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Planck length: Lost + found

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ABSTRACT

At mesoscopic scales, close to but somewhat larger than Planck length, one can describe the spacetime in terms of an effective geometry. The key feature of such an effective (quantum) geometry is the existence of a zero-point-length which, for example, modifies the propagator for a massive scalar field residing in that spacetime, in a specific manner. Such quantum gravitational effects arise, even in a globally flat spacetime, if one probes the spacetime at length scales close to Planck length. Principle of Equivalence demands that the effects of quantum spacetime observed in a freely-falling-frame (FFF) must be the same as those in a globally flat spacetime. But, in the FFF, gravity disappears and — along with it — the Newtonian gravitational constant G also disappears; therefore, operationally, the Planck length disappears in the FFF! So how can the quantum gravitational effects persist in the FFF, as they must? I show that the answer to this question is interesting and subtle. The Planck length reappears in FFF through the matter sector as the geometric mean $L_P = \sqrt{\lambda_c \lambda_g}$ of the Compton wavelength $\lambda_c = \hbar/m_i c$ (where m_i is the inertial mass) and the Schwarzschild radius $\lambda_g = G m_g/c^2$ (where m_g is the gravitational mass) when we invoke the Principle of Equivalence again, in the form $m_i = m_g$. So the Principle of Equivalence plays a crucial role in making the Planck length disappear and reappear to incorporate the effects of quantum spacetime in a FFF.

1. QG corrections to the propagator at mesoscopic scales

Consider a non-interacting massive scalar field $\phi(x)$ propagating in a curved spacetime described by a metric $g_{ab}(x)$ in a region of the spacetime manifold which we are interested in. I assume that the metric has no curvature singularities anywhere in the manifold; the length scale characterizing the curvature $L_{\rm curv}$, defined through, say, $L_{\rm curv}^{-2} \equiv \sqrt{R^{abcd}R_{abcd}}$ is bounded from below because the curvature is bounded from above. The quantum dynamics of the scalar field in such a classical background spacetime is completely captured by the Feynman propagator

$$G_{std}(x, y; m^2) = \int_0^\infty ds\, e^{-m^2 s} K_{std}(x, y; s) \tag{1}$$

where $K_{\rm std}$ is the standard, zero-mass, Schwinger (heat) kernel given by $K_{\rm std}(x, y; s) \equiv \langle x|e^{s\Box_g}|y\rangle$. Here $\Box_g$ is the Laplacian in the background space(time).[1] The heat kernel is a purely geometric object, entirely determined by the background geometry; all the information about the scalar field is contained in the single parameter m. The heat kernel has the structure (in $D = 4$):

$$K_{std}(x, y; s) \propto \frac{e^{-\bar{\sigma}^2(x,y)/4s}}{s^2}\,[1 + \text{curvature corrections}] \tag{2}$$

where $\bar{\sigma}^2(x, y)$ is the geodesic distance and the curvature corrections, encoded in the Schwinger-Dewitt expansion, will involve powers of (s/L^2_{curv}). The exponential $e^{-m^2 s}$ in Eq. (1) suppresses the contributions for $s \gtrsim \lambda_c^2$ — where $\lambda_c \equiv \hbar/mc$ is the Compton wavelength associated with mass m — in the integral in Eq. (1); when $\lambda_c \ll L_{curv}$, the curvature corrections will be small.

Such a description — based on quantum field theory in curved spacetime — is expected to breakdown when the quantum structure of spacetime becomes relevant. When exactly this will happen cannot be ascertained until we have a well established theory of quantum gravity (QG); the folklore belief, which I will take to be valid, is that these QG effects will be definitely important at Planck scales characterized by the Planck length, $L_P \equiv (G\hbar/c^3)^{1/2}$. That is, when we probe the spacetime at length scales $\lambda \lesssim L_P$ the description of QFT in CST will definitely breakdown. I will call this regime

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[1] I will work in a Euclidean space(time) for mathematical convenience and will assume that the results in spacetime arise through analytic continuation. This is *not* essential and one could have done everything in the Lorentzian spacetime itself; it just makes life easier.

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microscopic and the regime of QFT in CST (with $\lambda \gg L_P$) *macroscopic*.

I am interested in the intermediate scales, which I will call *mesoscopic*, at which one can describe the spacetime in the usual continuum language but still incorporate the effects of QG by modifying the propagator $G_{\rm std}$ to a quantum gravity corrected propagator $G_{\rm QG}$. (One could consider such a description to be operational at length scales $\lambda \gtrsim C L_P$, with $C = 10^3$, say, for definiteness. A factor of 10^3 could allow for the continuum description to emerge, but — at the same time — to be affected by the microscopic physics through a non-zero L_P. You could choose a larger value of C if you so feel.) Again, one cannot calculate $G_{\rm QG}$ (or C) from first principles, unless one has the full theory of quantum gravity. In the absence of such a luxury, I will use the following working hypothesis to make further progress.

It is possible to capture the most important effects of quantum gravity by introducing a zero-point-length to the spacetime. This is based on the idea [1–4] that the *dominant* effect of quantum gravity at mesoscopic scales can be captured by assuming that the (squared) path length $\sigma^2(x_2, x_1)$ has to be replaced by $\sigma^2(x_2, x_1) \to \sigma^2(x_2, x_1) + L^2$ where L^2 is of the order of Planck area $L_P^2 \equiv (G\hbar/c^3)$. This idea is decades old and has been explored extensively in the past and current literature (see e.g., [1–3]; for more recent explorations, see [4]). I will just accept it as a working hypothesis and proceed further. (A set of brief comments about this approach is given in the Appendix, for the sake of those who are unfamiliar with previous literature.)

More precisely stated, this postulate is equivalent to modifying $G_{\rm std}$ to the form

$$G_{QG}(x, y; m^2) = \int_0^\infty ds\, e^{-m^2 s - L^2/4s} K_{std}(s; x, y) \tag{3}$$

Recall that the leading order behaviour of the heat kernel is $K_{\rm std} \sim s^{-2} \exp[-\sigma^2(x, y)/4s]$ where σ^2 is the geodesic distance between the two events; so the modification in Eq. (3) amounts to the replacement $\sigma^2 \to \sigma^2 + L^2$ to the leading order.

It is possible to provide a nice geometric interpretation for this replacement of G_{std} by G_{QG}. Recall that the standard propagator G_{std} has world-line path integral representation involving the sum over amplitudes $\exp(-m\sigma)$:

$$\begin{aligned} G_{\rm std}(x_1, x_2; m^2) &= \sum_{\text{paths } \sigma} \exp -m\sigma(x_1, x_2) \\ &= \int_0^\infty ds\, e^{-m^2 s} K_{std}(s; x, y) \end{aligned} \tag{4}$$

where $\sigma(x_1, x_2)$ is the length of the path connecting the two events x_1, x_2 and the sum is over all paths connecting these two events. This path integral can be defined in the lattice and computed — with suitable measure — in the limit of zero lattice spacing [2,5] to give the standard result, indicated by the second equality. In a similar manner, one can obtain the modified propagator $G_{\rm QG}$ from a world line path integral representation [2]:

$$\begin{aligned} G_{QG}(x, y; m^2) &= \sum_\sigma \exp\left[-m\left(\sigma + \frac{L^2}{\sigma}\right)\right] \\ &= \int_0^\infty ds\, e^{-m^2 s - L^2/4s} K_{std}(s; x, y) \end{aligned} \tag{5}$$

where L is a length scale of the order of Planck length. The amplitude of the path integral for $G_{\rm QG}$ has the nice additional symmetry ('duality') of being invariant under $\sigma \to L^2/\sigma$. The path integral sum can again be computed by lattice regularization techniques [2] and will lead to the second equality.

In this approach the physics of the scalar field at mesoscopic scales is captured by a *purely geometrical* modification of the path integral amplitude by the replacement

$$\sigma \to \sigma + \frac{L^2}{\sigma} \tag{6}$$

I stress that this modification is universal and geometrical, independent of the parameters of the matter sector, viz., the mass m, which appears only as an overall multiplicative factor. So one could think of this modification as a relic of quantum structure of spacetime present at the mesoscopic scales.

2. Planck length is lost in a freely falling frame due to Principle of Equivalence

The modification of G_{std} to G_{QG}, given in Eq. (5) is valid in any curved spacetime. Therefore, as a special case, it should be also valid in a spacetime which is (globally) flat at macroscopic scales. If we probe such a spacetime at mesoscopic scales, the propagator G_{std} is corrected to the form in G_{QG}. This leads to some remarkable insights which I will now describe.

Consider a region of spacetime in which the curvature length scale $L_{\rm curv}$ is much larger[2] than Planck length: $L_{\rm curv} \gg L_P$. Concentrate on the modes of a quantum field which probe the several orders of magnitude between L_P and $L_{\rm curv}$. Let us start with modes which are far away from either extremities: $L_P \ll \lambda \ll L_{\rm curv}$, and study them in the freely falling frame (FFF) around an event $\mathcal{P}$ in this spacetime region. The *classical* effects due to spacetime curvature will now be absent to order $\mathcal{O}(\lambda^2/L_{\rm curv}^2)$. The Principle of Equivalence, which allows the choice of FFF around any even $\mathcal{P}$, has eliminated *classical* gravity. Let us now start decreasing λ. Since we are in FFF, no *classical* gravitational effects due to curvature can arise and the approximation of a flat spacetime becomes *more and more accurate* as λ becomes progressively smaller compared to $L_{\rm curv}$.

But when we start approaching Planck length (i.e., when $\lambda \approx C L_P$ where C, say, is about 10^3) *quantum* gravitational effects should start appearing. We will expect these effects to be described in the FFF by the *same* propagator G_{QG} with a zero-point-length $L = \mathcal{O}(G\hbar/c^3)^{1/2}$. This is, in fact, a direct consequence of Principle of Equivalence. One formulation of Principle of Equivalence will be to postulate that laws of classical special relativity will remain valid in a FFF around any event $\mathcal{P}$. But a classical, *globally flat,* spacetime will harbor quantum gravitational fluctuations, just as a classical electromagnetic vacuum will harbor quantum electrodynamical fluctuations. The Principle of Equivalence tells us that the quantum gravitational effects in FFF should be identical to the quantum gravitational effects in a (globally) flat spacetime. The effect of background curvature can be ignored to the order $\mathcal{O}(L_P^2/L_{\rm curv}^2)$.

But this leads to an (apparent) paradox: Recall that we are still in FFF in which there is no gravity! The spacetime is described by the metric $g_{ab} = \delta_{ab}$ and the Newtonian gravitational constant has disappeared. (For example, consider the Schwarzschild spacetime metric which, in standard coordinates, contains G; if you write the same metric in a FFF around any event, G will disappear at the lowest order.) This, in turn, means that the Planck length has no

[2] Of course, if you want to study situations in which $L_{\rm curv} \approx L_P$, you need the full machinery of quantum gravity; but when $L_{\rm curv} \gg L_P$ we can still meaningfully talk about quantum gravitational effects adding corrections to standard QFT in the mesoscopic regime with λ close — but not too close — to L_P.

operational significance in the FFF; the effective Planck length vanishes since $G = 0$ in the FFF. (No gravity in the FFF - Einstein's "most fortunate thought in my life"!).

So how can one introduce the correct G_{QG} in the FFF, which is needed for consistency? This will be possible, only if there is an alternative route to G_{QG} from the matter sector (since there is no gravity in the FFF) and indeed there is. Let me now describe how this comes about.

3. Planck length found from matter sector, again through Principle of Equivalence

The action for a relativistic particle of *inertial* mass m_i gives the factor $\exp(-A/\hbar)$ with $A/\hbar = -m_i c\sigma/\hbar = -\sigma/\lambda_c$ where σ is the length of the path and $\lambda_c = \hbar/m_i c$ is the Compton wavelength of the particle. The Compton wavelength $\lambda_c = \hbar/(m_i c)$ is defined in terms of the *inertial* mass of the particle. The part of the path integral amplitude $\exp[-(\sigma/\lambda_c)]$ comes from combining special relativity with quantum theory and does not depend on the existence of gravity. The path integral amplitude is exponentially suppressed for paths longer than the Compton radius $\lambda_c \equiv \hbar/m_i c$.

The description of scalar field as a test field, in terms of the propagator G_{std} defined in a fixed background spacetime, will cease to be valid when the self-gravity of the field is important. When the self-gravity of the matter field is introduced into the picture, another length scale, viz. the gravitational Schwarzschild radius $\lambda_g \equiv Gm_g/c^2$ where m_g is the gravitational mass of the particle, comes into play. The self-gravity of a particle of mass m_g will strongly curve the spacetime at length scales comparable to λ_g. At length scales comparable to λ_g, we can no longer think of a 'free field' even in flat spacetime.

So the propagator, and the world line path integral in Eq. (4) which defines it, need to be modified for path lengths $\sigma \lesssim \lambda_g$. *In fact, it makes absolutely no sense to sum over paths with $\sigma \lesssim \lambda_g$ in the path integral.* Just as paths with $\sigma \gtrsim \lambda_c$ are suppressed exponentially by the factor $\exp[-(\sigma/\lambda_c)]$, we should suppress the paths with $\sigma \lesssim \lambda_g$ by another dimensionless factor $F[(\lambda_g/\sigma)]$ which depends on the dimensionless ratio (λ_g/σ) and rapidly decreases for $\sigma \ll \lambda_g$. This will modify the amplitude for a path of length σ from $\exp[-(\sigma/\lambda_c)]$ to $F[(\lambda_g/\sigma)]\exp[-(\sigma/\lambda_c)]$. Writing, $F \equiv \exp -f$ for algebraic convenience, the modified propagator is now given by the path integral sum:

$$\mathcal{G}(x, y) \equiv \sum_{\text{paths } \sigma} \exp\left[-\frac{\sigma}{\lambda_c} - f[(\lambda_g/\sigma)]\right] = \sum_{\text{paths } \sigma} \exp -m_i\left[\sigma + \frac{1}{m_i} f[(\lambda_g/\sigma)]\right] \tag{7}$$

The crucial point is that, the self-gravity of the matter field is not eliminated in the FFF so that one can meaningfully talk about λ_g in the FFF.

We now have two completely independent ways of defining the propagator at mesoscopic scales.

(i) First, from the introduction of a zero-point-length in the spacetime, we argued that the propagator should be modified from G_{std} in Eq. (4) to G_{QG} in Eq. (5). This is equivalent to modifying the world line path integral amplitude by the duality relation in Eq. (6). This is purely geometrical. In this approach we introduced the Planck length by hand, through the postulate of zero-point-length.

(ii) Second, in the FFF (in which there is no operational definition of Planck length), we started from matter sector and — by incorporating the self gravity of a particle of mass m into the path integral— we have arrived at the modification of the propagator in Eq. (7). We have *not* introduced the notion of Planck length and have only used the two length scales (λ_c, λ_g) associated with the mass of the particle we are studying.

Consistency demands that these two propagators should be identical, which puts a nontrivial constraint on expression in Eq. (7). This constraint, in fact, allows us to fix the form of the function $F = \exp -f$ as follows. Since the result in Eq. (5) has a purely geometrical origin, the Eq. (7) can reproduce Eq. (5) only if the factor in the square bracket multiplying m_i in Eq. (7) is just a function of σ. That is, this factor cannot depend on the parameters of the scalar field like m_i, m_g. This, in turn, is possible only if (i) the Principle of Equivalence holds, allowing us to set $m_i = m_g$ *and* (ii) the function is given by $f[(\lambda_g/\sigma)] \propto (\lambda_g/\sigma)$. The proportionality constant will be of order unity; this is because the paths with lengths $\sigma < \lambda_g$ are now suppressed exponentially by the factor $F = \exp -f$ and we expect this suppression to happen for $\sigma \lesssim \lambda_g$. So the proportionality factor can be ignored with the understanding that we now redefine λ_g as $\mathcal{O}(1)(Gm/c^2)$. We can thus conclude that a natural and minimal modification of the path integral sum in Eq. (4), which incorporates the self gravity of a particle of mass $m = m_i = m_g$, will lead to the propagator:

$$\mathcal{G}(x, y) \equiv \sum_{\text{paths } \sigma} \exp\left[-\frac{\sigma}{\lambda_c}\right] \exp\left[-\frac{\lambda_g}{\sigma}\right] = \sum_{\sigma} \exp\left[-m\left(\sigma + \frac{L^2}{\sigma}\right)\right] \tag{8}$$

where $L = \mathcal{O}(1)L_P$. This modification, given by Eq. (8) is identical to the one in Eq. (5) and has the same [2] symmetry: viz. the amplitude is invariant under the duality transformation $\sigma \to L^2/\sigma$.

The result depends on the Principle of Equivalence in a subtle and interesting way. The Compton wavelength $\lambda_c = \hbar/(m_i c)$ is defined in terms of the *inertial* mass of the particle and gives part of the path integral amplitude $\exp[-(\sigma/\lambda_c)]$, which comes from combining special relativity with quantum theory; this factor does not depend on the existence of gravity. On the other hand, the gravitational radius $\lambda_g \equiv Gm_g/c^2$ is defined in terms of the *gravitational* mass of the particle and leads to the factor $\exp[-(\lambda_g/\sigma)]$. These two factors exist separately in the first equality of Eq. (8). But they can be expressed as in the second equality of Eq. (8) only because of the assumption $m_i = m_g$! If $m_i \neq m_g$ then we will end up with the argument of the exponential:

$$\frac{m_i\sigma}{\hbar c} + \frac{Gm_g}{c^2\sigma} = \frac{1}{\lambda_c}\left[\sigma + \left(\frac{m_g}{m_i}\right)\frac{L_P^2}{\sigma}\right] \tag{9}$$

Clearly, there is no universal, geometrical interpretation for such a factor in the square bracket, occurring in a path integral. The addition of a universal zero-point-length to the spacetime — which is independent of any parameters of the matter sector — will *not* be equivalent to the modification of the propagator due to its self-gravity if $m_i \neq m_g$. *Just as classical gravity admits a purely geometrical description only because $m_i = m_g$, the quantum geometry allows a universal description in terms of zero-point-length only because of $m_i = m_g$.* We now have Principle of Equivalence operating at Planck scales! So the duality symmetry for $\sigma \to L^2/\sigma$ is closely related to the Principle of Equivalence.[3]

[3] This result also tells us why the *exponential* form of the suppression $\exp[-(\lambda_g/\sigma)]$ — rather than some other functional form — in Eq. (8), for path lengths smaller than Schwarzschild radius, is uniquely selected. No other functional form will lead to the geometrical factor $[\sigma + (L^2/\sigma)]$, which is required.

The above argument, in a way, also "discovers" Planck length. The first equality in Eq. (8) gives two exponential suppression factors, based on two length scales λ_c and λ_g associated with the particle. *Both factors depend on the mass of the particle*. But when combined together, as in the second equality, the Planck length appears (essentially as the geometric mean $L_P = \sqrt{\lambda_c \lambda_g}$) which is independent of the mass of the particle and a universal constant. As a bonus, the duality structure, *with respect to* L_P, emerges.

4. Summary

Given the dominantly conceptual nature of this paper, it is worthwhile to summarize the flow of logic and the key results:

- At the mesoscopic scales — which are close to but not too close to Planck length — we expect an effective geometrical description to emerge with some quantum gravitational corrections. I take it as a working hypothesis that these corrections to the dynamics of a massive scalar field can be captured by modifying the standard propagator $G_{\rm std}$ to $G_{\rm QG}$ in Eq. (3). This postulate has been investigated several times in the past literature [1–4]; here I explore some further consequences of the same.
- The expression for $G_{\rm QG}$ in Eq. (3) is valid in any spacetime including flat spacetime. This shows that the flat spacetime — possibly considered as the ground state in quantum gravity — will exhibit quantum gravitational effects just as, for example, the electromagnetic ground state exhibits QED vacuum fluctuations. These effects are again captured through the zero-point-length in flat spacetime.
- In any arbitrary curved spacetime, around any non-singular event $\mathcal{P}$ one can introduce a Riemann normal coordinates, which reduces the metric to that of flat spacetime in a region small compared to the background curvature length scale $L_{\rm curv}$ at $\mathcal{P}$. I will assume that $L_P \ll L_{\rm curv}$ by several orders of magnitude. When the spacetime is probed at mesoscopic scales λ with $L_P \lesssim \lambda \ll L_{\rm curv}$ the QG effects will appear in the freely falling frame (FFF), but any effects due to the background curvature will be negligible. I call this the regime of *flat spacetime quantum gravity.*
- Principle of Equivalence demands that the quantum gravitational effects in this FFF must be identical to those in a globally flat spacetime and must be governed by the Planck length. In this FFF, described locally by flat metric, there is no gravity and no notion[4] of the gravitational constant G; therefore, there is no operational notion of Planck length in the FFF! How can we then understand the zero-point-length in this context? The Principle of Equivalence, allowing the construction of the FFF, seems to have made us 'lose' the Planck length!
- The answer to this paradox lies in the self-gravity of the matter sector. The standard propagator for the scalar field of mass m is described by the world-line path integral in Eq. (4) which does not take into account the self gravity of the scalar field. This expression for path integral cannot be valid for paths with lengths $\sigma \lesssim \lambda_g$ where $\lambda_g \approx G m_g/c^2$ is the Schwarzschild radius associated with mass m. Consistency demands that the suppression of the world-line path integral amplitude for paths with length $\sigma \lesssim \lambda_g$ must lead to the same modified propagator $G_{\rm QG}$. This is indeed possible if and only if the inertial mass m_i is equal to the gravitational mass m_g. When we use this version of Principle of Equivalence, we find that the Planck length reappears in the expression now as a geometric mean $L_P = \sqrt{\lambda_c \lambda_g}$ of the Compton wavelength and the Schwarzschild radius corresponding to a mass m. We rediscover the Planck length.

Thus, what the Principle of Equivalence taketh away, it giveth back!

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

I this appendix, I summarise some aspects of the approach, which incorporates zero-point-length into the propagator, for the sake of conceptual completeness. More details can be found in the cited references.

- The idea of introducing the zero-point-length by the modification $\sigma^2(x_2, x_1) \to \sigma^2(x_2, x_1) + L^2$ should be thought of as a working *hypothesis* which is postulated to make progress, in the absence of a complete theory of quantum gravity. Such an idea has been introduced and explored extensively in the past two decades or so in the literature [1–4] and I have explored some further consequences of this approach in this paper.
  In principle, one should be able to derive this form of the propagator from a more complete theory. For example, it can be obtained from the string theory [6] in a specific approximation; but for the purpose of this work, it is enough to consider it as a working hypothesis.
- Working directly with the propagator bypasses several nuances of standard QFT which may all require some kind of revision at mesoscopic scales. However, we know that both the dynamics *and the symmetries* of a free quantum field, propagating in a curved geometry, is completely encoded in the Feynman propagator. So, if we understand how QG effects modify the propagator, we get a *direct* handle on both the dynamics and the symmetries of the theory at mesoscopic scales. This is an efficient procedure which underlies this approach, viz., we work directly with the propagator containing QG corrections, without worrying about the (unknown) modifications to the standard formalism of QFT at mesoscopic scales.
- As an example of the economy involved in this approach, let me stress the notion of diffeomorphism invariance in a curved geometry and — as a special case, Lorentz symmetry — in flat spacetime. The prescription $\sigma^2(x_2, x_1) \to \sigma^2(x_2, x_1) + L^2$ is generally covariant when L is treated as a constant scalar number. In flat spacetime, this modification will replace $(x_2 - x_1)^2$ by $(x_2 - x_1)^2 + L^2$ which is clearly Lorentz invariant. The mere introduction of a constant, scalar, length scale into the propagator will *not* violate Lorentz invariance, as should be obvious from the fact that the propagator for the massive scalar field does depend on the length scale m^{-1} and is still perfectly Lorentz invariant. The results of detailed computations (see e.g., the extensive set of computations in the last paper

[4] The Planck length is built from three fundamental constants, $\hbar$ (describing quantum theory), c (describing relativity) and G (describing gravity). It is not possible to choose a coordinate system and make the effects of quantum theory or relativity to vanish. However, you can always choose a coordinate system in which the effects of gravity vanishes to the lowest order, showing that G has a rather amusing role to play in this discussion compared to $\hbar$ or c.

in ref. [3]) explicitly demonstrate the generally covariance and Lorentz invariance of the prescription.
This result is similar to that in, for example, LQG which contains a length scale but does not violate Lorentz invariance [7]. Moreover, in our approach, the general covariance (and Lorentz invariance) *is manifest* in the prescription $\sigma^2(x_2, x_1) \to \sigma^2(x_2, x_1) + L^2$; so no special demonstration of this fact is required unlike, for example, in the case of LQG [7]. Some other prescriptions in the literature for introducing a 'minimal length', do create problems for Lorentz invariance but *our* prescription is (manifestly) generally covariant.

- There are many (later) approaches, in the literature (see e.g., [8]) to modify the theory at mesoscopic scales. Some of these more recent attempts, rather intriguingly, share notions like duality first introduced in ref. [2]. Our approach has the advantage that (a) it is minimalistic and economical and (b) the basic prescription is strikingly simple to allow detailed and explicit computations. Since the other approaches use much more complex set of assumptions and prescriptions, it is not easy to compare these approaches. Roughly speaking, it appears that the introduction of zero-point-length is enough for some kind of duality to emerge but such a result is hard to prove rigorously.

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