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Research Article

Spinor Quintom Cosmology with Intrinsic Spin

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We consider a spinor quintom dark energy model with intrinsic spin, in the framework of Einstein-Cartan-Sciama-Kibble theory. After constructing the mathematical formalism of the model, we obtain the spin contributed total energy-momentum tensor giving the energy density and the pressure of the quintom model, and then we find the equation of state parameter, Hubble parameter, deceleration parameter, state finder parameter, and some distance parameter in terms of the spinor potential. Choosing suitable potentials leads to the quintom scenario crossing between quintessence and phantom epochs, or vice versa. Analyzing three quintom scenarios provides stable expansion phases avoiding Big Rip singularities and yielding matter dominated era through the stabilization of the spinor pressure via spin contribution. The stabilization in spinor pressure leads to neglecting it as compared to the increasing energy density and constituting a matter dominated stable expansion epoch.

1. Introduction

The astrophysical observations show that the universe is experiencing an accelerating expansion due to an unknown component of energy, named as the dark energy (DE) which is distributed all over the universe and having a negative pressure in order to drive the acceleration of the universe [1–9]. Various DE scenarios have been proposed: cosmological constant Λ is the oldest DE model which has a constant energy density filling the space homogeneously [10–13]. Equation of state (EoS) parameter of a cosmological fluid is $\omega = p/\rho$, where p and ρ are the pressure and energy density and the DE scenario formed by the cosmological constant refers to a perfect fluid with EoS $\omega_\Lambda = -1$.

Other DE scenarios can be constructed from the dynamical components, such as the quintessence, phantom, K-essence, or quintom [14–16]. Quintessence is considered as a DE scenario with EoS $\omega > -1$. Such a model can be described by using a canonical scalar field. Recent observational data presents the idea that the EoS of DE can be in a region where $\omega < -1$. The most common scenario generalizing this regime is to use a scalar field with a negative kinetic term. This DE model is known as the phantom scenario which is also named as a ghost [17]. This model experiences the shortcoming, such that its energy state is unbounded from below and this leads to the quantum instability problem [18]. If the potential value is not bounded from above, this scenario is even instable at the classical regime known as the Big Rip singularity [19]. Another DE scenario, K-essence, is constructed by using kinetic term in the domain of a general function in the field Lagrangian. This model can realize both $\omega > -1$ and $\omega < -1$ due to the existence of a positive energy density but cannot allow a consistent crossing of the cosmological constant boundary -1 [20].

The time variation of the EoS of DE has been restricted by the data obtained by Supernovae Ia (SNIa). According to the SNIa data, some attempts have come out to estimate the band power and density of the DE EoS as a function of the redshift [21]. There occur two main models for the variation of EoS with respect to time; Model A and Model B. While Model A is valid in low redshift, Model B suffers in low redshift; therefore, it works in high redshift values. These models imply that the evolution of the DE EoS begins from $\omega > -1$ in the past to $\omega < -1$ in the present time, namely, the observational and theoretical results allow the EoS ω of DE crossing the cosmological constant boundary or phantom divide during the evolution of the universe [22–25].

Crossing of the DE EoS the cosmological constant boundary is named as the quintom scenario and this can be constructed in some special DE models. For instance, if we consider a single perfect fluid or single scalar field DE constituent, this model does not allow the DE EoS to cross the $\omega = -1$ boundary according to the no-go theorem [26–31]. To overcome no-go theorem and to realize crossing phantom divide line, some modifications can be made to the single scalar field DE models. One can construct a quintom scenario by considering two scalar fields, such as quintessence and quintom [22]. The components cannot cross the -1 boundary alone but can cross it combined together. Another quintom scenario is achieved by constructing a scalar field model with nonlinear or higher order derivative kinetic term [27, 32] or a phantom model coupled to dark matter [33]. Also the scalar field DE models nonminimally coupled to gravity satisfy the crossing cosmological constant boundary [34, 35].

The aforementioned quintom models are constructed from the scalar fields providing various phantom behaviors, but the ghost field may cause some instable solutions. By considering the linearized perturbations in the effective quantum field equation at two-loop order one can obtain an acceleration phase [36–40]. On the other hand, there is another quintom model satisfying the acceleration of the universe, which is constructed from the classical homogeneous spinor field ψ [41–44]. In recent years, many studies for spinor fields in cosmology can be found [20], such as those for inflation and cyclic universe driven by spinor fields, for spinor matter in Bianchi Type I spacetime, and for a DE model with spinor matter [45–50].

The consistent quintom cosmology has been proposed by using spinor matter in Friedmann-Robertson-Walker (FRW) geometry, in Einstein's general relativity framework [51]. The spinor quintom scenario allows EoS crossing -1 boundary without using a ghost field. When the derivative of the potential term with respect to the scalar bilinear $\overline{\psi}\psi$ becomes negative, the spinor field shows a phantom-like behavior. But the spinor quintom exhibits a quintessence-like behavior for the positive definite potential derivative [20]. In this quintom model, there exist three categories of scenario depending on the choice of the type of potentials; one scenario is that the universe evolves from a quintessence-like phase $\omega > -1$ to a phantom-like phase $\omega < -1$, another scenario is for the universe evolving from a phantom-like phase $\omega < -1$ to a quintessence-like phase $\omega > -1$, and the third scenario is that the EoS of spinor quintom DE crosses the -1 boundary more than one time.

In this study, we consider the spinor quintom DE, in the framework of Einstein-Cartan-Sciama-Kibble (ECSK) theory which is a generalization of the metric-affine formulation of Einstein's general relativity with intrinsic spin [52–61]. Since the ECSK theory is the simplest theory including the intrinsic spin and avoiding the big-bang singularity [62], it is worth considering the spinor quintom in ECSK theory for investigating the acceleration phase of the universe with the phantom behavior. Therefore, we analyze the spinor quintom model with intrinsic spin in ECSK theory whether it provides the crossing cosmological constant boundary. Then if the model provides the crossing -1 boundary, we will find the suitable conditions on the potential for the crossing -1 boundary.

2. Algebra of Spinor Quintom with Intrinsic Spin

The most complicated example of the quantum field theories lying in curved spacetime is the theory of Dirac spinors. There occurs a conceptual problem related to obtaining the energy-momentum tensor of the spinor matter field from the variation of the matter field Lagrangian. For the scalar or tensor fields, energy-momentum tensor is the quantity describing the reaction of the matter field Lagrangian to the variations of the metric, while the matter field is held constant during the change of the metric. But for the spinor fields, the above procedure does not hold for obtaining the energy-momentum tensor from the variation with respect to metric only, because the spinor fields are the sections of a spinor bundle obtained as an associated vector bundle from the bundle of spin frames. The bundle of spin frames is double covering of the bundle of oriented and time-oriented orthonormal frames. For spinor fields, when one varies the metric, the components of the spinor fields cannot be held fixed with respect to some fixed holonomic frame induced by a coordinate system, as in the tensor field case [63]. Therefore, the intrinsic spin of matter field in curved spacetime requires ECSK theory which is the simplest generalization of the metric-affine formulation of general relativity.

According to the metric-affine formulation of the gravity, the dynamical variables are the tetrad (vierbein or frame) field e^i_a and the spin connection $\omega^a_{bk} = e^a_j e^j_{b;k} = e^a_j (e^j_{b,k} + \Gamma^j_{ik} e^i_b)$. Here comma denotes the partial derivative with respect to the x^k coordinate, while the semicolon refers to the covariant derivative with respect to the affine connection Γ^i_{jk}. The antisymmetric lower indices of the affine connection give the torsion tensor $S^i_{jk} = \Gamma^i_{[jk]}$. The tetrad gives the relation between spacetime coordinates denoted by the indices $i, j, k, \ldots$ and local Lorentz coordinates denoted by the indices $a, b, c, \ldots$, such that $V^a = V^i e^a_i$, where V^a is a Lorentz vector and V^i is a usual vector. Covariant derivative of a Lorentz vector is defined with respect to the spin connection and denoted by a bar: $V^a_{|i} = V^a_{,i} + \omega^a_{bi} V^b$ and $V_{a|i} = V_{a,i} - \omega^b_{ai} V_b$. Also the covariant derivative of a vector is defined in terms of the affine connection: $V^k_{;i} = V^k_{,i} + \Gamma^k_{li} V^l$ and $V_{k;i} = V_{k,i} - \Gamma^l_{ki} V_l$. Local Lorentz coordinates are lowered or raised by the Minkowski metric η_{ab} of the flat spacetime, while the spacetime coordinates are lowered or raised by the metric tensor g_{ik}. Metric compatibility condition $g_{ij;k} = 0$ leads to the definition of affine connection $\Gamma^k_{ij} = \left\{ {k \atop ij} \right\} + C^k_{ij}$ in terms of the Christoffel symbols $\left\{ {k \atop ij} \right\} = (1/2) g^{km} (g_{mi,j} + g_{mj,i} - g_{ij,m})$ and the contortion tensor $C^i_{jk} = S^i_{jk} + 2S^i_{(jk)}$. Throughout the paper, the $A_{(jk)} = (1/2)(A_{jk} + A_{kj})$ notation is used for symmetrization and $A_{[jk]} = A_{jk} - A_{kj}$ is used for the antisymmetrization. With the definitions $g_{ik} = \eta_{ab} e^a_i e^b_k$ and $S^j_{ik} =$

$\omega^j_{[ik]} + e^a_{[i,k]}e^j_a$, the metric tensor and the torsion tensor can be considered as the dynamical variable instead of the tetrad and spin connection.

A tensor density $\mathrm{A}^{ij\cdots}_{kl\cdots}$ is given in terms of the corresponding tensor $A^{ij\cdots}_{kl\cdots}$ as $\mathrm{A}^{ij\cdots}_{kl\cdots} = eA^{ij\cdots}_{kl\cdots}$, where $e = \det e^a_i = \sqrt{-\det g_{ik}}$. Therefore, we represent the spin density and the energy-momentum density, such as $\sigma_{ijk} = es_{ijk}$ and $\mathrm{T}_{ik} = eT_{ik}$. Here we call these tensors metric spin tensor and metric energy-momentum tensor, since the spacetime coordinate indices label these tensors and are obtained from the variation of the Lagrangian with respect to the torsion (or contortion) tensor C^{ij}_k and the metric tensor g^{ij}, respectively. The metric spin tensor is written as $s^k_{ij} = (2/e)(\delta\ell_m/\delta C^{ij}_k) = (2/e)(\partial\ell_m/\partial C^{ij}_k)$, while the metric energy-momentum tensor is given by $T_{ij} = (2/e)(\delta\ell_m/\delta g^{ij}) = (2/e)[\partial\ell_m/\partial g^{ij} - \partial_k(\partial\ell_m/\partial(g^{ij}_{,k}))]$. Here, the Lagrangian density of the source matter field is $\ell_m = eL_m$. When the local Lorentz coordinates are also used in these tensors as $\sigma^i_{ab} = es^i_{ab}$ and $\mathrm{T}^a_i = eT^a_i$, we call s^i_{ab} and T^a_i dynamical spin tensor and dynamical energy-momentum tensor, respectively, and they are obtained from the variation of the Lagrangian with respect to the tetrad e^i_a and the spin connection ω^{ab}_i. The dynamical spin tensor is $s^i_{ab} = (2/e)(\delta\ell_m/\delta\omega^{ab}_i) = (2/e)(\partial\ell_m/\partial\omega^{ab}_i)$, and energy-momentum tensor is $T^a_i = (1/e)(\delta\ell_m/\delta e^i_a) = (1/e)[\partial\ell_m/\partial e^i_a - \partial_j(\partial\ell_m/\partial(e^i_{a,j}))]$.

Total action of the gravitational field and the source matter in metric-affine ECSK theory is given in the same form with the classical Einstein-Hilbert action, such as $S = \kappa\int(\ell_g + \ell_m)d^4x$, where $\kappa = 8\pi G$ and $\ell_g = -(1/2\kappa)eR$ is the gravitational Lagrangian density. Here Ricci scalar is constructed from the spin connection containing curvature tensor, such that $R = R^b_j e^j_b$, where $R^b_j = R^{bc}_{jk}e^k_c$ is the Ricci tensor obtained from the curvature tensor R^{bc}_{jk} and finally this curvature tensor is expressed in terms of the spin connection, such that $R^a_{bij} = \omega^a_{bj,i} - \omega^a_{bi,j} + \omega^a_{ci}\omega^c_{bj} - \omega^a_{cj}\omega^c_{bi}$. Variation of the total action with respect to the contortion tensor gives Cartan equations $S^j_{ik} - S_i\delta^j_k + S_k\delta^j_i = -(\kappa/2e)\sigma^j_{ik}$ and with respect to the metric tensor gives Einstein equations in the form of $G_{ik} = \kappa(T_{ik} + U_{ik})$, where $G_{ik} = P^j_{ijk} - (1/2)P^{lm}_{lm}g_{ik}$ is the Einstein tensor and P^j_{ijk} is the Riemann curvature tensor satisfying $R^i_{klm} = P^i_{klm} + C^i_{km:l} - C^i_{kl:m} + C^j_{km}C^i_{jl} - C^j_{kl}C^i_{jm}$, where colon denotes the Riemannian covariant derivative with respect to the Levi-Civita connection, such as $V^k_{:i} = V^k_{,i} + \left\{{k \atop li}\right\}V^l$ and $V_{k:i} = V_{k,i} - \left\{{l \atop ki}\right\}V_l$. Also for torsion-free general relativity theory, curvature tensor turns out to be the Riemann tensor. Right hand side of Einstein equations contains an extra term U_{ik} which is the contribution to the energy-momentum tensor from the torsion and it is quadratic in the spin tensor, such as $U_{ik} = \kappa(-s^{ij}_{[l}s^{kl}_{j]} - (1/2)s^{ijl}s^k_{jl} + (1/4)s^{jli}s^k_{jl} + (1/8)g^{ik}(-4s^l_{j[m}s^{jm}_{l]} + s^{jlm}s_{jlm}))$. Therefore, the total energy-momentum tensor is $\Theta_{ik} = T_{ik} + U_{ik}$.

In metric-affine ECSK formulation of gravity, a spinor quintom field with intrinsic spin has a Lagrangian density of the form $\ell_m = e(i/2)(\overline{\psi}\gamma^k\psi_{;k} - \overline{\psi}_{;k}\gamma^k\psi) - eV$, where V is the potential of the spinor field ψ and the adjoint spinor $\overline{\psi} = \psi^+\gamma^0$. The covariant derivative of the spinor field is given as $\psi_{;k} = \psi_{,k} - \Gamma_k\psi$ and $\overline{\psi}_{;k} = \overline{\psi}_{,k} - \Gamma_k\overline{\psi}$, where $\Gamma_k = -(1/4)\omega_{abk}\gamma^a\gamma^b$ is the Fock-Ivanenko spin connection, and then γ^k and γ^a are the metric and dynamical Dirac gamma matrices satisfying $\gamma^k = e^k_a\gamma^a$, $\gamma^{(k}\gamma^{m)} = g^{km}I$, and $\gamma^{(a}\gamma^{b)} = \eta^{ab}I$. The covariant derivative of the spinor can be decomposed into the Riemannian covariant derivative plus a contortion tensor C_{ijk} containing term, such as $\psi_{;k} = \psi_{:k} + (1/4)C_{ijk}\gamma^{[i}\gamma^{j]}\psi$ and $\overline{\psi}_{;k} = \overline{\psi}_{:k} - (1/4)C_{ijk}\overline{\psi}\gamma^{[i}\gamma^{j]}$. The Riemannian covariant derivative of the spinor and adjoint spinor fields for quintom DE are given: $\psi_{:k} = \psi_{,k} + (1/4)g_{ik}\left\{{i \atop jm}\right\}\gamma^j\gamma^m\psi$ and $\overline{\psi}_{:k} = \overline{\psi}_{,k} - (1/4)g_{ik}\left\{{i \atop jm}\right\}\gamma^j\gamma^m\overline{\psi}$. These covariant derivatives including the contortion tensor C_{ijk} are embedded in the spinor quintom Lagrange density. However, the explicit form of the contortion tensor which can be obtained from the Cartan equations is needed. Since the right hand side of Cartan equations contains the spin tensor density, we obtain the spin tensor from the variation of the spinor Lagrangian with respect to the contortion tensor, such that $s^{ijk} = (1/e)\sigma^{ijk} = -(1/e)\varepsilon^{ijkl}s_l$, where ε^{ijkl} is the Levi-Civita symbol, $s^i = (1/2)\,\overline{\psi}\gamma^i\gamma^5\psi$ is the spin pseudovector, and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. Inserting the spin tensor for spinor quintom field in the Cartan equations gives the torsion tensor $S_{ijk} = C_{ijk} = (1/2)\kappa\varepsilon_{ijkl}s^l$ which will take place in the spinor quintom Lagrange density [52–62].

The variation of the spinor quintom matter Lagrangian density with respect to the adjoint spinor gives the ECSK Dirac equation, such as

$$i\gamma^k\psi_{:k} - \frac{\partial V}{\partial\overline{\psi}} + \frac{3}{8}\kappa\left(\overline{\psi}\gamma^k\gamma^5\psi\right)\gamma_k\gamma^5\psi = 0, \tag{1}$$

and the variation with respect to the spinor itself gives adjoint Dirac equation as

$$i\overline{\psi}_{:k}\gamma^k + \frac{\partial V}{\partial\psi} - \frac{3}{8}\kappa\left(\overline{\psi}\gamma^k\gamma^5\psi\right)\overline{\psi}\gamma_k\gamma^5 = 0. \tag{2}$$

Then the total energy-momentum tensor of the spinor quintom field is obtained from $\Theta_{ik} = T_{ik} + U_{ik}$. Here the metric energy-momentum tensor is obtained by the variation of spinor quintom Lagrange density with respect to the metric tensor, such as

$$\begin{aligned} T_{ik} &= \frac{2}{e}\left[\frac{\partial\ell_m/\partial g^{ik}}{-\partial_j\left(\partial\ell_m/\partial\left(g^{ik}_{,j}\right)\right)}\right] \\ &= \frac{i}{2}\left(\overline{\psi}\delta^j_{(i}\gamma_{k)}\psi_{;j} - \overline{\psi}_{;j}\delta^j_{(i}\gamma_{k)}\psi\right) \\ &\quad - \frac{i}{2}\left(\overline{\psi}\gamma^j\psi_{;j} - \overline{\psi}_{;j}\gamma^j\psi\right)g_{ik} + Vg_{ik}, \end{aligned} \tag{3}$$

and the spin contributing metric energy-momentum tensor is obtained by substituting the spin tensor for spinor quintom field in U_{ik}. Then the total metric energy-momentum tensor is found to be

$$\Theta_{ik} = \frac{i}{2}\left(\overline{\psi}\delta^{j}_{(i}\gamma_{k)}\psi_{:j} - \overline{\psi}_{:j}\delta^{j}_{(i}\gamma_{k)}\psi\right) + \frac{3}{4}\kappa s^{l} s_{l} g_{ik}. \quad (4)$$

Here, the semicolon covariant derivatives of the spinor field in (3) are decoupled into colon covariant derivatives in (4) and the contortion tensor containing parts of the decoupled covariant derivatives are suppressed in the spin pseudovector s^l by the contribution of U_{ik}. In order to rewrite (4) in a more convenient form for our further calculations, we multiply (1) by adjoint spinor $\overline{\psi}$ from the left and multiply (2) by spinor ψ from right, such that

$$i\overline{\psi}\gamma^{k}\psi_{:k} - V'\overline{\psi}\psi + \frac{3}{8}\kappa\left(\overline{\psi}\gamma^{k}\gamma^{5}\psi\right)\left(\overline{\psi}\gamma_{k}\gamma^{5}\psi\right) = 0, \quad (5)$$

$$i\overline{\psi}_{:k}\gamma^{k}\psi + V'\overline{\psi}\psi - \frac{3}{8}\kappa\left(\overline{\psi}\gamma^{k}\gamma^{5}\psi\right)\left(\overline{\psi}\gamma_{k}\gamma^{5}\psi\right) = 0, \quad (6)$$

where $V' = \partial V/\partial(\overline{\psi}\psi)$ for which $\overline{\psi}(\partial V/\partial\overline{\psi}) = (\partial V/\partial\psi)\psi = V'\overline{\psi}\psi$. By using (5) and writing the symmetrizations explicitly in (4), we obtain the total energy-momentum tensor Θ_{ik} of the spinor field dark energy in the form of

$$\begin{aligned}\Theta_{ik} = {} & \frac{i}{4}\left(\overline{\psi}\gamma_{i}\psi_{:k} + \overline{\psi}\gamma_{k}\psi_{:i} - \overline{\psi}_{:i}\gamma_{k}\psi - \overline{\psi}_{:k}\gamma_{i}\psi\right) \\ & + \frac{1}{2}\left(V'\overline{\psi}\psi - i\overline{\psi}\gamma^{l}\psi_{:l}\right) g_{ik}.\end{aligned} \quad (7)$$

We consider the spinor quintom DE model in FRW spacetime whose metric is given as

$$ds^2 = dt^2 - a^2(t)\left[dx^2 + dy^2 + dz^2\right], \quad (8)$$

and the corresponding tetrad components read

$$\begin{aligned} e^{i}_{0} &= \delta^{i}_{0}, \\ e^{i}_{a} &= \frac{1}{a(t)}\delta^{i}_{a}. \end{aligned} \quad (9)$$

Therefore, by performing the Riemannian covariant derivatives explicitly in (7), the time-like components Θ^{ψ}_{00} and the space-like components $\Theta^{\psi}_{\iota\iota}$ of the space independent spinor field dark energy energy-momentum tensor can be obtained, such as

$$\Theta_{00} = -\frac{i}{2}\dot{\overline{\psi}}\gamma_0\psi + \frac{1}{2}V'\overline{\psi}\psi - \frac{3i}{8}H\overline{\psi}\gamma_0\psi, \quad (10)$$

$$\Theta_{\iota\iota} = -\frac{i}{2}\overline{\psi}\gamma_0\dot{\psi}g_{\iota\iota} + \frac{1}{2}V'\overline{\psi}\psi g_{\iota\iota} - \frac{3i}{8}H\overline{\psi}\gamma_0\psi g_{\iota\iota}. \quad (11)$$

Here $H = \dot{a}(t)/a(t)$ is the Hubble parameter and it comes from the Levi-Civita connections in the Riemannian covariant derivatives. We now write ECSK Dirac equations (4) and (5) for a space independent spinor field as

$$\begin{aligned} & i\overline{\psi}\gamma^{0}\dot{\psi} + \frac{3i}{4}H\overline{\psi}\gamma^{0}\psi - V'\overline{\psi}\psi \\ & \quad + \frac{3}{8}\kappa\left(\overline{\psi}\gamma^{0}\gamma^{5}\psi\right)\left(\overline{\psi}\gamma_{0}\gamma^{5}\psi\right) = 0, \end{aligned} \quad (12)$$

$$\begin{aligned} & i\dot{\overline{\psi}}\gamma^{0}\psi + \frac{3i}{4}H\overline{\psi}\gamma^{0}\psi + V'\overline{\psi}\psi \\ & \quad - \frac{3}{8}\kappa\left(\overline{\psi}\gamma^{0}\gamma^{5}\psi\right)\left(\overline{\psi}\gamma_{0}\gamma^{5}\psi\right) = 0. \end{aligned} \quad (13)$$

The solution of (12) and (13) by adding them leads to

$$\overline{\psi}\dot{\psi} + \dot{\overline{\psi}}\psi + \frac{3}{2}H\overline{\psi}\psi = 0, \quad (14)$$

$$\overline{\psi}\psi = \frac{N}{a^{3/2}}. \quad (15)$$

Here N is the integration constant, and then, by using the scale factor $a \propto e^{\beta t}$ for a cosmological fluid [64], we can also obtain $\overline{\psi}\psi = Ne^{-3\beta t/2}$. Using (13) in (10) leads to the energy density

$$\rho = \Theta^{0}_{0} = V'\overline{\psi}\psi - \frac{3}{16}\kappa\left(\overline{\psi}\gamma^{0}\gamma^{5}\psi\right)\left(\overline{\psi}\gamma_{0}\gamma^{5}\psi\right), \quad (16)$$

and similarly using (12) in (11) leads to the pressure of the spinor field dark energy

$$p = -\Theta^{\iota}_{\iota} = -\frac{3}{16}\kappa\left(\overline{\psi}\gamma^{0}\gamma^{5}\psi\right)\left(\overline{\psi}\gamma_{0}\gamma^{5}\psi\right), \quad (17)$$

respectively. Then the EoS of the spinor field is given as

$$\omega = \frac{p}{\rho} = \frac{(3/16)\kappa\left(\overline{\psi}\gamma^{0}\gamma^{5}\psi\right)^2}{(3/16)\kappa\left(\overline{\psi}\gamma^{0}\gamma^{5}\psi\right)^2 - V'\overline{\psi}\psi}, \quad (18)$$

where $\gamma^0 = \gamma_0$ for a FRW metric. We rewrite the EoS in the form of

$$\omega = -1 + \alpha, \quad (19)$$

where

$$\alpha = \frac{6\kappa\left(\overline{\psi}\gamma^{0}\gamma^{5}\psi\right)^2 - 16V'\overline{\psi}\psi}{3\kappa\left(\overline{\psi}\gamma^{0}\gamma^{5}\psi\right)^2 - 16V'\overline{\psi}\psi}. \quad (20)$$

It is known that for $\alpha = 4/3$ the EoS of the spinor field is $\omega = 1/3$ and it behaves like radiation, but for $\alpha = 1$, $\omega = 0$, and it is normal matter. On the other hand, if $\alpha < 2/3$, the EoS $\omega < -1/3$ meaning that the spinor field behaves like a DE leading to the acceleration of universe. The $\alpha < 2/3$ region allows us to investigate the dynamical evolution of the spinor quintom DE described in ECSK formalism with intrinsic spin.

3. Dynamical Evolution of Spinor Quintom

From (20) we deduce that the spinor field can have an EoS of $-1 < \omega < -1/3$ for $0 < \alpha < 2/3$ and shows a quintessence-like behavior, but it has $\omega = -1$ cosmological constant value if $\alpha = 0$, and then it behaves like a phantom for $\omega < -1$ if $\alpha < 0$. Therefore, the spinor field exhibits a quintom picture by crossing the cosmological constant boundary $\omega = -1$ from above or below this boundary depending on the sign of α in (20).

There exist three categories of spinor quintom evolution depending on the behavior of the potential V. The quintom scenario may exhibit an evolution starting from $-1 < \omega$ quintessence phase to $\omega < -1$ phantom phase, called Quintom-A. Another scenario may evolve from $\omega < -1$ to $-1 < \omega$, Quintom-B scenario. The last quintom scenario contains the evolution for crossing $\omega = -1$ more than one time, and it is called Quintom-C model.

Considering the quintom scenario in which the spinor field comes from $-1 < \omega$ quintessence phase to $\omega < -1$ phantom phase, we first need to find the condition $0 < \alpha$. Since the energy density (16) must be positive definite, V' is positive. Therefore, the condition of occurring $-1 < \omega$ phase reads from (20) as

$$16V'\overline{\psi}\psi > 6\kappa\left(\overline{\psi}\gamma^0\gamma^5\psi\right)^2. \tag{21}$$

Similarly, $\omega = -1$ boundary occurs for

$$16V'\overline{\psi}\psi = 6\kappa\left(\overline{\psi}\gamma^0\gamma^5\psi\right)^2, \tag{22}$$

and $\omega < -1$ phantom phase occurs for

$$16V'\overline{\psi}\psi < 6\kappa\left(\overline{\psi}\gamma^0\gamma^5\psi\right)^2. \tag{23}$$

Since prime denotes the derivative with respect to $\overline{\psi}\psi$, the solution of (22) is found as $V_\Lambda = (6\kappa/16)(\overline{\psi}\gamma^0\gamma^5\psi)^2 \ln\overline{\psi}\psi$, in which the dynamical evolution of potential goes to the cosmological constant boundary.

In order to obtain Quintom-A scenario, we define the potential to be

$$V = \left(\frac{6\kappa}{16}\right)\left(\overline{\psi}\gamma^0\gamma^5\psi\right)^2 \ln\overline{\psi}\psi - \left(c - \overline{\psi}\psi\right)\overline{\psi}\psi, \tag{24}$$

for the early times of the universe. Then the potential leads the EoS from (20) as

$$\omega = -1 + \frac{16\left(2\overline{\psi}\psi - c\right)\overline{\psi}\psi}{16\left(2\overline{\psi}\psi - c\right)\overline{\psi}\psi + (3\kappa/16)\left(\overline{\psi}\gamma^0\gamma^5\psi\right)^2}, \tag{25}$$

and the term $(2\overline{\psi}\psi - c)$ in this potential satisfies $-1 < \omega$ quintessence scenario (21) with (15), since the scaling factor a is very small at the beginning of the evolution of the universe. When $\overline{\psi}\psi$ becomes equal to $c/2$, this potential leads the spinor field to approach $\omega = -1$ boundary (22). After that scaling factor evolves to a greater value, then $\overline{\psi}\psi$ reaches a value smaller than $c/2$. This gives the condition (23) phantom phase $\omega < -1$. We illustrate this behavior in Figure 1

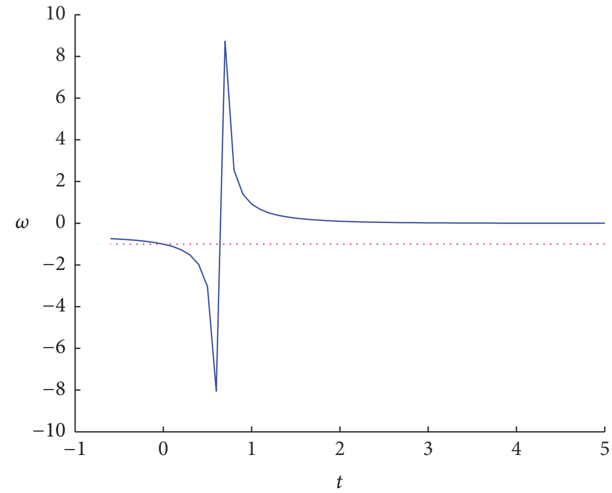

Figure 1: Evolution of ω with potential (24) as a function of time. For the numerical analysis we assume $N = 3$, $c = 6$, and $\beta = 1$. From [51].

by numerical analysis. According to the figure, ω starts its evolution from above -1 to below -1. We set the crossing cosmological constant boundary as at $t = 0$. After crossing the -1 boundary, spinor quintom picks up and is avoided from a Big Rip singularity and then enters a stable matter dominated expansion with $\omega = 0$ value.

We can also find other important cosmological quantities, such as luminosity distance, Hubble parameter, deceleration parameter, and jerk and state finder parameters. For this we use the Friedmann equations

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right), \tag{26}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho. \tag{27}$$

By using (16) for the potential (24), we obtain spinor energy density as

$$\rho = \left(\frac{3\kappa}{16}\right)\left(\overline{\psi}\gamma^0\gamma^5\psi\right)^2 + \left(2\overline{\psi}\psi - c\right)\overline{\psi}\psi, \tag{28}$$

and substituting (15) and (28) in (27) we obtain

$$\begin{aligned} H &= \frac{\dot{a}}{a} \\ &= \sqrt{\frac{8\pi G}{3}}\sqrt{\left(-\frac{3\kappa N^2}{8} + 2N^2\right)e^{-3\beta t} - cNe^{-3\beta t/2}}. \end{aligned} \tag{29}$$

Solving the differential equation in (29) gives the scale factor as

$$\begin{aligned} a = \exp\Bigg\{\sqrt{\frac{8\pi G}{3}}\Bigg[C_1 + t\sqrt{2N^2 - \frac{3\kappa N^2}{8} - cN} \\ + t^2\frac{3\beta\left(4N^2 - 3\kappa N^2/4 - cN\right)}{\sqrt{2N^2 - 3\kappa N^2/8 - cN}} + \cdots\Bigg]\Bigg\}, \end{aligned} \tag{30}$$

from which we obtain the redshift

$$z = -1 + \exp\left\{-\sqrt{\frac{8\pi G}{3}}\left[C_1 + t\sqrt{2N^2 - \frac{3\kappa N^2}{8} - cN} + t^2\frac{3\beta\left(4N^2 - 3\kappa N^2/4 - cN\right)}{\sqrt{2N^2 - 3\kappa N^2/8 - cN}} + \cdots\right]\right\}. \tag{31}$$

Therefore, we can find the Luminosity distance

$$d_L = H_0^{-1}\left[z + \frac{1}{2}\left(1 - q_0\right)z^2 + \cdots\right], \tag{32}$$

in terms of the measurable quantities present time Hubble parameter H_0 and deceleration parameter q_0. Moreover, to find the deceleration parameter, we use (16) and (17) for the potential (24) and obtain the term in (26), such that

$$\rho + 3p = \left(-\frac{3\kappa}{8}\right)\left(\overline{\psi}\gamma^0\gamma^5\psi\right)^2 + \left(2\overline{\psi}\psi - c\right)\overline{\psi}\psi, \tag{33}$$

and inserting (15) and (33) in (26) we find

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\left(\frac{3\kappa N^2}{4} + 2N^2\right)e^{-3\beta t} - cNe^{-3\beta t/2}\right]. \tag{34}$$

Then, the deceleration parameter is obtained from (29) and (34), such as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -H^{-2}\frac{\ddot{a}}{a} = \frac{1}{2}\frac{\left(3\kappa N^2/4 + 2N^2\right)e^{-3\beta t} - cNe^{-3\beta t/2}}{\left(-3\kappa N^2/8 + 2N^2\right)e^{-3\beta t} - cNe^{-3\beta t/2}}. \tag{35}$$

Also, we finally obtain the state finder parameters for spinor Quintom-A by taking time derivative of (34), such that

$$\left(\frac{\ddot{a}}{a}\right)^{\bullet} = 12\pi\beta G\left[\left(\frac{3\kappa N^2}{4} + 2N^2\right)e^{-3\beta t} - \frac{cN}{2}e^{-3\beta t/2}\right]. \tag{36}$$

Then the state finder parameter is

$$r = \frac{\dddot{a}}{aH^3} = \left(\frac{\ddot{a}}{a}\right)^{\bullet}H^{-3} - q = \frac{9\beta}{\sqrt{32\pi G}}\frac{\left(3\kappa N^2/4 + 2N^2\right)e^{-3\beta t} - (cN/2)e^{-3\beta t/2}}{\left(\left(-3\kappa N^2/8 + 2N^2\right)e^{-3\beta t} - cNe^{-3\beta t/2}\right)^{3/2}} - \frac{1}{2}\frac{\left(3\kappa N^2/4 + 2N^2\right)e^{-3\beta t} - cNe^{-3\beta t/2}}{\left(-3\kappa N^2/8 + 2N^2\right)e^{-3\beta t} - cNe^{-3\beta t/2}}, \tag{37}$$

and one can obtain the second state finder parameter $s = 2(r-1)/3(2q-1)$.

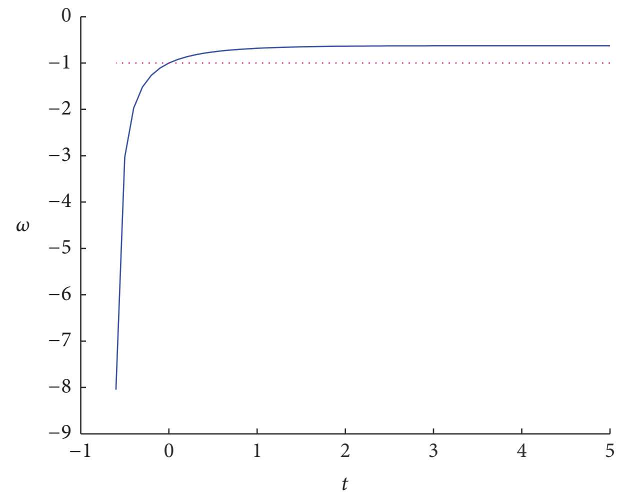

Figure 2: Evolution of ω with potential (38) as a function of time. For the numerical analysis we assume $N = 3$, $c = 3$, and $\beta = 1$. From [51].

For a Quintom-B model, the potential can be defined as

$$V = \left(\frac{6\kappa}{16}\right)\left(\overline{\psi}\gamma^0\gamma^5\psi\right)^2\ln\overline{\psi}\psi - \left(c - \overline{\psi}\psi\right)^2, \tag{38}$$

and then the EoS is obtained, such that

$$\omega = -1 + \frac{32\left(c - \overline{\psi}\psi\right)\overline{\psi}\psi}{32\left(c - \overline{\psi}\psi\right)\overline{\psi}\psi + (3\kappa/16)\left(\overline{\psi}\gamma^0\gamma^5\psi\right)^2}, \tag{39}$$

which lead to $\omega < -1$ phantom phase (23) at the beginning of the evolution of universe. With the increasing of the scale factor, $\overline{\psi}\psi$ decreases to c and the term $2(c - \overline{\psi}\psi)$ becomes zero. This gives $\omega = -1$ cosmological constant phase (22). As the evolution continues $\overline{\psi}\psi$ gets smaller than c and spinor quintom reaches a quintessence scenario $-1 < \omega$ in (21). The behavior of the spinor Quintom-B scenario is represented in Figure 2 which states that the spinor field starts the evolution from below $\omega = -1$ to above $\omega = -1$. Crossing from phantom to quintessence phase continues in this phase with an EoS value of $-1 < \omega < -1/3$ which imitates a stable de Sitter accelerated expansion for a scalar field dark energy model.

We proceed to find the other cosmological quantities; by using (16) for the potential (38), we get the quintom energy density

$$\rho = \left(\frac{3\kappa}{16}\right)\left(\overline{\psi}\gamma^0\gamma^5\psi\right)^2 + 2\left(c - \overline{\psi}\psi\right)\overline{\psi}\psi, \tag{40}$$

and inserting (15) and (40) in (27) we find

$$H = \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}}\sqrt{\left(-\frac{3\kappa N^2}{8} - 2N^2\right)e^{-3\beta t} + 2cNe^{-3\beta t/2}}. \tag{41}$$

By solving the differential equation in (41), we obtain the scale factor as

$$a = \exp\left\{\sqrt{\frac{8\pi G}{3}}\left[C_2 + t\sqrt{2cN - 2N^2 - \frac{3\kappa N^2}{8}} + t^2\frac{3\beta\left(2cN - 4N^2 - 3\kappa N^2/4\right)}{\sqrt{2cN - 2N^2 - 3\kappa N^2/8}} + \cdots\right]\right\}, \quad (42)$$

which gives the redshift as

$$z = -1 + \exp\left\{-\sqrt{\frac{8\pi G}{3}}\left[C_2 + t\sqrt{2cN - 2N^2 - 3\kappa N^2/8} + t^2\frac{3\beta\left(2cN - 4N^2 - 3\kappa N^2/4\right)}{\sqrt{2cN - 2N^2 - 3\kappa N^2/8}} + \cdots\right]\right\}. \quad (43)$$

Thus we obtain the Luminosity distance $d_L = H_0^{-1}[z + (1/2)(1-q_0)z^2 + \cdots]$ in terms of the measurable quantities H_0 and q_0. Then to find the deceleration parameter, we use (16) and (17) for the potential (24) and obtain the term in (26), such that

$$\rho + 3p = \left(-\frac{3\kappa}{8}\right)\left(\overline{\psi}\gamma^0\gamma^5\psi\right)^2 + 2\left(c - \overline{\psi}\psi\right)\overline{\psi}\psi, \quad (44)$$

and inserting (15) and (44) in (26) we obtain

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\left(\frac{3\kappa N^2}{4} - 2N^2\right)e^{-3\beta t} + 2cNe^{-3\beta t/2}\right]. \quad (45)$$

The deceleration parameter is obtained from (41) and (45) as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -H^{-2}\frac{\ddot{a}}{a} = \frac{1}{2}\frac{\left(3\kappa N^2/4 - 2N^2\right)e^{-3\beta t} + cNe^{-3\beta t/2}}{\left(-3\kappa N^2/8 - 2N^2\right)e^{-3\beta t} + cNe^{-3\beta t/2}}. \quad (46)$$

We can obtain the state finder parameters for spinor Quintom-B from the time derivative of (45) as

$$\left(\frac{\ddot{a}}{a}\right)^{\bullet} = 12\pi\beta G\left[\left(\frac{3\kappa N^2}{4} - 2N^2\right)e^{-3\beta t} + cNe^{-3\beta t/2}\right], \quad (47)$$

and the state finder parameter is

$$r = \frac{\dddot{a}}{aH^3} = \left(\frac{\ddot{a}}{a}\right)^{\bullet}H^{-3} - q = \frac{9\beta}{\sqrt{32\pi G}} \cdot \frac{\left(3\kappa N^2/4 - 2N^2\right)e^{-3\beta t} + cNe^{-3\beta t/2}}{\left(\left(-3\kappa N^2/8 - 2N^2\right)e^{-3\beta t} + 2cNe^{-3\beta t/2}\right)^{3/2}} - \frac{1}{2} \cdot \frac{\left(3\kappa N^2/4 - 2N^2\right)e^{-3\beta t} + cNe^{-3\beta t/2}}{\left(-3\kappa N^2/8 - 2N^2\right)e^{-3\beta t} + cNe^{-3\beta t/2}}, \quad (48)$$

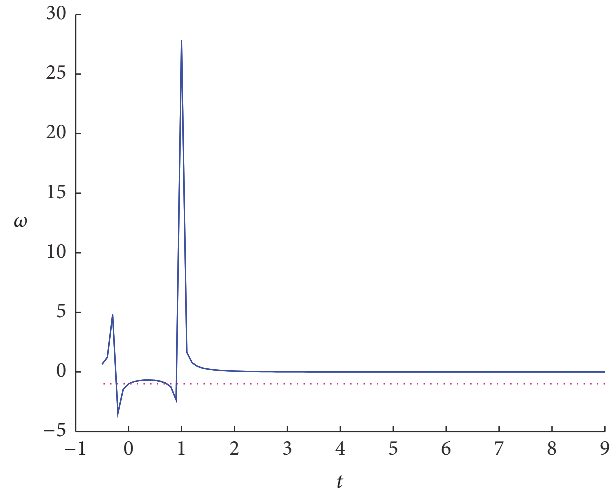

Figure 3: Evolution of ω with potential (49) as a function of time. For the numerical analysis we assume $N = 3$, $c = 3$, and $\beta = 1$. From [51].

from which we can obtain the second state finder parameter $s = 2(r-1)/3(2q-1)$.

Third case Quintom-C scenario can be obtained for the potential

$$V = \left(\frac{6\kappa}{16}\right)\left(\overline{\psi}\gamma^0\gamma^5\psi\right)^2 \ln\overline{\psi}\psi - \left(c - \overline{\psi}\psi\right)^2\overline{\psi}\psi, \quad (49)$$

which leads the EoS as

$$\omega = -1 + \frac{16\left(c - 3\overline{\psi}\psi\right)\left(\overline{\psi}\psi - c\right)\overline{\psi}\psi}{16\left(c - 3\overline{\psi}\psi\right)\left(\overline{\psi}\psi - c\right)\overline{\psi}\psi + (3\kappa/16)\left(\overline{\psi}\gamma^0\gamma^5\psi\right)^2}. \quad (50)$$

This potential provides two roots in $V'\overline{\psi}\psi$ for crossing the $\omega = -1$ boundary. The term coming from the derivative of V is $-3(\overline{\psi}\psi)^2 + 4c\overline{\psi}\psi - c^2$ which determines the sign of $16V'\overline{\psi}\psi$ in (21)–(23). During the evolution of universe with the increase in scale factor, $\overline{\psi}\psi$ decreases firstly to the value c which is the bigger root. This is a transition from phantom phase to quintessence phase by crossing -1 boundary. After continuing the evolution $\overline{\psi}\psi$ decreases to the second root $c/3$ which is recrossing the -1 boundary as a transition from quintessence phase to phantom phase again. This scenario is obviously a Quinton-C scenario and is illustrated in Figure 3. We see from the figure that the EoS of the quintom model crosses the $\omega = -1$ boundary twice, firstly from below $\omega = -1$ to above $\omega = -1$ and secondly from above to below $\omega = -1$, then it picks up and then is avoided from Big Rip singularities, and finally it asymptotically evolves to a stable matter dominated expansion epoch with a value of $\omega = 0$.

Although considering the phantom scenarios normally leads to the Big Rip singularities due to the unbound of EoS from below $\omega = -1$, our spinor quintom model with intrinsic spin in ECSK theory is avoided from the Big Rip singularities by picking up to a bound value and approaching a stable value, as seen in Figures 1 and 3. Diverging EoS of a dark fluid from a constant bound toward a lower singularity refers to continuous increase in the pressure of the fluid. This scenario is avoided in spinor quintom with intrinsic spin, which may be interpreted as the intrinsic spin of the fluid quanta leads to a bound pressure value. The increase of the pressure with the energy density is bounded due to the effect of intrinsic spin, then singular values of EoS are avoided, and the universe enters a stable expansion in the final era.

We now obtain other cosmological quantities; by using (16) for the potential (49), we get the Quintom-C energy density as

$$\rho=\left(\frac{3\kappa}{16}\right)\left(\overline{\psi}\gamma^0\gamma^5\psi\right)^2+\left(c-3\overline{\psi}\psi\right)\left(\overline{\psi}\psi-c\right)\overline{\psi}\psi, \tag{51}$$

and inserting (15) and (51) in (27) we find

$$\begin{aligned}H&=\frac{\dot{a}}{a}\\&=\sqrt{\frac{8\pi G}{3}}\sqrt{\left(-\frac{3\kappa N^2}{8}+4cN^2\right)e^{-3\beta t}-c^2Ne^{-3\beta t/2}-3N^3e^{-9\beta t/2}}.\end{aligned} \tag{52}$$

We now solve the differential equation in (52) to obtain the scale factor, such that

$$\begin{aligned}a=\exp\Bigg\{\sqrt{\frac{8\pi G}{3}}\Bigg[&C_{3+}t\sqrt{4cN^2-c^2N-3N^3-3\kappa N^2/8}\\&+t^2\frac{3\beta\left(8cN^2-c^2N+9N^3-3\kappa N^2/4\right)}{\sqrt{4cN^2-c^2N-3N^3-3\kappa N^2/8}}+\cdots\Bigg]\Bigg\},\end{aligned} \tag{53}$$

and this gives the redshift, such as

$$\begin{aligned}z=-1+\exp\Bigg\{-\sqrt{\frac{8\pi G}{3}}\Bigg[&C_3\\&+t\sqrt{4cN^2-c^2N-3N^3-3\kappa N^2/8}\\&+t^2\frac{3\beta\left(8cN^2-c^2N+9N^3-3\kappa N^2/4\right)}{\sqrt{4cN^2-c^2N-3N^3-3\kappa N^2/8}}+\cdots\Bigg]\Bigg\}.\end{aligned} \tag{54}$$

Therefore, we can find the luminosity distance as $d_L=H_0^{-1}[z+(1/2)(1-q_0)z^2+\cdots]$ in terms of the quantities H_0 and q_0. Here the deceleration parameter is again obtained by using (16) and (17) for the potential (49) and obtaining the term in (26), such that

$$\begin{aligned}\rho+3p=&\left(-\frac{3\kappa}{8}\right)\left(\overline{\psi}\gamma^0\gamma^5\psi\right)^2\\&+\left(c-3\overline{\psi}\psi\right)\left(\overline{\psi}\psi-c\right)\overline{\psi}\psi,\end{aligned} \tag{55}$$

and by substituting (15) and (55) in (26) we obtain

$$\begin{aligned}\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}\Bigg[&\left(-\frac{3\kappa N^2}{4}+4cN^2\right)e^{-3\beta t}-c^2Ne^{-3\beta t/2}\\&-3N^3e^{-9\beta t/2}\Bigg].\end{aligned} \tag{56}$$

The deceleration parameter is obtained from (52) and (56) as

$$\begin{aligned}q=-\frac{a\ddot{a}}{\dot{a}^2}=-H^{-2}\frac{\ddot{a}}{a}=\frac{1}{2}\\\cdot\frac{\left(-3\kappa N^2/4+4cN^2\right)e^{-3\beta t}-c^2Ne^{-3\beta t/2}-3N^3e^{-9\beta t/2}}{\left(-3\kappa N^2/8+4cN^2\right)e^{-3\beta t}-c^2Ne^{-3\beta t/2}-3N^3e^{-9\beta t/2}}.\end{aligned} \tag{57}$$

By using the time derivative of (56), we obtain the state finder parameters for Quintom-C model as

$$\begin{aligned}\left(\frac{\ddot{a}}{a}\right)^{\bullet}=12\pi\beta G\Bigg[&\left(-\frac{3\kappa N^2}{4}+4cN^2\right)e^{-3\beta t}\\&-\frac{c^2N}{2}e^{-3\beta t/2}+\frac{9N^3}{2}e^{-9\beta t/2}\Bigg],\end{aligned} \tag{58}$$

and the state finder parameter is

$$\begin{aligned}r=\frac{\dddot{a}}{aH^3}=&\frac{9\beta}{\sqrt{32\pi G}}\frac{\left(-3\kappa N^2/4+4cN^2\right)e^{-3\beta t}-\left(c^2N/2\right)e^{-3\beta t/2}+\left(9N^3/2\right)e^{-9\beta t/2}}{\left(\left(-3\kappa N^2/8+4cN^2\right)e^{-3\beta t}-c^2Ne^{-3\beta t/2}-3N^3e^{-9\beta t/2}\right)^{3/2}}\\&-\frac{1}{2}\frac{\left(-3\kappa N^2/4+4cN^2\right)e^{-3\beta t}-c^2Ne^{-3\beta t/2}-3N^3e^{-9\beta t/2}}{\left(-3\kappa N^2/8+4cN^2\right)e^{-3\beta t}-c^2Ne^{-3\beta t/2}-3N^3e^{-9\beta t/2}},\end{aligned} \tag{59}$$

from which we can obtain the second state finder parameter $s=2(r-1)/3(2q-1)$.

4. Conclusion

By using the spinor field dark energy in a FRW geometry, a consistent quintom model, in which EoS crosses -1 boundary without using a ghost field, has recently been obtained in the framework of general relativity [51]. Here, we consider the spinor field dark energy with intrinsic spin in the formalism of metric-affine ECSK theory. We first introduce the ECSK formalism and then define the model Lagrangian whose variations with respect to the tetrad field and torsion tensor give the total energy-momentum tensor consisting of metric and spin contributions. Also from the variation of Lagrangian with respect to the spinor field we obtain the ECSK Dirac equation. By using the total energy-momentum tensor and ECSK Dirac equation, the energy density and the pressure values of the spinor quintom DE are obtained, from which the EoS of the model is obtained for an arbitrary potential. The dependence of the potential on the spinor field leads to the evolution of potential with the change of scale factor, since the scale factor increases by time. Constructing the ECSK spinor potential suitably the quintom scenario is reached, for three different cases as Quintom-A, Quintom-B, and Quintom-C models. We also obtained the redshift values for three quintom scenarios from the scale factors of each quintom model. Then, we find other cosmological parameters, such as the Hubble parameter, deceleration parameter, and state finder parameters for three different potential values of each quintom scenario, respectively.

The Quintom-A case exhibits the transition of EoS from quintessence phase to phantom phase, evolving to a stable matter dominated expansion with $\omega \rightarrow 0$. This scenario is avoided from the Big Rip singularities due to the balancing of energy density and pressure of spinor DE by intrinsic spin. Similarly, in the Quintom-B scenario, the EoS of the model evolves from phantom region $\omega < -1$ to quintessence region $\omega > -1$ and approaches an EoS value of $-1 < \omega < -1/3$ referring to a stable de Sitter accelerated expansion for a scalar field dark energy model. On the other hand, the Quintom-C scenario exhibits the evolution of EoS which crossed the cosmological constant boundary $\omega = -1$ more than one time. The spinor Quintom-C firstly crosses the −1 boundary from phantom epoch, and then it again enters the phantom epoch from quintessence epoch. Then it converges to $\omega = 0$ stable matter dominated expansion phase by picking up from avoiding the singularities.

The proposed ECSK spinor quintom model differs from the spinor Quintom model in the framework of general relativity with the existence of matter dominated expansion phases in cases A and C. In both Quintom-A and Quintom-C cases, after the spinor field crosses the −1 boundary from a quintessence epoch toward the phantom epoch, it suddenly picks and enters the stable matter dominated expansion with $\omega = 0$. This can be interpreted as the intrinsic spin causes to fix the pressure of the fluid to a certain value as the energy density increases. After the spinor field reaches a very large energy density value, this allows neglecting the pressure relative to energy density value, which imitates a pressure-free matter dominated era with zero EoS.

Competing Interests

The author declares that he has no competing interests.

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