Molecular charmed baryons and pentaquarks from light-meson exchange saturation

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The spectrum of the cqq baryons contains a few states whose nature is not clearly a three-quark composite and which might have a sizable baryon-meson component. Examples include the $\Sigma_c(2800)$ or the $\Lambda_c(2940)$. Here we explore the spectrum of two-body systems composed of a light, octet baryon and a charmed meson (or antimeson) within a simple contact-range theory in which the couplings are saturated by light-meson exchanges. This results in the prediction of a series of composite anticharmed pentaquarks ($\bar{c}qqqq$) and singly charmed baryons ($c\bar{q}qqq$). Among the latter we find $J = \frac{1}{2}$ ΞD and $J = \frac{3}{2}$ ΞD^* bound states with masses matching those of the recently observed $\Omega_c(3185)$ and $\Omega_c(3327)$ baryons.

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I. INTRODUCTION

The discovery of a plethora of new heavy hadrons in experimental facilities calls for their theoretical description and classification [1–4]. While a large number of them are standard three-quark baryons or quark-antiquark mesons, others do not easily fit into this explanation and are suspected to be exotic. If we consider charmed baryons, a few of them do not conform to the theoretical expectations for cqq states. For instance, the mass of the $\Lambda_c(2940)$ [5–7] is somewhat large compared with quark model predictions [8–11] and is really close to the $D^{*0}p$ threshold, two factors which have in turn prompted its molecular interpretation [12–17]. The case of the $\Sigma_c(2800)$ [18] is similar [17,19] and there are a few excited Ξ_c states (e.g., the $\Xi_c(3055)$ and $\Xi_c(3123)$ [20]) which might also be amenable for a bound state explanation owing to their closeness to meson-baryon thresholds (though the most common theoretical explanation of the $\Xi_c(3055)$ and $\Xi_c(3123)$ is that they are compact hadrons [21–25]). The $\Omega_c(3185)$ and $\Omega_c(3327)$ states recently observed by the LHCb [26] might provide another example, being really close to the ΞD and ΞD^* thresholds (check, for instance, Ref. [27] for a molecular explanation of their decays or Refs. [28,29] for their interpretations within the quark model). Much of the theoretical speculations are driven by the lack of detailed experimental information about these charmed baryons. Often, we do not even know their quantum numbers or whether a particular charmed, nonstrange baryon is a Λ_c or a Σ_c (i.e., the case of the $\Lambda_c(2765)$, which is considered to be a Λ_c in Ref. [30], but it could also be a Σ_c or a superposition of Λ_c and Σ_c [31]).

In view of the previous situation, the investigation of the bound state spectrum of a light baryon and a charmed meson is well justified. Identifying in which configurations to expect molecular charmed baryons could improve our priors when confronted with a new experimental discovery or our posteriors when analyzing previous observations. To deal with the spectroscopy of these states, we formulate here a contact-range theory where the couplings are saturated from light-meson exchanges in the line of what we previously did in Refs. [32,33]. This approach is indeed able to reproduce a few hadrons which are often though to be exotic, such as the $X(3872)$ [34] or the $Y(4230)$ [35], and in the present manuscript we will particularize it to the case at hand (i.e., charmed meson and light baryon).

Regarding the aforementioned $\Sigma_c(2800)$ and $\Lambda_c(2940)$, it turns out that they might be identified with two of the most attractive $ND^{(*)}$ configurations within our model, giving support to the idea that they might be molecular after all [12–17,19], though more so for the $\Lambda_c(2940)$ than for the $\Sigma_c(2800)$. If we turn our attention to the $\Omega_c(3185)$ and $\Omega_c(3327)$, their masses are easily reproduced if they are ΞD and ΞD^* states with spin $J = \frac{1}{2}$ and $\frac{3}{2}$, respectively (in the latter case coinciding with the preferred spin of the $\Omega_c(3327)$ according to Ref. [27]). Yet, besides these candidates, we are able to identify other attractive configurations that may lead to a molecular singly charmed

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baryon or anticharmed pentaquark (though in this latter case there are no experimental candidates).

II. SATURATION OF THE CONTACT-RANGE COUPLINGS

We describe the charmed meson–octet baryon interaction with a contact-range theory of the type

$$V_C(\vec{q}) = C_0 + C_1 \vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2}, \tag{1}$$

where C_0 and C_1 are coupling constants, $\vec{\sigma}_{L1}$ and $\vec{\sigma}_{L2}$ are the light-spin operators for the meson and baryon, respectively, where for the octet baryon the light spin is identical to the total spin (as it does not contain a heavy quark), and $\vec{q}$ is the momentum exchanged between the particles.

This description is valid provided the following conditions are met: (i) the typical momentum of the particles is considerably smaller than the mass of the scalar and vector mesons (σ, ρ, ω), and (ii) pion exchanges are perturbative. We remind that this potential is singular though (it corresponds to a Dirac-delta in r-space) and will have to be regularized with a regularization scale or cutoff Λ, as we will explain later.

To determine the value of the couplings C_0 and C_1 we exploit the first of the previous conditions—that $|\vec{q}| < m_\sigma$, m_ρ, m_ω—and combine it with a specific choice of the regularization scale Λ. At low enough momenta the finite-range potential generated by the exchange of a meson can be effectively approximated by a contact-range potential. If the regularization scale is of the order of the mass of the aforementioned meson, i.e., $\Lambda \sim m_\sigma$, m_ρ, m_ω, the value of the contact-range couplings will be saturated by light-meson exchanges [36,37]. The scalar meson, which generates the potential

$$V_S(\vec{q}) = -\frac{g_{S1} g_{S2}}{m_S^2 + \vec{q}^2}, \tag{2}$$

will contribute to the couplings C_0 and C_1 as follows

$$C_0^S(\Lambda \sim m_S) \propto -\frac{g_{S1} g_{S2}}{m_S^2}, \tag{3}$$

$$C_1^S(\Lambda \sim m_S) \propto 0, \tag{4}$$

where g_{S1}, g_{S2} are the scalar couplings of hadron 1 and 2 and m_S the mass of the scalar meson. For the vector mesons, the potential reads

$$V_V(\vec{q}) = \frac{g_{V1} g_{V2}}{m_V^2 + \vec{q}^2} + \frac{f_{V1} f_{V2}}{6M^2} \frac{m_V^2}{m_V^2 + \vec{q}^2} \vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2} + \ldots, \tag{5}$$

where the dots indicate either higher partial wave operators or Dirac-delta contributions. This leads to the saturated couplings

$$C_0^V(\Lambda \sim m_V) \propto \frac{g_{V1} g_{V2}}{m_V^2}, \tag{6}$$

$$C_1^V(\Lambda \sim m_V) \propto \frac{f_{V1} f_{V2}}{6M^2}, \tag{7}$$

where we have obviated isospin or flavor factors for simplicity and with g_{V1}, g_{V2} the electriclike couplings, f_{V1}, f_{V2} the magneticlike ones, m_V the mass of the vector meson and M a scaling mass which is often taken to be the nucleon mass ($M = m_N$, with $m_N \approx 940$ MeV). Here we notice that the higher partial wave operators do not contribute to the saturation of the S-wave couplings, while the Dirac-delta contributions are regularized by the finite size of hadrons 1 and 2 and only contribute to the saturation of the couplings at the regularization scale $\Lambda \sim M_H$, with M_H the characteristic momentum scale of the finite size effects for a hadron H. In general $M_H \gg m_V$, which is why we ignore the Dirac-delta contributions [32].

At this point we encounter a problem: saturation is expected to work for a regularization scale similar to the mass of the light-meson being exchanged, yet the masses of the scalar and vector mesons are different. This means that there is a small mismatch in the ideal saturation scale for scalar ($\Lambda \sim m_S$) and vector ($\Lambda \sim m_V$) mesons. This is however easily solvable from the renormalization group (RG) evolution of the saturated couplings, which can be derived from the condition that the matrix elements of the contact-range potential are independent of the cutoff [38]

$$\frac{d}{d\Lambda} \langle \Psi | V_C(\Lambda) | \Psi \rangle = 0. \tag{8}$$

If the wave function has a power-law behavior $\Psi(r) \sim r^{\alpha/2}$ at distances $r \sim 1/\Lambda$, the RG equation above leads to

$$\frac{C(\Lambda_1)}{\Lambda_1^\alpha} = \frac{C(\Lambda_2)}{\Lambda_2^\alpha}, \tag{9}$$

from which we can combine the scalar and vector meson contributions as

$$C(m_V) = C^V(m_V) + \left(\frac{m_V}{m_S}\right)^\alpha C^S(m_S). \tag{10}$$

The intuitive meaning of this equation is that the relative strength of the contribution of a lighter meson scales as $1/m^{2+\alpha}$ (instead of $1/m^2$ if we do not consider their RG evolution). For the exponent α we use the semiclassical approximation together with the Langer correction [39], leading to $\Psi(r) \sim \sqrt{r}$ or $\alpha = 1$.

Finally, if we plug in the expected values of the coupling constants from saturation we end up with

$$C^{\rm sat}(m_V) \propto \frac{g_{\rho 1} g_{\rho 2}}{m_V^2}\left(1 + \kappa_{\rho 1}\kappa_{\rho 2}\frac{m_V^2}{6M^2}\hat{S}_{L12}\right)\hat{T}_{12} + \frac{g_{\omega 1} g_{\omega 2}}{m_V^2}\left(1 + \kappa_{\omega 1}\kappa_{\omega 2}\frac{m_V^2}{6M^2}\hat{S}_{L12}\right)\zeta + \left(\frac{m_V}{m_\phi}\right)\frac{g_{\phi 1} g_{\phi 2}}{m_\phi^2}\left(1 + \kappa_{\phi 1}\kappa_{\phi 2}\frac{m_\phi^2}{6M^2}\hat{S}_{L12}\right)\zeta - \left(\frac{m_V}{m_S}\right)\frac{g_{S1} g_{S2}}{m_S^2}, \tag{11}$$

where we have now included isospin factors ($\hat{T}_{12} = \hat{\vec{T}}_1 \cdot \hat{\vec{T}}_2$, with $\hat{\vec{T}} = \vec{T}/T$ a normalized isospin operator and T the isospin of the particle), defined $\hat{S}_{L12} = \vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2}$ and taken into account that $\alpha = 1$. In the previous equation we use the decomposition $f_V = \kappa_V g_V$ for the magneticlike couplings and introduce the G-parity sign ζ, which is $\zeta = +1$ or -1 for molecular anticharmed pentaquarks and charmed baryons, respectively. The ρ and ω contributions are kept separate because for the nucleon we have $g_\rho \neq g_\omega$. For the masses of the vector mesons we take $m_V = (m_\rho + m_\omega)/2 = 775$ MeV for $V = \rho, \omega$ (i.e., the average of the ρ and ω masses) and $m_\phi = 1020$ MeV. The only thing left is the proportionality constant, which can be determined from the condition of reproducing the binding energy of a known molecular candidate.

III. QUALITATIVE FEATURES OF THE SPECTRUM

From the previous formalism we can already determine the qualitative characteristics of the two-body light baryon and charmed (anti)meson bound state spectrum.

First, we need the couplings of the scalar and vector mesons to the light baryons and charmed mesons, for which we will refer to Table I. For the vector mesons (ρ, ω and ϕ) we have simply made use of the mixing of these mesons with the electromagnetic current (vector meson dominance [40–42]) as a way to determine the g_V and κ_V (E0 and M1) couplings: we can match g_V and κ_V to the charge and magnetic moment of the particular hadron we are interested in. The κ_V couplings are written in terms of the magnetic moments of the constituent quarks, μ_q, in units of the nuclear magneton (we take $\mu_u = 1.9\mu_N$, $\mu_d = -\mu_u/2$, $\mu_s = -0.6\mu_N$ with μ_N the nuclear magneton). For the scalar meson the linear sigma model [43] predicts $g_S = \sqrt{2}m_N/f_\pi \simeq 10.2$ for the nucleon, where m_N is the nucleon mass and $f_\pi \simeq 132$ MeV the pion weak decay constant. For the charmed meson, which contains one light-quark instead of three, we assume the quark model [44] relation $g_{Sqq} = g_S/3$, i.e., that the coupling of the sigma is proportional to the number of light-quarks within the hadron. In the strange sector we will assume that the coupling of the scalar to the s quark is approximately the same as to the u and d quarks: $g_{Suu} = g_{Sdd} = g_{Sss}$. This assumption works well when comparing the $D\bar{D}$ and $D_s\bar{D}_s$ systems predicted in the lattice and for the 27-plet dibaryons (i.e., the NN, ΣN, $\Sigma\Sigma$, $\Sigma\Xi$, and $\Xi\Xi$ in the $^1S_0$ partial wave and in their respective maximum isospin configurations). The only exception to this rule will be the Λ hyperon, for which a coupling $g_{S\Lambda\Lambda} \simeq 0.75 g_S$ is necessary for reproducing the $N\Lambda$ and $\Lambda\Lambda$ scattering lengths correctly. A more complete explanation of our choice can be found in the Appendix.

TABLE I. Choice of couplings for light-meson exchange saturation in this work. For their concrete values we take $g_S = 10.2$, $g_V = 2.9$, $\mu_u = 1.9$ and $\mu_s = -0.6$.

Hadron	g_σ	g_ρ	g_ω	g_ϕ	κ_ρ	κ_ω	κ_ϕ
D, D^*	$\frac{1}{3}g_S$	g_V	g_V	0	$\frac{3}{2}\mu_u$	$\frac{3}{2}\mu_u$	0
D_s, D_s^*	$\frac{1}{3}g_S$	0	0	$\sqrt{2}g_V$	0	0	$-3\mu_s$
N	g_S	g_V	$3g_V$	0	$\frac{5}{2}\mu_u$	$\frac{1}{2}\mu_u$	0
Λ	$0.75g_S$	0	$2g_V$	$\sqrt{2}g_V$	0	0	$-3\mu_s$
Σ	g_S	$2g_V$	$2g_V$	$\sqrt{2}g_V$	μ_u	μ_u	$-\mu_s$
Ξ	g_S	g_V	g_V	$2\sqrt{2}g_V$	$-\frac{1}{2}\mu_u$	$-\frac{1}{2}\mu_u$	$2\mu_s$

Second, for simplicity in the discussion that follows we will use the SU(3)-symmetric limit of the vector meson masses and the previous couplings. That is, now we will assume $m_\rho = m_\omega = m_{K^*} = m_\phi$, $\mu_s = -\mu_u/2$, and $g_{S\Lambda\Lambda} = g_S$. In contrast, for the actual quantitative predictions of the next section, we will use the values of Table I and the vector meson masses specified below Eq. (11).

Third, the light baryons and charmed mesons belong to the 8 and $\bar{3}$ representations of SU(3)-flavor. Conversely, the two-hadron interaction can be decomposed in a sum of contributions from different irreducible representations of SU(3):

$$V_C = \sum_R \lambda^R V_C^R, \tag{12}$$

where R indicates a particular representation and λ^R is a numerical factor (actually, the square of the relevant SU(3) Clebsch-Gordan coefficient, which we take from [45]). For the scalar meson contribution, the decomposition will be trivial

$$C_S^R = -\frac{1}{3}\frac{g_S^2}{m_S^2}, \tag{13}$$

independently of the representation R.

For the vector mesons the decomposition is not trivial, but it is still straightforward. If we consider the baryon—charmed meson two-body system, the SU(3) decomposition is $8 \otimes \bar{3} = \bar{3} \oplus 6 \oplus \overline{15}$. The electric-type vector meson contributions are

$$C_{V0}^{\bar{3}} = -8\,\frac{g_V^2}{m_V^2}, \tag{14}$$

$$C_{V0}^{6} = -4\,\frac{g_V^2}{m_V^2}, \tag{15}$$

$$C_{V0}^{\overline{15}} = 0, \tag{16}$$

while the magnetic-type ones are

$$C_{V1}^{\bar{3}} = -8 g_V^2 \frac{m_V^2}{6M^2}\kappa_q^2, \tag{17}$$

$$C_{V1}^{6} = +\frac{4}{3} g_V^2 \frac{m_V^2}{6M^2}\kappa_q^2, \tag{18}$$

$$C_{V1}^{\overline{15}} = 0, \tag{19}$$

where $\kappa_q = \frac{3}{2}(\mu_u/\mu_N)$, i.e., the value of κ_V for a light-quark in the SU(3)-symmetric limit. If we consider the baryon–charmed antimeson two-body system instead, the SU(3) decomposition is $8 \otimes 3 = 3 \oplus \bar{6} \oplus 15$. In this case, the E0 vector meson contributions are

$$C_{V0}^{3} = -4\,\frac{g_V^2}{m_V^2}, \tag{20}$$

$$C_{V0}^{\bar{6}} = 0, \tag{21}$$

$$C_{V0}^{15} = +4\,\frac{g_V^2}{m_V^2}, \tag{22}$$

while the M1 are

$$C_{V1}^{3} = 0, \tag{23}$$

$$C_{V1}^{\bar{6}} = -4 g_V^2 \frac{m_V^2}{6M^2}\kappa_q^2, \tag{24}$$

$$C_{V1}^{15} = +\frac{8}{3} g_V^2 \frac{m_V}{6M^2}\kappa_q^2. \tag{25}$$

The SU(3) decomposition of the light baryon and charmed (anti)meson potential is shown in Tables II and III. While the strength of scalar meson exchange is the same for all the baryon-meson molecules in the SU(3) symmetric limit, this is not the case for vector meson exchange, which is the factor deciding what are the most attractive molecules. If we consider the baryon-meson case, the total strength of the central and spin-spin pieces of vector meson exchange is shown in Table II. For the molecules involving the D and D_s pseudoscalar charmed mesons the spin-spin interaction does not contribute and, provided all configurations are attractive enough to bind, we will expect the following hierarchy for the binding energies

TABLE II. SU(3) decomposition of the light octet baryon and charmed meson system, which can be decomposed into the $8 \otimes \bar{3} = \bar{3} \oplus 6 \oplus \overline{15}$ representations. "System" refers to the two-body system under consideration, λ_R the numerical flavor factor for the V_R contribution to the potential (where $R = \bar{3}, 6$, or $\overline{15}$), C_0^V and C_1^V the relative strength of the electric- and magnetic-type piece of vector meson exchange and M_{th}, M_{th}^* the threshold (in MeV) for the system containing a ground (D or D_s) or excited state (D^* or D_s^*) charmed meson.

System	S	I	$\lambda^{\bar{3}}$	λ^{6}	$\lambda^{\overline{15}}$	C_0^V	C_1^V	$M_{\rm th}$	$M_{\rm th}^*$
ND_s	+1	$\frac{1}{2}$	0	0	1	0	0	2907.3	3051.1
ND	0	0	$\frac{3}{4}$	0	$\frac{1}{4}$	−6	−6	2806.2	2947.5
ΛD_s	0	0	$\frac{1}{4}$	0	$\frac{3}{4}$	−2	−2	3084.1	3227.9
ND	0	1	0	$\frac{1}{2}$	$\frac{1}{2}$	−2	$+\frac{2}{3}$	2806.2	2947.5
ΣD_s	0	1	0	$\frac{1}{2}$	$\frac{1}{2}$	−2	$+\frac{2}{3}$	3161.5	3305.4
ΛD	−1	$\frac{1}{2}$	$\frac{1}{16}$	$\frac{3}{8}$	$\frac{9}{16}$	−2	0	2983.0	3124.3
ΣD	−1	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$	−6	−4	3060.4	3201.7
ΞD_s	−1	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	−4	$-\frac{8}{3}$	3286.7	3430.5
ΣD	−1	$\frac{3}{2}$	0	0	1	0	0	3060.4	3201.7
ΞD	−2	0	0	1	0	−4	$+\frac{4}{3}$	3185.5	3326.9
ΞD	−2	1	0	0	1	0	0	3185.5	3326.9

$$\begin{aligned} B_{\rm mol}&\left(ND(0), \Sigma D\left(\frac{1}{2}\right)\right) \\ &> B_{\rm mol}(\Xi D, \Xi D_s) \\ &> B_{\rm mol}(\Lambda D_s, ND(1), \Lambda D, \Sigma D_s) \\ &> B_{\rm mol}\left(ND_s, \Sigma D\left(\frac{3}{2}\right), \Xi D(1)\right), \end{aligned} \tag{26}$$

where $B_{\rm mol}$ if defined as positive (such that the mass of a two-hadron bound state is given by $M = m_1 + m_2 - B_{\rm mol}$, with m_1, m_2 the masses of the hadrons) and the number in parentheses refers to the isospin of a given molecule (if there is more than one isospin configuration). If we change the pseudoscalar charmed mesons by antimesons, the hierarchy will be instead

$$\begin{aligned} B_{\rm mol}&\left(\Sigma\bar{D}\left(\frac{1}{2}\right), \Xi\bar{D}(0)\right) \\ &> B_{\rm mol}(N\bar{D}(0), N\bar{D}_s) \\ &> B_{\rm mol}(\Lambda\bar{D}, \Lambda\bar{D}_s, \Xi\bar{D}(1), \Sigma\bar{D}_s) \\ &> B_{\rm mol}\left(N\bar{D}(1), \Sigma\bar{D}\left(\frac{3}{2}\right), \Xi\bar{D}_s\right), \end{aligned} \tag{27}$$

though it should be noted that the molecules with charmed antimesons are in general less attractive than the ones

TABLE III. SU(3) decomposition of the light octet baryon and anticharmed meson system, which can be decomposed into the $8 \otimes 3 = 3 \oplus \bar{6} \oplus 15$ representations. We refer to Table II for the conventions used here.

System	S	I	λ^3	$\lambda^{\bar{6}}$	λ^{15}	C_0^V	C_1^V	$M_{\rm th}$	$M^*_{\rm th}$
$N\bar{D}$	0	0	0	1	0	0	−4	2806.2	2947.5
$N\bar{D}$	0	1	0	0	1	+4	$+\frac{8}{3}$	2806.2	2947.5
$N\bar{D}_s$	−1	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	0	0	2907.3	3051.1
$\Lambda\bar{D}$	−1	$\frac{1}{2}$	$\frac{1}{16}$	$\frac{3}{8}$	$\frac{9}{16}$	+2	0	2982.9	3124.3
$\Sigma\bar{D}$	−1	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$	−2	$-\frac{3}{2}$	3060.4	3201.7
$\Sigma\bar{D}$	−1	$\frac{3}{2}$	0	0	1	+4	$+\frac{8}{3}$	3060.4	3201.7
$\Lambda\bar{D}_s$	−2	0	$\frac{1}{4}$	0	$\frac{3}{4}$	+2	+2	3084.1	3227.9
$\Xi\bar{D}$	−2	0	$\frac{3}{4}$	0	$\frac{1}{4}$	−2	$+\frac{2}{3}$	3185.5	3326.9
$\Xi\bar{D}$	−2	1	0	$\frac{1}{2}$	$\frac{1}{2}$	+2	$-\frac{2}{3}$	3185.5	3326.9
$\Sigma\bar{D}_s$	−2	1	0	$\frac{1}{2}$	$\frac{1}{2}$	+2	$-\frac{2}{3}$	3161.5	3305.4
$\Xi\bar{D}_s$	−3	$\frac{1}{2}$	0	0	1	+4	$+\frac{8}{3}$	3286.7	3430.5

containing charmed mesons, owing to the sign of ω and ϕ exchange.

For the molecules containing a D^* ($\bar{D}^*$) or D_s^* ($\bar{D}_s^*$) vector charmed (anti)meson, the spin-spin interaction generates a hyperfine splitting between the $J = \frac{1}{2}$ and $\frac{3}{2}$ configurations. The sign of this splitting will depend on the sign of C_{V1}, where we will have

$$\begin{aligned} M\left(J=\frac{1}{2}\right) &< M\left(J=\frac{3}{2}\right) \quad \text{for } C_{V1} > 0, \\ M\left(J=\frac{1}{2}\right) &> M\left(J=\frac{3}{2}\right) \quad \text{for } C_{V1} < 0, \quad \text{and} \\ M\left(J=\frac{1}{2}\right) &= M\left(J=\frac{3}{2}\right) \quad \text{for } C_{V1} = 0. \end{aligned} \tag{28}$$

We find examples of these three types of hyperfine splitting in Tables II and III.

IV. CALIBRATION AND QUANTITATIVE PREDICTIONS

For calibrating the proportionality constant of the $C^{\rm sat}$ coupling we need a *reference state*, i.e., a molecular candidate from which we can calculate the coupling by reproducing its mass. Two suitable choices are the $\Sigma_c(2800)$ and $\Lambda_c(2940)$ charmed baryons, which have been proposed to be molecular:

(i) Of the two states, the $\Lambda_c(2940)$ fits the molecular interpretation better and is usually interpreted as a $J^P = \frac{3}{2}^-$ ND^* bound state [12–17] (though it should be noticed that its J^P is not completely established yet).

(ii) For the $\Sigma_c(2800)$ its interpretation as a molecular state is that of a $J^P = \frac{1}{2}^-$ ND bound/virtual state or resonance [17,19], but it is more contested [16,46].

First, for the calculation of the binding energies we begin by regularizing the contact-range potential:

$$\langle \vec{p}\,'|V_C|\vec{p}\rangle = C^{\rm sat}_{\rm mol}(\Lambda_H) f\left(\frac{p'}{\Lambda_H}\right) f\left(\frac{p}{\Lambda_H}\right), \tag{29}$$

where $f(x)$ is a regularization function and Λ_H the regularization scale. We choose a Gaussian $f(x) = e^{-x^2}$ and a cutoff $\Lambda_H = 0.75$ GeV (i.e., close to the vector meson mass). This potential is inserted into the bound state equation

$$1 + 2\mu_{\rm mol} C^{\rm sat}_{\rm mol}(\Lambda_H) \int_0^\infty \frac{q^2 dq}{2\pi^2} \frac{f^2(q/\Lambda_H)}{q^2 + \gamma^2_{\rm mol}}, \tag{30}$$

that is, the Lippmann-Schwinger equation as particularized for the poles of the scattering amplitude. Within the bound state equation, $\mu_{\rm mol}$ is the two-body reduced mass and $\gamma_{\rm mol}$ the wave number of the bound state, which is related to its binding energy $B_{\rm mol}$ by $\gamma_{\rm mol} = \sqrt{2\mu_{\rm mol} B_{\rm mol}}$. Notice that we define $B_{\rm mol} > 0$ for bound states and that the mass of the molecular state will be given by $M_{\rm mol} = M_{\rm th} - B_{\rm mol}$, with $M_{\rm th}$ the two-body threshold. For the regulator we are using, $f(x) = e^{-x^2}$, the loop integral is given by

$$\begin{aligned} &\int_0^\infty \frac{q^2 dq}{2\pi^2} \frac{f^2(q/\Lambda_H)}{q^2 + \gamma^2_{\rm mol}} \\ &= \frac{1}{8\pi^2}\left[\sqrt{2\pi}\Lambda_H - 2e^{2\gamma^2_{\rm mol}/\Lambda^2_H} \pi \gamma_{\rm mol} \,\mathrm{erfc}\left(\frac{\sqrt{2}\gamma_{\rm mol}}{\Lambda_H}\right)\right], \end{aligned} \tag{31}$$

with erfc(x) the complementary error function. Depending on the choice of sign for $\gamma_{\rm mol}$, we will talk about bound ($\gamma_{\rm mol} > 0$) or virtual ($\gamma_{\rm mol} < 0$) states.

The calibration of $C^{\rm sat}_{\rm mol}$ involves its calculation for the reference state (for which the mass is known), i.e., we take "mol = ref." For the $\Sigma_c(2800)$ and $\Lambda_c(2940)$ cases, this results in $C^{\rm sat}_{\rm ref} = -1.76$ fm^2 and -1.74 fm^2, respectively (where we use the couplings of Table I). For other molecules we define the ratio

$$R_{\rm mol} = \frac{\mu_{\rm mol} C^{\rm sat}_{\rm mol}}{\mu_{\rm ref} C^{\rm sat}_{\rm ref}}, \tag{32}$$

which can be determined from Eq. (11) or its SU(3)-flavor extension. After this, we find the mass of the molecule by solving

$$1 + (2\mu_{\rm ref} C^{\rm sat}_{\rm ref}) R_{\rm mol} \int_0^\infty \frac{q^2 dq}{2\pi^2} \frac{f^2(q/\Lambda_H)}{q^2 + \gamma^2_{\rm mol}} = 0. \tag{33}$$

This leads to the spectrum we show in Tables IV and V for the molecular charmed baryons and anticharmed pentaquarks, respectively.

TABLE IV. Molecular charmed baryons predicted in our model. “System” refers to the octet baryon–charmed meson pair under consideration, S, I, J^P to their strangeness, isospin and spin-parity, $R_{\rm mol}$ to the relative strength (central value) of the saturated coupling with respect to the $\Lambda_c(2940)$ or $\Sigma_c(2800)$ as $ND^{(*)}$ molecules, $B_{\rm mol}$ to the binding energy (central value), $M_{\rm mol}$ to the mass of the molecule (includes uncertainties), “Candidate” to a possible molecular candidate corresponding to the configuration we are calculating, and $M_{\rm cand}$ to the mass of this candidate. A superscript V over the binding energy or mass indicates a virtual state solution. The uncertainties in $M_{\rm mol}$ come from varying the scalar meson mass in the (400–550) MeV range (while a change in the sheet, e.g., from virtual to bound, is indicated with a B or V superscript in parentheses and next to the error). All binding energies and masses are in units of MeV.

System	S	I	J^P	$R_{\rm mol}(\Lambda_c^*)$	$B_{\rm mol}$	$M_{\rm mol}$	$R_{\rm mol}(\Sigma_c^*)$	$B_{\rm mol}$	$M_{\rm mol}$	Candidate	$M_{\rm cand}$
ND_s	+1	$\frac{1}{2}$	$\frac{1}{2}^-$	0.60	$(2.2)^V$	$(2905.0^{+2.2(B)}_{-6.9})^V$	0.91	2.8	2904.5 ± 1.4	⋯	⋯
ND_s^*	+1	$\frac{1}{2}$	$\frac{1}{2}^-$	0.62	$(1.7)^V$	$(3049.4^{+1.7(B)}_{-6.3})^V$	0.95	3.4	$3047.7^{+1.6}_{-1.5}$	⋯	⋯
ND_s^*	+1	$\frac{1}{2}$	$\frac{3}{2}^-$	0.62	$(1.7)^V$	$(3049.4^{+1.7(B)}_{-6.4})^V$	0.95	3.4	$3047.7^{+1.6}_{-1.5}$	⋯	⋯
ND	0	0	$\frac{1}{2}^-$	0.79	0.6	$2805.6^{+0.5}_{-1.2}$	1.20	17.7	$2788.5^{+4.9}_{-6.8}$	$\Lambda_c(2765)$	2766.6 ± 2.4 [31]
ND^*	0	0	$\frac{1}{2}^-$	0.44	$(16)^V$	$(2932^{+14}_{-34})^V$	0.66	1.2	$2946.3^{+1.0}_{-10.5}$	$\Lambda_c(2940)$	$2939.6^{+1.3}_{-1.5}$ [31]
ND^*	0	0	$\frac{3}{2}^-$	1 (Input)	7.9	2939.6	1.51	42	2906^{+15}_{-22}	$\Lambda_c(2940)$	$2939.6^{+1.3}_{-1.5}$ [31]
ND	0	1	$\frac{1}{2}^-$	0.66	$(0.6)^V$	$(2805.6^{+0.6(B)}_{-3.1})^V$	1 (Input)	6.2	2800.0	$\Sigma_c(2800)$	∼2800 [31]
ND^*	0	1	$\frac{1}{2}^-$	0.72	$(0.0)^V$	$(2947.5^{+0.0(B)}_{-1.1})^V$	1.09	10.4	$2937.1^{+1.3}_{-1.7}$		
ND^*	0	1	$\frac{3}{2}^-$	0.66	$(0.7)^V$	$(2946.8^{+0.7(B)}_{-3.7})^V$	0.99	5.7	$2941.8^{+0.6}_{-0.5}$		
ΛD_s	0	0	$\frac{1}{2}^-$	0.54	$(5.0)^V$	$(3079.0^{+3.7}_{-1.9})^V$	0.82	0.4	3083.6 ± 0.2		
ΛD_s^*	0	0	$\frac{1}{2}^-$	0.51	$(7.0)^V$	$(3220.9^{+5.2}_{-2.1})^V$	0.77	0.0	$3227.9^{+0.0(V)}_{-0.3}$		
ΛD_s^*	0	0	$\frac{3}{2}^-$	0.59	$(3.0)^V$	$(3224.9^{+2.5}_{-2.4})^V$	0.87	1.4	$3226.5^{+0.0(V)}_{-0.3}$		
ΣD_s	0	1	$\frac{1}{2}^-$	0.74	0.0	$3164.5^{+0.0(V)}_{-1.9}$	1.12	10.6	$3150.9^{+1.6}_{-1.3}$	⋯	⋯
ΣD_s^*	0	1	$\frac{1}{2}^-$	0.74	0.0	$3305.3^{+0.0(V)}_{-2.1}$	1.13	10.7	$3294.7^{+2.1}_{-1.8}$	⋯	⋯
ΣD_s^*	0	1	$\frac{3}{2}^-$	0.77	0.2	$3305.2^{+0.2(V)}_{-2.4}$	1.16	12.5	$3292.9^{+1.4}_{-1.2}$	⋯	⋯
ΛD	−1	$\frac{1}{2}$	$\frac{1}{2}^-$	0.57	$(3.4)^V$	$(2979.6^{+2.5}_{-4.3})^V$	0.87	1.3	$2981.7^{+0.2}_{-0.3}$	⋯	⋯
ΛD^*	−1	$\frac{1}{2}$	$\frac{1}{2}^-$	0.59	$(2.6)^V$	$(3121.6^{+2.1}_{-3.9})^V$	0.89	1.8	$3122.5^{+0.3}_{-0.4}$	$\Xi_c(3123)$	3122.9 ± 1.3 [31]
ΛD^*	−1	$\frac{1}{2}$	$\frac{3}{2}^-$	0.59	$(2.6)^V$	$(3121.6^{+2.1}_{-3.9})^V$	0.89	1.8	$3122.5^{+0.3}_{-0.4}$	$\Xi_c(3123)$	3122.9 ± 1.3 [31]
ΣD	−1	$\frac{1}{2}$	$\frac{1}{2}^-$	0.92	3.8	$3056.6^{+1.9}_{-2.5}$	1.40	28.1	$3023.3^{+6.1}_{-8.2}$	$\Xi_c(3055)$	3055.9 ± 0.4 [31]
ΣD^*	−1	$\frac{1}{2}$	$\frac{1}{2}^-$	0.66	$(0.6)^V$	$(3201.1^{+0.6(B)}_{-5.8})^V$	0.99	5.0	$3196.8^{+3.0}_{-3.2}$	⋯	⋯
ΣD^*	−1	$\frac{1}{2}$	$\frac{3}{2}^-$	1.10	11.5	$3190.3^{+1.1}_{-1.2}$	1.66	47	3155^{+14}_{-18}	⋯	⋯
ΣD	−1	$\frac{3}{2}$	$\frac{1}{2}^-$	0.69	$(0.1)^V$	$(3060.3^{+0.1(B)}_{-3.4})^V$	1.05	7.3	$3053.1^{+2.3}_{-2.1}$	⋯	⋯
ΣD^*	−1	$\frac{3}{2}$	$\frac{1}{2}^-$	0.71	$(0.0)^V$	$(3201.7^{+0.0(B)}_{-2.7})^V$	1.08	8.4	$3193.3^{+2.5}_{-2.2}$	⋯	⋯
ΣD^*	−1	$\frac{3}{2}$	$\frac{1}{2}^-$	0.71	$(0.0)^V$	$(3201.7^{+0.0(B)}_{-2.7})^V$	1.08	8.4	$3193.3^{+2.5}_{-2.2}$	⋯	⋯
ΞD_s	−1	$\frac{1}{2}$	$\frac{1}{2}^-$	0.82	1.0	$3286.7^{+0.9}_{-3.2}$	1.25	16.6	$3270.0^{+0.4}_{-0.3}$	⋯	⋯
ΞD_s^*	−1	$\frac{1}{2}$	$\frac{1}{2}^-$	0.91	3.1	$3427.4^{+2.3}_{-3.8}$	1.38	24.2	$3406.3^{+2.0}_{-2.5}$	⋯	⋯
ΞD_s^*	−1	$\frac{1}{2}$	$\frac{3}{2}^-$	0.81	0.8	$3429.8^{+0.6}_{-3.4}$	1.23	15.3	$3415.2^{+1.6}_{-1.4}$	⋯	⋯
ΞD	−2	0	$\frac{1}{2}^-$	0.90	2.8	$3182.7^{+2.1}_{-3.2}$	1.36	23.9	$3161.6^{+3.0}_{-3.9}$	$\Omega_c(3185)$	$3185.1^{+7.6}_{-1.9}$ [26]
ΞD^*	−2	0	$\frac{1}{2}^-$	1.03	7.6	$3319.3^{+2.5}_{-3.0}$	1.56	36.7	$3290.2^{+7.5}_{-10.0}$	⋯	⋯
ΞD^*	−2	0	$\frac{3}{2}^-$	0.87	2.0	$3324.8^{+1.9}_{-3.6}$	1.32	20.9	$3306.0^{+1.1}_{-1.4}$	$\Omega_c(3327)$	$3327.1^{+1.2}_{-1.8}$ [26]
ΞD	−2	1	$\frac{1}{2}^-$	0.73	0.0	$3185.5^{+0.0(V)}_{-2.1}$	1.11	9.8	$3175.8^{+2.6}_{-2.3}$	⋯	⋯
ΞD^*	−2	1	$\frac{1}{2}^-$	0.76	0.1	$3326.8^{+0.1(V)}_{-2.6}$	1.15	11.1	$3315.8^{+2.8}_{-2.5}$	⋯	⋯
ΞD^*	−2	1	$\frac{1}{2}^-$	0.76	0.1	$3326.8^{+0.1(V)}_{-2.6}$	1.15	11.1	$3315.8^{+2.8}_{-2.5}$	⋯	⋯

TABLE V. Molecular anticharmed pentaquarks predicted in our model. We refer to Table IV for the conventions used, where the only significant difference with the aforementioned table is that here there are no experimental candidates (and hence we do not include the "Candidate" and $M_{\rm cand}$ columns). All binding energies and masses are in units of MeV.

System	S	I	J^P	$R_{\rm mol}(\Lambda_c^*)$	$B_{\rm mol}$	$M_{\rm mol}$	$R_{\rm mol}(\Sigma_c^*)$	$B_{\rm mol}$	$M_{\rm mol}$
$N\bar{D}$	0	0	$\frac{1}{2}^-$	0.59	$(2.7)^V$	$(2803.4^{+2.6}_{-7.4})^V$	0.90	2.3	$2803.8^{+1.2}_{-1.3}$
$N\bar{D}^*$	0	0	$\frac{1}{2}^-$	0.36	$(30)^V$	$(2918^{+24}_{-68})^V$	0.54	$(7.0)^V$	$2940.5^{+6.9}_{-35.9}$
$N\bar{D}^*$	0	0	$\frac{3}{2}^-$	0.73	0.0	$2947.5^{+0.0(V)}_{-0.9}$	1.11	11.7	$2935.8^{+1.8}_{-2.4}$
$N\bar{D}$	0	1	$\frac{1}{2}^-$	0.46	$(13)^V$	$(2793^{+11}_{-26})^V$	0.70	$(0.4)^V$	$(2805.8^{+0.4(B)}_{-5.7})^V$
$N\bar{D}^*$	0	1	$\frac{1}{2}^-$	0.64	$(1.1)^V$	$(2946.4^{+1.1(B)}_{-4.6})^V$	0.97	4.7	$2942.8^{+1.0}_{-0.9}$
$N\bar{D}^*$	0	1	$\frac{3}{2}^-$	0.39	$(23)^V$	$(2924^{+19}_{-51})^V$	0.59	$(4.0)^V$	$(2943.4^{+4.0(B)}_{-22.7})^V$
$N\bar{D}_s$	−1	$\frac{1}{2}$	$\frac{1}{2}^-$	0.60	$(2.2)^V$	$(2905.0^{+2.2(B)}_{-6.9})^V$	0.92	2.7	2904.5 ± 1.4
$N\bar{D}_s^*$	−1	$\frac{1}{2}$	$\frac{1}{2}^-$	0.62	$(1.7)^V$	$(3049.4^{+1.7(B)}_{-6.3})^V$	0.94	3.4	3047.7 ± 1.5
$N\bar{D}_s^*$	−1	$\frac{1}{2}$	$\frac{3}{2}^-$	0.62	$(1.7)^V$	$(3049.4^{+1.7(B)}_{-6.3})^V$	0.94	3.4	3047.7 ± 1.5
$\Lambda\bar{D}$	−1	$\frac{1}{2}$	$\frac{1}{2}^-$	0.42	$(16)^V$	$(2967^{+11}_{-20})^V$	0.64	$(1.6)^V$	$(2981.4^{+1.4}_{-4.3})^V$
$\Lambda\bar{D}^*$	−1	$\frac{1}{2}$	$\frac{1}{2}^-$	0.44	$(14)^V$	$(3110^{+10}_{-19})^V$	0.66	$(1.1)^V$	$(3123.2^{+1.1(B)}_{-3.8})^V$
$\Lambda\bar{D}^*$	−1	$\frac{1}{2}$	$\frac{3}{2}^-$	0.44	$(14)^V$	$(3110^{+10}_{-19})^V$	0.66	$(1.1)^V$	$(3123.2^{+1.1(B)}_{-3.8})^V$
$\Sigma\bar{D}$	−1	$\frac{1}{2}$	$\frac{1}{2}^-$	0.77	0.2	$3060.2^{+0.2(V)}_{-2.1}$	1.17	13.2	3047.2 ± 0
$\Sigma\bar{D}^*$	−1	$\frac{1}{2}$	$\frac{1}{2}^-$	0.69	$(0.1)^V$	$(3201.6^{+0.1(B)}_{-3.6})^V$	1.05	7.2	$3194.5^{+2.8}_{-2.6}$
$\Sigma\bar{D}^*$	−1	$\frac{1}{2}$	$\frac{3}{2}^-$	0.84	1.3	$3200.4^{+1.3(V)}_{-2.9}$	1.27	19.0	$3182.7^{+1.7}_{-2.2}$
$\Sigma\bar{D}$	−1	$\frac{3}{2}$	$\frac{1}{2}^-$	0.54	$(5.1)^V$	$(3055.3^{+5.1(B)}_{-18.1})^V$	0.81	0.4	$3060.0^{+0.4(V)}_{-3.0}$
$\Sigma\bar{D}^*$	−1	$\frac{3}{2}$	$\frac{1}{2}^-$	0.75	0.1	$3201.7^{+0.1(B)}_{-2.0}$	1.13	11.1	$3190.6^{+1.5}_{-1.3}$
$\Sigma\bar{D}^*$	−1	$\frac{3}{2}$	$\frac{1}{2}^-$	0.45	$(12)^V$	$(3190^{+11}_{-38})^V$	0.69	$(0.5)^V$	$(3201.2^{+0.5(B)}_{-13.6})^V$
$\Lambda\bar{D}_s$	−2	0	$\frac{1}{2}^-$	0.47	$(10.1)^V$	$(3073.9^{+7.1}_{-13.0})^V$	0.72	$(0.1)^V$	$(3083.9^{+0.1(B)}_{-1.1})^V$
$\Lambda\bar{D}_s^*$	−2	0	$\frac{1}{2}^-$	0.53	$(5.5)^V$	$(3222.4^{+4.2}_{-7.8})^V$	0.80	0.3	$3227.6^{+0.2}_{-0.3}$
$\Lambda\bar{D}_s^*$	−2	0	$\frac{3}{2}^-$	0.46	$(10.9)^V$	$(3217.0^{+7.9}_{-15.0})^V$	0.70	$(0.3)^V$	$(3227.6^{+0.3(B)}_{-1.9})^V$
$\Sigma\bar{D}_s$	−2	1	$\frac{1}{2}^-$	0.67	$(0.4)^V$	$(3161.2^{+0.4(B)}_{-4.7})^V$	1.02	5.9	$3155.6^{+3.6}_{-2.8}$
$\Sigma\bar{D}_s^*$	−2	1	$\frac{1}{2}^-$	0.70	$(0.0)^V$	$(3305.3^{+0.0(B)}_{-3.1})^V$	1.07	7.8	$3297.5^{+3.1}_{-2.7}$
$\Sigma\bar{D}_s^*$	−2	1	$\frac{3}{2}^-$	0.68	$(0.2)^V$	$(3305.1^{+0.2(B)}_{-4.5})^V$	1.03	6.3	$3299.0^{+3.2}_{-3.1}$
$\Xi\bar{D}$	−2	0	$\frac{1}{2}^-$	0.82	0.8	$3184.2^{+0.8(V)}_{-3.0}$	1.24	16.3	3169.2 ± 0
$\Xi\bar{D}^*$	−2	0	$\frac{1}{2}^-$	0.89	2.6	$3324.3^{+2.1}_{-3.6}$	1.35	22.7	$3304.1^{+1.8}_{-2.4}$
$\Xi\bar{D}^*$	−2	0	$\frac{3}{2}^-$	0.82	0.8	$3326.1^{+0.6}_{-3.3}$	1.23	15.7	$3311.1^{+1.0}_{-0.8}$
$\Xi\bar{D}$	−2	1	$\frac{1}{2}^-$	0.65	$(0.6)^V$	$(3184.9^{+0.6(B)}_{-6.7})^V$	0.99	4.6	$3181.0^{+3.6}_{-3.9}$
$\Xi\bar{D}^*$	−2	1	$\frac{1}{2}^-$	0.62	$(1.3)^V$	$(3325.5^{+0.8}_{-10.2})^V$	0.94	2.9	$3324.0^{+3.1}_{-4.8}$
$\Xi\bar{D}^*$	−2	1	$\frac{1}{2}^-$	0.70	$(0.1)^V$	$(3326.8^{+0.1(B)}_{-4.1})^V$	1.06	7.1	$3319.8^{+3.7}_{-3.5}$
$\Xi\bar{D}_s$	−3	$\frac{1}{2}$	$\frac{1}{2}^-$	0.68	$(0.2)^V$	$(3286.4^{+0.2(B)}_{-5.3})^V$	1.03	6.0	$3280.7^{+3.3}_{-3.8}$
$\Xi\bar{D}_s^*$	−3	$\frac{1}{2}$	$\frac{1}{2}^-$	0.76	0.1	$3430.4^{+0.1(B)}_{-9.7}$	1.15	11.5	$3419.0^{+2.8}_{-4.5}$
$\Xi\bar{D}_s^*$	−3	$\frac{1}{2}$	$\frac{3}{2}^-$	0.66	$(0.4)^V$	$(3430.1^{+0.4(B)}_{-2.6})$	1.00	5.0	3425.5 ± 3.7

For the uncertainties, we will do as follows: the largest source of error in the saturated couplings is the σ meson, the parameters and nature of which are not particularly well known. Besides, the RG-improved saturated coupling is most sensitive to the contribution of the σ meson owing to its lighter mass when compared to the vector mesons. Thus we will vary the scalar meson mass within its RPP window of $m_\sigma = (400\text{–}550)$ MeV as a practical method to estimate the uncertainties of our model. In addition to this uncertainty there is of course the uncertainty coming from the choice of a reference state, which results in two different sets of predictions depending on whether we use the $\Lambda_c(2940)$ or $\Sigma_c(2800)$.

Regarding the predictions for the molecular baryons in Table IV, we find it worth commenting the following:

(i) Predictions derived from the $\Sigma_c(2800)$ are considerably more attractive than the ones derived from the $\Lambda_c(2940)$.

(ii) We find molecular matches of the $\Xi_c(3055)$ (ΣD), $\Xi_c(3123)$ (ΛD^*) and the $\Omega_c(3185)$ (ΞD) and $\Omega_c(3327)$ ($J = \frac{3}{2}$ ΞD^*) [26].

(iii) The recent LHCb manuscript in which the $\Omega_c(3185/3327)$ have been discovered [26] also indicates that no structures have been observed in $\Xi_c^+ K^+$. $\Sigma D_s^{(*)}$ molecules can decay into this channel via a short-range operator (exchange of a light-baryon). Though only expected to generate a narrow width, the size of this matrix element grows with the binding energy,[1] disfavoring the use of $\Sigma_c(2800)$ as a reference state because of the large bindings it entails for $\Sigma D_s^{(*)}$.

(iv) Curiously, if $\Sigma_c(2800)$ is the reference state, we predict two $I = 0$ ND^* bound states that might correspond to the $\Lambda_c(2940)$ (but now appearing as a $J = \frac{1}{2}$ state) and the recently discovered $\Lambda_c(2910)$ [47] (as a $J = \frac{3}{2}$ state). This interpretation coincides with the one proposed in [48], but not with Refs. [46,49] that consider the $\Lambda_c(2910)$ as compact or at least nonmolecular.

If we consider the anticharmed pentaquarks predicted in Table V, the first problem we are confronted with is the lack of candidates. Nonetheless, there is experimental information about $I = 0$ $N\bar{D}$ scattering at low energies from the ALICE collaboration [50], which constrained the values of the inverse scattering length[2] of this system to the following range:

$$f_0^{-1}(I = 0) \in [-0.4, 0.9]\ \mathrm{fm}^{-1}. \tag{34}$$

The calculation of f_0 in our formalism is given by

$$-\frac{1}{f_0} = \frac{2\pi}{\mu_{\rm ref} C_{\rm ref}^{\rm sat}} \frac{1}{R_{\rm mol}} + \frac{2}{\pi} \int_0^\infty dq f^2\left(\frac{q}{\Lambda_H}\right), \tag{35}$$

and, depending on the reference state used, we arrive at

$$f_0^{-1}(I = 0) = +0.34\ \mathrm{fm}^{-1} \quad \text{for ref} = \Lambda_c(2940),$$
$$f_0^{-1}(I = 0) = -0.24\ \mathrm{fm}^{-1} \quad \text{for ref} = \Sigma_c(2800). \tag{36}$$

That is, from the prediction of the inverse scattering length we conclude that both reference states comply with this experimental constraint.

Alternatively, we might compare the spectrum in Table V with previous theoretical predictions. The first predictions of a $\bar{c}qqqq$ pentaquark are maybe the ones by Gignoux *et al.* [51] and Lipkin [52], who calculated that the anticharmed-strange pentaquark configurations could be stable and located below the $N\bar{D}_s$ threshold. Here the $N\bar{D}_s$ system shows a remarkable amount of attraction, but binding is subordinate to our choice of reference state: from the $\Sigma_c(2800)$ we indeed find a shallow bound state, but if we use the $\Lambda_c(2940)$ instead, we end up with a virtual state (albeit close to threshold). Hofmann and Lutz [53] proposed that the $N\bar{D}_s$-$\Lambda\bar{D}$-$\Sigma\bar{D}$ and $\Lambda\bar{D}_s$-$\Xi\bar{D}$ systems might generate bound states at 2.78 and 2.84 GeV, respectively (and also a hidden-charmed pentaquark at 3.52 GeV, probably one of the first predictions of these states). Even though we find considerably less attraction for the aforementioned systems than in [53], these systems are still attractive and able to bind within our model. More recently, Yalikun and Zou [54] have studied possible $\Sigma\bar{D}$ and $\Sigma\bar{D}^*$ bound states within the one boson exchange model. We find three possible near-threshold states in these configurations in agreement with [54]. That is, in general the qualitative features of the spectrum we predict align with previous results, though there are differences at the quantitative level, which will only be elucidated once we have further experimental results.

V. ISOSPIN BREAKING EFFECTS AND THE $\Omega_c(3185/3327)$

The previous predictions have been done in the isospin symmetric limit, i.e., our calculations use the isospin averages of the charmed meson and light octet baryon masses.

The inclusion of explicit isospin breaking effects will have different effects depending on the particular two-body system under consideration. The effects are trivial in meson-baryon systems for which there is only one particle channel per isospin state (e.g., ΞD_s for which the two isospin states are $|\frac{1}{2}\frac{1}{2}\rangle_I = |\Xi^0 D_s^+\rangle$ and $|\frac{1}{2} -\frac{1}{2}\rangle_I = |\Xi^- D_s^+\rangle$). Here isospin breaking only entails a shift in the mass of the molecule equal to the shift of the physical and isospin symmetric thresholds (e.g., ± 3.4 MeV for the $\Xi^- D_s^+$ and $\Xi^0 D_s^+$ molecules, with respect to the ΞD_s calculations of Table IV).

More interesting is the case of the ND and ΞD systems, for which isospin mixing of the $I = 0$ and $I = 1$ states is possible (or the ΣD system, where mixing happens between the $I = \frac{1}{2}$ and $I = \frac{3}{2}$ configurations, though we will not consider this case in detail here). For ND and ΞD with $M_I = 0$ (with M_I the third component of the isospin wave function) we have a *light* and *heavy* particle channel

[1] More binding implies a larger probability of the two particles being close to each other, which for a short-range operator would be a necessary condition for having a non-negligible matrix element.

[2] In [50] the sign convention of the scattering length is $f_0 > 0$ ($f_0 < 0$) for a two-body system with a virtual (bound) state.

$$|00\rangle_I = \frac{1}{\sqrt{2}}[|L\rangle - |H\rangle], \tag{37}$$

$$|10\rangle_I = \frac{1}{\sqrt{2}}[|L\rangle + |H\rangle], \tag{38}$$

where $|L\rangle = |pD^0\rangle$ or $|\Xi^0 D^0\rangle$ and $|H\rangle = -|nD^+\rangle$ or $-|\Xi^- D^+\rangle$,[3] depending on the system. This decomposition implies that the contact-range potential now becomes a matrix in the $\{|L\rangle, |H\rangle\}$ basis. The identity and product isospin operators change to

$$1 \to \begin{pmatrix} +1 & 0 \\ 0 & +1 \end{pmatrix} \quad \text{and} \quad \hat{T}_{12} \to \begin{pmatrix} +1 & -2 \\ -2 & +1 \end{pmatrix}, \tag{39}$$

from which the explicit expression of the saturated contact-range potential reads

$$\begin{aligned} C^{\rm sat}(m_V) \propto & \begin{pmatrix} +1 & -2 \\ -2 & +1 \end{pmatrix} \frac{g_{\rho 1} g_{\rho 2}}{m_V^2} \left(1 + \kappa_{\rho 1}\kappa_{\rho 2} \frac{m_V^2}{6M^2} \hat{S}_{L12}\right) \\ & + \begin{pmatrix} +\zeta & 0 \\ 0 & +\zeta \end{pmatrix} \frac{g_{\omega 1} g_{\omega 2}}{m_V^2} \left(1 + \kappa_{\omega 1}\kappa_{\omega 2} \frac{m_V^2}{6M^2} \hat{S}_{L12}\right) \\ & + \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \left(\frac{m_V}{m_S}\right) \frac{g_{S1} g_{S2}}{m_S^2}, \end{aligned} \tag{40}$$

where it is apparent that the isospin breaking effects derive from ρ exchange between the L and H channels.

The bound state equation becomes now a two-channel linear system

$$\phi_A + 2\mu_{\rm ref} \sum_B \phi_B C^{\rm sat}_{\rm ref}(\Lambda) R^{AB}_{\rm mol} \int \frac{q^2 dq}{2\pi^2} \frac{f^2(\frac{q}{\Lambda})}{q^2 + \gamma^2_{{\rm mol}(A)}} = 0, \tag{41}$$

where $A, B = L,\ H$ are indices denoting the channels, ϕ_A the vertex function for channel A, $\gamma_{{\rm mol}(A)} = \sqrt{2\mu_A (M_{{\rm th}(A)} - M_{\rm mol})}$ with $M_{\rm mol}$ the mass of the predicted molecule, $M_{{\rm th}(A)}$ the mass of threshold A and μ_A the reduced mass of channel A. The ratio $R^{AB}_{\rm mol}$ is given by

$$R^{AB}_{\rm mol} = \frac{\mu_A C^{\rm sat}_{{\rm mol}(AB)}(\Lambda)}{\mu_{\rm ref} C^{\rm sat}_{\rm ref}(\Lambda)}, \tag{42}$$

where the indices AB in the saturated coupling refer to the components of $C^{\rm sat}$ in matrix form for a given molecule "mol." For simplicity, $C^{\rm sat}_{\rm ref}$ will refer to the coupling of the reference state in the isospin symmetric limit. For the $M_I = 0$ ND and ΞD systems—i.e., the states with third component of their isospin wave function $|IM_I\rangle$ equal zero, check Eqs. (37) and (38)—the $I = 0$ and $I = 1$ configurations correspond to the vertex functions

$$\phi(I=0) = (\phi_L, \phi_H) = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \tag{43}$$

$$\phi(I=1) = (\phi_L, \phi_H) = \left(\frac{1}{\sqrt{2}}, +\frac{1}{\sqrt{2}}\right). \tag{44}$$

Owing to the different masses of the L and H channels, the $I = 0$ and $I = 1$ configurations will mix. In turn, this will entail changes in the predicted masses. Naively, the size of this effect is expected to be of the order of the ratio of the binding energy over the mass gap of the L and H channels. However, in practice what we find is that if in the isospin symmetric limit the molecular state is predicted below the threshold of the L channel, the impact of isospin breaking in its mass will be rather small.

With the previous formalism we can estimate the effects of isospin breaking in the two reference states:

(i) We first calculate $C^{\rm sat}_{\rm ref}$ in the isospin limit for a given reference state.
(ii) Then we recalculate the mass of said reference state after the inclusion of isospin breaking in the masses of the hadrons.

From this, the reference states are now postdicted at

(a) For the $\Lambda_c(2940)$, the new mass is 2939.2 MeV (previously: 2939.6) and the L and H vertex functions are now $(\phi_L, \phi_H) = (0.76, -0.65)$, indicating a small deviation with respect to a pure $I = 0$ state.
(b) For the $\Sigma_c(2800)$, the mass is $\sim$2800.6 MeV (before: $\sim$2800 MeV) and $(\phi_L, \phi_H) = (0.41, 0.91)$, i.e., a larger deviation from the isospin symmetric limit when compared with the $\Lambda_c(2940)$.

That is, for the masses of the two previous molecular states isospin symmetry breaking seems to be a perturbative correction over the isospin symmetric limit. But this is only true provided the mass of the molecular state is predicted below the L channel threshold: for predictions above the L threshold, which is what happens in the $D\Xi$ and $D^*\Xi$ systems, there will be significant changes in the predicted masses.

In the particular case of the $D\Xi$ and $D^*\Xi$ molecules, the two particles channels corresponding to the $I = 0, 1, M_I = 0$ configurations are relatively far away from each other

$$m(D^0\Xi^0) = 3179.3 \text{ MeV}, \tag{45}$$

$$m(D^+\Xi^-) = 3191.4 \text{ MeV}, \tag{46}$$

$$m(D^{*0}\Xi^0) = 3321.8 \text{ MeV}, \tag{47}$$

[3]Here we are making use of the existence of a relative sign for the isospin states of the light antiquarks: $|\bar{d}\rangle = -|\frac{1}{2}\frac{1}{2}\rangle_I$ and $|\bar{u}\rangle = |\frac{1}{2} -\frac{1}{2}\rangle_I$. If we extend this convention to the charmed mesons, which contain an antiquark, we arrive at the minus sign for the definition of the $|H\rangle$ state.

TABLE VI. Predictions for the Ω_c molecular baryons when isospin breaking effects in the masses of the $\Xi^0 D^{0(*)}$ and $\Xi^- D^{+(*)}$ are taken into account. "System" refers to the particular $\Xi D^{(*)}$ molecule under consideration, J^P to its spin and parity, $R_{\rm mol}$ is the relative strength of the contact-range interaction as defined in Eq. (42), (ϕ_L, ϕ_H) the vertex function for the lower and higher mass channels, $M_{\rm mol}$ the mass of the predicted state and $M_{\rm cand}$ the mass of the Ω_c candidate states. The uncertainties in $M_{\rm mol}$ come from varying the scalar meson mass in the (400–550) MeV range. All masses are in units of MeV.

System	J^P	$R_{\rm mol}(\Lambda_c^*)$	(ϕ_L, ϕ_H)	$M_{\rm mol}$	$R_{\rm mol}(\Sigma_c^*)$	(ϕ_L, ϕ_H)	$M_{\rm mol}$	$M_{\rm cand}$
$\Xi^0 D^0 - \Xi^- D^+$	$\frac{1}{2}^-$	$\begin{pmatrix} 0.82 & -0.08 \\ -0.08 & 0.82 \end{pmatrix}$	$(-0.86, 0.51)$ $(0.07 - 0.30i, 0.95)$	$3178.5^{+1.2}_{-3.0}$ $3190.7^{+0.8}_{-3.0}$	$\begin{pmatrix} 1.23 & -0.12 \\ -0.12 & 1.24 \end{pmatrix}$	$(-0.86, 0.51)$ $(0.28, 0.96)$	$3159.7^{+2.1}_{-3.4}$ $3177.6^{+2.1}_{-1.4}$	... $3185.1^{+7.6}_{-1.9}$ [26]
$\Xi^0 D^{*0} - \Xi^- D^{*+}$	$\frac{1}{2}^-$	$\begin{pmatrix} 0.89 & -0.14 \\ -0.14 & 0.89 \end{pmatrix}$	$(-0.86, 0.51)$ $(0.07 - 0.30i, 0.95)$	$3317.1^{+2.3}_{-3.1}$ $3330.3^{+2.2}_{-3.7}$	$\begin{pmatrix} 1.35 & -0.21 \\ -0.21 & 1.36 \end{pmatrix}$	$(-0.79, 0.62)$ $(0.45, 0.89)$	$3289.3^{+7.0}_{-9.8}$ $3316.5^{+2.4}_{-1.8}$	... ...
$\Xi^0 D^{*0} - \Xi^- D^{*+}$	$\frac{3}{2}^-$	$\begin{pmatrix} 0.81 & -0.06 \\ -0.06 & 0.82 \end{pmatrix}$	$(-0.97, 0.26)$ $(0.00 - 0.16i, 0.99)$	$3320.8^{+0.8}_{-3.3}$ $3331.2^{+0.7}_{-3.2}$	$\begin{pmatrix} 1.23 & -0.09 \\ -0.09 & 1.24 \end{pmatrix}$	$(-0.89, 0.45)$ $(0.27, 0.96)$	$3303.9^{+0.4}_{-0.9}$ $3317.8^{+2.2}_{-1.7}$	... $3327.1^{+1.2}_{-1.8}$ [26]

$$m(D^{*+}\Xi^-) = 3332.0 \text{ MeV}, \tag{48}$$

where the predictions of the saturation model fall in between the two thresholds when the reference state is the $\Lambda_c(2940)$. In this latter case, concrete calculations show that the $I = 0$ and $I = 1$ ΞD states we originally predicted in Table IV now become a pair of predominantly $\Xi^0 D^{(*)0}$ and $\Xi^- D^{(*)+}$ states, as shown in Table VI. For ΞD (with $\Lambda_c(2940)$ as the reference state) we predict the masses

$$m(\Xi^0 D^0(L)) = 3178.5 \text{ MeV}, \tag{49}$$

$$m(\Xi^- D^+(H)) = 3190.7 \text{ MeV}, \tag{50}$$

where the higher energy state is relatively close to the experimental mass ($M = 3185^{+7.6}_{-1.9}$ MeV). Conversely, for $J = \frac{3}{2}$ $D^*\Xi$ we predict now

$$m(\Xi^0 D^{*0}(L)) = 3320.8 \text{ MeV}, \tag{51}$$

$$m(\Xi^- D^{*+}(H)) = 3331.2 \text{ MeV}. \tag{52}$$

Again, the heavier molecule is not far away from the experimental mass ($M = 3327.1^{+1.2}_{-1.8}$ MeV). The vertex functions for the L and H channels, ϕ_L and ϕ_H, are also listed in Table VI, where it is apparent that isospin is badly broken at the level of the wave function and neither of the two states can be interpreted as a $I = 0$ or $I = 1$ state. However, when we use the $\Sigma_c(2800)$ as the reference state, which implies more attraction, and the prediction of the $I = 0$ state happens below the L threshold in the isospin symmetric limit, then the changes in the mass after including isospin breaking in the masses are relatively small, check Tables IV and VI.

As a consequence, if the $\Omega_c(3185/3327)$ are molecular they should appear as a double peak: (i) a peak close to the $D^{(*)+}\Xi^-$ threshold, roughly corresponding to what is seen in the experiment, and (ii) a second, lighter peak close to the $D^{(*)0}\Xi^0$ threshold. *Prima facie*, this seems to contradict the experimental findings, as there is no Ω_c listed with the mass of the lighter peak. Yet, regarding the $\Omega_c(3185)$, Ref. [26] states: *"A two-peak structure also describes the data well in the mass region around 3185 MeV, hence the presence of two states in this region cannot be excluded."*

Unfortunately, the masses of the two-peak solution are not given, neither it is said explicitly whether this also applies to the $\Omega_c(3327)$. For the latter, in Table VI we predict that the $J = \frac{1}{2}$ and $\frac{3}{2}$ $\Xi^- D^{*+}$ peaks are almost at the same mass, which (within the two-peak hypothesis) might explain why the uncertainties in the $\Omega_c(3327)$ mass are much smaller ($3327.1^{+1.2}_{-1.8}$ MeV) than those of the $\Omega_c(3185)$ ($3185.1^{+7.6}_{-1.9}$ MeV).

Moreover, if the $\Omega_c(3185)$ and $\Omega(3327)$ were double peaks, this factor could indeed explain their large observed widths in [26]. A pure molecular explanation will result in a narrow state because the $\Xi D \to \Xi_c \bar{K}$ transition depends on short-range dynamics (e.g., the exchange of a light-baryon). These dynamics are expected to be suppressed if the wave function has a large size. Within this scenario, the relatively large width (for a molecular state) of the experimental peaks could be a consequence of its double peak nature.

More insight might be gained from a comparison with the compact hadron hypothesis.[4] From the equal spacing rule, we expect the partners of the $\Omega_c(3185)$ and $\Omega(3327)$ to have a similar spacing to that of the lowest mass charmed baryon sextet, that is, $M(\Omega_c) - M(\Xi_c') \sim 125$ MeV and $M(\Xi_c') - M(\Sigma_c) \sim 125$ MeV. Thus we might expect the sextets:

$$\Sigma_c(2940), \qquad \Xi_c(3060), \qquad \Omega_c(3185), \tag{53}$$

[4]Regarding this hypothesis, we mention in passing that recently Ref. [28] has proposed that the $\Omega_c(3327)$ is a compact $1D_{\frac{5}{2}}$ state, while Ref. [29] interprets the $\Omega_c(3185)$ and $\Omega_c(3327)$ as $2S_{\frac{1}{2}}$ and $2S_{\frac{3}{2}}$ states.

$$\Sigma_c(3075), \qquad \Xi_c(3200), \qquad \Omega_c(3327). \qquad (54)$$

Currently, only the $\Xi_c(3055)$ fits within the previous pattern. However, the identification of the $\Xi_c(3055)$ as a partner of the $\Omega_c(3185)$ is problematic in what regards the widths: the $\Xi_c(3055)$ has a width of a few MeV ($\Gamma = 7.8 \pm 1.9$ MeV [31]), while for the $\Omega_c(3185)$ it is of a few tens of MeV ($\Gamma = 50 \pm 7^{+10}_{-20}$ MeV [26]), a difference of one order of magnitude. This suggests that they are not partners, though confounding factors might exist: a compact $\Xi_c(3055)$ and $\Omega_c(3185)$ could both mix with the nearby meson-baryon thresholds, muddling the comparison between the two.

VI. CONCLUSIONS

We have considered the spectroscopy of charmed meson and octet baryon molecules within a phenomenological model. This model is a contact-range theory in which the couplings are saturated by the exchange of the light scalar and vector mesons (σ, ρ, ω, K^*, and ϕ). The choice of a contact-range interaction is motivated by the difference in scales between the range of light-meson exchange (short-range) and the size of the molecular states predicted (long-range). The saturation of the couplings exploits their RG evolution to combine the contributions from light-mesons with different masses. The couplings are determined up to a proportionality constant that has to be calibrated by reproducing a given reference state, i.e., a known state with a plausible molecular interpretation. For this we use the $\Lambda_c(2940)$ (as an $I=0$, $J=\frac{3}{2}$ ND^* molecule) and the $\Sigma_c(2800)$ ($I=1$, $J=\frac{1}{2}$ ND molecule). Each reference state leads to quantitative differences in the charmed baryon and anticharmed pentaquark spectra.

Among the molecular charmed baryons we predict, there are ΣD and ΛD_s^* bound states that might correspond with the $\Xi_c(3055)$ and $\Xi_c(3123)$ baryons. Yet, the more interesting result might be the prediction of ΞD and ΞD^* bound states with masses matching those of the recently observed $\Omega_c(3185)$ and $\Omega_c(3327)$. For this molecular interpretation to be valid it would be required that the $\Omega_c(3185)$ is composed of two narrow peaks with a mass difference of about 10 MeV (i.e., the gap between the $\Xi^- D^+$ and $\Xi^0 D^0$ thresholds). It is noteworthy that the $\Omega_c(3185)$ indeed accepts a two peak description [26], though the masses of each of the peaks is not mentioned. For the $\Omega_c(3327)$ the situation might be more complex because the two spin configurations ($J=\frac{1}{2}$ and $\frac{3}{2}$) of the ΞD^* system bind, meaning that there could be up to four peaks (though this might depend on the magnitude of the isospin splitting). Yet, the $J=\frac{1}{2}$ and $\frac{3}{2}$ $\Xi^- D^{*+}$ bound states are predicted about the same mass, representing a simplification with respect to the four peak scenario. In this latter case, if the $\Omega_c(3327)$ turns out to contain two nearby peaks with a mass difference smaller than the $\Xi^- D^{*+}$ and $\Xi^0 D^0$ thresholds gap, this would support a molecular interpretation.

Finally, we predict a few molecular anticharmed pentaquarks. In this case there are no experimental candidates and the only comparison left is with other theoretical models [51–54], which in general do agree on the qualitative features of the molecular spectrum (for instance, the possibility of $N\bar{D}_s^{(*)}$ [51–53] or $\Sigma\bar{D}^{(*)}$ [54] states). Yet, there is experimental information about the $I=0$ $N\bar{D}$ interaction from the ALICE collaboration [50]: its inverse scattering length. This datum is reproduced by our RG saturation model independently of the input [$\Lambda_c(2940)$ or $\Sigma_c(2800)$].

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APPENDIX: LIGHT-MESON COUPLINGS

Here we explore in more detail our choice of couplings for the light baryons. We begin with the vector meson couplings, which are derived from the mixing with the electromagnetic current. We continue with the scalar couplings, whose choice requires a more careful comparison with molecular predictions in a few system. This leads to the conclusion that this coupling is weaker for the Λ than for the other octet baryons.

1. Vector couplings

For the vector couplings, we determined them from the fact that the neutral vector mesons can mix with the photon current (because they have the same charge and quantum numbers $J^{PC} = 1^{--}$), i.e., from vector meson dominance [40–42]. For this we consider the nonrelativistic interaction between a hadron h and a vector meson V as given by the Lagrangian

$$\mathcal{L}_{hhV} = g_V h^\dagger \left[\partial_0 V_0 + \frac{\kappa_V}{2M} \epsilon_{ijk} \hat{S}_i \partial_j V_k \right] h, \qquad (A1)$$

with g_V and κ_V the electric- and magneticlike couplings, $\hat{S}_i$ the $i = 1, 2, 3$ spatial component of the reduced spin operator $\hat{\vec{S}} = \vec{S}/S$ (with S the spin of hadron h) and M a scaling mass, for which we choose the nucleon mass.

For simplicity we have not indicated the isospin or flavor indices explicitly. Next we make the substitutions

$$\rho^0_\mu \to \rho^0_\mu + \frac{1}{2}\frac{e}{g}A_\mu, \tag{A2}$$

$$\omega_\mu \to \omega_\mu + \frac{1}{6}\frac{e}{g}A_\mu, \tag{A3}$$

$$\phi_\mu \to \phi_\mu - \frac{1}{3\sqrt{2}}\frac{e}{g}A_\mu, \tag{A4}$$

which depend on whether the vector meson is a neutral ρ (ρ^0), an ω or a ϕ, μ is a Lorentz index and A_μ the photon field; e is the proton charge and $g = m_V/2f_\pi \simeq 2.9$ the universal vector meson coupling (in the $f_\pi = 132$ MeV normalization). We match the A_μ piece of the previous substitution to the nonrelativistic electromagnetic Lagrangian for the light-quark content of the hadron H

$$\mathcal{L}_{hh\gamma} = e h^\dagger \left[Q_L \partial_0 A_0 + \frac{\mu_L}{2M}\epsilon_{ijk}\hat{S}_i \partial_j A_k \right] h, \tag{A5}$$

where Q_L is the charge of the light quarks within the hadron h (in units of e) and μ_L its magnetic moments in units of $e/(2M)$ (or nuclear magnetons, if M is chosen to be the nucleon mass). Of course, if isospin or flavor are explicitly considered, Q_L and μ_L will become matrices. The g_V couplings depend on the charges of the isospin components of the hadrons, while κ_V on their (light) magnetic moments. By using the quark model calculation of the magnetic moments of the octet baryons and the part of the charmed meson magnetic moments that come from the light-quarks, we arrive at the κ_V couplings of Table I.

2. Scalar coupling

Here we explore in more detail the couplings of the scalar meson in the strange sector for the light baryons and charmed mesons. Our baseline scenario is that this couplings is given by $g_{Sqq} = 3.4$ for $q = u$, d, s, as derived from the linear sigma model [43], the quark model [44] and the additional assumption that the coupling to the s quark is similar to the u and d quarks. We will decide whether this baseline value requires corrections or not by calculating the spectra of a few two-hadron systems and comparing with experimental information or other theoretical models.

For the coupling of the scalar meson to the light baryons, we calculate a few two light baryon systems using the two-nucleon $^1S_0$ virtual state as the reference state (or, equivalently, by using the $^1S_0$ scattering length as input, $a_0(^1S_0) = -23.7$ fm). First, we notice that in terms of SU(3) symmetry, the two-nucleon $^1S_0$ configuration and a series of other configurations:

$$|NN(^1S_0, I=1)\rangle = |27\rangle, \tag{A6}$$

$$\left|\Sigma N\left(^1S_0, I=\frac{3}{2}\right)\right\rangle = |27\rangle, \tag{A7}$$

$$|\Sigma\Sigma(^1S_0, I=2)\rangle = |27\rangle, \tag{A8}$$

$$\left|\Xi\Sigma\left(^1S_0, I=\frac{3}{2}\right)\right\rangle = |27\rangle, \tag{A9}$$

$$|\Xi\Xi(^1S_0, I=1)\rangle = |27\rangle, \tag{A10}$$

are all in the 27-plet SU(3)-flavor representation, from which the potential should be the same in the flavor-symmetric limit. Indeed, all of these systems happen to show large scattering lengths that are in a few cases positive (indicating a bound state). Following [33] we use a softer cutoff in the light sector, $\Lambda = 0.5$ GeV, in which case saturation yields

$$B_2\left(\Sigma N, I=\frac{3}{2}\right) = 1.1 \text{ MeV}, \tag{A11}$$

$$B_2(\Sigma\Sigma, I=2) = 1.6(0-0.01) \text{ MeV}, \tag{A12}$$

$$B_2\left(\Xi\Sigma, I=\frac{3}{2}\right) = 1.0(0.58-0.19) \text{ MeV}, \tag{A13}$$

$$B_2(\Xi\Xi, I=1) = 2.1(0.40-1.0) \text{ MeV}, \tag{A14}$$

where the values in parentheses correspond to chiral EFT results when terms up to order Q^2 are included in the potential [55], where we notice that (i) for the $^1S_0$ ΣN no bound state is predicted in [55], though there is considerable attraction if we look at the scattering length and (ii) that the order Q^0 results would be more similar to our $\Xi\Sigma$ [(2.23–6.18) MeV in [55] versus our 1.0 MeV result] and $\Xi\Xi$ predictions [(2.56–7.27) MeV in [55] versus 2.1 MeV]. Taking into account that we are not considering exchange of pseudoscalar mesons, which lead to less attraction in the strangeness $S = -1$ and -2 system relative to the singlet, the results we obtain are sensible. We could have also compared the scattering lengths, in which case we would have had

$$a_0(\Sigma N) = 6.4 \text{ fm}, \tag{A15}$$

$$a_0(\Sigma\Sigma) = 5.2(60.6-(-286.0)) \text{ fm}, \tag{A16}$$

$$a_0(\Xi\Sigma) = 6.2(8.4-13.8) \text{ fm}, \tag{A17}$$

$$a_0(\Xi\Xi) = 4.4(9.7-6.5) \text{ fm}, \tag{A18}$$

where results in parentheses are again from Ref. [55].

Alternatively, we can compare instead to the lattice QCD results of Ref. [56], which after extrapolation to the physical pion mass lead to a bound $^1S_0$ system with binding energy

$$B_2^{\mathrm{lin\ (quad)}}(NN, I=0) = 6.4^{+6.3}_{-6.5}(9.9^{+4.6}_{-4.5})\ \mathrm{MeV}, \quad (A19)$$

depending on whether a linear or quadratic (in parentheses) extrapolation is used to reach $m_\pi = 138$ MeV. By using the linear extrapolation as input we obtain

$$B_2\left(\Sigma N, I=\frac{3}{2}\right) = 14.9(8.4^{+7.8}_{-6.6})\ \mathrm{MeV}, \quad (A20)$$

$$B_2(\Sigma\Sigma, I=2) = 15.4(1.0\pm 6.1)\ \mathrm{MeV}, \quad (A21)$$

$$B_2\left(\Xi\Sigma, I=\frac{3}{2}\right) = 12.8(5.9^{+5.7}_{-5.8})\ \mathrm{MeV}, \quad (A22)$$

$$B_2(\Xi\Xi, I=1) = 16.2(9.6^{+4.5}_{-4.7})\ \mathrm{MeV}, \quad (A23)$$

while if we use the quadratic extrapolation as input

$$B_2\left(\Sigma N, I=\frac{3}{2}\right) = 20.1(11.5^{+5.7}_{-4.8})\ \mathrm{MeV}, \quad (A24)$$

$$B_2(\Sigma\Sigma, I=2) = 20.4(5.8^{+4.2}_{-4.3})\ \mathrm{MeV}, \quad (A25)$$

$$B_2\left(\Xi\Sigma, I=\frac{3}{2}\right) = 17.2(9.5^{+3.8}_{-4.0})\ \mathrm{MeV}, \quad (A26)$$

$$B_2(\Xi\Xi, I=1) = 21.1(12.4^{+3.0}_{-3.1})\ \mathrm{MeV}, \quad (A27)$$

where the results in parentheses are from Ref. [56]. In this case our predictions tend to bind more than the extrapolated lattice results. The point is though that the naive choice of couplings works (within reason) in this particular case, and thus we do not modify it for the N, Σ, and Ξ baryons.

Yet, for the Λ baryon we actually have to modify its coupling to the scalar meson in order to reproduce current theoretical estimations of the ΛN and $\Lambda\Lambda$ scattering length. If we use $g_{\sigma\Lambda\Lambda} = g_{\sigma NN}$ (and the $^1S_0$ virtual state as a reference state), in general we find excessive attraction, where the scattering lengths are

$$a_0(\Lambda N, {}^1S_0) = 53.1\ \mathrm{fm}, \quad (A28)$$

$$a_0(\Lambda N, {}^1S_0) = 346.8\ \mathrm{fm}, \quad (A29)$$

$$a_0(\Lambda\Lambda) = 16.2\ \mathrm{fm}, \quad (A30)$$

where the positive scattering lengths indicate the existence of bound states, in disagreement with other theoretical models. In contrast, for $g_{\sigma\Lambda\Lambda} = (3/4)g_{\sigma NN}$ we obtain

$$a_0(\Lambda N, {}^1S_0) = -3.1\ \mathrm{fm}, \quad (A31)$$

$$a_0(\Lambda N, {}^1S_0) = -2.9\ \mathrm{fm}, \quad (A32)$$

$$a_0(\Lambda\Lambda) = -1.3\ \mathrm{fm}, \quad (A33)$$

which compare well (though not perfectly) with other models: (i) for the ΛN case, we have $a_0(\Lambda N, {}^1S_0) = -2.9$, -2.6, and -2.6 fm and $a_0(\Lambda N, {}^3S_1) = -1.7$, -1.7 and -1.7 fm with chiral potentials at the next-to-leading order (NLO) [57], the Jülich 04 model [58] and the Nijmegen soft core potential [59], respectively, while (ii) for the $\Lambda\Lambda$ case, $a_0(\Lambda\Lambda) = -(0.33\text{–}0.85)$ fm in chiral NLO [60], $a_0(\Lambda\Lambda) = -0.81 \pm 0.23^{+0.0}_{-0.13}$ fm in the lattice [61]. Even though it is possible to further fine-tune the parameters to match better the results of other models, we consider that the current change ($g_{S\Lambda\Lambda} = (3/4)g_{SNN}$) is enough for our purposes.

The charmed meson case is simpler. Here we will use the recent lattice QCD prediction of $J^{PC} = 0^{++}$ $D\bar{D}$ and $D_s\bar{D}_s$ bound states [62] as pseudodata. We notice in passing that the $X(3960)$ [63] is interpreted as a $D_s\bar{D}_s$ molecular state too [64,65], where the location of the pole (bound or virtual) is usually not far away from the lattice result. For this prediction, we use the $X(3872)$ as the reference state (interpreted as a $I=0$, $J^{PC}=1^{++}$ $D^*\bar{D}$ bound state) and a cutoff of $\Lambda = 1.0$ GeV as in [33]. The binding energies of the $D\bar{D}$ state is calculated to be

$$B_2(D\bar{D}) = 4.0^{+5.0}_{-3.7}\ \mathrm{MeV}, \quad (A34)$$

and we will use it as input in our calculations. For the $D_s\bar{D}_s$ state there are two calculations, a single channel one in which a bound state is found

$$B_2^{\mathrm{SC}}(D_s\bar{D}_s) = 6.2^{+2.0}_{-3.8}\ \mathrm{MeV}, \quad (A35)$$

and a coupled channel one, in which we have a resonance instead with energy

$$E_2^{\mathrm{CC}}(D_s\bar{D}_s) = -0.2^{+0.17}_{-4.9} - \frac{i}{2}(0.27^{+2.5}_{-0.15})\ \mathrm{MeV}, \quad (A36)$$

where E_2 is the energy of the state with respect to the $D_s\bar{D}_s$ threshold. If we assume $g'_S = g_S$, we will predict this state to be at

$$B_2^{\mathrm{SC}}(D_s\bar{D}_s) = (1.0)^V\ \mathrm{MeV}, \quad (A37)$$

$$E_2^{\mathrm{CC}}(D_s\bar{D}_s) = \left(-2.4 - \frac{i}{2}1.5\right)^V\ \mathrm{MeV}, \quad (A38)$$

with both solutions corresponding to a virtual state (where in the coupled channel case this specifically means a pole in the (I,II) Riemann sheet). Even though outside the error bands of the lattice predictions, these two results are still in line with them. From this point of view, it might not be necessary to tweak the scalar coupling. If we take $g_{SD_sD_s} = 1.15 g_{SDD}$ instead, we will predict

$$B_2^{\rm SC}(D_s\bar{D}_s) = 1.5\ \text{MeV}, \tag{A39}$$

$$E_2^{\rm CC}(D_s\bar{D}_s) = \left(-0.25 - \frac{i}{2}0.42\right)\ \text{MeV}, \tag{A40}$$

where the single channel calculation is now a bound state, in agreement with Ref. [62], and the coupled channel calculation a resonance in the (II,I) Riemann sheet (we notice that in this case, Ref. [62] finds that this state is in the (II,I) sheet in 70% of the bootstrap samples and in (I,II) in the rest). However, even though this change improves the agreement with lattice, we do not consider that it is necessary to include it (the improvement is marginal) and will opt instead for the more simple $g_{SD_sD_s} = g_{SDD}$ choice. Finally, we also notice that the reproduction of the $Z_{cs}(3985)$ as a $D^*\bar{D}_s$-$D\bar{D}_s^*$ molecule from the $Z_c(3900)$ ($D^*\bar{D}$) also requires $g_{SD_sD_s} \geq g_{SDD}$ [66].

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