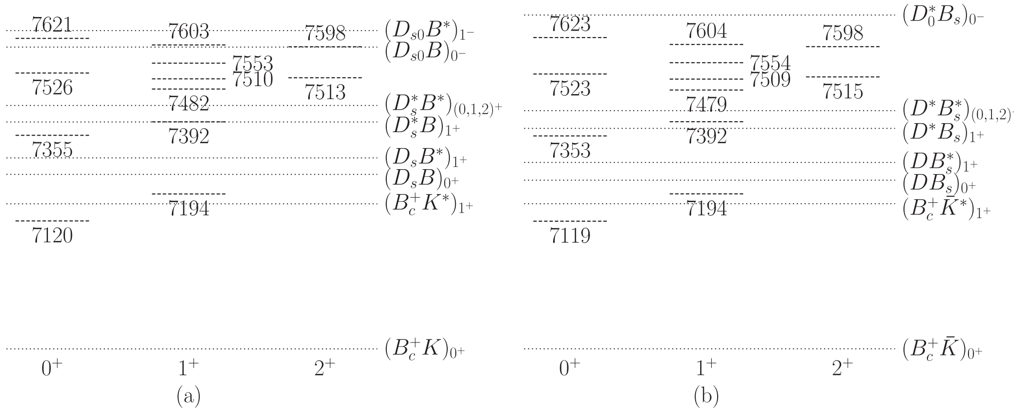

FIG. 5. Relative positions for B_c-like heavy tetraquarks (black dashed lines): (a) $cn\bar{b}\,\bar{s}$ and (b) $cs\bar{b}\,\bar{n}$ and various meson-meson thresholds (black dotted lines). The masses are given in units of MeV. The subscripts of threshold symbols are J^{PC} in the S-wave case.

$X(5568)$, it will be very difficult to understand its nature either in the tetraquark picture or molecule picture. One gets a similar conclusion for the $I=1$ $cn\bar{b}\bar{n}$ case.

IV. DISCUSSIONS AND SUMMARY

In this study, we systematically analyze the spectra of the possible $Q_1q_2\bar{Q}_3\bar{q}_4$ ($Q=b$, c and $q=n$, s with $n=u$, d) tetraquark states by using the CMI model. We use the diquark-antiqiquark bases to construct the wave functions and calculate the CMI matrices. After diagonalizing the matrices, the eigenvalues irrelevant with base choice are obtained. Such values determine the mass splittings between states with the same quark content. Since the present model does not involve dynamics, one cannot determine the absolute masses by solving the bound state problem. To get numerical results of masses, we tried several estimation approaches: (1) with Eq. (2), (2) with the $(Q_1\bar{Q}_3)(q_2\bar{q}_4)$ type meson-meson threshold as a reference, (3) with the $(Q_1\bar{q}_4)(q_2\bar{Q}_3)$ type meson-meson threshold as a reference, and (4) with a postulated mass scale relating to $X(4140)$ as a reference. In the first approach, the obtained masses are always larger than those in other approaches. For conventional hadrons, the obtained masses are usually higher than the experimental measurements (see Table IV of Ref. [54]). This means that the additional attraction effects are actually needed in this approach and we may treat tetraquark masses in this approach as theoretical upper limits. In the second approach, the obtained masses are always smaller than those in other approaches. Therefore, we may treat masses in this approach as theoretical lower limits. In the third approach, the obtained masses are moderate. Now we analyze the reason why the masses in this approach are larger than those in the second approach. In fact, when we estimating masses with the modified Eq. (11), we are making the following replacements from Eq. (2),

$$\begin{aligned} m_1+m_3 &= M^{\rm Th}_{(Q_1\bar{Q}_3)} - \langle H_{\rm CM}\rangle_{(Q_1\bar{Q}_3)} \\ &\to M^{\rm Ex}_{(Q_1\bar{Q}_3)} - \langle H_{\rm CM}\rangle_{(Q_1\bar{Q}_3)}, \\ m_2+m_4 &= M^{\rm Th}_{(q_2\bar{q}_4)} - \langle H_{\rm CM}\rangle_{(q_2\bar{q}_4)} \\ &\to M^{\rm Ex}_{(q_2\bar{q}_4)} - \langle H_{\rm CM}\rangle_{(q_2\bar{q}_4)} \end{aligned} \tag{25}$$

in the second approach and

$$\begin{aligned} m_1+m_4 &= M^{\rm Th}_{(Q_1\bar{q}_4)} - \langle H_{\rm CM}\rangle_{(Q_1\bar{q}_4)} \\ &\to M^{\rm Ex}_{(Q_1\bar{q}_4)} - \langle H_{\rm CM}\rangle_{(Q_1\bar{q}_4)}, \\ m_2+m_3 &= M^{\rm Th}_{(q_2\bar{Q}_3)} - \langle H_{\rm CM}\rangle_{(q_2\bar{Q}_3)} \\ &\to M^{\rm Ex}_{(q_2\bar{Q}_3)} - \langle H_{\rm CM}\rangle_{(q_2\bar{Q}_3)} \end{aligned} \tag{26}$$

in the third approach. Here, $M^{\rm Th}_{(Q_1\bar{Q}_3)}$ ($M^{\rm Ex}_{(Q_1\bar{Q}_3)}$) means the calculated (measured) mass for the $(Q_1\bar{Q}_3)$ meson, etc. Then the compensated attraction in the second approach is represented by $(M^{\rm Th}_{(Q_1\bar{Q}_3)} - M^{\rm Ex}_{(Q_1\bar{Q}_3)}) + (M^{\rm Th}_{(q_2\bar{q}_4)} - M^{\rm Ex}_{(q_2\bar{q}_4)})$ and that in the third approach is $(M^{\rm Th}_{(Q_1\bar{q}_4)} - M^{\rm Ex}_{(Q_1\bar{q}_4)}) + (M^{\rm Th}_{(q_2\bar{Q}_3)} - M^{\rm Ex}_{(q_2\bar{Q}_3)})$. From Table IV of Ref. [54], the former value is usually larger than the latter value and their difference is the mass difference for tetraquarks between the two approaches. For a tetraquark state, its size should be larger than that of a conventional meson, which means that the compensated attraction should not be so strong like the value in the second approach. Although the tetraquark masses in the third approach are larger, it seems that they are still smaller than the realistic case. From Ref. [63], additional kinetic energy may contribute and lead to larger masses. The dynamical calculation in Ref. [64] also favors the argument that the estimated masses with the reference thresholds are still small. Therefore, the values obtained in the forth approach seem to be more realistic. The masses are about 80–105 MeV higher than those in the third approach. Treating the $X(4140)$ as the lowest 1^{++} $cs\bar{c}\bar{s}$ tetraquark state, the problem of mass estimation becomes the problem to determine quark mass differences in hadrons. Here, we assume that the largest values in the fourth approach are closest to the realistic tetraquark masses and perform discussions. Searching for the various predicted states in this approach may help to test the assumptions we adopt.

For the color-spin interactions between quark components in multiquark states, the complicated structure mixing effects may change their original properties, from attractive to repulsive or from repulsive to attractive. For the tetraquark states studied in this work, attractive diquarks while repulsive quark-antiquark pairs are helpful for relatively stable states. To understand this property for the quark interactions, we evaluated the measure, K_{ij} defined in Ref. [57], for various states.

According to the numerical results, we performed the discussions in the previous section. Our results on the exotic XYZ states may be summarized as follows:

(i) From the qualitative features of both mass and width, the newly observed $Z_c(4100)$ by LHCb seems to be a 0^{++} $cn\bar{c}\bar{n}$ tetraquark state.
(ii) From the consistency of mass and width with the $Z_c(4100)$, the $X(3860)$ observed by Belle may be another 0^{++} $cn\bar{c}\bar{n}$ tetraquark state.
(iii) The $Z_c(4200)$ observed by Belle is probably a 1^{+-} $cn\bar{c}\bar{n}$ tetraquark state.
(iv) The $Z_c(4250)$ can be a tetraquark but the quantum numbers cannot be assigned.
(v) The $Z_c(3900)$, $X(3940)$, and $X(4160)$ are unlikely compact tetraquark states.
(vi) The $Z_c(4020)$ is unlikely a compact tetraquark, but seems to be the hidden-charm correspondence of the $Z_b(10650)$ with $J^{PC} = 1^{+-}$.

Our predictions on possible tetraquarks can be found in Fig. 1 of Ref. [52] and Figs. 1–5. There should exist relatively narrow tetraquarks, such as the lowest 0^{++} $cs\bar{c}\bar{s}$ and $cn\bar{c}\bar{n}$. In particular, from the signs of measure for effective quark interactions, we find that for the highest 2^+ and the second highest 0^+ states, the structures probably are more stable than other partner states with the same J^P, because the quark-quark interactions in them are effectively attractive while the quark-antiquark interactions are effectively repulsive. For the case having C-parity, the highest 1^{++} states also have such a property. The remaining states having such a property are the highest 1^+ $cn\bar{c}\bar{s}$ and $bn\bar{b}\bar{s}$. The widths of these mentioned states are probably not very broad although their masses are not low.

In the modified estimation method, the dominant uncertainties for the tetraquark masses are partly remedied, but the uncertainties in coupling parameters still exist, although the effects on mass splittings may be small. It seems that one cannot solve this problem without dynamical calculations. We wait for experimental measurements to answer whether the extracted C_{ij}'s from the conventional hadrons are actually applicable to multiquarks or not and how large the induced uncertainties for mass splittings are.

In our study, we did not consider the generally mixed isoscalar states of $Q_1 n\bar{Q}_3\bar{n}$ and $Q_1 s\bar{Q}_3\bar{s}$, but considered the states similar to the ω and ϕ case. The mixing between $Q_1 n\bar{Q}_3\bar{n}$ and $Q_1 s\bar{Q}_3\bar{s}$ surely affects the spectrum. Once the predicted tetraquarks could be confirmed, one may study this case if necessary.

Here we consider the compact tetraquark states. In the literature, there are studies of various charmonium-, bottomonium-, and B_c-like meson-meson molecules [113–115]. Apparently, the two configurations are difficult to distinguish just from the quantum numbers. Since the distances between quark components in these two configurations are different, the masses are not always the same. To identify the inner structure to which an observed meson belongs, the decay properties should be helpful.

An inconsistency about decay width might exist in our arguments. If our argument about stability of states and the assignments for tetraquark states are correct, the qualitative consistency between the widths of $X(4140)$ and $X(4274)$ is satisfied. That for $Z_c(4100)$, $X(3860)$, and $Z_c(4200)$ is also observed. However, if we compare the widths of $cn\bar{c}\bar{n}$ tetraquarks and those of $cs\bar{c}\bar{s}$ tetraquarks, the consistency seems a problem. The widths of the former states are larger than 100 MeV while those for the latter are at most tens of MeV. It is worthwhile to study more on the decay widths of tetraquarks [116,117] in future works in order to check or confirm the assumptions used here.

To summarize, by studying the chromomagnetic interaction between quark components, we calculated the mass splittings between the $Q_1 q_2\bar{Q}_3\bar{q}_4$ tetraquark states. With the assumption that the $X(4140)$ is the lowest 1^{++} $cs\bar{c}\bar{s}$ tetraquark, we estimated all the $Q_1 q_2\bar{Q}_3\bar{q}_4$ masses, which can be tested in future experimental measurements. According to the numerical results, we discussed possible assignments for several exotic XYZ mesons.

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