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Can the triple-parton scattering be observed in open charm meson production at the LHC?

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ABSTRACT

We investigate whether the triple-parton scattering effects can be observed in open charm production in proton–proton collisions at the LHC. We use so-called factorized Ansatz for calculations of hard multiple-parton interactions. The numerical results for each parton interaction are obtained within the k_T-factorization approach. Predictions for one, two and three $c\bar{c}$ pairs production are given for $\sqrt{s} = 7$ TeV and $\sqrt{s} = 13$ TeV. Quite large cross sections, of the order of milibarns, for the triple-parton scattering mechanism are obtained. We suggest a measurement of three D^0 or three $\bar{D}^0$ mesons by the LHCb Collaboration. Confronting our results with recent LHCb experimental data for single and double D^0 (or $\bar{D}^0$) meson production we present our predictions for triple meson final state: $D^0D^0D^0$ or $\bar{D}^0\bar{D}^0\bar{D}^0$. We present cross sections for the LHCb fiducial volume as well as distributions for D^0 meson transverse momentum and three-D^0 meson invariant mass. The predicted visible cross sections, including the detector acceptance, hadronization effects and $c \to D^0$ branching fraction, is of the order of a few nanobarns. The counting rates including $D^0 \to K^-\pi^+$ branching fractions are given for known or expected integrated luminosities.

1. Introduction

The multi-parton scattering effects got new impulse with the start of the LHC operation [1,2]. There are several ongoing studies of different processes. So far theoretical studies concentrated on double-parton scattering (DPS). Some time ago we have shown that charm production should be one of the best reaction to study double-parton scattering effects [3] (see also Ref. [4]). This was confirmed by the LHCb experimental data [5] and their subsequent interpretation [6–8].

Very recently also triple parton scattering (TPS) was discussed in the context of multiple production of $c\bar{c}$ pairs [9]. Inspiringly large cross sections were presented there.

We decided to follow this first analysis with application to triple D meson production. We try to stay in contact with experimental data for single-parton scattering (SPS) D^0 ($\bar{D}^0$) and DPS D^0D^0 ($\bar{D}^0\bar{D}^0$) production to minimize the theoretical uncertainties. Theoretical uncertainties of the SPS charm cross sections get amplified by about factor 2 and 3 for the DPS and TPS predictions, respectively. An alternative way is to absolutely normalize the theoretical predictions to the LHCb data. For this purpose we follow the k_t-factorization approach which was successfully used previously for SPS single D, SPS $D\bar{D}$ pair as well as for DPS double D meson production [8,10]. Within this approach a very good description of the LHC experimental data was obtained with the central value of the standard uncertainty bands, *i.e.* with the standard set of pQCD parameters (such as factorization/renormalization scales or charm quark mass).

Experimentally one measures rather D meson (or nonphotonic leptons). We wish to answer the question whether the triple-parton scattering could be seen in three D^0 or three $\bar{D}^0$ production. In order to answer the question one has to obtain cross section for meson production, taking into account $c \to D$ hadronization and acceptance of the existing detectors. Reliable predictions for TPS triple D meson production should check consistency of model predictions for SPS single and DPS double D meson production with already existing experimental data.

2. A sketch of the model calculations

The triple-parton scattering mechanism for $pp \to c\bar{c}c\bar{c}c\bar{c}X$ reaction is schematically illustrated in Fig. 1. The corresponding inclu-

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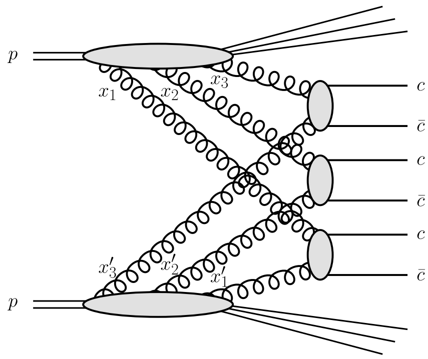

Fig. 1. A diagrammatic illustration of the triple-parton scattering mechanism for triple-$c\bar{c}$ production in proton–proton scattering. Only the dominant at high-energies gluon–gluon fusion partonic subprocesses are taken into account.

sive TPS cross section in a general form [11–13] can be written as follows:

$$\begin{aligned}
&\sigma^{TPS}_{pp\to c\bar{c}c\bar{c}c\bar{c}} \\
&= \left(\frac{1}{3!}\right)\int \Gamma^{ggg}_p(x_1,x_2,x_3;\vec{b}_1,\vec{b}_2,\vec{b}_3;\mu_1^2,\mu_2^2,\mu_3^2) \\
&\quad\times \hat{\sigma}^{gg}_{c\bar{c}}(x_1,x_1',\mu_1^2)\,\hat{\sigma}^{gg}_{c\bar{c}}(x_2,x_2',\mu_2^2)\,\hat{\sigma}^{gg}_{c\bar{c}}(x_3,x_3',\mu_3^2) \\
&\quad\times \Gamma^{ggg}_p(x_1',x_2',x_3';\vec{b}_1-\vec{b},\vec{b}_2-\vec{b},\vec{b}_3-\vec{b};\mu_1^2,\mu_2^2,\mu_3^2) \\
&\quad\times dx_1\,dx_2\,dx_3\,dx_1'\,dx_2'\,dx_3'\,d^2b_1\,d^2b_2\,d^2b_3\,d^2b,
\end{aligned} \tag{2.1}$$

where x_i, x_i' are the longitudinal momentum fractions, μ_i are the renormalization/factorization scales, $\hat{\sigma}^{gg}_{c\bar{c}}(x_i,x_i',\mu_i^2)$ are the partonic cross sections for $gg\to c\bar{c}$ mechanism and $\frac{1}{3!}$ is the combinatorial factor relevant for the case of the three identical final states. The above TPS hadronic cross section is expressed in terms of the so-called triple-gluon distribution functions $\Gamma^{ggg}_p(x_1,x_2,x_3;\vec{b}_1,\vec{b}_2,\vec{b}_3;\mu_1^2,\mu_2^2,\mu_3^2)$ that contain additional informations about positions $\vec{b}_i$ of the three corresponding partons in the transverse plane of the colliding protons.

The triple parton distribution functions (triple PDFs) shall account for all possible correlations between the partons, not only kinematical and spatial one, but also including spin and color/flavor correlations. The MPI theory in this general form is well established (see *e.g.* Refs. [14,15]) but not yet fully applicable for phenomenological studies. The double PDFs in the case of DPS are under intense theoretical studies but their adoption to real process calculations is still limited. On the other hand, the objects like triple PDFs for TPS were discussed so far only in Ref. [13].

Therefore, in practice one usually follows the so-called factorized Ansatz, where the correlations between partons are neglected and longitudinal and transverse degrees of freedom are separated. According to these approximations the triple-gluon PDFs from Eq. (2.1) take the following form:

$$\begin{aligned}
&\Gamma^{ggg}_p(x_1,x_2,x_3;\vec{b}_1,\vec{b}_2,\vec{b}_3;\mu_1^2,\mu_2^2,\mu_3^2) \\
&= D^{ggg}_p(x_1,x_2,x_3;\mu_1^2,\mu_2^2,\mu_3^2)F(\vec{b}_1)F(\vec{b}_2)F(\vec{b}_3),
\end{aligned} \tag{2.2}$$

where $D^{ggg}_p(x_1,x_2,x_3;\mu_1^2,\mu_2^2,\mu_3^2) = g(x_1;\mu_1^2)g(x_2;\mu_2^2)g(x_3;\mu_3^2)$ is the product of single gluon PDFs and $F(\vec{b}_i)$ describe the gluon distributions in transverse plane. The factors $F(\vec{b}_i)$ of the two triple-gluon PDFs in Eq. (2.2) are connected with the proton–proton overlap function via the following relation:

$$T(\vec{b}) = \int F(\vec{b}_i)F(\vec{b}_i-b)\,d^2b_i, \text{ with } \int d^2b\,T(\vec{b}) = 1, \tag{2.3}$$

and are usually assumed to be universal for all types of partons.

Taking all together, the formula for inclusive TPS cross section (Eq. (2.1)) can be simplified to the pocket form [9]:

$$\sigma^{TPS}_{pp\to c\bar{c}c\bar{c}c\bar{c}} = \left(\frac{1}{3!}\right)\frac{\sigma^{SPS}_{pp\to c\bar{c}}\cdot\sigma^{SPS}_{pp\to c\bar{c}}\cdot\sigma^{SPS}_{pp\to c\bar{c}}}{\sigma^2_{\text{eff,TPS}}}, \tag{2.4}$$

where the triple-parton scattering normalization factor $\sigma_{\text{eff,TPS}}$ contains only information about proton transverse profile and can be related to the overlap function from Eq. (2.3) via the following expression:

$$\sigma^2_{\text{eff,TPS}} = \left[\int d^2b\,T^3(\vec{b})\right]^{-1}. \tag{2.5}$$

Its pure geometrical interpretation comes from the practical approximations of the factorized Ansatz mentioned above. In principle, taking into account various parton correlations as well as multi-parton PDF sum rules [16] or including perturbative-parton-splitting contributions [17–19] may lead to a breaking of the pocket-formula. However, most of the violation sources are expected to vanish for processes driven by small-x partons (see *e.g.* Refs. [20,21]), that is exactly the case of charm production at high energies. As was shown by us, *e.g.* in Ref. [7], the factorized framework seems to be sufficient to explain the LHCb double charm data. Therefore, we think that it can be safely used, at least as a starting point, to draw practical conclusions also in the case of triple charm production.

In principle, the DPS normalization factor $\sigma_{\text{eff,DPS}}$ was extracted experimentally from several Tevatron and LHC measurements (see *e.g.* Refs. [1,2] and references therein) and its world average value is $\sigma_{\text{eff,DPS}} \simeq 15 \pm 5$ mb.[1] Such experimental inputs are not available for $\sigma_{\text{eff,TPS}}$ in studies of triple-parton scattering. However, as was shown in Ref. [9] for proton–proton collisions, the latter quantity can be expressed in terms of their more known DPS counterpart:

$$\sigma_{\text{eff,TPS}} = k \times \sigma_{\text{eff,DPS}}, \text{ with } k = 0.82 \pm 0.11\,. \tag{2.6}$$

The relation is valid for different (typical) parton transverse profiles of proton. In the numerical calculations below we take $\sigma_{\text{eff,DPS}} = 21$ mb which is a rather conservative choice but it corresponds to the average value extracted by the LHCb experiment only from the double charm data [5]. This input gives us the value of $\sigma_{\text{eff,TPS}} \simeq 17$ mb.

In this paper, each of the single-parton scattering cross sections $\sigma^{SPS}_{pp\to c\bar{c}}$ in Eq. (2.4) is calculated in the k_T-factorization approach [23] where part of (real) higher-order QCD corrections are effectively included. In this framework exact kinematics is kept from the very beginning and additional hard dynamics coming form transverse momenta of incident partons is taken into account. It was shown in Ref. [10] that within this approach one can get a good description of the LHC inclusive charm data, similar to the case of next-to-leading order (NLO) collinear calculations. Likewise, the successful theoretical analyses of double charm production from Refs. [6–8] were also based on the k_T-factorization.

In this approach the differential SPS cross section for inclusive single $c\bar{c}$ pair production can be written as:

[1] A detailed study of the $\sigma_{\text{eff,DPS}}$ can be found in Ref. [22].

$$\frac{d\sigma^{SPS}_{pp\to c\bar{c}}}{dy_1 dy_2 d^2p_{1,t} d^2p_{2,t}} = \frac{1}{16\pi^2 (x_1 x_2 S)^2} \int \frac{d^2k_{1t}}{\pi}\frac{d^2k_{2t}}{\pi} \overline{|\mathcal{M}_{g*g*\to c\bar{c}}|^2} \times \delta^2\left(\vec{k}_{1t}+\vec{k}_{2t}-\vec{p}_{1t}-\vec{p}_{2t}\right)\mathcal{F}_g(x_1,k^2_{1t},\mu^2)\mathcal{F}_g(x_2,k^2_{2t},\mu^2), \tag{2.7}$$

where the extra, compared to collinear factorization, integrals over transverse momenta k_{it} of initial state particles appear. Here, $\mathcal{M}_{g^*g^*\to c\bar{c}}$ is the well-known gauge-invariant off-shell matrix element for $g^*g^* \to c\bar{c}$ partonic subprocess and $\mathcal{F}_g(x_i, k^2_{it}, \mu^2)$ are the so-called unintegrated (transverse momentum dependent) gluon PDFs (uPDFs).

The pocket-formula for TPS (Eq. (2.4)) can then be written in the differential form:

$$\frac{d\sigma^{TPS}_{pp\to c\bar{c}c\bar{c}c\bar{c}}}{d\xi_{12}\, d\xi_{34}\, d\xi_{56}} = \left(\frac{1}{3!}\right)\frac{1}{\sigma^2_{\mathrm{eff,TPS}}}\frac{d\sigma^{SPS}_{pp\to c\bar{c}}}{d\xi_{12}}\cdot\frac{d\sigma^{SPS}_{pp\to c\bar{c}}}{d\xi_{34}}\cdot\frac{d\sigma^{SPS}_{pp\to c\bar{c}}}{d\xi_{56}}, \tag{2.8}$$

where for simplicity $d\xi_{ij}$ stand for $dy_i dy_j d^2p_{i,t} d^2p_{j,t}$.

In the present paper we use the Kimber–Martin–Ryskin (KMR) uPDFs [24], generated from the LO set of an up-to-date Martin–Motylinski–Harland–Lang–Thorne (MMHT2014) collinear gluon PDFs [25] fitted also to the LHC data. In the perturbative part of the calculations, for the central predictions, we use a running $\alpha_S^{LO}(\mu)$ provided with the MMHT2014 PDFs and the charm quark mass $m_c = 1.5$ GeV. We set both the renormalization and factorization scales equal to the averaged transverse mass $\mu^2 = \frac{m^2_{i,t}+m^2_{j,t}}{2}$, where $m_{i,t} = \sqrt{p^2_{i,t} + m^2_c}$.

The parton-level cross sections for triple charm quark (or charm antiquark) production are further corrected for the $c \to D$ (or $\bar{c} \to \bar{D}$) hadronization effects. This is done with the help of the fragmentation function (FF) technique. The quark cross section is transformed to D-meson hadron-level via the following procedure:

$$\frac{d\sigma^{TPS}_{pp\to DDD}}{d\xi^D} \approx \int \frac{D_{c\to D}(z_1)}{z_1}\cdot\frac{D_{c\to D}(z_2)}{z_2}\cdot\frac{D_{c\to D}(z_3)}{z_3} \cdot \frac{d\sigma^{TPS}_{pp\to ccc}}{d\xi^c} dz_1 dz_2 dz_3\,, \tag{2.9}$$

where $d\xi^a$ stand for $dy^a_1 dy^a_2 dy^a_3 d^2p^a_{1,t} d^2p^a_{2,t} d^2p^a_{3,t}$ taking $a = c$ quark or D meson and $p^c_{i,t} = \frac{p^D_{i,t}}{z_i}$ with meson momentum fractions $z_i \in (0, 1)$. The usual approximation here is that the quark rapidities y^c_1, y^c_2, y^c_3 are unchanged in the fragmentation process which is known to be especially legitimate in the case of heavy flavors and meson transverse momenta larger than its mass (see *e.g.* Ref. [26]). In the numerical calculations here we use the commonly used in the literature scale-independent Peterson FF [27] with the parameter $\varepsilon_c = 0.05$, which is the averaged value extracted from different e^+e^- experiments. In the last step the obtained cross sections for triple-meson production in the way sketched above are normalized with the corresponding fragmentation fraction BR$(c \to D^0) = 0.565$ [28].

3. Numerical results

We start with our predictions for multiple $c\bar{c}$ production. In order to compare our results to the results of Ref. [9] in Table 1 we show central predictions for cross sections for the full phase space for two different collision energies. We get considerably larger cross section for triple $c\bar{c}$ production compared to the numbers read off from Fig. 1 of Ref. [9]. It is not obvious how to understand the difference, as rather different approaches were used in both cases. There are obvious uncertainties related to the choice of factorization/renormalization scales in both approaches. In our approach the region of very small transverse momenta of c or $\bar{c}$ quarks is the least certain, which is related to uncertain region of very small gluon transverse momenta. Small uncertainties for single-parton scattering are considerably magnified for triple-parton scattering. Large differences of cross section in the full phase space do not necessarily involve such differences for fiducial volume. We think that agreement of theoretical inclusive cross sections for D meson production in a fiducial volume is a necessary (and sufficient) condition for the best estimating of DPS and TPS effects.

Table 1
Central predictions for total charm SPS, DPS and TPS cross sections (in mb) in pp-collisions at the LHC, calculated in the k_T-factorization approach and using the factorized Ansatz.

Final state	$\sqrt{s} = 7$ TeV	$\sqrt{s} = 13$ TeV
SPS: $\sigma(c\bar{c} + X)$	9.84	17.30
DPS: $\sigma(c\bar{c}c\bar{c} + X)$	2.31	7.13
TPS: $\sigma(c\bar{c}c\bar{c}c\bar{c} + X)$	0.55	2.99

To avoid the model uncertainties one has to describe the experimental data for SPS single D^0 ($\bar{D}^0$) for the LHCb acceptance. As already mentioned in the introduction, the k_T-factorization approach is very useful in this context, since it provides a good parametrization of the LHCb charm data with the standard set of factorization/renormalization scales and charm quark mass (defined in the previous section). As will be shown below, the usual variation of these parameters is not necessary in order to stay in touch with the LHCb experimental data. Then such a "parametrization" of the data can be used for the predictions for DPS double and TPS triple D^0 ($\bar{D}^0$) production. Then only uncertainties of the SPS result will be propagated to the two and three meson production. Uncertainties related to $\sigma_{\mathrm{eff,DPS}}$ are not taken into account since again we get good description of the LHCb double charm data within its central value (extracted from the experiment). We also use the central value of the relation between $\sigma_{\mathrm{eff,DPS}}$ and $\sigma_{\mathrm{eff,TPS}}$ (see Eq. (2.6)).

In Fig. 2 we present our results for SPS inclusive single D^0 meson production for the LHCb experiment at $\sqrt{s} = 7$ and 13 TeV. In both cases we get good description of the measured transverse momentum distributions for different intervals of rapidity. Here we present our k_t-factorization result and $\pm 40\%$ uncertainty band. The choice of the uncertainty band is motivated by our previous studies of Ref. [10] where a detailed analysis of the uncertainties related to factorization/renormalization scales and to quark mass for SPS charm production at the LHC was presented. For the LHCb fiducial volume considered here, the chosen p_T-independent uncertainty at the level of 40% seems to be sufficient. Similar studies in the case of DPS double charm production can be found in Ref. [8]. Experimental data are within such an uncertainty band. The good agreement with the LHCb data is a good starting point for calculation of double and triple D meson production. We will propagate such an uncertainty band through the factorization formula for DPS and TPS calculations.

In Table 2 we show our predicted cross sections for two and three mesons within the fiducial volume of the LHCb detector. The DD pairs were already measured by the LHCb Collaboration. In addition to the k_t-factorization result we show the lower and upper limits obtained from the lower and upper limits for SPS mechanism. The predicted value at $\sqrt{s} = 7$ TeV for $D^0D^0 + \bar{D}^0\bar{D}^0$ final

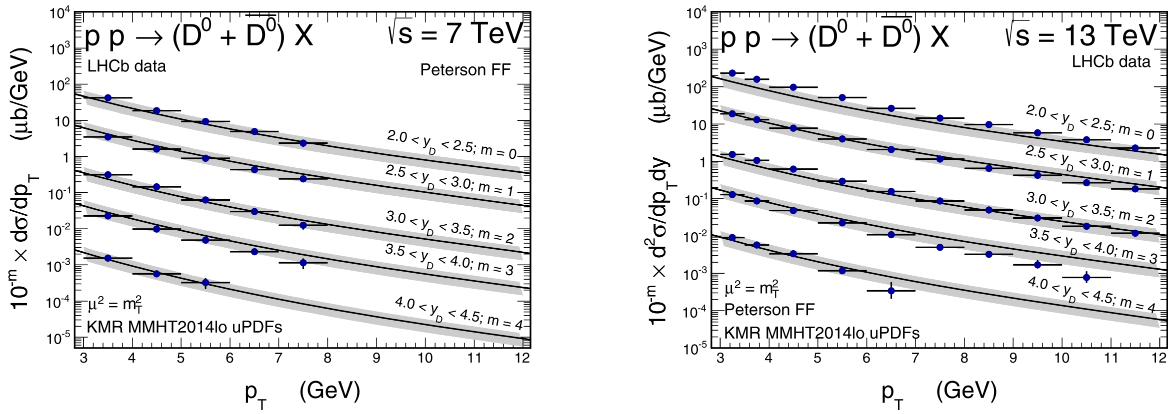

Fig. 2. Transverse momentum distributions of D^0 meson in the case of inclusive single meson production measured by the LHCb experiment at $\sqrt{s} = 7$ TeV [29] (left panel) and $\sqrt{s} = 13$ TeV [30] (right panel). The solid lines correspond to our theoretical predictions based on the k_T-factorization approach with the KMR uPDFs. The results and experimental data points are shown for different rapidity bins defined in the figure. The discussed in the text uncertainty bands are shown in addition.

Table 2
The integrated cross sections for double and triple D^0 meson production (in nb) within the LHCb acceptance: $2 < y_{D^0} < 4$ and $3 < p_T^{D^0} < 12$ GeV, calculated in the k_T-factorization approach. The numbers include also the charge conjugate states.

Final state	$\sqrt{s} = 7$ TeV		$\sqrt{s} = 13$ TeV	
DPS: $\sigma(D^0D^0 + X)$	784.74	max: 1538.09 min: 282.51	2992.91	max: 5866.10 min: 1077.45
TPS: $\sigma(D^0D^0D^0 + X)$	2.38	max: 6.53 min: 0.51	17.71	max: 48.59 min: 3.83

state is consistent with the measured one: $\sigma_{LHCb} = 690 \pm 40 \pm 70$ (see Table 12 in Ref. [5]). Whether the three mesons can be measured at the LHCb will be discussed in the following.

Now we wish to discuss some differential distributions for double and triple D^0 meson production. In Fig. 3 we show transverse momentum distribution of one of the two or one of the three D^0 mesons (all measured by the LHCb detector). In the used multiple parton scattering formalism the distributions for single, double and triple production have the same shape/slope and differ only by normalization. The distributions for triple D^0 production are about two orders of magnitude smaller than for double D^0 production, consistent with Table 2. In the left panel, for $\sqrt{s} = 7$ TeV, we also show for reference the LHCb experimental data [5]. As for single scattering case (see Fig. 2) we get a good agreement also here within the uncertainty band shown in Fig. 3, so we hope our predictions for triple D^0 production (lowest curves) are (should be) reliable. The chosen uncertainty procedure leads us to the conclusion that our result for TPS is known within a factor ≈ 3.

In Fig. 4 we show our predictions for invariant mass distributions for two (left panel) and three (right panel) D^0 mesons. The lower curves are for $\sqrt{s} = 7$ TeV and the upper curves for $\sqrt{s} = 13$ TeV. We show uncertainty band only for the lower energy. Again we show the LHCb experimental data for $\sqrt{s} = 7$ TeV (left panel). The distributions shown in the right panel are waiting for experimental verification.

Finally in Table 3 we show the number of counts for different realistic values of the integrated luminosity for the LHCb experiment. The predicted numbers of events for DPS double- and TPS triple-D^0 production correspond to the central predictions for cross sections from Table 2. Here we have included in addition the relevant decay branching fraction BR($D^0 \to K^-\pi^+$) = 0.0393 [31]. As for differential distributions those numbers are estimated with a precision of a factor ≈ 3.

4. Conclusions

In this letter we have presented a first estimation of the cross sections for triple D^0 production within the LHCb fiducial volume in order to verify triple parton scattering effects for triple $c\bar{c}$ production.

Any pQCD calculation, including k_t-factorization, inherently has uncertainties related to the choice of parton distributions, of the

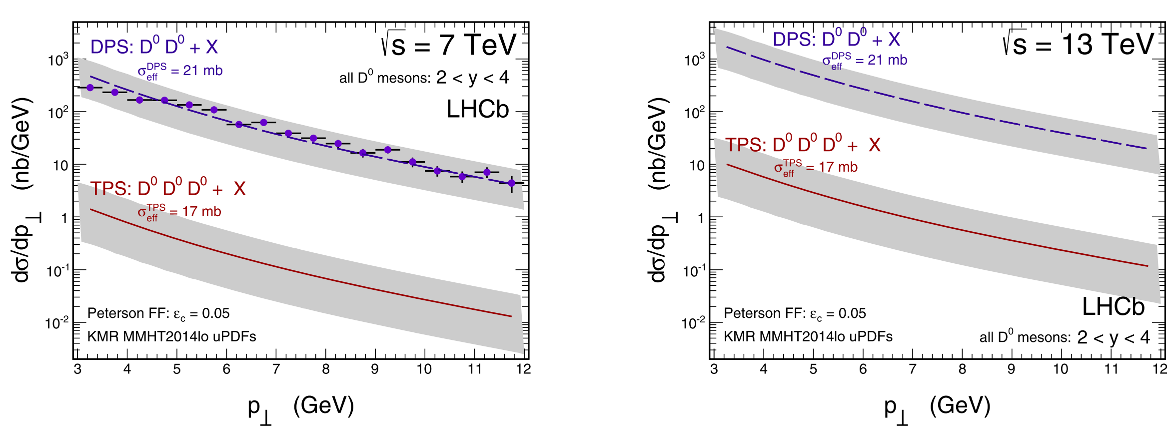

Fig. 3. Transverse momentum distributions of one of the measured D^0 mesons for double-D^0 (upper long-dashed lines) and triple-D^0 (lower solid lines) production for $\sqrt{s} = 7$ (left panel) and 13 TeV (right panel). Details are specified in the figure. The data points for double-D^0 production in the left panel are taken from Ref. [5]. The lower and upper of uncertainty bands were obtained from the lower and upper limits for single D^0 ($\bar{D}^0$) production.

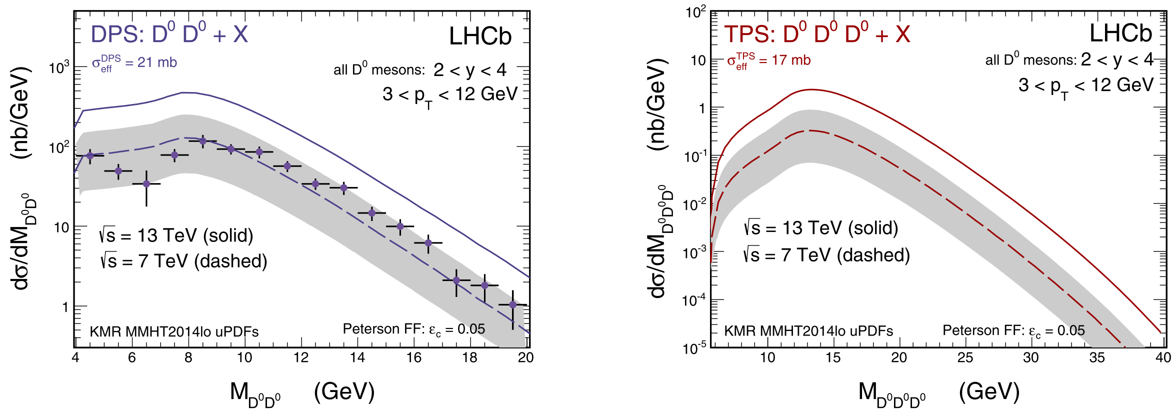

Fig. 4. Invariant mass distributions (corresponding to the LHCb acceptance) of the di-meson D^0D^0 system for the DPS mechanism (left panel) and tri-meson $D^0D^0D^0$ system for the TPS mechanism (right panel), for $\sqrt{s} = 7$ TeV (lower long-dashed lines) and 13 TeV (upper solid lines). Details are specified in the figure. The data points for double-D^0 production in the left panel are taken from Ref. [5]. The lower and upper limits of uncertainty bands were obtained from the lower and upper limits for single D^0 ($\bar{D}^0$) production.

Table 3
Number of events for different values of the feasible integrated luminosity in the LHCb experiment for the central predictions of cross sections from Table 2. The branching fractions for $D^0 \to K^-\pi^+(\bar{D}^0 \to K^+\pi^-)$ are included here.

$\sqrt{s}$	Integrated luminosity	DPS (D^0D^0)	TPS ($D^0D^0D^0$)
7 TeV	355 pb^{-1}	0.43×10^6	51
	1106 pb^{-1}	1.34×10^6	159
13 TeV	1665 pb^{-1}	7.70×10^6	1789
	5000 pb^{-1}	23.11×10^6	5374

factorization and renormalization scales, parameters of hadronization etc. In this paper we have used the k_t-factorization approach to parametrize experimental data for single D^0 ($\bar{D}^0$) meson production. The best fit of the LHCb experimental data is obtained with the standard set of factorization/renormalization scale and charm quark mass choices. This means we can avoid all uncertainties of the k_t-factorization or any pQCD calculation in general. Therefore only model-independent uncertainty band, related to the quality of the description of the experimental data, was used in our analysis. The uncertainty band for single meson production was propagated to double and triple meson production. Our estimate should be more precise than within a factor of 3.

We have obtained rather large cross sections for triple $c\bar{c}$ production, larger than predicted very recently in Ref. [9] but still consistent within uncertainties of the two approaches. We have checked, however, our approach against inclusive single D^0 and double D^0D^0 production as measured by the LHCb Collaboration. In both cases we have obtained a fairly good agreement which gives us confidence for triple D^0 production.

We have presented both integrated cross section as well as differential distributions for double and triple D^0 production for $\sqrt{s} = 7$ and 13 TeV. We have presented also predicted number of counts for different realistic values of integrated luminosity for the LHCb experiment. In the case of triple D^0 production we have predicted about 100 counts at $\sqrt{s} = 7$ TeV and a few thousands of counts at $\sqrt{s} = 13$ TeV for realistic integrated luminosities. We hope the LHCb Collaboration will be able to verify our predictions soon.

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