DK, *DDK*, and *DDDK* molecules–understanding the nature of the $D_{s0}^*(2317)$

Tian-Wei Wu,[1] Ming-Zhu Liu,[1] Li-Sheng Geng,[1,2,3,*] Emiko Hiyama,[4,5,†] and Manuel Pavon Valderrama[1,‡]

[1]*School of Physics and Nuclear Energy Engineering, Beihang University, Beijing 100191, China*
[2]*Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 100191, China*
[3]*School of Physics and Engineering, Zhengzhou University, Zhengzhou, Henan 450001, China*
[4]*Department of Physics, Kyushu University, Fukuoka 819-0395, Japan*
[5]*Nishina Center for Accelerator-Based Science, RIKEN, Wako 351-0198, Japan*

(Received 5 July 2019; published 29 August 2019)

The DK interaction is strong enough to form a bound state, the $D_{s0}^*(2317)$. This in turn begs the question of whether there are bound states composed of several charmed mesons and a kaon. Previous calculations indicate that the three-body DDK system is probably bound, where the quantum numbers are $J^P = 0^-$, $I = \frac{1}{2}$, $S = 1$ and $C = 2$. The minimum quark content of this state is $cc\bar{q}\,\bar{s}$ with $q = u, d$, which means that, if discovered, it will be an explicitly exotic tetraquark. In the present work. we apply the Gaussian expansion method to study the $DDDK$ system and show that it binds as well. The existence of these three and four body states is rather robust with respect to the DD interaction and subleading (chiral) corrections to the DK interaction. If these states exist, it is quite likely that their heavy quark symmetry counterparts exist as well. These three-body DDK and four-body $DDDK$ molecular states could be viewed as counterparts of atomic nuclei, which are clusters of nucleons bound by the residual strong force, or chemical molecules, which are clusters of atoms bound by the residual electromagnetic interaction.

DOI: 10.1103/PhysRevD.100.034029

I. INTRODUCTION

In 2003 the *BABAR* collaboration discovered the $D_{s0}^*(2317)$ [1],[1] a strange-charmed scalar meson, the observation of which was subsequently confirmed by CLEO [2] and Belle [3]. Its mass is about 160 MeV below the one predicted for the lightest $c\bar{s}$ scalar state in the naive quark model, which makes it difficult to interpret the D_{s0}^* as a conventional $q\bar{q}$ state [4–17].

On the other hand, the D_{s0}^* can be easily explained as a dynamically generated state arising from the Weinberg-Tomozawa (WT) DK interaction [18–38]. This has led to the prevailing idea that the $D_{s0}^*(2317)$ is a molecular state, a hypothesis which has been further supported by a series of Lattice QCD simulations [39–43]. For a recent brief summary of all the experimental, lattice QCD, and theoretical supports for such an assignment, see, e.g., Ref. [44].

If the DK interaction is strongly attractive, a natural question to ask is what happens when one adds one extra D meson to the system.[2] The answer seems to be that it binds [48,49]. In Ref. [48] it was noticed that the DD_{s0}^* system can exchange a kaon near the mass shell, leading to a relatively long-range attractive Yukawa potential that is strong enough to bind. This conclusion is left unchanged if one explicitly considers the composite nature of the D_{s0}^*, which simply leads to more binding [48]. A later, more complete calculation in Ref. [49] leads to a binding energy of about 90 MeV for the DDK three-body system.

In the present manuscript we revisit the calculation of the DDK bound state and extend it to the $DDDK$ system by using the Gaussian expansion method (GEM), which offers a number of advantages compared to previous studies [48,49]. First, a main advantage of the GEM is that it can be easily extended to four (or even five [50]) body systems, which allows us to study the likely existence of a $DDDK$ bound state. Second, with the GEM one can calculate directly the density distribution of the three (four) body system, which then gives a transparent picture for their spacial distributions. Third, it has enough flexibility so

*lisheng.geng@buaa.edu.cn
†hiyama@riken.jp
‡mpavon@buaa.edu.cn

[1]From now on, we will simply refer to it as D_{s0}^* unless specified otherwise.

[2]It has been shown that the $D\bar{D}^*K$ system binds as well in two recent works [45,46], though the dynamics in these two frameworks are quite different, as well as the DKK and $DK\bar{K}$ systems [47].

that one can study the impact of the existence of a repulsive core. Indeed, the chiral potential kernel up to the next to leading order with the low-energy constants determined by the corresponding lattice QCD data shows that this may indeed be the case [33].

The outcome of the exploration presented in this work is that both the DDK and $DDDK$ systems bind, with binding energies of the order of 65–70 and 90–100 MeV in each case. While the DDK bound state, owing to its $cc\bar{q}\bar{s}$ quark content, might be produced in experiments in the future, the $DDDK$ bound state is more likely to be observed on the lattice instead.[3]

This article is organized as follows. In Sec. II, we explain how we parametrize and determine the two-body DK and DD interactions. In Sec. III, we explain how to construct the three- and four-body DDK and $DDDK$ wave functions and solve the corresponding Schrödinger equation using the GEM. In Sec. IV, we present our predictions for the DDK and $DDDK$ bound states and discuss their sensitivity to a series of possible corrections. Finally, we summarize the results of this manuscript in Sect. V.

II. THE S-WAVE DK AND DD POTENTIALS

The calculation of the DDK and $DDDK$ bound states depends on the DK and DD two-body interactions. While the DK interaction can be well constrained directly from the assumption that the $D_{s0}^*(2317)$ is a DK bound state, and indirectly from chiral perturbation theory, the DD interaction is far from being well determined and we will have to resort to phenomenological models instead. In this section we will explain the type of potentials we will use to model these two-body interactions. For the masses and spin parties of the D, K, and $D_{s0}^*(2317)$, please refer to Table I.

A. The DK interaction

The most important contribution to the DK interaction is the WT term between a D meson and a kaon.[4] In the nonrelativistic limit we can write this interaction as a standard quantum mechanical potential,

$$V_{DK}(\vec{q}) = -\frac{C_W(I)}{2f_\pi^2} \tag{1}$$

where the pion decay constant $f_\pi \approx 130$ MeV and $C_W(I)$ represents the strength of the WT interaction, which is

$$C_W(0) = 2 \quad \text{and} \quad C_W(1) = 0, \tag{2}$$

TABLE I. Mass and spin-parity of the D, K and $D_{s0}^*(2317)$ mesons.

Particles	Mass(MeV)	$I(J^P)$
$D^\pm$	1869.65	$\frac{1}{2}(0^-)$
D^0	1864.83	$\frac{1}{2}(0^-)$
$K^\pm$	493.677	$\frac{1}{2}(0^-)$
K^0	497.611	$\frac{1}{2}(0^-)$
$D_{s0}^*(2317)$	2317.7	$0(0^+)$

depending on whether we are considering the isospin $I = 0$ or $I = 1$ configuration of the DK system. The Fourier-transform of the previous potential in coordinate space is

$$V_{DK}(\vec{r}) = -\frac{C_W(I)}{2f_\pi^2}\delta^{(3)}(\vec{r}), \tag{3}$$

which has to be regularized before being used within the Schrödinger equation. A possible choice is to use a local Gaussian regulator of the type

$$V_{DK}(r;R_c) = -\frac{C_W(I)}{2f_\pi^2}\frac{e^{-(r/R_c)^2}}{\pi^{3/2}R_c^3}, \tag{4}$$

where R_c is the cutoff we use to smear the delta function. For sensible choices of the cutoff, this potential reproduces the D_{s0}^* pole. Nowadays we consider the WT interaction as the leading order (LO) term in the chiral expansion of the DK potential [33,34]. In this regard it is interesting to notice that even though LO chiral perturbation theory (ChPT) indeed indicates that the $I = 0$ DK interaction in S-wave is attractive, it happens that the next-to-leading order (NLO) correction is weakly repulsive, see, e.g., Ref. [33]. This motivates the inclusion of a short-range repulsive core in the DK interaction, as we will explain in the next paragraph.

For the present purposes a more practical approach will be to consider the DK interaction in a contact-range effective field theory, in which at LO we have the (already regularized) potential

$$V_{DK}(r;R_c) = C(R_C)\frac{e^{-(r/R_c)^2}}{\pi^{3/2}R_c^3}, \tag{5}$$

with R_c the cutoff and where the $C(R_c)$ is now a running coupling constant. The differences with a unitarized WT term are (i) that we let the cutoff R_c to float and (ii) that we consider the strength of the interaction to run with the cutoff. In this way by varying the cutoff within a sensible range, for which we choose $R_c = 1 - 3$ fm in this work, we can estimate the uncertainty in the calculations coming from subleading corrections. We advance that the cutoff variation will be tiny. Besides the variation of the cutoff, we will consider a second method to assess the error in our

[3]It must be pointed out that multimeson systems have been studied before, see, e.g., Refs. [51–55], though from different perspectives.

[4]In the framework of Ref. [49], we have checked that coupled channel interactions are small, namely, have basically no impact on the binding energy of the DDK state. Therefore, in the present work we would work in the single-channel scenario.

calculations. Inspired by the fact that ChPT predicts a repulsive core in the DK interaction at NLO (as previously mentioned), we can explicitly include this core in the potential

$$V_{DK}(\vec{r};R_c) = C_S \frac{e^{-(r/R_S)^2}}{\pi^{3/2}R_S^3} + C(R_C)\frac{e^{-(r/R_c)^2}}{\pi^{3/2}R_c^3} = C'_S e^{-(r/R_S)^2} + C'_L e^{-(r/R_c)^2}, \quad (6)$$

where C_S is a coupling constant that we set as to provide a repulsive core, i.e., we take $C'_S > |C'_L|$, and R_S is a second cutoff which fulfills the condition $R_S < R_c$. For concreteness we take $R_S = 0.5$ fm.

B. The DD interaction

The DD interaction is not known experimentally, but there are phenomenological models for it. Here we will consider the one boson exchange (OBE) potential, which provides a very simple and intuitive description of the hadron-hadron interactions. The first qualitatively successful description of the two-nucleon potential used the OBE model [56,57], and the same is true for the first speculations about the existence of heavy hadron molecules [58]. The particular version of the OBE model that we will use is the one in Ref. [59], developed for the description of heavy meson-meson and heavy meson-antimeson systems.

In the particular case of the DD two-body system, the OBE potential involves the exchange of the σ, ρ and ω mesons:

$$V_{DD}(r;\Lambda) = V_\rho(r;\Lambda) + V_\omega(r;\Lambda) + V_\sigma(r;\Lambda) \quad (7)$$

where the contribution of each light meson is regularized by means of a form factor and Λ is a cutoff. The particular contribution of each meson can be written as [59]

$$V_\sigma(r;\Lambda) = -g_\sigma^2 m_\sigma W_C\left(m_\sigma r, \frac{\Lambda}{m_\sigma}\right), \quad (8)$$

$$V_\rho(r;\Lambda) = +\vec{\tau}_1\cdot\vec{\tau}_2 g_\rho^2 m_\rho W_C\left(m_\rho r, \frac{\Lambda}{m_\rho}\right), \quad (9)$$

$$V_\omega(r;\Lambda) = +g_\omega^2 m_\omega W_C\left(m_\omega r, \frac{\Lambda}{m_\omega}\right), \quad (10)$$

where

$$W_C(x,\lambda) = \frac{e^{-x}}{4\pi x} - \lambda\frac{e^{-\lambda x}}{4\pi\lambda x} - \frac{(\lambda^2-1)}{2\lambda}\frac{e^{-\lambda x}}{4\pi}. \quad (11)$$

The masses of the bosons we use are $m_\rho = 0.770$ GeV, $m_\omega = 0.780$ GeV, $m_\sigma = 0.6$ GeV, and the couplings are $g_\rho = g_\omega = 2.6$, $g_\sigma = 3.4$. The cutoff is set by reproducing the $X(3872)$ pole, yielding $\Lambda = 1.01^{+0.19}_{-0.10}$ GeV [59]. Here for the sake of simplicity we will set the cutoff to $\Lambda = 1.0$ GeV, where we note that the cutoff dependence is weak.

III. GAUSSIAN EXPANSION METHOD TO SOLVE THE 3-BODY DDK AND 4-BODY $DDDK$ SYSTEMS

In this section we briefly explain the Gaussian expansion method (GEM) [60,61] as applied to the DDK and $DDDK$ systems. In the past the GEM has been successfully applied in hypernuclear as well as heavy-hadron systems. The focus of the manuscript is on the one hand to confirm the previous theoretical studies about the existence of a DDK bound state and to explore whether there are also bound $DDDK$ tetramers. Regarding the DDK system, it was investigated in Ref. [48] first as a DD^*_{s0} two-body system, a description which is valid provided that the size of the DDK trimer is larger than its components (in particular the D^*_{s0} meson), and second as a genuine three-body system by solving the Faddeev equations. In each case the bound state is at about $(50-60)$ MeV and $(60-100)$ MeV below the DDK threshold, respectively. Later a more complete study appeared in Ref. [49], which uses the method developed by the Valencia group [62–70] to solve the Faddeev equation [71] for the DDK system, predicting a bound state at about 90 MeV below the DDK threshold.

A. Three-body DDK system

The Schrödinger equation of the DDK 3-body system is

$$H\Psi^{\rm total}_{JM} = E\Psi^{\rm total}_{JM}, \quad (12)$$

with the corresponding Hamiltonian

$$\hat{H} = \sum_{i=1}^{3}\frac{p_i^2}{2m_i} - T_{\rm c.m.} + \sum_{1=i<j}^{3} V(r_{ij}), \quad (13)$$

where $T_{\rm c.m.}$ is the kinetic energy of the center of mass and $V(r_{ij})$ is the potential between the ith and the jth particle pair. The three Jacobi coordinates for the DDK system are shown in Fig. 1. The total wave function is a sum of the amplitudes of the three possible rearrangement of the Jacobi coordinates, i.e., of the channels ($c = 1$–3) shown in Fig. 1

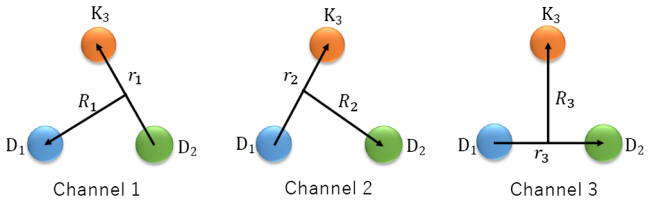

FIG. 1. The three permutations of the Jacobi coordinates for the DDK system.

$$\Psi^{\text{total}}_{JM} = \sum_{c,\alpha} C_{c,\alpha} \Psi^{c}_{JM,\alpha}(\mathbf{r}_c, \mathbf{R}_c), \tag{14}$$

where $\alpha = \{nl, NL, \Lambda, tT\}$ and $C_{c,\alpha}$ are the expansion coefficients. Here l and L are the orbital angular momenta for the coordinates r and R, t is the isospin of the two-body subsystem in each channel, Λ and T are the total orbital angular momentum and isospin, n and N are the numbers of Gaussian basis function corresponding to coordinates r and R, respectively. For the DD and DK two-body potentials we refer to Sec. II. The eigenenergy E and coefficients are determined by the Rayleigh-Ritz variational principle. Considering that the two D mesons are identical, the total wave function should be symmetric with respect to the exchange of the two D mesons, which requires that

$$P_{12}\Psi^{\text{total}}_{JM} = \Psi^{\text{total}}_{JM}, \tag{15}$$

and P_{12} is the exchange operator of particles 1 and 2. The wave function of each channel has the following form

$$\Psi^{c}_{JM,\alpha}(\mathbf{r}_c, \mathbf{R}_c) = H^{c}_{T,t} \otimes [\Phi^{c}_{lL,\Lambda}]_{JM}, \tag{16}$$

where $H^{c}_{T,t}$ is the isospin wave function, and $\Phi^{c}_{lL,\Lambda}$ the spacial wave function. The total isospin wave function reads as

$$\begin{aligned} H^{c=1}_{T,t} &= [[\eta_{\frac{1}{2}}(D_2)\eta_{\frac{1}{2}}(K_3)]_{t_1}\eta_{\frac{1}{2}}(D_1)]_{\frac{1}{2}}, \\ H^{c=2}_{T,t} &= [[\eta_{\frac{1}{2}}(D_1)\eta_{\frac{1}{2}}(K_3)]_{t_2}\eta_{\frac{1}{2}}(D_2)]_{\frac{1}{2}}, \\ H^{c=3}_{T,t} &= [[\eta_{\frac{1}{2}}(D_1)\eta_{\frac{1}{2}}(D_2)]_{t_3}\eta_{\frac{1}{2}}(K_3)]_{\frac{1}{2}}, \end{aligned} \tag{17}$$

where η is the isospin wave function of each particle. The spacial wave function $\Phi^{c}_{lL,\Lambda}$ is given in terms of the Gaussian basis functions

$$\Phi^{c}_{lL,\Lambda}(\mathbf{r}_c, \mathbf{R}_c) = [\phi^{G}_{n_c l_c}(\mathbf{r}_c)\psi^{G}_{N_c L_c}(\mathbf{R}_c)]_{\Lambda}, \tag{18}$$

$$\phi^{G}_{nlm}(\mathbf{r}_c) = N_{nl} r_c^{l} e^{-\nu_n r_c^2} Y_{lm}(\hat{r}_c), \tag{19}$$

$$\psi^{G}_{NLM}(\mathbf{R}_c) = N_{NL} R_c^{L} e^{-\lambda_n R_c^2} Y_{LM}(\hat{R}_c). \tag{20}$$

Here $N_{nl}(N_{NL})$ are the normalization constants of the Gaussian basis and the range parameters ν_n and λ_n are given by

$$\begin{aligned} &\nu_n = 1/r_n^2, \qquad r_n = r_{\min} a^{n-1} \quad (n = 1, n_{\max}), \\ &\lambda_N = 1/R_N^2, \qquad R_N = R_{\min} A^{N-1} \quad (N = 1, N_{\max}), \end{aligned} \tag{21}$$

in which $\{n_{\max}, r_{\min}, a \text{ or } r_{\max}\}$ and $\{N_{\max}, R_{\min}, A \text{ or } R_{\max}\}$ are Gaussian basis parameters. After the basis expansion, the Schrödinger equation of this system is transformed into a generalized matrix eigenvalue problem:

TABLE II. Quantum numbers of different Jacobi coordinate channels $(c = 1 - 3)$ of the DDK $I(J^P) = \frac{1}{2}(0^-)$ state and the number of Gaussian basis used. Note that channel 1 and channel 2 are the same.

c	l	L	Λ	t	T	J	P	$n_{\max}$	$N_{\max}$
1(2)	0	0	0	0	$\frac{1}{2}$	0	$-$	10	10
1(2)	0	0	0	1	$\frac{1}{2}$	0	$-$	10	10
3	0	0	0	1	$\frac{1}{2}$	0	$-$	10	10

$$[T^{ab}_{\alpha\alpha'} + V^{ab}_{\alpha\alpha'} - EN^{ab}_{\alpha\alpha'}]C_{b,\alpha'} = 0. \tag{22}$$

Here, $T^{ab}_{\alpha\alpha'}$ is the kinetic matrix element, $V^{ab}_{\alpha\alpha'}$ is the potential matrix element and $N^{ab}_{\alpha\alpha'}$ is the normalization matrix element.

The quantum numbers of all the allowed configurations are determined by angular momentum conservation, isospin conservation, parity conservation, and Bose-Einstein statistics. Given that we only consider S-wave interactions, and only the DK interaction in $I = 0$ is dominant, we obtain the allowed configurations shown in Table II, where we have given explicitly the values of $n_{\max}$ and $N_{\max}$, which are sufficiently large to ensure convergence. The DDK system that we are interested in has isospin $1/2$ and spin parity 0^-.

B. Four-body *DDDK* system

A generic four-body system has 18 Jacobi coordinates. In the $DDDK$ system, owing to the fact that there are three identical D mesons, the possible configurations of the Jacobi coordinates reduce to three K-type channels and one H-type channel, see Fig. 2. There are 4 identical Jacobi coordinates for each K-type channel and 6 identical Jacobi coordinates for the H-type channel. The total wave function of this $DDDK$ system is

$$\Psi^{\text{total}}_{I(J^P)} = \sum_{c,\alpha} A_{c,\alpha} \Psi^{c}_{\alpha}(\boldsymbol{r}_c, \boldsymbol{R}_c, \boldsymbol{\rho}_c), \qquad c = 1 - 18, \tag{23}$$

and the wave function in each Jacobi channel reads

$$\Psi^{c}_{\alpha}(\boldsymbol{r}_c, \boldsymbol{R}_c, \boldsymbol{\rho}_c) = H^{c}_{t,T,I} \otimes \Phi^{c,JP}_{lL\lambda,\sigma\Lambda}. \tag{24}$$

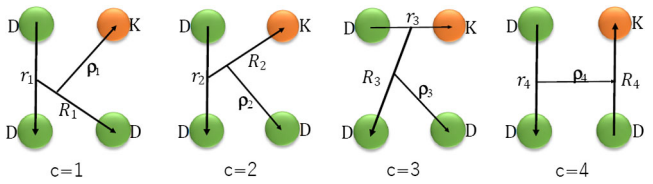

FIG. 2. Jacobi coordinates for the rearrangement channels $(c = 1 - 4)$ of the $DDDK$ system. The three D mesons are to be symmetrized.

TABLE III. Quantum numbers of different Jacobi coordinate channels ($c = 1 - 4$) of the $DDDK$ $I(J^P) = 1(0^+)$ state and the number of Gaussian basis used. The identical channels have the same configuration. The number in the brackets denotes the alternative value.

c	l	L	λ	σ	L	t	T	I	J	P	$n_{\max}$	$N_{\max}$	$\nu_{\max}$
1	0	0	0	0	0	1	$\frac{1}{2}(\frac{3}{2})$	1	0	+	10	10	10
2	0	0	0	0	0	1	$\frac{1}{2}(\frac{3}{2})$	1	0	+	10	10	10
3	0	0	0	0	0	0(1)	$\frac{1}{2}(\frac{3}{2})$	1	0	+	10	10	10
4	0	0	0	0	0	1	0 (1)	1	0	+	10	10	10

Here t, T, I are the isospin of the coordinates r, R and ρ in each channel; l, L and λ are the orbital angular momenta for the coordinates r, R and ρ, while σ is the coupling of l and L, Λ is the coupling of σ and λ, and J, P is the total angular momentum and parity. The Gaussian basis and parameters are in the same form as those in the 3-body system, which are

$$\Phi^c_{lL\lambda,\sigma\Lambda} = [\phi^G_{n_c l_c}(\boldsymbol{r}_c)\psi^G_{N_c L_c}(\boldsymbol{R}_c)]_{\sigma_c}\varphi^G_{\nu_c\lambda_c}(\boldsymbol{\rho}_c)]_\Lambda, \quad (25)$$

$$\phi^G_{nlm}(\boldsymbol{r}_c) = N_{nl} r_c^l e^{-\nu_n r_c^2} Y_{lm}(\hat{r}_c), \quad (26)$$

$$\psi^G_{NLM}(\boldsymbol{R}_c) = N_{NL} R_c^L e^{-\lambda_N R_c^2} Y_{LM}(\hat{R}_c), \quad (27)$$

$$\varphi^G_{\nu\lambda\mu}(\boldsymbol{\rho}_c) = N_{\nu\lambda}\rho_c^\lambda e^{-\omega_\nu \rho_c^2} Y_{\lambda\mu}(\hat{\rho}_c). \quad (28)$$

Here $N_{nl}(N_{NL})$ are the normalization constants of the Gaussian basis and the range parameters ν_n, λ_n, and ω_ν are given by

$$\begin{aligned} \nu_n &= 1/r_n^2, \qquad r_n = r_{\min} a^{n-1}(n = 1, n_{\max}), \\ \lambda_N &= 1/R_N^2, \qquad R_N = R_{\min} A^{N-1}(N = 1, N_{\max}), \\ \omega_\nu &= 1/\rho_\nu^2, \qquad \rho_\nu = \rho_{\min}\alpha^{\nu-1}(\nu = 1, \nu_{\max}). \end{aligned} \quad (29)$$

Since we are considering only S-wave interactions, we have $J = l = L_\lambda = \sigma = \Lambda = 0$, and the parity is +. The procedure to determine the allowed configurations for the $DDDK$ system is the same as the DDK case. The 4-body $DDDK$ configurations are shown in Table III, together with the number of Gaussian basis used.

IV. PREDICTIONS

In this section we discuss the predictions we make for the DDK and $DDDK$ bound states. With the two-body inputs of Sec. II and the three(four)-body configurations detailed in Sec. III, we can predict the existence of DDK and $DDDK$ bound states. The outcome is that the DDK trimer will bind by about 70 MeV and the $DDDK$ tetramer by about 100 MeV, with variations of a few MeV at most, stemming from the uncertainties in the DK and DD potentials.

A. Solving the DDK and $DDDK$ systems

The two basic input blocks for the calculation of the DDK and $DDDK$ systems are the DK and DD interactions, of which the DK one is the most important factor when it comes to binding. The DK potential contains the running coupling $C(R_c)$ and the cutoff R_c, where $R_c = (1 - 3)$ fm and $C(R_c)$ is determined from the condition of reproducing the well-known $D^*_{s0}(2317)$ as a DK bound state with a binding energy of 45 MeV. In addition there are two additional parameters, the coupling C_S and the short-range radius $R_s = 0.5$ fm, which are used to estimate the uncertainties in the DK potential. We study three combinations of R_S and R_c, which can be consulted in Table IV,

TABLE IV. Binding energies (in units of MeV) of DDK and $DDDK$ systems with and without the DD interaction for different combinations of R_S, R_c, C'_S, and C'_L. The couplings are in units of MeV.

C'_S	C'_L	E_2	E_3 (only V_{DK})	$E_3(V_{DK} + V_{DD})$	E_4 (only V_{DK})	$E_4(V_{DK} + V_{DD})$
		$R_S = 0.5$ fm		$R_c = 1$ fm		
0	−320.1	−45.0	−65.8	−71.2	−89.4	−106.8
500	−455.4	−45.0	−65.8	−70.4	−89.2	−103.5
1000	−562.6	−45.0	−65.7	−69.7	−88.8	−101.4
3000	−838.7	−45.0	−65.0	−68.4	−87.0	−97.3
		$R_S = 0.5$ fm		$R_c = 2$ fm		
0	−149.1	−45.0	−66.0	−68.8, −45.1	−88.7, −66.3	−97.6, −70.7
500	−178.4	−45.0	−65.9	−68.2, −45.5	−88.5, −66.7	−95.5, −70.9
1000	−195.0	−45.0	−65.8, −45.2	−67.9, −45.8	−88.2, −66.9	−94.5, −71.2
3000	−225.9	−45.0	−65.3, −45.6	−67.2, −46.6	−87.0, −67.0	−92.6, −71.7
		$R_S = 0.5$ fm		$R_c = 3$ fm		
0	−107.0	−45.0	−66.2, −47.3	−68.0, −48.3	−88.8, −70.2	−94.4, −74.3
500	−119.4	−45.0	−66.2, −48.2	−67.7, −49.3	−88.7, −71.0	−93.2, −74.8
1000	−125.6	−45.0	−66.1, −48.7	−67.5, −49.8	−88.4, −71.3	−92.5, −75.2
3000	−136.2	−45.0	−65.8, −49.4	−67.1, −50.7	−87.6, −71.7	−91.4, −75.7

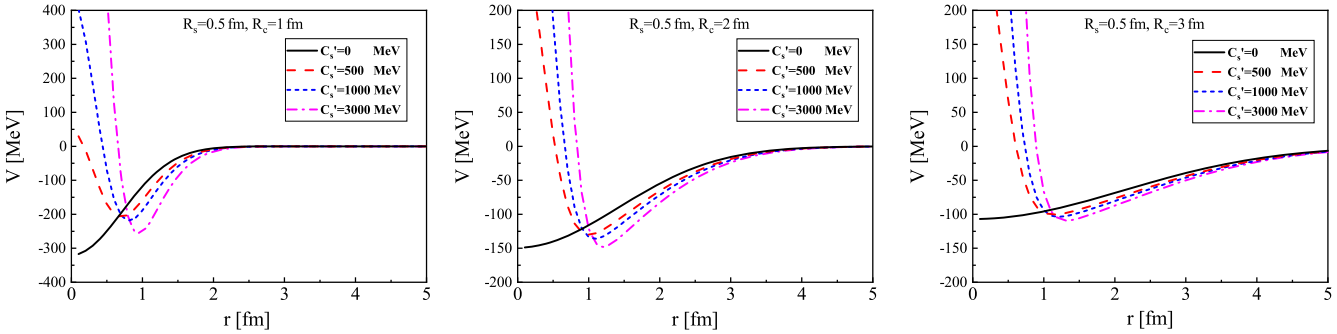

FIG. 3. Isospin $t = 0$ DK potential as a function of the distance between D and K for different R_S, R_c, and C'_S. The coupling C'_L in each case is determined by reproducing the D^*_{s0}.

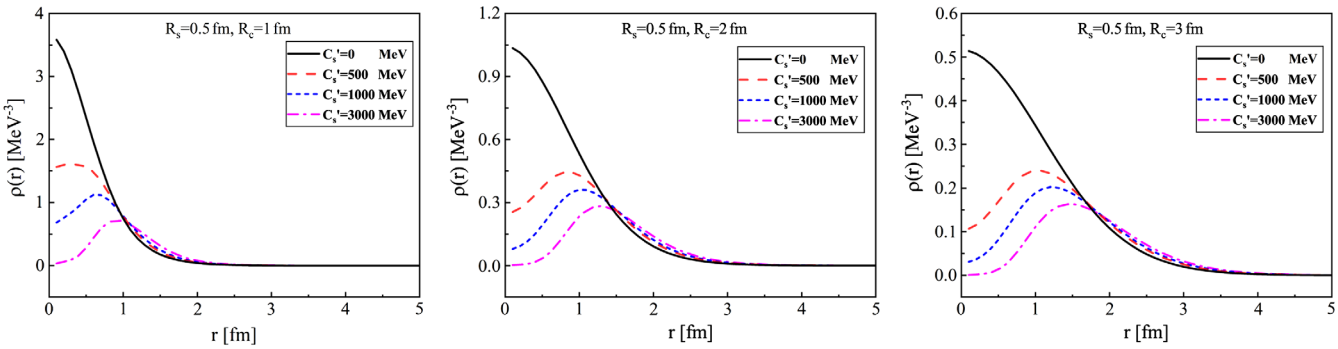

FIG. 4. Density profile of the DK molecule corresponding to the potentials of Fig. 3.

where we also list the values of the couplings C_R and $C(R_c)$ and the binding energies of the DDK and $DDDK$ systems. The different potentials investigated are shown in Fig. 3 and the probability density distributions of the DK pair corresponding to the potentials are shown in Fig. 4.

A few comments about the results of Table IV are in order. The first thing we notice is that the impact of the DD interaction is mild. It makes the DDK and $DDDK$ systems more bound, but only by a few MeV. This is a bit relieving as the DD interaction is not well known. The second interesting observation is that the existence of the DDK and $DDDK$ bound states is rather robust with respect to the likely existence of a short-range repulsive core. In other words, the existence of the DDK and $DDDK$ bound states is almost guaranteed as long as the D^*_{s0} is dominantly a DK bound state (we will later check that this will still be the case even if the D^*_{s0} is a compact $\bar{c}s$ state). The third observation is that as the range of the attraction becomes larger, two bound state solutions appear instead of one, with the deepest bound one becoming slightly shallower.

In Table V we show the root mean square (RMS) radius of the DK and DDK systems as well as the expectation values of the kinetic and potential terms. The RMS radius of the D^*_{s0}, which ranges from 1.2 to 2.6 fm, increases with the cutoff R_c and with the coupling C_S of the short-range repulsive core. In the DDK system, the RMS radius of the DK pair is slightly larger than its counterpart in the D^*_{s0}. The RMS radius of the DD system also increases if we increase the cutoff R_c or the coupling C_S. We notice that the geometry of the DDK system is more or less of a proper triangle, which agrees qualitatively with the findings of Ref. [49]. From the last two columns of Table V, it is clear that the DD interaction is weakly attractive, accounting for only a few MeV of the total potential energy.

B. Solving the $DDDK$ system as an equivalent DDD^*_{s0} system

If the separation of the DK pair within the DDK trimer and $DDDK$ tetramer is comparable to or larger than the expected size of the D^*_{s0}, in a first approximation it will be possible to treat the D^*_{s0} as a pointlike particle, with its compound structure providing subleading corrections to this pointlike approximation. From Table V we can see that the RMS of the DK subsystem in the DDK and $DDDK$ systems is similar to that of the D^*_{s0} as a DK molecule. In this regard we notice that in Ref. [48] the D^*_{s0} is approximated as pointlike, where the interaction between the D and D^*_{s0} is mediated by one kaon exchange and is strong enough to form a bound state. This DD^*_{s0} molecule is predicted to be 50–60 MeV below the DDK threshold, to be compared with 65 MeV when we consider it as a

TABLE V. Root mean square (RMS) radius (in units of fm) of DK and DDK systems, the expectation values (in units of MeV) of the kinetic term, DK and DD interactions with various parameters R_S. R_c, C'_S, and C'_L. The couplings are in units of MeV.

C'_S	C'_L	$r_2(DK)$	$r_3(DK)$	$r_3(DD)$	$\langle T\rangle$	$\langle V_{DK}\rangle$	$\langle V_{DD}\rangle$
		$R_S = 0.5$ fm		$R_c = 1$ fm			
0	−320.1	1.28	1.32	1.36	124.37	−189.61	−5.98
500	−455.4	1.39	1.44	1.47	99.51	−164.83	−5.03
1000	−562.6	1.46	1.53	1.54	91.43	−156.67	−4.51
3000	−838.7	1.61	1.69	1.68	93.24	−157.80	−3.82
		$R_S = 0.5$ fm		$R_c = 2$ fm			
0	−149.1	1.74	1.80	1.80	60.20	−125.74	−3.23
500	−178.4	1.91	1.98	1.96	51.00	−116.59	−2.64
1000	−195.0	1.99	2.07	2.04	50.63	−116.12	−2.43
3000	−225.9	2.13	2.22	2.15	53.61	−118.59	−2.24
		$R_S = 0.5$ fm		$R_c = 3$ fm			
0	−107.0	2.13	2.19	2.17	39.49	−105.35	−2.13
500	−119.4	2.31	2.38	2.34	34.80	−100.73	−1.77
1000	−125.6	2.37	2.47	2.42	34.90	−100.77	−1.65
3000	−136.2	2.53	2.61	2.53	36.66	−102.24	−1.54

genuine DDK three-body state and ignore the DD interaction (see Table IV). This indicates that the predictions of the pointlike approximation are reasonably good (for such a simple approximation) and that the compound structure of the D^*_{s0} provides additional attraction. In the following lines we will extend the ideas of Ref. [48] to the $DDDK$ tetramer, i.e., we will treat it as a three-body DDD^*_{s0} system where the D^*_{s0} is assumed to be a compact meson. To do this, we first reproduce the two-body calculation of Ref. [48], but in coordinate space, and then study the three-body DDD^*_{s0} system using the GEM.

The interaction of DD^*_{s0} is attractive and reads as

$$V_{OKE}(\vec{q}) = -h^2\frac{\omega_K^2}{f_\pi^2}\frac{1}{\mu_K^2+\vec{q}^2}, \tag{30}$$

where $\omega_K = m_{D^*_{s0}} - m_D$ and the effective kaon mass $\mu_K = \sqrt{m_K^2 - \omega_K^2}$. As in Ref. [48], we take $h = 0.7$ and $f_\pi = 130$ MeV. We regularize the potential by multiplying it with a dipole form factor of the type:

$$F_D(q^2) = \frac{(\Lambda^2 - m_K^2)^2}{(\Lambda^2 - q^2)^2}. \tag{31}$$

TABLE VI. Binding energies (in units of MeV) of DD^*_{s0} and DDD^*_{s0} systems with different cutoff Λ' (in units of GeV).

Λ'	$B_{DD^*_{s0}}$	$B_{DDD^*_{s0}}$ (only $V_{DD^*_{s0}}$)	$B_{DDD^*_{s0}}$ ($V_{DD}+V_{DD^*_{s0}}$)
0.8	−5.1	−11.5	−13.9
1.0	−8.5	−18.9	−22.5
1.2	−11.7	−25.8	−30.3
1.4	−14.5	−31.9	−37.2
1.6	−17.0	−37.2	−43.3

After the inclusion of this form factor, the DD^*_{s0} potential in coordinate space reads

$$V_{DD^*_{so}}(r) = -h^2\frac{\omega_K^2}{f_\pi^2}\left(\frac{e^{-\mu_K r}}{4\pi r} - \frac{e^{-\Lambda' r}}{4\pi r} - \frac{(\Lambda'^2 - \mu_K^2)e^{-\Lambda' r}}{8\pi\Lambda'}\right), \tag{32}$$

where we define Λ' as

$$\Lambda'^2 = \Lambda^2 - q_0^2 = \Lambda^2 - \omega_K^2. \tag{33}$$

Using the above DD^*_{s0} potential and the DD potential provided by the OBE model, we can check whether the three-body DDD^*_{s0} system binds. The binding energies we obtain with different cutoffs are tabulated in Table VI.

With the effective cutoff Λ' ranging from 0.8–1.6 GeV, the results of Table. VI indicate that the DDD^*_{s0} bound state is located about (65 – 90) MeV below the $DDDK$ threshold. This is to be compared with 100 MeV for the full four-body calculation, see Table IV for details. That is, as happened with the DD^*_{s0}/DDK system, the approximation that the D^*_{s0} is a compact state results in underbinding for the $DDD^*_{s0}/DDDK$ system, but not much.

V. SUMMARY

In this manuscript we argued that the DK interaction is attractive enough as to generate DK, DDK, and $DDDK$ bound states. For this we began by assuming that the $D^*_{s0}(2317)$ is a DK molecule, which determines in turn the DK interaction. Then, by means of the Gaussian expansion method [60,61] (a method for few-body calculations), we have addressed the question of whether one can build up multi-component molecular states, similar to the formation of atomic nuclei from clusters of nucleons bound by the

nucleon-nucleon interaction. The answer is yes. We find a bound DDK trimer and a $DDDK$ tetramer. The prediction of this trimer confirms the previous calculations of Refs. [48,49], while the prediction of the tetramer is novel to the present work.

We have checked the robustness of these predictions against a series of uncertainties. While the DK interaction is well constrained by the existence of the $D_{s0}^*(2317)$ and chiral perturbation theory, the DD interaction is considerably less well-known. Yet it also enters the calculations. We chose to describe the DD potential in terms of the OBE model, in which the DD interaction turns out to be mildly attractive and has a minor impact on the binding energy of the trimer and tetramer states. The DK potential, though well-known, is still subject to subleading corrections, which we take into account by varying the exact form of this potential. As expected from the fact that we are dealing with subleading corrections, the predictions are almost left unchanged by these variations.

In addition, we have studied a rather unlikely scenario that the $D_{s0}^*(2317)$ is dominantly a genuine $c\bar{s}$ state. Nonetheless, even in such a case, we still predict DD_{s0}^* and DDD_{s0}^* bound states with the same quantum numbers as the DDK trimer and $DDDK$ tetramer, but this time located at approximately $(50-62)$ and $(60-90)$ MeV below the DDK and $DDDK$ thresholds (instead of 70 and 100 MeV when the D_{s0}^* is a molecular meson). The binding mechanism is the long-range one-kaon-exchange potential in the DD_{s0}^* system: owing to the mass difference between the D and D_{s0}^* mesons, the kaon is exchanged near the mass shell, leading to an enhancement in the range of the potential [48].

Although the existence of the DDK and $DDDK$ bound states seems to be quite robust, the question of where to find them is much more challenging. If we now focus on the DDK state, the experimental discovery of the $D_{s0}^*(2317)$ gives a clue. As already argued in Ref. [49], but awaiting for a concrete study, the DDK state can decay into DD_s^* or D^*D_s in P-wave. Therefore one may look for inclusive combinations of three particles $DD_s\pi$ and search for structures in the corresponding invariant mass distributions. Given enough statistics, there should be a possibility to discover it in the e^+e^- collision data collected by Belle or BelleII or in the pp collision data collected at the LHC.

It is well known that heavy quark spin and flavor symmetries relate the DK interaction to those of D^*K, $B\bar{K}$, and $B^*\bar{K}$. This is consistent with the existence of the $D_{s1}(2460)$. The bottom counterparts of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ have been predicted in a number of studies [20,33,72] and confirmed by lattice QCD simulations [73]. As a result, we naively expect the existence of the heavy quark symmetry partners of the DDK and $DDDK$ states. At this moment, given the accessible center of mass energies at current facilities, and the simplification that both the D and K are 0^- mesons that only decay weakly, we believe that they should be of top priority both experimentally and theoretically.

ACKNOWLEDGMENTS

This work is partly supported by the National Natural Science Foundation of China under Grants No. 11975041 and No. 11735003, the Fundamental Research Funds for the Central Universities, and the Thousand Talents Plan for Young Professionals.

[1] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **90**, 242001 (2003).
[2] D. Besson *et al.* (CLEO Collaboration), Phys. Rev. D **68**, 032002 (2003); **75**, 119908(E) (2007).
[3] P. Krokovny *et al.* (Belle Collaboration), Phys. Rev. Lett. **91**, 262002 (2003).
[4] W. A. Bardeen, E. J. Eichten, and C. T. Hill, Phys. Rev. D **68**, 054024 (2003).
[5] M. A. Nowak, M. Rho, and I. Zahed, Acta Phys. Pol. B **35**, 2377 (2004).
[6] E. van Beveren and G. Rupp, Phys. Rev. Lett. **91**, 012003 (2003).
[7] Y.-B. Dai, C.-S. Huang, C. Liu, and S.-L. Zhu, Phys. Rev. D **68**, 114011 (2003).
[8] S. Narison, Phys. Lett. B **605**, 319 (2005).
[9] A. P. Szczepaniak, Phys. Lett. B **567**, 23 (2003).
[10] T. E. Browder, S. Pakvasa, and A. A. Petrov, Phys. Lett. B **578**, 365 (2004).
[11] T. Barnes, F. E. Close, and H. J. Lipkin, Phys. Rev. D **68**, 054006 (2003).
[12] H.-Y. Cheng and W.-S. Hou, Phys. Lett. B **566**, 193 (2003).
[13] Y.-Q. Chen and X.-Q. Li, Phys. Rev. Lett. **93**, 232001 (2004).
[14] V. Dmitrasinovic, Phys. Rev. Lett. **94**, 162002 (2005).
[15] J.-R. Zhang, Phys. Lett. B **789**, 432 (2019).
[16] K. Terasaki, Phys. Rev. D **68**, 011501 (2003).
[17] L. Maiani, F. Piccinini, A. Polosa, and V. Riquer, Phys. Rev. D **71**, 014028 (2005).
[18] E. E. Kolomeitsev and M. F. M. Lutz, Phys. Lett. B **582**, 39 (2004).
[19] J. Hofmann and M. F. M. Lutz, Nucl. Phys. **A733**, 142 (2004).

[20] F.-K. Guo, P.-N. Shen, H.-C. Chiang, R.-G. Ping, and B.-S. Zou, Phys. Lett. B **641**, 278 (2006).
[21] D. Gamermann, E. Oset, D. Strottman, and M. J. Vicente Vacas, Phys. Rev. D **76**, 074016 (2007).
[22] F.-K. Guo, C. Hanhart, S. Krewald, and U.-G. Meissner, Phys. Lett. B **666**, 251 (2008).
[23] F.-K. Guo, C. Hanhart, and U.-G. Meissner, Eur. Phys. J. A **40**, 171 (2009).
[24] M. Cleven, F.-K. Guo, C. Hanhart, and U.-G. Meissner, Eur. Phys. J. A **47**, 19 (2011).
[25] A. Martinez Torres, L. R. Dai, C. Koren, D. Jido, and E. Oset, Phys. Rev. D **85**, 014027 (2012).
[26] D.-L. Yao, M.-L. Du, F.-K. Guo, and U.-G. Meißner, J. High Energy Phys. 11 (2015) 058.
[27] Z.-H. Guo, U.-G. Meißner, and D.-L. Yao, Phys. Rev. D **92**, 094008 (2015).
[28] M. Albaladejo, P. Fernandez-Soler, F.-K. Guo, and J. Nieves, Phys. Lett. B **767**, 465 (2017).
[29] M.-L. Du, F.-K. Guo, U.-G. Meißner, and D.-L. Yao, Eur. Phys. J. C **77**, 728 (2017).
[30] X.-Y. Guo, Y. Heo, and M. F. M. Lutz, Phys. Rev. D **98**, 014510 (2018).
[31] M. Albaladejo, P. Fernandez-Soler, J. Nieves, and P. G. Ortega, Eur. Phys. J. C **78**, 722 (2018).
[32] M. Altenbuchinger and L.-S. Geng, Phys. Rev. D **89**, 054008 (2014).
[33] M. Altenbuchinger, L. S. Geng, and W. Weise, Phys. Rev. D **89**, 014026 (2014).
[34] L. S. Geng, N. Kaiser, J. Martin-Camalich, and W. Weise, Phys. Rev. D **82**, 054022 (2010).
[35] P. Wang and X. G. Wang, Phys. Rev. D **86**, 014030 (2012).
[36] Y.-R. Liu, X. Liu, and S.-L. Zhu, Phys. Rev. D **79**, 094026 (2009).
[37] Z.-H. Guo, L. Liu, U.-G. Meißner, J. A. Oller, and A. Rusetsky, Proc. Sci., LATTICE2018 (**2018**) 055.
[38] Z.-H. Guo, L. Liu, U.-G. Meißner, J. A. Oller, and A. Rusetsky, Eur. Phys. J. C **79**, 13 (2019).
[39] L. Liu, K. Orginos, F.-K. Guo, C. Hanhart, and U.-G. Meissner, Phys. Rev. D **87**, 014508 (2013).
[40] D. Mohler, C. B. Lang, L. Leskovec, S. Prelovsek, and R. M. Woloshyn, Phys. Rev. Lett. **111**, 222001 (2013).
[41] C. B. Lang, L. Leskovec, D. Mohler, S. Prelovsek, and R. M. Woloshyn, Phys. Rev. D **90**, 034510 (2014).
[42] A. Martínez Torres, E. Oset, S. Prelovsek, and A. Ramos, J. High Energy Phys. 05 (2015) 153.
[43] G. S. Bali, S. Collins, A. Cox, and A. Schäfer, Phys. Rev. D **96**, 074501 (2017).
[44] F.-K. Guo, EPJ Web Conf. **202**, 02001 (2019).
[45] L. Ma, Q. Wang, and U.-G. Meißner, Chin. Phys. C **43**, 014102 (2019).
[46] X.-L. Ren, B. B. Malabarba, L.-S. Geng, K. P. Khemchandani, and A. Martínez Torres, Phys. Lett. B **785**, 112 (2018).
[47] V. R. Debastiani, J. M. Dias, and E. Oset, Phys. Rev. D **96**, 016014 (2017).
[48] M. Sanchez Sanchez, L.-S. Geng, J.-X. Lu, T. Hyodo, and M. P. Valderrama, Phys. Rev. D **98**, 054001 (2018).
[49] A. Martinez Torres, K. Khemchandani, and L.-S. Geng, Phys. Rev. D **99**, 076017 (2019).
[50] E. Hiyama, M. Kamimura, A. Hosaka, H. Toki, and M. Yahiro, Phys. Lett. B **633**, 237 (2006).
[51] L. Roca and E. Oset, Phys. Rev. D **82**, 054013 (2010).
[52] J. Yamagata-Sekihara, L. Roca, and E. Oset, Phys. Rev. D **82**, 094017 (2010); **85**, 119905(E) (2012).
[53] C. W. Xiao, M. Bayar, and E. Oset, Phys. Rev. D **86**, 094019 (2012).
[54] E. Oset, A. Martinez Torres, K. P. Khemchandani, L. Roca, and J. Yamagata, Prog. Part. Nucl. Phys. **67**, 455 (2012).
[55] E. Oset *et al.*, Acta Phys. Pol. B **47**, 357 (2016).
[56] R. Machleidt, K. Holinde, and C. Elster, Phys. Rep. **149**, 1 (1987).
[57] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).
[58] M. Voloshin and L. Okun, JETP Lett. **23**, 333 (1976).
[59] M.-Z. Liu, T.-W. Wu, M. Pavon Valderrama, J.-J. Xie, and L.-S. Geng, Phys. Rev. D **99**, 094018 (2019).
[60] M. Kamimura, Phys. Rev. A **38**, 621 (1988).
[61] E. Hiyama, Y. Kino, and M. Kamimura, Prog. Part. Nucl. Phys. **51**, 223 (2003).
[62] A. Martinez Torres, K. P. Khemchandani, and E. Oset, Phys. Rev. C **77**, 042203 (2008).
[63] K. P. Khemchandani, A. Martinez Torres, and E. Oset, Eur. Phys. J. A **37**, 233 (2008).
[64] A. Martinez Torres, K. P. Khemchandani, L. S. Geng, M. Napsuciale, and E. Oset, Phys. Rev. D **78**, 074031 (2008).
[65] A. Martinez Torres, K. P. Khemchandani, and E. Oset, Phys. Rev. C **79**, 065207 (2009).
[66] A. Martinez Torres, K. Khemchandani, D. Gamermann, and E. Oset, Phys. Rev. D **80**, 094012 (2009).
[67] A. Martinez Torres, K. P. Khemchandani, U.-G. Meissner, and E. Oset, Eur. Phys. J. A **41**, 361 (2009).
[68] A. Martinez Torres and D. Jido, Phys. Rev. C **82**, 038202 (2010).
[69] A. Martinez Torres, K. P. Khemchandani, D. Jido, and A. Hosaka, Phys. Rev. D **84**, 074027 (2011).
[70] A. Martinez Torres, D. Jido, and Y. Kanada-En'yo, Phys. Rev. C **83**, 065205 (2011).
[71] L. D. Faddeev, Sov. Phys. JETP **12**, 1014 (1961) [Zh. Eksp. Teor. Fiz. **39**, 1459 (1960)].
[72] F.-K. Guo, P.-N. Shen, and H.-C. Chiang, Phys. Lett. B **647**, 133 (2007).
[73] C. B. Lang, D. Mohler, S. Prelovsek, and R. M. Woloshyn, Phys. Lett. B **750**, 17 (2015).