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Tree level Majorana neutrino mass from Type-1 × Type-2 Seesaw mechanism with Dark Matter

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ABSTRACT

We propose a type of hybrid Seesaw model that combines Type-1 and Type-2 Seesaw mechanism in multiplicative way to generate tree level Majorana neutrino mass and provides a Dark Matter candidate. The model extends the Standard Model by extra gauge symmetry $U(1)_D$ and hidden sector consisted of chiral fermions and additional scalar fields. After spontaneous symmetry breaking, light neutrino masses are generated not only by exchange of the new heavy fermions as Type-1 Seesaw, but also by coupling to the naturally small induced vacuum expectation value of new heavy scalar as Type-2 Seesaw. An unbroken residue of $U(1)_D$ protects the lightest Dirac fermion required by anomaly cancellation in hidden sector from decaying, therefore giving rise to a Dark Matter candidate. Due to strong enough Seesaw suppression from our hybridization, new physics scale can be as low as TeV in this model and discovering signal from LHC data is possible in near future.

1. Introduction

Neutrino oscillation indicates nonzero neutrino masses and mixings and that the Standard Model (SM) must be extended. Minimal approaches generating Majorana neutrino mass at tree level such as the Type-1 [1–4], Type-2 [5–8], and Type-3 [9] Seesaw models require new degrees of freedom (DOFs) that are too heavy to test in near future, although these models are well motivated by UV physics such as the Grand Unified Theory. Hybridization of Seesaw mechanisms can provide stronger suppression on neutrino mass therefore lowering the mass scale of new physics (NP), for example, by considering cancellation on neutrino masses between Type-1 and Type-2 contributions in "Type-1 + Type-2" Seesaw models [10–12], or by doubling the Seesaw suppression with an extended sterile fermion sector in Inverse Seesaw models [13].

Particularly, Seesaw suppression of different types can be combined multiplicatively in a class of models [14–19] where Type-3 Seesaw is generalized by replacing the fermion triplets and Higgs doublet in Yukawa coupling with higher dimensional representations. The resulting neutrino masses are then suppressed not only by the intermediate fermions as in usual Type-3 models, but also by a naturally small induced vacuum expectation value (VEV) of the scalar multiplet as in Type-2 models. We may refer this class of models "Type-3 × Type-2" models. Beside technical success, UV origin of such hybridization will be curious to ask.

Another evidence of NP is the existence of Dark Matter (DM) and it is intriguing to understand both neutrino mass and DM with a common theory. For example, in Type-1 Seesaw models, DM can be given by introducing a keV-scale sterile neutrino [20], but it is not apparent the theoretical necessity of both neutrino mass and DM to each other. In contrast, DM can be an inevitable consequence of neutrino mass due to specific representation of fields under the known gauge symmetry. For example, in Minimal Dark Matter model [21], neutral component of exotic multiplets introduced for neutrino mass generation can be accidentally stable thus can account for DM in the Universe. Moreover, symmetry can be the common reason behind stability of DM and tininess of neutrino mass. In some radiative models [22,23], both DM decay and tree level neutrino mass are forbidden by a new global symmetry, rendering loop correction from dark particles the leading source of neutrino mass. These approaches establish theoretical connection between both phenomena, however, they cannot provide further information about dark sector such as the number and couplings of the dark species.

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Table 1
Particle content of the model. L_i and Φ the SM lepton doublet and Higgs scalar doublet respectively.

	$L_{1,2,3}$	$\xi_{(1)}$	$\xi_{(2)_{1,2}}$	$\xi_{(-3)_{1,2,3}}$	$\xi_{(4)}$	Φ	η	S_3	S_6
$SU(2)_L$	**2**	**1**	**1**	**1**	**1**	**2**	**2**	**1**	**1**
$U(1)_Y$	-1/2	0	0	0	0	1/2	1/2	0	0
$U(1)_D$	0	1	2	-3	4	0	-3	3	6

In this work, we study a TeV-scale SM extension to address the origins of neutrino mass and DM by a hybrid "Type-1 × Type-2" Seesaw mechanism and an inevitable DM candidate, respectively, all originated from a set of new DOFs of which both number and representations are well motivated by symmetry. Specifically, the SM is extended with a hidden sector consisted of a new gauge symmetry $U(1)_D$, a set of SM-singlet chiral fermions, and additional scalar fields for symmetry breaking and communication to the SM sector. Because of the chirality of fermions, anomaly cancellation must come into play. The $U(1)_D$ charge assignment $z_1, z_2, ..., z_N$ of the N chiral fermions must satisfy equations $\sum_i z_i = 0$ and $\sum_i z_i^3 = 0$, for cancellation of mixed gauge-gravitational anomaly [24–26] and triangular gauge anomaly [27–29], respectively. As a consequence, both number and couplings of hidden fermions are well determined. Such approach of introducing a gauged chiral sector as the source of NP has advantage on predictive power and its analog to the SM [30–32]. In general, infinite number of solutions can be found satisfying the equations above. Some of them have been reported in literature [32–35]. In this work, the charge assignment of hidden fermions is given by following anomaly-free solution reported previously [32][1]:

$$1, 2, 2, -3, -3, -3, 4. \tag{1}$$

After the SSB of $U(1)_D$ at TeV scale by the VEVs of two scalar singlets, the chiral fermions merge into two sets of mass eigenstates. One set consisted of three Majorana fermions will be responsible for neutrino mass generation, another set consisted of two Dirac fermions acquires a residual symmetry thus by which the lightest Dirac fermion is stable and becomes the DM candidate. Subsequently, when the Higgs doublet VEV is developed and the SM gauge symmetry is broken, the messenger scalar doublet that communicates between hidden sector and SM sector will acquire a naturally small VEV induced by its trilinear coupling with other scalars. Consequently, light neutrino masses are generated by exchange of the TeV-scale Majorana fermions and Yukawa couplings proportional to the small induced VEV. We refer such model the "Type-1 × Type-2" Seesaw model. Because of the stronger suppression effect, the induced VEV can be as large as $\mathcal{O}(10\ \text{GeV})$, with all constraints satisfied. It gives rise to interesting signature in colliders observable in the 3000 fb^{-1} 14 TeV LHC data.

We discuss the basic structure and features of this model in sec. 2, followed by explanation on neutrino mass generation in sec. 3, and other phenomenologies including DM and LHC signature in sec. 4. We conclude in sec. 5.

2. Model

The SM is extended by a new gauge symmetry $U(1)_D$ and a hidden sector that contains a set of chiral fermions $\xi_{(z_i)}$ singlet under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ while carrying respective $U(1)_D$ charge z_i given by eq. (1). Without lost of generality, all chiral fermions are assumed right-handed. The model also considers an extended scalar sector. Beside the SM-like Higgs doublet Φ, we also introduce a messenger scalar doublet η that connects the hidden sector and the SM lepton sector, and two scalar singlets S_3 and S_6 responsible for spontaneous breaking of $U(1)_D$ and generates masses to all fermionic DOFs.[2] The scalar fields are parametrized by

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{v+\phi_R^0+i\phi_I^0}{\sqrt{2}} \end{pmatrix}, \quad \eta = \begin{pmatrix} \eta^+ \\ \frac{w+\eta_R^0+i\eta_I^0}{\sqrt{2}} \end{pmatrix}, \quad S_3 = \frac{u_3+s_{3R}+is_{3I}}{\sqrt{2}}, \quad S_6 = \frac{u_6+s_{6R}+is_{6I}}{\sqrt{2}} \tag{2}$$

The particle content is shown in Table 1.

The mass spectrum of the fermions is determined by following Yukawa sector

$$-\mathcal{L} \supset g_{ij}\overline{\xi^c_{(-3)_i}}\xi_{(-3)_j}S_6 + h_l\overline{\xi^c_{(2)_l}}\xi_{(1)}S_3^* + k_l\overline{\xi^c_{(2)_l}}\xi_{(4)}S_6^* + h.c., \tag{3}$$

where $i, j = 1, 2, 3$ and $l = 1, 2$. The coupling matrix g_{ij} is symmetric and can be defined real and diagonal $\text{diag}(g_i)$. At low energy, the nonzero vacuum expectation values (VEVs) $\langle S_3\rangle = u_3/\sqrt{2}$ and $\langle S_6\rangle = u_6/\sqrt{2}$ lead to three Majorana fermion mass eigenstates $N_i = \xi_{(-3)_i} + \xi^c_{(-3)_i}$ with masses $m_{N_i} = \sqrt{2}g_i u_6$. Moreover, the chiral fermions $\xi_{(1)}$, $\xi_{(2)_{1,2}}$, and $\xi_{(4)}$ are mixed via

$$-\mathcal{L} \supset \frac{1}{\sqrt{2}}\begin{pmatrix} \overline{\xi^c_{(2)_1}} & \overline{\xi^c_{(2)_2}} \end{pmatrix}\begin{pmatrix} h_1u_3 & k_1u_6 \\ h_2u_3 & k_2u_6 \end{pmatrix}\begin{pmatrix} \xi_{(1)} \\ \xi_{(4)} \end{pmatrix} + h.c. \tag{4}$$

to generate two Dirac fermions Ψ_1 and Ψ_2. The arbitrary complex 2-by-2 mass matrix in eq. (4) can be diagonalized with a bi-unitary transformation consisted of unitary matrices U_L and U_R, giving rise $(\Psi_1, \Psi_2)^T = U_L^\dagger(\xi^c_{(2)_1}, \xi^c_{(2)_2})^T + U_R^\dagger(\xi_{(1)}, \xi_{(4)})^T$.

These Dirac fermions experience flavor off-diagonal interaction stemmed from two sources. One from Yukawa interactions with the CP-even scalar bosons s_{3R} and s_{6R}, and CP-odd scalar bosons s_{3I} and s_{6I} in

$$-\sqrt{2}\mathcal{L} \supset \begin{pmatrix} \overline{\Psi_{1L}} & \overline{\Psi_{2L}} \end{pmatrix}\left[U_L^\dagger\begin{pmatrix} h_1 & 0 \\ h_2 & 0 \end{pmatrix}U_R(s_{3R}+is_{3I}) + U_L^\dagger\begin{pmatrix} 0 & k_1 \\ 0 & k_2 \end{pmatrix}U_R(s_{6R}+is_{6I})\right]\begin{pmatrix} \Psi_{1R} \\ \Psi_{2R} \end{pmatrix} + h.c. \tag{5}$$

[1] Also appears in [30].
[2] Instead, introducing two scalar singlets S_1 and S_2 carrying one and two units of $U(1)_D$ charge respectively can play the same role on symmetry breaking and mass generation, while it leads to only one Majorana fermion and three Dirac fermions after symmetry breaking.

Another source of flavor off-diagonal interaction is from gauge interaction between the right-handed component of Ψ_i and the new gauge boson X_μ:

$$\mathcal{L} \supset g_D X_\mu \left(\overline{\Psi_{1R}} \quad \overline{\Psi_{2R}}\right) \gamma^\mu \left[U_R^\dagger \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} U_R\right] \begin{pmatrix} \Psi_{1R} \\ \Psi_{2R} \end{pmatrix} \tag{6}$$

where g_D and X_μ are the coupling and gauge field corresponding to $U(1)_D$, respectively. In general, the Yukawa coupling matrices in eq. (5) and the charge matrix $U_R^\dagger \text{diag}(1,4) U_R$ in eq. (6) are not diagonal. Therefore, the heavier Dirac fermion Ψ_2 can decay into the lighter one Ψ_1 plus an on/off-shell scalar s_{3R}, s_{3I}, s_{6R}, s_{6I}, or vector boson X^μ.

The most general renormalizable scalar potential is given by[3]

$$\begin{aligned} \mathcal{V} = & -\mu_1^2|\Phi|^2 + \mu_2^2|\eta|^2 - \mu_3^2|S_3|^2 - \mu_4^2|S_6|^2 \\ & + \lambda_1|\Phi|^4 + \lambda_2|\eta|^4 + \lambda_3|S_3|^4 + \lambda_4|S_6|^4 \\ & + \lambda_{12}|\Phi|^2|\eta|^2 + \lambda'_{12}|\Phi^\dagger\eta|^2 + \lambda_{13}|\Phi|^2|S_3|^2 + \lambda_{14}|\Phi|^2|S_6|^2 \\ & + \lambda_{23}|\eta|^2|S_3|^2 + \lambda_{24}|\eta|^2|S_6|^2 + \lambda_{34}|S_3|^2|S_6|^2 \\ & + \kappa_1 \Phi^\dagger \eta\, S_3 + \kappa_2 S_3\, S_3\, S_6^* + \lambda \Phi^\dagger \eta\, S_3^*\, S_6 + h.c. \end{aligned} \tag{7}$$

The non-Hermitian operators in the last line break the global $[U(1)]^4$ symmetry in the scalar potential to $U(1)$, thus one may expect the parameters κ_1, κ_2, and λ are naturally small. Moreover, only two out of three complex phases of these parameters can be absorbed by the scalar fields, so one of them is complex in principle, providing possibility of explicit CP violation.

This scalar potential provides two interesting scenarios about the VEVs of the scalar fields. In the first scenario, only $-\mu_1^2$ and $-\mu_3^2$ among all four quadratic terms are negative, giving rise to nonzero v and u_3. However, the coupling $\mathcal{L} \supset \kappa_2 S_3\, S_3\, S_6^*$ induces the VEV $u_6 \sim \kappa_2 u_3^2/\mu_4^2$ that could be Type-2 "Seesaw-ed" if κ_2 is regarded a small parameter. Furthermore, together with v, coupling $\mathcal{L} \supset \kappa_1 \Phi^\dagger \eta\, S_3$ induces the fourth VEV w that is also "Seesaw-ed" as $w \sim \kappa_1 v u_3/\mu_2^2$. This scenario gives rise to hierarchy $u_3 \gg u_6$ and $u_3, v \gg w$. An apparent consequence is that the first column in mass matrix of Ψ_i's in eq. (4) will be significantly larger than the second column, resulting in large mass splitting between Ψ_1 and Ψ_2. More explicitly, one can have $m_{\Psi_2} \gg m_{\Psi_1}, m_{N_i}$.

Another scenario assumes all four quadratic couplings are negative except μ_2^2, giving rise to nonzero v, u_3, and u_6. The operator $\mathcal{L} \supset \kappa_1 \Phi^\dagger \eta\, S_3 + \lambda \Phi^\dagger \eta\, S_3^*\, S_6$ then induces the Seesaw VEV $w \sim (\kappa_1 v u_3 + \lambda v u_3 u_6)/\mu_2^2$. If we assume $u_3 \sim u_6$, we should have all m_{Ψ_1}, m_{Ψ_2}, and m_{N_i} at the similar mass scale. In this work, we adopt the second scenario assuming both u_3 and u_6 are generated by the negative quadratic terms and are at TeV scale. The only hierarchically small VEV is w, and it will play important role in neutrino mass generation.

The more complete expression of the VEVs can be calculated by the first-order derivative to the scalar potential. To simplify the result, we can assume $(\kappa_1, \kappa_2, \lambda u_3, \lambda u_6) \ll (u_3, u_6)$, therefore the leading order (expanded in respect to v) VEVs are

$$u_3 \simeq \sqrt{\frac{4\lambda_4\mu_3^2 - 2\lambda_{34}\mu_4^2}{4\lambda_3\lambda_4 - \lambda_{34}^2}} \tag{8a}$$

$$u_6 \simeq \sqrt{\frac{4\lambda_3\mu_4^2 - 2\lambda_{34}\mu_3^2}{4\lambda_3\lambda_4 - \lambda_{34}^2}} \tag{8b}$$

$$v \simeq \sqrt{\frac{2\mu_1^2 - \lambda_{13}u_3^2 - \lambda_{14}u_6^2}{2\lambda_1}} \tag{8c}$$

$$w \simeq \frac{-(\sqrt{2}\kappa_1 + \lambda u_6) v u_3}{2\mu_2^2 + (\lambda_{12} + \lambda'_{12})v^2 + \lambda_{23}u_3^2 + \lambda_{24}u_6^2} \tag{8d}$$

This model therefore suggests an almost complete breaking of the gauge $U(1)_D$ symmetry. The second scalar doublet η, scalar singlets S_3 and S_6, and the heavy Majorana fermions N_i are not charged under any residual symmetry thus all of them mix with SM DOFs of the same quantum numbers. The charged scalars $\phi^\pm$ and $\eta^\pm$ are mixed into the physical charged scalar $H^\pm$ and the charged Goldstone bosons $G^\pm$ being absorbed by $W^\pm$. The CP-odd neutral scalars ϕ_I^0, η_I^0, s_{3I} and s_{6I} are mixed into two neutral Goldstone bosons G_1 and G_2 being absorbed by the two neutral gauge bosons, leaving two physical pseudoscalar bosons A_1 and A_2. The CP-even neutral scalars ϕ_R^0, η_R^0, s_{3R} and s_{6R} are mixed into four physical scalar bosons H_i with $i = 1, 2, 3, 4$. The lightest state H_1 can be interpreted as the scalar boson of mass 125 GeV discovered at the LHC [37]. Finally, the three Majorana fermions N_i mix with the three active neutrinos, giving rise three more mass eigenstates, as discussed in next section. The only exception is the Dirac fermions $\Psi_{1,2}$. They acquire accidentally conversed global $U(1)$ quantum number, forming a dark sector where the lightest state (Ψ_1) is stable and will play the role of Weakly Interactive Mass Particle (WIMP) DM.

3. Neutrino mass

The messenger scalar doublet η connects the SM lepton doublet L and the chiral hidden fermions $\xi_{(3)_i}$ via Yukawa couplings:

$$\mathcal{L} = -y_{ij}\overline{L_i}\tilde{\eta}\xi_{(3)\,j} + h.c. \xrightarrow{SSB} -\frac{y_{ij}w}{\sqrt{2}}\overline{\nu_{iL}}N_j + h.c. \tag{9}$$

[3] A similar but not identical scalar potential has been studied in [36].

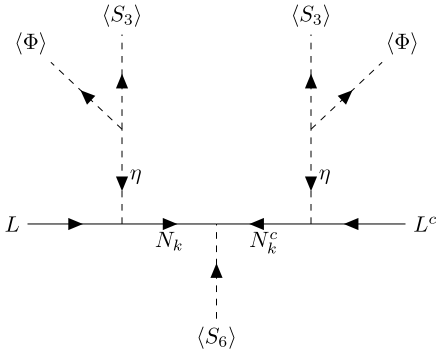

Fig. 1. Neutrino mass generated at tree level. Only contribution from $\Phi^\dagger \eta S_3$ operator is shown.

With the Majorana mass terms $\mathcal{L} \supset -(m_{N_i}/2)\overline{N_i}N_i$, the active neutrinos obtain the Type-1 Seesaw Majorana neutrino masses after SSB:

$$m^\nu_{ij} = \sum_k \frac{y^*_{ik} y^*_{jk} w^2}{m_{N_k}} \tag{10}$$

Recall that w is induced by operators $\kappa_1 \Phi^\dagger \eta S_3$ and $\lambda \Phi^\dagger \eta S_3^* S_6$ in scalar potential, the tree level neutrino mass generation can be given by Feynman diagram shown in Fig. 1. This process corresponds to a dim-8 operator $(1/\Lambda^4)\overline{L}L^c\Phi^\dagger\Phi S_3 S_3 S_6^*$. There is also contribution from two loop-level that can be constructed by connecting the two S_3 legs with the S_6 leg in Fig. 1 through operator $\kappa_2 S_3\, S_3\, S_6^*$. Although such diagram corresponds to dim-5 Weinberg operator, loop suppression, propagators, and κ_2 suggests it a subleading contribution.

It is worth to compare the new physics scale suggested by this model and that by the conventional Seesaw mechanisms. In Type-1 and Type-3 Seesaw, neutrino masses are determined by propagator of heavy right-handed neutrinos of mass scale M and Yukawa coupling proportional to the VEV v of Higgs doublet; in Type-2 Seesaw, neutrino masses are determined directly by Yukawa coupling proportional to the Seesaw-suppressed VEV $\mu_{\rm tri} v^2/M^2$ of a heavy scalar triplet of mass scale M, where $\mu_{\rm tri}$ denotes the trilinear coupling in scalar potential; in this model, neutrino masses are determined not only by propagator of the heavy right-handed neutrinos and Yukawa coupling proportional to VEV w, but also the Seesaw-suppression subjected by $w \sim (\kappa_1 + \lambda u_6) v u_3/\mu_2^2$. To make its analogy to Type-2 Seesaw models more apparent, we denote $w \sim \mu_{\rm tri} v/M$ in our comparison. If we assume all relevant Yukawa coupling about $\mathcal{O}(1)$, $v \sim \mathcal{O}(100\ {\rm GeV})$, and neutrino mass about $\mathcal{O}(0.01\ {\rm eV})$, the new physics scale M in each type of Seesaw mechanism can be estimated:

$$\text{Type-1/3:} \quad M \sim 10^{15}\ {\rm GeV} \tag{11a}$$

$$\text{Type-2:} \quad M \sim 10^{15}\ {\rm GeV}\ \frac{\mu_{\rm tri}}{M} \tag{11b}$$

$$\text{Type-1} \times \text{Type-2 (this model):} \quad M \sim 10^{15}\ {\rm GeV}\ \left(\frac{\mu_{\rm tri}}{M}\right)^2 \tag{11c}$$

Assuming a small trilinear coupling $\mu_{\rm tri} \sim \mathcal{O}(1\ {\rm MeV})$, we have $M \sim 10^6$ GeV for Type-2 Seesaw, while $M \sim 1$ TeV for Type-1 $\times$ Type-2 Seesaw (this model) due to the double suppression from twice appearance of the Type-2 Seesaw suppression (on w). This estimation shows that this model is more testable than the conventional Seesaw models in near future experiments.

Therefore, this model shares part of phenomenologies with the Type-1 Seesaw models, such as neutrino oscillation [38], non-unitarity in leptonic flavor mixings [39], and collider signatures [40], except the vanilla leptogenesis due to too light new physics scale [41,42].

4. Phenomenology

4.1. Rho parameter

In Two Higgs Doublet Model, the condensate of the second scalar doublet does not affect ρ parameter at tree level if it shares the same EW charges with the SM Higgs doublet. In this model, however, the present of the gauge $U(1)_D$ under which η is charged brings about mass mixing between the two massive neutral gauge vector bosons Z and Z', even though ignoring the kinetic mixing $\mathcal{L} \supset (-\epsilon/2) X^{\mu\nu} B_{\mu\nu}$ where $B_{\mu\nu}$ the field strength of hypercharge gauge field. The $\rho = 1 + \delta\rho = m_W/m_Z \cos\theta_{\rm weak}$ is then deviated from unity at tree level as

$$\delta\rho_{\rm tree} = \frac{(w/v)^4}{1+(w/v)^2}\left[\left(\frac{u_3 z_{S_3}}{v z_\eta}\right)^2 + \left(\frac{u_6 z_{S_6}}{v z_\eta}\right)^2 - \frac{g_1^2+g_2^2}{4 g_D^2 z_\eta^2}\right]^{-1} \tag{12}$$

where z_a is the $U(1)_D$ charge of particle $a = \eta, S_3, S_6$. The global fit of EW Precision Tests in PDG [37] gives rise $\rho_{\rm exp,\,fit} = 1.00038 \pm 0.00020$. Within 1σ uncertainty, w can take values as large as $\mathcal{O}(10\ {\rm GeV})$ if $v = 246$ GeV, $u_3 = u_6 = 1$ TeV, and $g_D = 0.2$.

4.2. LFV processes

The charged scalar bosons $H^\pm$ mediate Lepton Flavor Violating (LFV) decays of charged leptons, e.g., $\mu \to e\gamma$, at one loop-level via Yukawa coupling $y_{ij}\overline{\ell_i} N_j \eta^-$. Relevant calculation is the same as in the scotogenic models [23,43]. In the large w-regime where one may

consider $w = 10$ GeV, eq. (10) suggests a generic Yukawa coupling $y \sim \mathcal{O}(10^{-5})$, that leads to tiny $\mathrm{BR}(\mu^+ \to e^+\gamma) \sim 10^{-28}$ far away from the MEG bound [44]. LFV processes such as $\mu \to e\gamma$ in this model become relevant to near future experiments only in small w-regime where assuming $w = 10 - 100$ MeV, leading to $y \sim 10^{-1} - 10^{-2}$ thus $\mathrm{BR}(\mu^+ \to e^+\gamma) \sim 10^{-13} - 10^{-16}$.

4.3. DM

Dirac fermions Ψ_1 and Ψ_2 are charged under an unbroken residual $U(1)$ symmetry after SSB, therefore the lightest state (i.e., Ψ_1) is stable and plays the role of WIMP DM. DM is thermally produced in the early universe through its couplings with gauge boson X and scalar bosons S_3 and S_6 that further mix with the SM DOFs. However, these portals cannot play the dominant role on DM relic density because the direct DM search experiments have put stringent constraints. Following analysis in [45,46], the coupling constants for Higgs-portal operator $\overline{\Psi_1}\Psi_1 H_1$ and for Z-portal operator $\overline{\Psi_1}\gamma^\mu\Psi_1 X_\mu$ have upper limits about 10^{-3} and 10^{-5}, respectively, if DM mass is relatively light (100 GeV). In this model, the new scalar boson s_{3R} mixes with ϕ_R^0 through the 4-by-4 symmetric mass-squared matrix

$$\begin{pmatrix} \lambda_1 v^2 & \frac{1}{2}u_3(\sqrt{2}\kappa_1 + \lambda u_6) & \lambda_{13} v u_3 & \lambda_{14} v u_6 \\ \dots & \frac{-1}{2}\frac{v}{w}u_3(\sqrt{2}\kappa_1 + \lambda u_6) & \frac{1}{2}v(\sqrt{2}\kappa_1 + \lambda u_6) & \frac{1}{2}\lambda v u_3 \\ \dots & \dots & 2\lambda_3 u_3^2 & u_3(\sqrt{2}\kappa_2 + \lambda_{34} u_6) \\ \dots & \dots & \dots & 2\lambda_4 u_6^2 - \frac{1}{\sqrt{2}}\frac{\kappa_2}{u_6}u_3^2 \end{pmatrix} \tag{13}$$

in the basis of $(\phi_R^0, \eta_R^0, s_{3R}, s_{6R})$. The DM candidate Ψ_1 interacts with Higgs-like bosons H_1 through $\mathcal{L} \supset a\overline{\Psi_1}\Psi_1 s_{3R} \sim a\sin\theta\,\overline{\Psi_1}\Psi_1 H_1$, where a denotes the Yukawa coupling from the 1-1 component of the first non-diagonal coupling matrix shown in eq. (5) after the bi-unitary transformation U_L and U_R; and θ the mixing angle between ϕ_R^0 and s_{3R} that can be estimated from eq. (13) is about $\theta \sim (\lambda_{13} v)/(2\lambda_3 u_3)$, thus $a\sin\theta \lesssim 10^{-3}$ is expected given $v = 246$ GeV and $u_3 = 1$ TeV and appropriate values for a and the λ's. Similar result can be obtained in case for S_6. Moreover, the mixing angle between Z and Z' is given by:

$$\theta_Z = \frac{2\sqrt{g_1^2 + g_2^2}\, g_D z_\eta w^2}{4g_D^2(w^2 z_\eta^2 + u_3^2 z_{S_3}^2 + u_6^2 z_{S_6}^2) - (g_1^2 + g_2^2)(v^2 + w^2)} + \mathcal{O}(w^4) \tag{14}$$

and it can be estimated $\theta_Z \sim (w/u_3)^2 \sim 10^{-4}$ as $w \simeq 10$ GeV and $u_3 \simeq 1$ TeV, giving rise to $g_D\theta_Z \lesssim \mathcal{O}(10^{-5})$ that satisfies the relevant bound. In this sense, the large w-regime of this model can be made compatible with Higgs- and Z-portal constraints from direct DM searches if DM mass is 100 GeV. The constraints are even looser with heavier DM candidate.

DM relic abundance is then dominantly determined by its annihilation into the unstable new particles in this model. For example, $\Psi_1\overline{\Psi_1} \to N_i N_i$ mediated by s_{6R} in s-channel can give rise to thermal averaged cross-section

$$\langle\sigma v\rangle(\Psi_1\overline{\Psi_1} \to N_1 N_1) \sim \frac{\lambda_S^4 m_{\Psi_1}^2}{m_S^4} \tag{15}$$

estimated by dimensional analysis, where m_S the mass of the mediator scalar boson s_{6R} and λ_S the coupling of the scalar to the fermions. A large enough cross-section $\langle\sigma v\rangle \simeq 1$pb demanded by observational result $\Omega_{\mathrm{DM}}h^2 \simeq 0.1$ from Planck [47] through relation $\Omega_{\Psi_1}h^2 = 0.1\mathrm{pb}/\langle\sigma v\rangle$ will be satisfied if $\lambda_S \simeq 0.5$, $m_S = 1$ TeV, and DM mass $m_{\Psi_1} = 200$ GeV.

The same annihilation process $\Psi_1\overline{\Psi_1} \to N_i N_i$ also happens in the present-day universe in zero-momentum limit and provides possible signal to indirect DM searches. If considering RHN decays to SM species through only active-sterile mixing, DM mass of about 200 GeV satisfies constraints from gamma-ray [48,49], see also that including anti-proton data [50].

4.4. LHC signal

Given the merit of low energy scale for new physics, new particles introduced in this model are more hopeful to produce and detect in collider experiments. Especially, the crucial trilinear coupling κ_1 that helps to determine the neutrino mass scale can be directly probed. Because η does not carry any residual symmetry from the hidden sector, it can be singly produced in the LHC. For example, η_R^0 can be singly produced via Vector-Boson-Fusion(VBF)-like process in which a pair of gauge bosons, either $W^\pm$ or Z^0, fuse into η_R^0. Trilinear coupling $\kappa_1\Phi^\dagger\eta S_3$ then mediates the subsequent decay of $\eta_R^0 \to \phi_R^0 s_{3R}$. The s_{3R} is assumed heavy and decays dominantly into a pair of DM particle $\Psi_1\overline{\Psi_1}$, giving rise to large missing transverse energy ($\not{E}_T$, MET), while the $\phi_R^0 \simeq H_1$ is then highly boosted and decays into a pair of bottom quark and antiquark. The overall signature seen in detectors will be consisted of a boosted "Higgs-jet" identified by the constituent b and $\bar{b}$ jets, plus large MET, plus two quark jets. The process is depicted in Fig. 2.

The cross-section of such process can be roughly estimated. Assuming all intermediate states are produced on-shell and their widths are negligible (Narrow Width Approximation), in flavor basis, we have

$$\sigma(pp \to b\bar{b}q\bar{q}\not{E}_T) \simeq 0.051\ \mathrm{fb}\left(\frac{\sigma(pp \to q\bar{q}\eta_R^0)}{106\ \mathrm{fb}\left(\frac{10}{246}\right)^2}\right)\left(\frac{\mathrm{BR}(\eta_R^0 \to \phi_R^0 s_{3R})}{0.5}\right)\left(\frac{\mathrm{BR}(\phi_R^0 \to b\bar{b})}{0.58}\right)\left(\frac{\mathrm{BR}(s_{3R} \to \Psi_1\overline{\Psi}_1)}{1}\right) \tag{16}$$

The VBF-like production cross-section $\sigma(pp \to q\bar{q}\eta_R^0)$ can be estimated from the similar process of the SM Higgs boson [51], scaled by $(w/v)^2$. In the large w-regime where $w = 10$ GeV, we can have $\sigma(pp \to q\bar{q}\eta_R^0) \sim 0.18$ fb if $m_{\eta_R^0} = 1$ TeV is assumed. We assume the branching ratios $\mathrm{BR}(\eta_R^0 \to \phi_R^0 s_{3R}) \sim \mathcal{O}(0.1)$ and $\mathrm{BR}(s_{3R} \to \Psi_1\overline{\Psi}_1) \sim 1$ as they are controlled by model parameters. The branching ratio $\mathrm{BR}(\phi_R^0 \to b\bar{b}) \simeq \mathrm{BR}(H_1 \to b\bar{b})$ is known about 0.58. In high luminosity run of the LHC, this process could generate about 153 events if

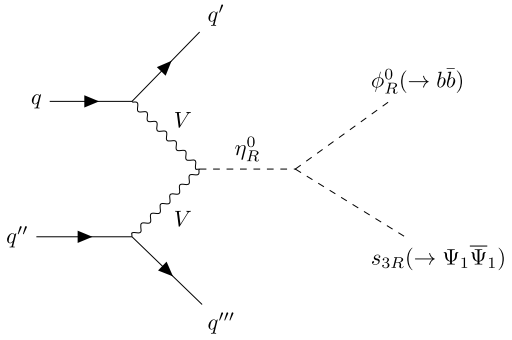

Fig. 2. Single production of the messenger scalar boson η_R^0 and its subsequent decay into boosted $b\bar{b}$ pair and large missing transverse energy, where V stands for either W or Z boson of the SM.

integrated luminosity is assumed 3000 fb^{-1}, and such level of event samples may be possible for a discovery in near future. This process is informative for model parameters. The invariant mass of the MET can be found just the mass of s_{3R}; the invariant mass of the $b\bar{b}$-pair, on the other hand, is just the mass of ϕ_R^0 which is assumed known as 125 GeV. The invariant mass of the $b\bar{b}$-pair and the MET together is just the mass of the η_R^0. Moreover, the total cross-section depends on κ_1, w, and BR($s_{3R} \to \Psi_1\overline{\Psi}_1$). Once discovery, these parameters can be further determined and/or constrained.

Similar but not identical event signature consisted of a boosted Higgs decaying into $b\bar{b}$ pair plus large MET has been searched for in the LHC and reported by ATLAS [52] and CMS [53]. In these searches, pair of DM particles are generated in final state associated with a boosted Higgs boson. Ref. [52] has given rise model-independent upper limit on cross-section between 0.05–3.26 fb, depending on MET. However, the process we have discussed contains two extra quark jets appear in final state, that is not considered in the experimental searches above, so the quoted limit is not directly applicable in this case.

5. Conclusion

To go beyond the minimal Seesaw models for additional features such as testability and Dark Matter while still being UV-motivated, we have extended the SM by a new gauge symmetry $U(1)_D$, a set of anomaly-free SM singlet chiral fermions, and an enlarged scalar sector. After SSB of $U(1)_D$ and EW symmetry, the new chiral fermions compose two sets of massive fermions that play important role in neutrino physics and cosmology. One set of fermions consisted of three Majorana fermions play role as the right-handed neutrinos and are responsible for generating tree level neutrino masses via the Type-1 Seesaw mechanism; another set of fermions consisted of two Dirac fermions carry a residual unbroken symmetry of the $U(1)_D$ and provides a DM candidate. In contrast to put-in-by-hand, both sets of fermions rely on existence of each other intrinsically as required by anomaly cancellation.

Unlike conventional Type-1 Seesaw models, the mass scale of the right-handed neutrinos can naturally be around TeV scale thanks to the additional suppression originated from Type-2-like Seesaw in scalar sector gives rise to a tiny induced VEV that lowers the Dirac mass terms between left- and right-handed neutrinos. Therefore, such hybrid Seesaw mechanism is referred to Type-1 $\times$ Type-2 Seesaw model since the suppressions from both sources are combined multiplicatively. The model suggests a lower mass scale of new physics, and allows a relatively large value of the induced VEV w. With $w \simeq 10$ GeV, constraints from rho parameter, LFV decays of charged leptons, DM relic density and direct/indirect searches are all satisfied, while an interesting collider signature is possible for discovery in the upcoming High-Luminosity run of the LHC at CERN. Decay of a singly produced scalar boson of η from VBF-like process in colliders can give rise to final state consisted of a highly boosted $b\bar{b}$ jet, large missing transverse energy, and two quark jets. Once measured, such process can reveal information including the couplings and masses of the relevant new particles. With a small w, instead, the interactions between SM sector and the new sector such as Z-Z' mixing and new particle production in colliders become more difficult, while LFV decays of charged leptons will be closer to experimental sensitivity in near future.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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