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Thermodynamics of novel charged dilatonic BTZ black holes

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ABSTRACT

In this paper, the three-dimensional Einstein–Maxwell theory in the presence of a dilatonic scalar field has been studied. It has been shown that the dilatonic potential must be considered as the linear combination of two Liouville-type potentials. Two new classes of charged dilatonic BTZ black holes, as the exact solutions to the coupled scalar, vector and tensor field equations, have been obtained and their properties have been studied. The conserved charge and mass of the new black holes have been calculated, making use of the Gauss's law and Abbott–Deser proposal, respectively. Through comparison of the thermodynamical extensive quantities (i.e. temperature and entropy) obtained from both, the geometrical and the thermodynamical methods, the validity of the first law of black hole thermodynamics has been confirmed for both of the new black holes we just obtained. A black hole thermal stability or phase transition analysis has been performed, making use of the canonical ensemble method. Regarding the black hole heat capacity, it has been found that for either of the new black hole solutions there are some specific ranges in such a way that the black holes with the horizon radius in these ranges are locally stable. The points of type one and type two phase transitions have been determined. The black holes, with the horizon radius equal to the transition points are unstable. They undergo type one or type two phase transitions to be stabilized.

1. Introduction

Although the Einstein's tensorial theory of gravitation is in agreement with a large amount of observational tests, but it fails regarding some important issues [1–5]. Modification of the Einstein's theory of gravity is one of the main approaches to overcome the related failures. Among the various proposed modifications [6–13], the so-called scalar-tensor theories [14], as the modification arisen from string theory, have provided interesting results [15]. The Einstein's action is naturally modified by the scalar-tensor superstring terms at the high energy regime. In the low energy limit of the string theory, the Einstein's theory of gravity is recovered which is coupled to a dilatonic scalar field [16].

Black holes with scalar hair are interesting solutions of Einstein's theory of gravity and also of certain types of modified gravity theories. These solutions have been investigated by theoretical physicists in four and higher dimensional space times for a long time (see [17] and references therein). The first studies on the three-dimensional black holes, as the interesting predictions of Einstein's theory of relativity in lower dimensional space times, have been done by Banados, Teitelboim, and Zanelli (BTZ) [18]. Investigation of the three-dimensional black holes is one of the interesting subjects for recent gravitational studies [19]. Chan and Mann [20], are the first authors who investigated the charged three-dimensional dilatonic black holes in the presence of a minimally coupled logarithmic dilaton field.

It is a commonly believed that study of three-dimensional solutions help us to find a deeper insight into the fundamental ideas in comparison to higher dimensional black holes. Also, according to (A)dS/CFT correspondence, there is a dual between quantum gravity on A(dS) space and Euclidean conformal field theory on the lower dimensional space times [21,22]. From this point of view, study of physics in $(2+1)$-dimensional space times can be useful for understanding of quantum field theory on A(dS) spacetimes. Although this subject area has been considered extensively [23], it still has many unknown and interesting parts to be studied [24].

On the other hand, after the discoveries of Bekenstein, Bardeen, Carter and Hawking, it is well-known that black holes can be considered as the thermodynamical systems with a temperature proportional to the surface gravity and having pure geometrical entropy equal to one-fourth of the horizon area [25–27]. When a dilatonic scalar field is coupled to the three-dimensional Einstein–Maxwell theory, it is expected to produce new and interesting consequences for the black hole solutions. Thus, it is worth to find exact solutions of Einstein–Maxwell theory in the presence of a

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dilatonic scalar field with an arbitrary coupling constant, and investigate how the thermodynamical properties of black holes are modified. Also it is interesting to investigate the black holes remnant and find out the impacts of dilatonic field on the thermal stability of the black hole solutions.

The main object of this paper is to introduce new charged dilatonic BTZ black holes as the exact solutions to the coupled scalar, vector and tensor field equations, and provide a detailed analysis of the thermodynamical properties as well as the thermal stability of the new three-dimensional electrically charged black holes in the presence of a dilatonic scalar field.

The paper is outlined in the following order. In Sec. 2, by varying the proper Einstein–Maxwell action coupled to a dilatonic scalar field, the related scalar, vector and tensor field equations have been obtained. By introducing a static spherically symmetric geometry two new classes of charged dilatonic BTZ black holes, as the exact solution to the field equations, have been obtained. The asymptotic behavior of the new black holes are neither flat nor like anti-de Sitter (AdS) black holes. Sec. 3 is dedicated to study of thermodynamical properties of the new charged dilatonic black hole solutions obtained in the previous section. The conserved masses and charges of the black holes have been calculated based on the Gauss's law and Abbott–Deser proposal, respectively. Also, the black holes temperature, entropy and electric potential have been calculated from both, the geometrical an thermodynamical approaches. The compatibility of the results of these two alternative approaches confirms the validity of the first law of black hole thermodynamics for both of the new black holes obtained here. Sec. 4 is devoted to study of thermal stability or phase transition of the new charged dilatonic BTZ black holes introduced here. A black hole stability analysis has been performed, making use of the canonical ensemble method and regarding the black hole heat capacity with the black hole charge as a constant. It has been found that the black holes under consideration are stable or may undergo phase transition if some simple conditions are satisfied. Some concluding remarks and discussions have been presented in Sec. 5.

2. Basic equations and black hole solutions

The action for three-dimensional charged hairy black holes can be written in the following general form [20,28]

$$I=-\frac{1}{16\pi}\int\sqrt{-g}d^3x\left[\mathcal{R}-U(\phi)-2g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi-\mathcal{F}e^{-2\alpha\phi}\right]. \tag{2.1}$$

Here, $\mathcal{R}$ is the Recci scalar. ϕ is the scalar field coupled to itself via the functional form $U(\phi)$. The parameter α is the scalar-electromagnetic coupling constant and $\mathcal{F}=F^{\mu\nu}F_{\mu\nu}$ being the Maxwell invariant. $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ and A_μ is the electromagnetic potential. By varying the action (2.1) with respect to the gravitational, electromagnetic and scalar fields, we get the related field equations as

$$\mathcal{R}_{\mu\nu}-\frac{1}{2}\mathcal{R}g_{\mu\nu}+\frac{1}{2}g_{\mu\nu}U(\phi)=T^{(s)}_{\mu\nu}+T^{(em)}_{\mu\nu}, \tag{2.2}$$

$$T^{(s)}_{\mu\nu}=2\nabla_\mu\phi\nabla_\nu\phi-g_{\mu\nu}(\nabla\phi)^2,$$

$$T^{(em)}_{\mu\nu}=-\frac{1}{2}\mathcal{F}e^{-2\alpha\phi}g_{\mu\nu}+2e^{-2\alpha\phi}F_{\mu\alpha}F_\nu{}^\alpha,$$

$$\nabla_\mu\left[e^{-2\alpha\phi}F^{\mu\nu}\right]=0, \tag{2.3}$$

$$4\Box\phi=\frac{dU(\phi)}{d\phi}-2\alpha\mathcal{F}e^{-2\alpha\phi},\qquad \phi=\phi(r). \tag{2.4}$$

Assuming as a function of r, the only non-vanishing component of the electromagnetic field is $F_{tr}=-E(r)=h'(r)$, and we have

$$\mathcal{F}=-2E^2(r)=-2(h'(r))^2. \tag{2.5}$$

In overall the paper, prime means derivative with respect to the argument. The gravitational field equations (2.2) can be rewritten as

$$\mathcal{R}_{\mu\nu}=U(\phi)g_{\mu\nu}+2\nabla_\mu\phi\nabla_\nu\phi-\left(\mathcal{F}g_{\mu\nu}-2F_{\mu\alpha}F_\nu^{\ \alpha}\right)e^{-2\alpha\phi}. \tag{2.6}$$

We consider the following ansatz as the three-dimensional spherically symmetric solution to the gravitational field equations (2.6)

$$ds^2=-\Psi(r)dt^2+\frac{1}{\Psi(r)}dr^2+r^2R(r)^2d\theta^2. \tag{2.7}$$

It leads to the following independent differential equations

$$E_{00}\equiv\Psi''+\left(\frac{1}{r}+\frac{R'}{R}\right)\Psi'+2U=0, \tag{2.8}$$

$$E_{11}\equiv E_{00}+2\Psi\left(\frac{R''}{R}+\frac{2R'}{rR}+2\phi'^2\right)=0, \tag{2.9}$$

$$E_{22}\equiv\left(\frac{1}{r}+\frac{R'}{R}\right)\Psi'+\left(\frac{R''}{R}+\frac{2R'}{rR}\right)\Psi+U+2F_{tr}^2e^{-2\alpha\phi}=0. \tag{2.10}$$

Noting Eqs. (2.8) and (2.9) we obtain

$$\frac{R''}{R}+\frac{2}{r}\frac{R'}{R}+2\phi'^2=0. \tag{2.11}$$

The differential equation (2.11) can be written in the following form

$$\frac{2}{r}\frac{d}{dr}\ln R(r)+\frac{d^2}{dr^2}\ln R(r)+\left(\frac{d}{dr}\ln R(r)\right)^2+2\phi'^2=0. \tag{2.12}$$

From Eq. (2.12), one can argue that $R(r)$ must be an exponential function of $\phi(r)$. Therefore, we can write $R(r)=e^{2\beta\phi}$, in Eq. (2.12), and show that $\phi=\phi(r)$ satisfies the following differential equation

$$\beta\phi''+(1+2\beta^2)\phi'^2+\frac{2\beta}{r}\phi'=0. \tag{2.13}$$

The case of $\beta=\alpha$ has been considered in a previous work [29]. Here, we are interested on the case $\beta\neq\alpha$.

It is easy to write the solution of Eq. (2.13) in terms of a positive constant b as $\phi(r)=\gamma\ln\left(\frac{b}{r}\right)$, with $\gamma=\beta(1+2\beta^2)^{-1}$. Similar solutions have been used by Hendi et al. [28]. The authors of ref. [20], have started with a power law of the form $R(r)\propto r^n$ and $\phi(r)\propto\ln r$, and showed that black hole solutions can exist if n is restricted in some ranges. In the following subsection, we proceed to obtain the solution of the field equations with the condition $\beta\neq\alpha$.

2.1. Solutions with $\beta\neq\alpha$

We start with the scalar field

$$\phi=\gamma\ln\left(\frac{b}{r}\right),\qquad\text{and}\quad\gamma=\frac{\beta}{1+2\beta^2}. \tag{2.14}$$

Making use of these solutions together with Eqs. (2.3) and (2.7), we have

$$\begin{cases}h(r)=-\frac{q}{A}r^{-A}, & \text{and}\quad A=2\gamma(\alpha-\beta),\\ F_{tr}=q\,r^{-(1+A)}, & \end{cases} \tag{2.15}$$

where, q is an integration constant related to the total electric charge on black hole. It will be calculated in the following section. Note that in the case of $\beta=\alpha$, $h(r)$ is a logarithmic function of r [29]. In order to the potential function $h(r)$ be physically reasonable (i.e. zero at infinity), the statement $A=2\gamma(\alpha-\beta)$ must be positive. Thus we suppose that $\alpha>\beta$.

Now, Eq. (2.10) can be rewritten as

$$\Psi'-\frac{2\beta\gamma}{r}\Psi+\frac{r}{1-2\beta\gamma}\left[U(\phi)+2F_{tr}^2e^{-2\alpha\phi}\right]=0. \tag{2.16}$$

For solving this equation for the metric function $\Psi(r)$, we need to calculate the functional form of $U(\phi(r))$ as the function of radial coordinate. For this purpose we proceed to solve the scalar field equation (2.4). It can be written as

$$\frac{dU(\phi)}{d\phi}-4\beta U(\phi)-4(2\beta-\alpha)F_{tr}^2e^{-2\alpha\phi}=0. \tag{2.17}$$

Noting Eq. (2.15), the first order differential (2.17) can be solved as

$$U(\phi)=2\Lambda e^{4\beta\phi}+2\Lambda_0 e^{4\beta_0\phi}, \tag{2.18}$$

where

$$\Lambda_0=\frac{q^2(\Upsilon-1)}{b^{2(A+1)}}\quad\text{and}\quad \Upsilon=(1+\alpha\beta-2\beta^2)^{-1}\quad\text{and}$$
$$\beta_0=\frac{1+\alpha\beta}{2\beta}. \tag{2.19}$$

It is notable that the solution given by Eq. (2.18) can be considered as the generalized form of the Liouville scalar potential. Also, it must be noted that in the absence of dilatonic field ϕ, we have $U(\phi=0)=2\Lambda=-2\ell^{-2}$ and the action (2.1) reduces to that of Einstein-Λ-Maxwell theory.

Now, making use of Eqs. (2.15), (2.16) and (2.18) the metric function $\Psi(r)$ can be obtained as

$$\Psi(r)=\begin{cases}-m\,r^{2/3}-3\left(\frac{r}{b}\right)^{2/3}\left[2b^2\Lambda\ln\left(\frac{r}{\ell}\right)-\frac{3q^2}{(\alpha-1)^2}(br)^{\frac{2}{3}(1-\alpha)}\right],\\ \quad\text{for } \beta=1,\ \alpha>1\\ -m\,r^{2\beta\gamma}-(1+2\beta^2)^2\left[\frac{\Lambda r^2}{1-\beta^2}\left(\frac{b}{r}\right)^{4\beta\gamma}\right.\\ \quad\left.+\frac{q^2\Upsilon b^{-2A}}{\beta(\beta-\alpha)}\left(\frac{b}{r}\right)^{2\gamma(\alpha-2\beta)}\right],\\ \quad\text{for } \beta\neq1.\end{cases} \tag{2.20}$$

The plots of metric functions $\Psi(r)$, presented in Eq. (2.20), for $\beta=1$ and $\beta\neq1$ cases have been shown in Figs. 1 and 2, respectively. The effects of α, Q and b on the metric function $\Psi(r)$ have been shown in Fig. 1 for the case $\beta=1$. Plots of Fig. 2 show the effects of parameters α, β and b on the metric function $\Psi(r)$ for the case $\beta\neq1$ by considering the condition $\alpha>\beta$. From the curves of Figs. 1 and 2, it is understood that the metric function $\Psi(r)$ can produce two horizon, extreme and naked singularity black holes for both of $\beta=1$ and $\beta\neq1$ cases.

Now, we investigate the curvature singularities. As a matter of calculation, one can show that the Ricci and Kretschmann scalars can be written in the following forms

$$R=\begin{cases}\frac{2}{9r^2}+6\Lambda\left(\frac{b}{r}\right)^{4/3}+\frac{2q^2}{b^{2(1+2\alpha)/3}}\left(\frac{2\alpha-5}{\alpha-1}\right)\left(\frac{b}{r}\right)^{2(1+\alpha)/3},\\ \quad\text{for } \beta=1,\\ 6\Lambda\left(\frac{b}{r}\right)^{\frac{4\beta^2}{1+2\beta^2}}+\frac{2\beta^2}{b^2(1+2\beta^2)^2}\left(\frac{b}{r}\right)^2+\frac{2q^2(3\Upsilon-2)}{b^{2(1+A)}}\left(\frac{b}{r}\right)^{\frac{2(1+\alpha\beta)}{1+2\beta^2}},\\ \quad\text{for } \beta\neq1,\end{cases} \tag{2.21}$$

$$R^{\mu\nu\rho\lambda}R_{\mu\nu\rho\lambda}=\begin{cases}r^{-2\delta_1}\left(\zeta_0+\zeta_1r^{\delta}+\zeta_2r^{2\delta}\right)+r^{-\delta_1}\left[\zeta_3+r^{\delta}(\zeta_4+\zeta_5\ln r)\right]\ln r,\\ \quad\text{for } \beta=1,\\ r^{-4}\left[B_1r^{\frac{4\gamma}{\beta}}+B_2r^{4\gamma(\alpha-2\beta)}+B_3r^{4\gamma(\alpha-\beta)}+B_4r^{4\gamma\beta}\right.\\ \quad+B_5r^{\frac{2\gamma}{\beta}(1-\alpha\beta+2\beta^2)}+B_6r^{\frac{2\gamma}{\beta}(1+\alpha\beta-\beta^2)}+B_7r^{\frac{2\gamma}{\beta}(1+\beta^2)}\\ \quad+B_8r^{2\gamma\beta}+B_9r^{2\gamma(3\beta-\alpha)}+B_{10}r^{2\gamma(4\beta-\alpha)}\\ \quad\left.+B_{11}r^{2\gamma(\beta+\alpha)}+B_{12}r^{2\gamma\alpha}\right],\\ \quad\text{for } \beta\neq1,\end{cases} \tag{2.22}$$

where $\delta=\frac{2}{3}(\alpha-1)$, $\delta_1=\frac{2}{3}(\alpha+1)$, ζ_i's and B_i's are functions of Λ, q, m, β, α and b. From Eqs. (2.21) and (2.22), one can argue that there is an essential singularity located at $r=0$. Also, the black holes asymptotic behavior are neither flat nor AdS. Recently, three and higher dimensional asymptotically Lifshitz black holes have been studied by many authors [30].

3. Thermodynamics

In this section, we would like to check the validity of the first law of black hole thermodynamics for the new dilatonic black holes we just introduced. At first it must be noted that the conserved charge of the black hole can be obtained by calculating the total electric flux measured by an observer located at infinity with respect to the horizon (i.e. $r\to\infty$) [31–33]. Making use of Eq. (2.15) together with the help of Gauss's law, after some simple calculations we arrived at

$$q=\begin{cases}2Q\,b^{\frac{2}{3}(\alpha-1)}, & \text{for } \beta=1,\\ 2Q\,b^{A}, & \text{for } \beta\neq1,\end{cases} \tag{3.1}$$

which reduces to that of charged BTZ black holes in the absence of dilatonic field.

The other conserved quantity to be calculated is the black hole mass. As mentioned before, it can be obtained in terms of the mass parameter m. The Abbott–Deser total mass of the charged dilatonic BTZ black holes introduced here can be obtained as [28,35]

$$m=\begin{cases}24M\,b^{-2/3}, & \text{for } \beta=1,\\ 8M(1+2\beta^2)b^{-2\beta\gamma}, & \text{for } \beta\neq1,\end{cases} \tag{3.2}$$

which is compatible with the mass of charged BTZ black hole when the dilatonic potential disappears.

We can obtain the Hawking temperature associated with the black hole horizon $r=r_+$, which is the root(s) of $\Psi(r_+)=0$, in terms of the surface gravity κ as

$$T=\frac{\kappa}{2\pi}=\frac{1}{4\pi}\frac{d}{dr}\Psi(r)|_{r=r_+}$$
$$=\begin{cases}-\frac{3}{2\pi}\left(\frac{b}{r_+}\right)^{1/3}\left[b\Lambda+\frac{q^2b^{(1-4\alpha)/3}}{\alpha-1}\left(\frac{b}{r_+}\right)^{\frac{2}{3}(\alpha-1)}\right], & \text{for } \beta=1,\\ -\frac{1+2\beta^2}{2\pi r_+}\left[\Lambda r_+^2\left(\frac{b}{r_+}\right)^{4\beta\gamma}+\frac{q^2\Upsilon}{b^{2A}}\left(\frac{b}{r_+}\right)^{2\gamma(\alpha-2\beta)}\right], & \text{for } \beta\neq1.\end{cases} \tag{3.3}$$

Since, the terms in the brackets have opposite sign ($\Lambda<0$), from thermodynamical point of view, the physical and un-physical black holes can appear. Also, it must be noted that extreme black holes occur if q and r_+ be chosen such that $T=0$. Now, making use of Eq. (3.3) we can obtain the horizon radius of the extreme black holes as

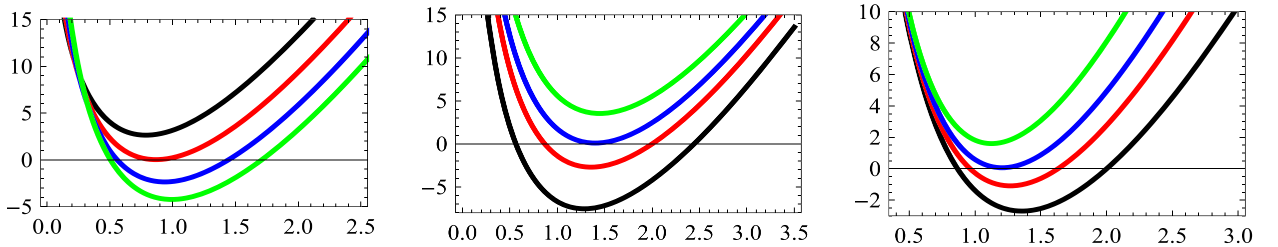

Fig. 1. $\Psi(r)$ versus r for $\beta=1$ and $\Lambda=-1$. Left: $Q=1$, $b=3$, $M=2.5$ and $\alpha=2.1, 2.195, 2.3, 2.4$ for black, red, blue and green curves, respectively. Middle: $M=3$, $b=3$, $\alpha=2.5$ and $Q=1.1, 1.2, 1.255, 1.32$ for black, red, blue and green curves, respectively. Right: $Q=1.2$, $M=3$, $\alpha=2.5$ and $b=3.0, 3.1, 3.18, 3.3$ for black, red, blue and green curves, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

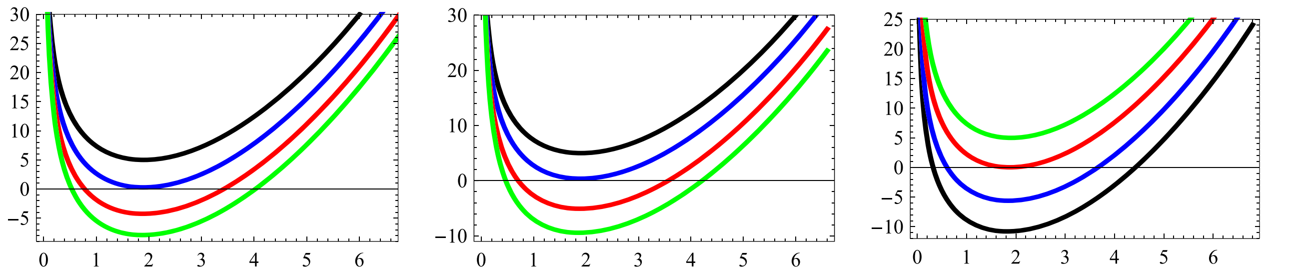

Fig. 2. $\Psi(r)$ versus r for $M=2$, $Q=1$ and $\Lambda=-1$. Left: $b=2$, $\beta=0.06$ and $\alpha=2.0, 2.3, 2.8, 3.4$ for black, blue, red and green curves, respectively. Middle: $b=2$, $\alpha=2$, and $\beta=0.06, 0.0702, 0.084, 0.1$ for black, blue, red and green curves, respectively. Right: $\alpha=2$, $\beta=0.06$, and $b=1.2, 1.4, 1.68, 2$ for black, blue, red and green curves, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$r_{ext}=\begin{cases}\left(\frac{q^2\ell^2}{\alpha-1}\right)^{\frac{3}{2(\alpha-1)}} b^{\frac{\alpha+2}{1-\alpha}}, & \text{for } \beta=1,\ \alpha>1,\\ b\left(\frac{q^2\ell^2\Upsilon}{b^{2(1+A)}}\right)^{\frac{\Upsilon}{2}(1+2\beta^2)}, & \text{for } \beta\neq 1.\end{cases} \tag{3.4}$$

It is evident that, in the case $\beta\neq 1$, extreme black holes exist if $\Upsilon>0$. In order to investigate the effects of scalar hair on the horizon temperature of the black holes correspond to $\beta=1$ the plot of black hole temperature versus horizon radius, for different values of α, has been shown in Fig. 3 (left). The physical black holes with positive temperature are those for which $r_+>r_{ext}$ and un-physical black holes, having negative temperature, occur if $r_+<r_{ext}$. The plot of T versus r_+, with $\beta\neq 1$, for different values of α and β has been shown in Fig. 4. They show that both, the physical (having positive temperature) and un-physical black holes (having negative temperature), will occur if the parameters are fixed, properly.

Next, we calculate the entropy of the black holes. It can be obtained from Hawking–Bekenstein entropy-area law, that is

$$S=\frac{A}{4}=\begin{cases}\frac{\pi b}{2}\left(\frac{r_+}{b}\right)^{1/3}, & \text{for } \beta=1,\\ \frac{\pi b}{2}\left(\frac{r_+}{b}\right)^{1-2\beta\gamma}, & \text{for } \beta\neq 1.\end{cases} \tag{3.5}$$

The black hole's electric potential Φ, measured by an observer located at infinity with respect to the horizon, can be obtained by using the following standard relation [17,31–33]

$$\Phi=A_\mu\chi^\mu|_{\text{reference}}-A_\mu\chi^\mu|_{r=r_+}, \tag{3.6}$$

where, $\chi=C\partial_t$ is the null generator of the horizon and C is an arbitrary constant [34]. Noting Eqs. (2.15) and (3.6) we can obtain the black hole's electric potential on the horizon. That is

$$\Phi=\begin{cases}\frac{3Cq}{2(\alpha-1)}r_+^{-\frac{2}{3}(\alpha-1)}, & \text{for } \beta=1,\\ \frac{Cq}{A}r_+^{-A}, & \text{for } \beta\neq 1,\end{cases} \tag{3.7}$$

in terms of the constant coefficient C, which will be determined in the following.

In order to investigate the consistency of these quantities with the thermodynamical first law, from Eqs. (2.20), (3.1) and (3.5), we can obtain the black hole mass as the function of extensive parameters S and Q. For this purpose we use the relation $\Psi(r_+)=0$. The corresponding Smarr-type mass formula is obtained as

$$M(S,Q)=\begin{cases}\frac{1}{4}\left[\frac{b^2}{\ell^2}\ln\left(\frac{\ell}{r_+(S)}\right)+\frac{6Q^2}{(\alpha-1)^2}\left(\frac{b}{r_+(S)}\right)^{\frac{2}{3}(\alpha-1)}\right],\\ \quad\text{for } \beta=1,\\ -\frac{1+2\beta^2}{8}\left[\frac{\Lambda b^2}{1-\beta^2}\left(\frac{b}{r_+(S)}\right)^{2(3\beta\gamma-1)}+\frac{4\Upsilon Q^2}{\beta(\beta-\alpha)}\left(\frac{b}{r_+(S)}\right)^{A}\right],\\ \quad\text{for } \beta\neq 1.\end{cases} \tag{3.8}$$

We can calculate the intensive parameters T and Φ, conjugate to the black hole entropy and charge, respectively. It is a matter of calculation to show that

$$\left(\frac{\partial M}{\partial S}\right)_Q=T \qquad \text{for both} \quad \beta=1 \quad \text{and} \quad \beta\neq 1, \tag{3.9}$$

and

$$\left(\frac{\partial M}{\partial Q}\right)_S=\Phi, \tag{3.10}$$

provided that [34]

$$C=\begin{cases}(\alpha-1)^{-1}, & \text{for } \beta=1,\\ (1+\alpha\beta-2\beta^2)^{-1}, & \text{for } \beta\neq 1.\end{cases} \tag{3.11}$$

Therefore, we proved that the first law of black hole thermodynamics is valid, for both classes of the charged dilatonic BTZ black holes, in the following form

$$dM(S,Q)=TdS+\Phi dQ. \tag{3.12}$$

4. Thermal stability analysis in the canonical ensemble method

In this section, we would like to analyze the stability or phase transition of the either of the black hole solutions, regarding

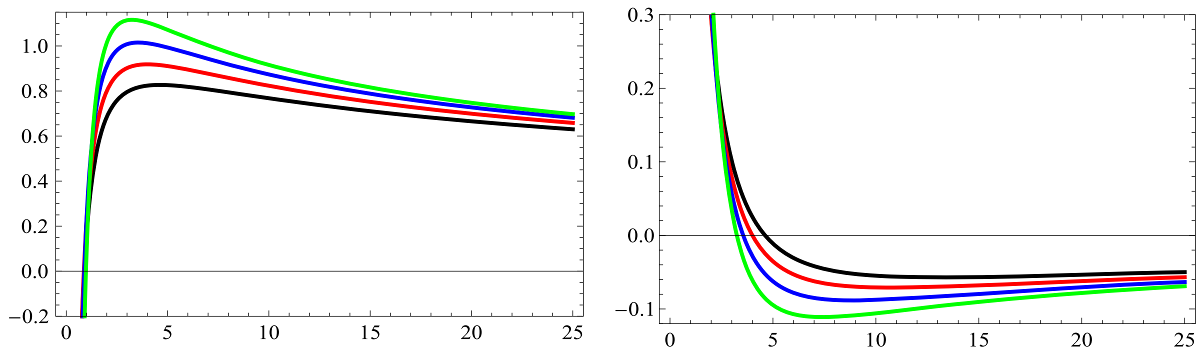

Fig. 3. Assuming $\beta = 1$, $\Lambda = -1$, $Q = 1$, $b = 3$ and $\alpha = 2, 2.2, 2.5, 3$, for black, red, blue, and green curves, respectively. Left: T versus r_+. Right: $(\partial^2 M/\partial S^2)_Q$ versus r_+. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

the black hole heat capacity with the fixed black hole charge, separately. It is well known that the positivity of heat capacity $C_Q = T\,(\partial S/\partial T)_Q = T/\left(\partial^2 M/\partial S^2\right)_Q$ or equivalently the positivity of $(\partial S/\partial T)_Q$ or $\left(\partial^2 M/\partial S^2\right)_Q$ with $T > 0$ are sufficient to ensure the local stability of the black hole. The unstable black holes undergo phase transitions to be stabilized. Type one phase transition takes place at the points where the black hole heat capacity vanishes. On the other hand, an unstable black hole undergoes type two phase transition at the divergent points of black hole heat capacity [17,31–33]. With these issues in mind, we proceed to analyze the thermal stability or phase transition of both of the new black hole solutions we just obtained here.

4.1. Black holes with $\beta = 1$

Making use of Eq. (3.7), the denominator of the black hole heat capacity can be calculated as

$$\left(\frac{\partial^2 M}{\partial S^2}\right)_Q = \frac{3}{\pi^2}\left(\frac{b}{r_+}\right)^{2/3}\left[\Lambda + q^2\left(\frac{2\alpha-1}{\alpha-1}\right)b^{-\frac{2}{3}(2\alpha+1)}\left(\frac{b}{r_+}\right)^{\frac{2}{3}(\alpha-1)}\right]. \tag{4.1}$$

Since $\alpha > 1$ and $\Lambda = -\ell^{-2}$, the terms in the brackets have opposite sign and it is understood from Eq. (4.1) that if

$$r_+ \equiv r_0 = \left[q^2\ell^2\left(\frac{2\alpha-1}{\alpha-1}\right)\right]^{\frac{3}{2(\alpha-1)}} b^{\frac{\alpha+2}{1-\alpha}}, \tag{4.2}$$

the denominator of the black hole heat capacity vanishes and black holes with the size satisfying this condition undergo type two phase transition. It must be noted that $r_0 > r_{ext}$ (see Eq. (3.4)). At the $r_+ = r_{ext}$ the black hole heat capacity vanishes and the type one phase transition takes place. In addition, for $r_+ > r_0$ the denominator of the heat capacity is negative and the physical black holes (black holes with positive temperature) will be thermodynamically unstable. On the other hand, for $r_{ext} < r_+ < r_0$ the denominator of the black hole heat capacity as well as the black hole temperature are positive and as a result the heat capacity of the black holes with the horizon in this range are positive and they will be locally stable. The plot of $\left(\partial^2 M/\partial S^2\right)_Q$ versus r_+ is shown in Fig. 3 (right). The plots show that the physical black holes with the horizon radius in the range $r_{ext} < r_+ < r_0$ are locally stable. Otherwise they are thermally unstable and can undergo phase transition to be stabilized.

4.2. Black holes with $\beta \neq 1$

It is a matter of calculation to show that

$$\left(\frac{\partial^2 M}{\partial S^2}\right)_Q = \frac{1+2\beta^2}{\pi^2}\left[\Lambda(2\beta^2-1)\left(\frac{b}{r_+}\right)^{2\beta\gamma} + \frac{q^2(1+\alpha\beta\Upsilon)}{b^{2(1+A)}}\left(\frac{b}{r_+}\right)^{\frac{2\gamma}{\beta}(1+\alpha\beta-\beta^2)}\right]. \tag{4.3}$$

The statement given in Eq. (4.3) denotes the denominator of the black hole heat capacity. It is consisted of two terms which, apart from scalar hair, show the contributions from Λ and from black hole charge, separately. One can argue that charged dilatonic BTZ black holes are unstable and undergo type two phase transition at the real roots of Eq. (4.3), which are located at

$$r_+ \equiv r_1 = b\left[\frac{q^2\ell^2(1+\alpha\beta\Upsilon)}{(2\beta^2-1)b^{2(1+A)}}\right]^{\frac{\Upsilon}{2}(1+2\beta^2)}. \tag{4.4}$$

Also, the type one phase transition takes place at the point $r_+ = r_{ext}$, given by Eq. (3.4), where the black hole heat capacity vanishes. The plots of $\left(\partial^2 M/\partial S^2\right)_Q$ versus r_+ are shown in Fig. 4 for various α and β values. They show that for the properly fixed parameters the three following cases are distinguishable

- r_{ext} exist but r_1 does not exist (Fig. 4 (left)). In this case $\left(\partial^2 M/\partial S^2\right)_Q$ does not vanish and no type two phase transition takes place. The type one phase transition takes place at $r_+ = r_{ext}$, where the temperature and hence the black hole heat capacity vanishes. Both T and $\left(\partial^2 M/\partial S^2\right)_Q$ are positive for $r_+ > r_{ext}$ and the charged dilatonic BTZ black holes with $\beta \neq 1$ are stable if their horizon radius, r_+, is greater than r_{ext}.
- r_1 exist but r_{ext} does not exist (Fig. 4 (middle)). The black holes have positive temperature and therefore are reasonable, thermodynamically. The black hole heat capacity does not vanish and no type one phase transition takes place. It diverges at $r_+ = r_1$ and black holes with the horizon radius equal to r_1 are unstable. They will undergo type two phase transition to be stabilized. The black hole heat capacity is positive for the charged dilatonic BTZ black holes with the horizon radius, r_+, greater than r_1 and they are thermodynamically stable.
- Both r_{ext} and r_1 are exist (Fig. 4 (right)). The black hole temperature vanishes at $r_+ = r_{ext}$. Therefore, the charged dilatonic BTZ black holes with $\beta \neq 1$ undergo type one phase transition at this point. The black holes with $r_+ < r_{ext}$, having negative temperature, are not physically reasonable. The denominator of the black hole heat capacity vanishes at $r_+ = r_1$. Therefore, $r_+ = r_1$ is the point of type two phase transition. For the

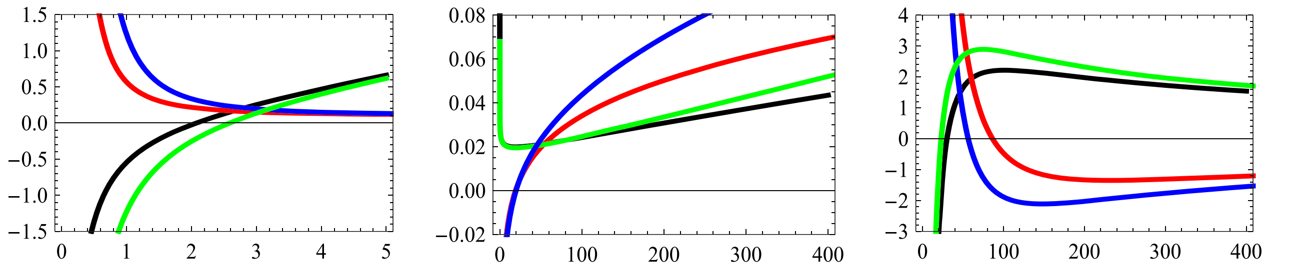

Fig. 4. Left: T (green $\alpha = 3.5$ and black $\alpha = 0.5$) and $\left(\partial^2 M/\partial S^2\right)_Q$ (blue $\alpha = 3.5$ and red $\alpha = 0.5$) versus r_+ for $\Lambda = -1$, $\beta = 0.04$, $b = 3$ and $Q = 1$. r_{ext} exist but r_1 does not exist. Middle: $0.02T$ (green $\alpha = 1.2$ and black $\alpha = 1$) and $\left(\partial^2 M/\partial S^2\right)_Q$ (blue $\alpha = 1.2$ and red $\alpha = 1$) versus r_+ for $\Lambda = -1$, $\beta = -0.8$, $b = 3$ and $Q = 1$. r_1 exist but r_{ext} does not exist. Right: $20T$ (green $\alpha = 3.2$ and black $\alpha = 3.1$) and $\left(\partial^2 M/\partial S^2\right)_Q$ (blue $\alpha = 3.2$ and red $\alpha = 3.1$) versus r_+ for $\Lambda = -1$, $\beta = 1.5$, $b = 3$ and $Q = 1$. Both r_{ext} and r_1 are exist. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

charged dilatonic BTZ black holes with the horizon radius in the range $r_{ext} < r_+ < r_1$, both T and $\left(\partial^2 M/\partial S^2\right)_Q$, are positive and they are locally stable.

5. Conclusion

Here, we studied the charged dilatonic BTZ black holes as the exact solutions to the Einstein–Maxwell theory coupled to a dilatonic scalar field. We solved the coupled scalar, electromagnetic and gravitational field equations in a static spherically symmetric space time and obtained two new classes of charged dilatonic BTZ black hole solutions. The solutions contain an essential (not coordinate) singularity located at the origin. Also, they do not behave asymptotically like the flat or AdS black holes. Furthermore, we showed that both of the new black hole solutions (correspond to $\beta = 1$ and $\beta \neq 1$) present naked singularity, extreme and two horizon black holes if the parameter α and β are chosen suitably (see Figs. 1 and 2).

We calculated the electric charges and masses of the black holes, as conserved quantities, making use of the Gauss's law and Abbott–Deser proposal, respectively. Also, we calculated the entropy, temperature and electric potential using the geometrical methods. On the other hand, through a Smarr-type mass formula, we constructed out the black holes masses as functions of both charge and entropy, as the thermodynamical extensive quantities. Making use of the Smarr-type mass formula we calculated the electric potential and temperature, as the thermodynamical intensive quantities, for both classes of the new BTZ black holes. We found that the thermodynamical quantities obtained from geometrical and thermodynamical approaches are identical for either of the black hole classes. It confirms the validity of the first law of black hole thermodynamics for both of the new black hole solutions in the form of Eq. (3.11).

Finally, making use of the canonical ensemble method, we performed the thermal stability analysis of both classes of the new BTZ black holes. Regarding the black hole heat capacity with constant black hole charge, we found that the black holes correspond to the case of $\beta = 1$ are stable if their horizon radius is in the range $r_{ext} < r_+ < r_0$ (see Eqs. (3.4) and (4.2)). This class of BTZ black holes undergo type one phase transition at $r_+ = r_{ext}$ and type two phase transition at $r_+ = r_0$ (see Fig. 3). In order to discuss the thermal stability of the second class of the black hole solutions according to the plots of Fig. 4 three possibilities are considered, separately. (i) If the parameters are chosen such that r_{ext} is exist but r_1 does not exist, there is type one phase transition located at r_{ext}. No type two phase transition can occur and black holes with the horizon radius greater than r_{ext} are stable. (ii) when the parameters are fixed in such a way that r_1 exist but r_{ext} does not exist, no type one phase transition takes place. There is type two phase transition point located at $r_+ = r_{ext}$. The black holes with the horizon radius greater than r_1 are locally stable. (iii) It is possible to fix the parameters such that both r_{ext} and r_1 are exist. In this case $r_+ = r_{ext}$ and $r_+ = r_1$ are points of type one and type two phase transitions, respectively. The black holes are locally stable if their horizon radiuses are in the range $r_{ext} < r_+ < r_0$.

The dynamical stability and quasi-normal modes of these novel dilatonic black hole solutions will be investigated in the forthcoming papers.

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