Tests of the Z_c-like Laplace sum rule results using finite energy sum rule at NLO

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(Received 15 April 2022; accepted 16 May 2022; published 21 June 2022)

In this note, we use local duality finite energy sum rule to test the validity of the Laplace sum rules results truncated at the dimension-six condensates for the estimates of the masses and couplings of the Z_c-like ground states in Albuquerque *et al.* [Phys. Rev. D **103**, 074015 (2021)] by taking the example of the D^*D molecule configuration. We confirm the existence of an eventual $(D^*D)_1$ radial excitation with a mass around 5700 MeV and coupling of 197(25) keV to the current which may mask the eventual $Z_c(4430)$ radial excitation candidate [named $(D^*D)_0$ in Albuquerque *et al.*] having a relatively small coupling $f_{(D^*D)_0} = 46(56)$ keV. We add more explanations on the estimates in Albuquerque *et al.* from Laplace sum rules and comment on the results in Wang [arXiv:2202.06058; Commun. Theor. Phys. **63**, 325 (2015)].

DOI: 10.1103/PhysRevD.105.114035

I. INTRODUCTION

In Ref. [1], we have estimated the masses and couplings of Z_c-like states within different configurations of their eventual nature using Laplace sum rules (LSR) [2–5] *à la* Shifman-Vainshtein-Zakharov (SVZ) [6,7] and their ratios at Next-to-Leading Order (NLO) of Perturbative (PT) series:

$$\mathcal{L}_n^c(\tau,\mu) = \int_{t_0}^{t_c} dt t^n e^{-t\tau} \frac{1}{\pi} \mathrm{Im}\Pi^{(1)}_{\mathcal{H}}(t,\mu),$$
$$\mathcal{R}_n^c(\tau) = \frac{\mathcal{L}_{n+1}^c}{\mathcal{L}_n^c}. \quad (1)$$

m_c is the charm quark mass, τ is the LSR variable, $n = 0, 1$ is the degree of moments, and t_0 is the quark/hadronic threshold. t_c is the threshold of the "QCD continuum" that parametrizes, from the discontinuity of the Feynman diagrams, the spectral function $\mathrm{Im}\Pi^{(1)}_{\mathcal{H}}(t, m_c^2, \mu^2)$. $\Pi^{(1)}_{\mathcal{H}}(t, m_c^2, \mu^2)$ is the transverse scalar correlator corresponding to a spin one hadron:

$$\Pi^{\mu\nu}_{\mathcal{H}}(q^2) = i \int d^4x e^{-iqx} \langle 0|\mathcal{T}\mathcal{O}^{\mu}_{\mathcal{H}}(x)(\mathcal{O}^{\nu}_{\mathcal{H}}(0))^{\dagger}|0\rangle,$$
$$\equiv -\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\Pi^{(1)}_{\mathcal{H}}(q^2) + \frac{q^\mu q^\nu}{q^2}\Pi^{(0)}_{\mathcal{H}}(q^2), \quad (2)$$

where, e.g., in the case of the D^*D configuration, the hadronic current reads:

$$\mathcal{O}^{\nu}_{\mathcal{H}} = (\bar{c}\gamma_\mu q)(\bar{u} i\gamma_5 c). \quad (3)$$

We have used the usual minimal duality ansatz:

$$\frac{1}{\pi}\mathrm{Im}\Pi_{\mathcal{H}} \simeq f^2_{\mathcal{H}} M^8_{\mathcal{H}} \delta(t - M^2_{\mathcal{H}}) + \Theta(t - t_c)\text{"Continuum"}, \quad (4)$$

for parametrizing the molecule/four-quark state spectral function. $M_{\mathcal{H}}$ and $f_{\mathcal{H}}$ are the lowest ground state mass and coupling analogue to $f_\pi = 131$ MeV. The "continuum"

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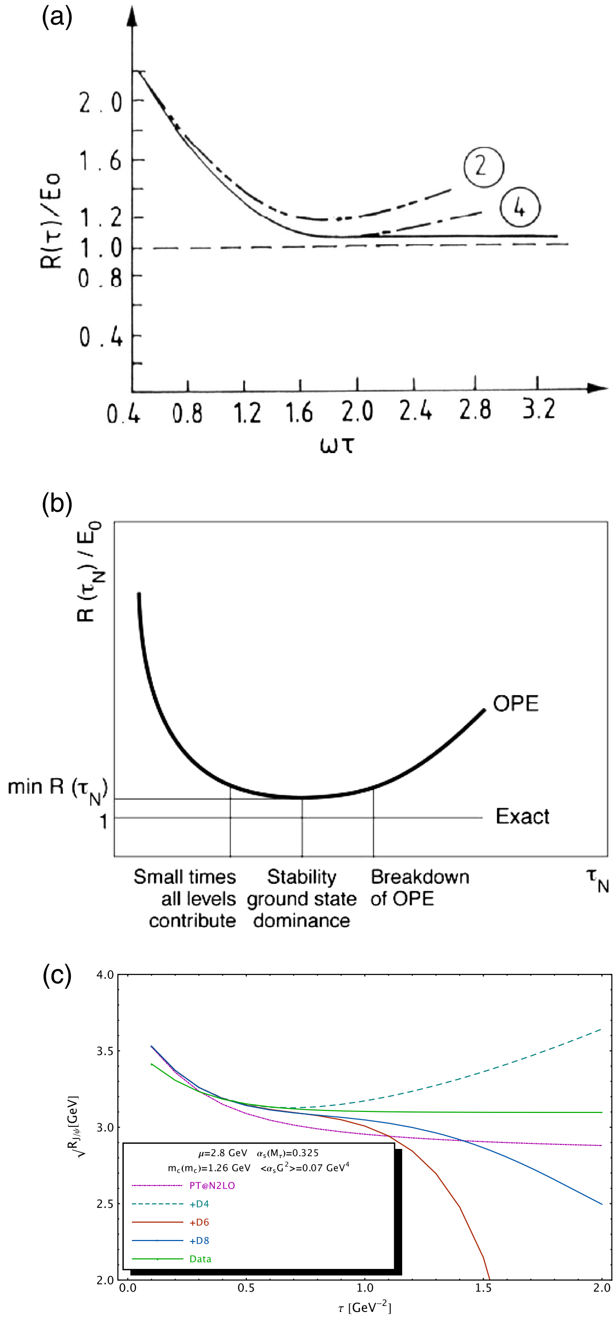

FIG. 1. (a) Harmonic oscillator state for each given truncation of the series compared to the exact solution (horizontal line). (b) Schematic presentation of stability of the charmonium ratio of moments. (c) Explicit analysis of the J/ψ systems moment for different truncation of the OPE from, e.g., [11,12].

or "QCD continuum" is the imaginary part of the QCD correlator [as mentioned after Eq. (1)] from the threshold t_c which is assumed to smear all higher states contributions. This parametrization insures that both sides of the sum rules have the same large t asymptotic behavior that leads to the LSR in Eq. (1). Within a such parametrization, one obtains

$$\mathcal{R}_n^c \equiv \mathcal{R}_H \simeq M_{\mathcal{H}}^2, \tag{5}$$

indicating that the ratio of moments appears to be a useful tool for extracting the mass of the hadron ground state [8–10]. The corresponding value of t_c corresponds approximately to the mass of the first radial excitation. However, one should bear in mind that a such parametrization cannot distinguish two nearby resonances but instead will consider them as one "effective resonance."

II. OPTIMIZATION CRITERIA

As τ (LSR variable), t_c (QCD continuum threshold), and μ (subtraction constant of the PT series) are free external parameters, we shall use stability criteria (minimum sensitivity on the variation of these parameters) to extract the hadron masses and couplings.

A. τ stability

This optimization procedure for the case of the τ variable has been explicitly illustrated for the harmonic oscillator in quantum mechanics [2,3] and from charmonium LSR analysis [11,12] (see Fig. 1) where the optimal result is obtained at the minimum or inflexion point of the approximate series in τ. These optimal values of τ are equivalent to the so-called plateau used in the literature using the Borel $M^2 \equiv 1/\tau$ variable. However, one should note, e.g., in the case of Z_c, that the values of M^2 in Ref. [13] move in a relatively small range $M^2 \simeq (2.7 \sim 3.3)$ GeV$^2 \equiv \tau \simeq (0.30 \sim 0.37)$ GeV^{-2} compared to the range of τ values analyzed in [1].

B. The t_c stabiity

The QCD continuum threshold t_c is (in principle) a free parameter in the analysis though one (intuitively) expects it to be around the mass of the first excitation which cannot be accurate as the QCD continuum is supposed to smear all higher radial excitations contributions to the spectral function.

To be conservative we take t_c from the beginning of τ stability until the beginning of t_c stability [8–10] where the t_c-stability region corresponds to a complete dominance of the lowest ground state in the QSSR analysis. This conservative range of t_c values is larger and wider than the usual choice done in the current literature where t_c is taken at lower values of t_c often below the beginning of the τ-stability region. For the present case of Z_c, we obtain [1]

$$t_c = (22 \sim 38) \text{ GeV}^2, \tag{6}$$

where the first value of t_c corresponds to the beginning of τ minimum of the coupling and the second one to the t_c stability.

C. The μ stability

μ stability is used to fix in a rigorous optimal way, the arbitrary substraction constant appearing in the PT calculation of the Wilson coefficients and in the QCD input renormalized parameters. We have obtained for Z_c [1]

$$\mu_c \simeq 4.65(5)\ \text{GeV}, \tag{7}$$

which has the same value as the one in our different analysis for the four-quark and molecule states [14–18].

Alternatively, one can also eliminate the μ dependence of the result by working with the resummed quantity after applying the homogeneous renormalization group equation obeyed the QCD expression of the LSR, which is superconvergent:

$$\left\{-\frac{\partial}{\partial t}+\beta(\alpha_s)\alpha_s\frac{\partial}{\partial\alpha_s}-\sum_i(1+\gamma_m(\alpha_s)\right.\\ \left.\times x_i\frac{\partial}{\partial x_i}\right\}\mathcal{L}_n^c(e^t\tau,\alpha_s,x_i,\mu)=0, \tag{8}$$

where $t\equiv(1/2)L_\tau$, $x_i\equiv m_i/\mu$, β is the β function, and γ_i is the quark mass anomalous dimension. The renormalization group improved solution is

$$\mathcal{L}_n^c(e^t\tau,\alpha_s,x_i)=\mathcal{L}_n^c(t=0,\bar{\alpha}_s(\tau),\bar{x}_i(\tau)), \tag{9}$$

where $\bar{\alpha}_s(\tau)$ and $\bar{x}_i(\tau)$ are the running QCD coupling and mass. However, the renormalization group equation solution $\mu^2=1/\tau$ would correspond to a lower value of $\mu\approx 1.6$ GeV where the convergence of the PT series can be questionable. An explicit comparison of the results from these two ways can be found in [19].

Results based on these stability criteria have lead to successful predictions in the current literature (see [8–10] and original papers). In the case of Z_c, we have obtained

$$f_{Z_c}=140(15)\ \text{keV},\qquad M_{Z_c}=3912(61)\ \text{MeV}, \tag{10}$$

where M_{Z_c} is in a remarkable agreement with the data $Z_c(3900)$ [20].

III. THE Z_c GROUND STATE FROM FESR

In Ref. [1] for a D^*D molecule description of the Z_c, the optimal values of the coupling and mass have been extracted [see Eq. (10)] inside the conservative range of t_c given in Eq. (6). The value of the mass does not present a τ minimum but a τ-inflexion point where its value is about the same as the one of the τ minimum of the coupling for the same value of t_c.

To test the consistency of the values of the ground state coupling and mass extracted in this way from Laplace (global duality) sum rule, we use local duality finite energy sum rule (FESR). This approach has been extensively discussed in Ref. [21] in the case of the ρ meson where to NLO of the PT series, the lowest moment gives the constraint

$$\frac{M_\rho^2}{4\gamma_\rho^2}=\frac{t_c}{8\pi^2}\left[1+\left(\frac{\alpha_s(t_c)}{\pi}\right)+\mathcal{O}(\alpha_s^2)\right], \tag{11}$$

for a minimal duality ansatz one resonance $\oplus$ QCD continuum. Using the experimental mass $M_\rho=775$ MeV and coupling $\gamma_\rho=2.55$, one obtains for $\alpha_s=0.39$:

$$\sqrt{t_c}\simeq 1.27\ \text{GeV}, \tag{12}$$

which is slightly lower than the mass 1465(25) MeV of the first radial excitation ρ' of the ρ meson. This value of t_c is inside the stability region of the LSR analysis [8,9].

We extend this analysis to the case of the Z_c meson assumed to be a D^*D molecule. Using as input (in a first iteration) the mass prediction: $M_{Z_c}=3912$ MeV in Table III of [1], we estimate f_{Z_c}. Then, we extract M_{Z_c} using the ratio of moments at the τ minimum of f_{Z_c} and at the corresponding value of t_c. In the second iteration, we use this value of M_{Z_c} to reextract f_{Z_c}. We repeat this procedure for different t_c for LSR. The results are shown in Fig. 2 where a common stability region in t_c is obtained for the coupling and the mass. We notice that in the stability region, the experimental Z_c mass is well reproduced from the LSR analysis. A similar procedure is done for FESR where, unlike LSR, the result increases with t_c. A similar behavior has been obtained in the case of the ρ meson [see

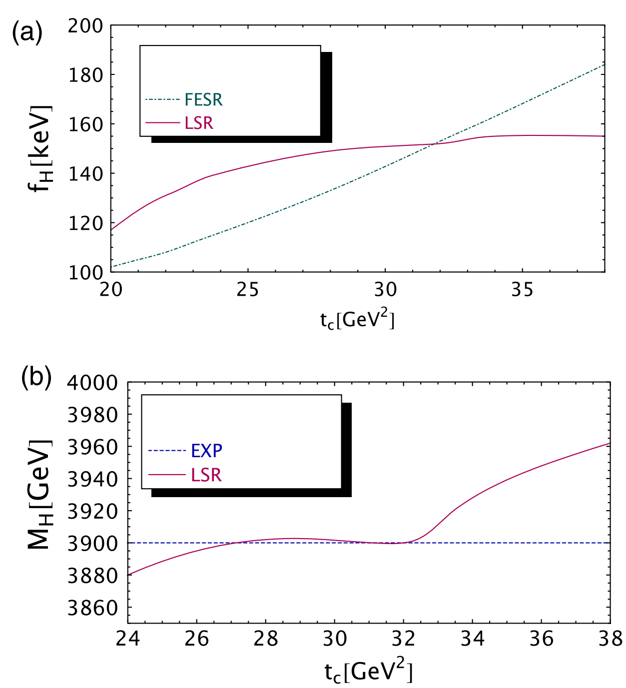

FIG. 2. Z_c parameters from LSR and FESR as a function of t_c at NLO for $\mu=4.65$ GeV.

the constraint in Eq. (11)]. The t_c stability for FESR needs a complete data parametrization of the spectral function [21] which is not yet possible for the Z_c. One can also note that FESR overestimates the mass of Z_c which is due to the fact that the second moment entering in the ratio for extracting the mass is more affected by the higher mass radial excitations. The corresponding curve is not shown in Fig. 2.

One can notice that the LSR and FESR predictions for the coupling meet at

$$t_c \simeq 32\ \mathrm{GeV}^2, \quad (13)$$

which is inside the conservative range in Eq. (6) where

$$f_{Z_c} \simeq 153(16)\ \mathrm{keV}, \qquad M_{Z_c} \simeq 3900(60)\ \mathrm{MeV}. \quad (14)$$

These values reproduce (within the errors) the ones in Ref. [1].

IV. Z_c RADIAL EXCITATIONS

If one attempts to identify the value of t_c in Eq. (13) with the mass squared of the first radial excitation, then one would obtain

$$M_{(D^*D)_1} \simeq 5657\ \mathrm{MeV}, \quad (15)$$

which we can identify with $M_{(D^*D)_1} = 5709(70)$ extracted directly from LSR in Ref. [1] with

$$f_{(D^*D)_1} = 197(25)\ \mathrm{keV}. \quad (16)$$

In Ref. [1], we have also attempted to assume that the $Z_c(4430)$ is the first radial excitation of the $Z_c(3900)$. Then we have estimated its coupling to the current to be

$$f_{(D^*D)_0} = 46(56)\ \mathrm{keV}, \quad (17)$$

which is much smaller than the one of $(D^*D)_1$ in Eq. (16).

We conclude from the previous study that the $(D^*D)_0$ and $(D^*D)_1$ states can be the radial excitations of the $Z_c(3900)$ having the parameters in Eqs. (15)–(17). However, the $(D^*D)_0$ might have been masked by the $(D^*D)_1$ from the direct extraction using LSR due to its weaker coupling to the current.

V. ON THE FOUR-QUARK CONDENSATES

Earlier estimates of the four-quark condensates:

$$\langle 0|\bar\psi\Gamma_1\psi\bar\psi\Gamma_2\psi|0\rangle \quad (18)$$

(Γ_i is a generic notation for γ matrices) from $e^+e^- \to$ Hadrons data [21,22], τ decays [23], and light baryon systems [24–26] have indicated a deviation of about a factor of 3–4 of their value from vacuum saturation.

In Ref. [13], the author claims that, in the light meson systems, the effect of the four-quark condensate is relatively small compared to the lower dimension condensates one appearing in the Operator Product Expansion (OPE) as it is multiplied by α_s. This argument is not correct because, due to the anomalous dimension, the quantity $\alpha_s\langle\bar\psi\psi\rangle^2$ has a weak $\log^{1/9}(Q/\Lambda)$ behavior for, e.g., three light flavors (see, e.g., [8,9]).

The author in Ref. [13] also claims that the corrections to the vacuum saturation is obviously negligible using an argument based on the eventual smallness of the $1/N_c$ corrections. In order to validate his claim, the author should compute explicitly the coefficient of such $1/N_c$ perturbative corrections and show that the nonperturbative contributions of hadronic intermediate states $|\pi\rangle\langle\pi|, |\rho\rangle\langle\rho|\ldots$ are negligible. He should also invalidate all previous phenomenological estimates of this quantity.

VI. THE OPE AND PT SERIES

In Ref. [1], the OPE is truncated at the dimension-six condensate contributions where the systematic error related to this truncation has been estimated by rescaling the dimension-six condensate contributions using the typical exponential factor $m_c^2\tau/3$ where the size of this estimate is about the one of the dimension-8 $\langle\bar q q\rangle\langle\bar q G q\rangle$ condensate contributions obtained in [14,15]. However, one should have in mind that this contribution is only a part of the complete $d = 8$ condensate ones while the validity of the vacuum saturation used for its estimate is also questionable. Therefore, a valuable claim on the convergence of the OPE requires an evaluation of the complete dimension-8 contributions and a nonuse of factorization for estimating these high-dimension condensates that should mix under renormalization [27].

Alternatively, we use FESR to test the validity of the LSR results truncated at the dimension-six condensates [1]. Unlike the LSR where the OPE is done in terms of the τ variable, the OPE for FESR is done in terms of t_c where its large value [see Eq. (13)] guarantees a much better convergence of the OPE which we illustrate for the coupling shown in Table I. As expected, we notice that the contributions of the high-dimension condensates are negligible while the one of the four-quark condensate is relatively large in this channel. This result from FESR consolidates the one obtained from LSR in Ref. [1] at a lower scale.

TABLE I. Perturbative (PT) ($d = 0$) at NLO and nonperturbative condensate contributions of dimension $d \le 6$ to the Z_c coupling from local duality FESR. $d_{0-n} \equiv$ contributions of dimensions $d = 0 + (d \equiv n-1) + (d \equiv n)$ condensates.

Z_c	$t_c[\mathrm{GeV}]^2$	d_0	d_{0-4}	d_{0-5}	d_{0-6}
f_{Z_c} [keV]	32	113.2	149.9	149.5	152.5

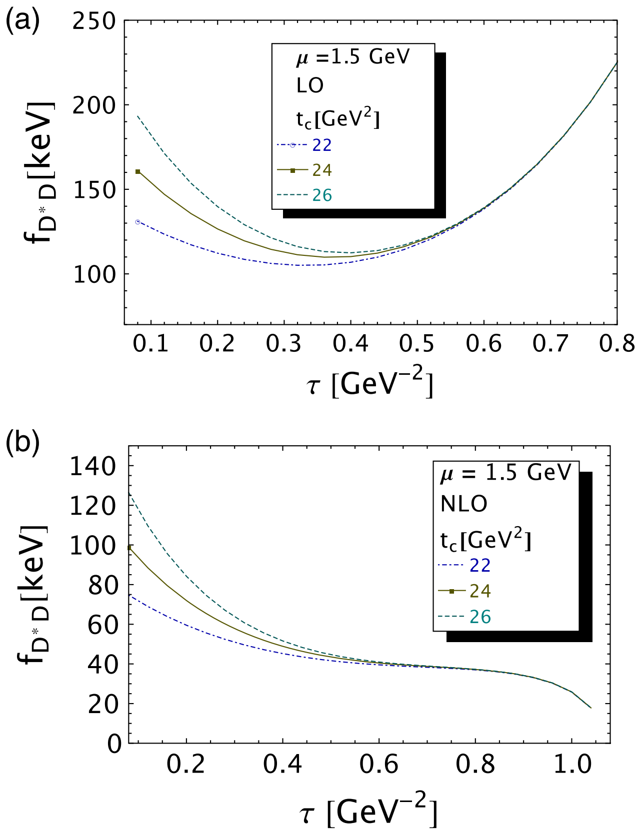

FIG. 3. $f_{(D^*D)}$ as a function of τ at (a) LO and (b) NLO for different values of t_c and for $\mu = 1.5$ GeV in the case of factorization of the four-quark condensate.

At the scale $\mu = 4.65$ GeV, we also test the convergence of the PT series. For $t_c = 32$ GeV^2, we obtain

$$f^{LO}_{Z_c} = 149.4 \text{ keV}, \qquad f^{\text{NLO}}_{Z_c} = 152.5 \text{ keV}, \tag{19}$$

where the effect of the NLO correction is (almost) negligible.

VII. ON THE ANALYSIS IN REF. [13]

The author in Ref. [13] uses LSR within his optimization procedure to estimate the mass of ground state $Z_c(3900)$ and of the first radial excitation $Z_c(4430)$. He obtains

$$M_{Z_c} = 3.91^{+0.21}_{-0.17} \text{ GeV}, \qquad M_{Z'_c} = 4.51^{+0.17}_{-0.09} \text{ GeV} \tag{20}$$

using the following favored choice of parameters.

A. Continuum threshold

The author chooses the value $t_c = (22 \sim 24)$ GeV^2 for extracting the $Z_c(3900)$ and $Z_c(4430)$ masses and couplings. Hopefully, this value of t_c is inside the conservative stability region given in Eq. (6).

B. Plateau region and optimal results

The "plateau region" is taken in the range $1/\tau \equiv T^2 = (2.7 \sim 3.3)$ GeV^2, which is narrower [$\tau \sim (0.30 \sim 0.37)$ GeV^{-2}] than the one in Ref. [1] and in Fig. 3. One should remark that the scale of the figure in Ref. [13] (and in some papers in the literature) is (exaggeratedly) enlarged, which gives the impression of a large plateau.

Taking the example of $f_{(D^*D)}$ in Fig. 3, one can remark that the minimum in τ obtained at LO becomes an inflexion point at NLO in the case of the vacuum saturation estimate of the four-quark condensate. It shows that the extraction of the optimal value does not necessary need a large plateau contrary to the claim in Ref. [13]. The existence of a minimum or/and an inflexion point is sufficient for an approximate OPE and PT series according to the example of harmonic oscillator and charmonium channel discussed in Sec. II.

C. Subtraction point μ and PT series convergence

The author favors the choice $\mu = 1.5(2.7)$ GeV of the subtraction point for extracting the $Z_c(Z'_c)$ masses and couplings. We check explicitly in Fig. 3 the convergence of the PT series for extracting the Z_c coupling and mass at

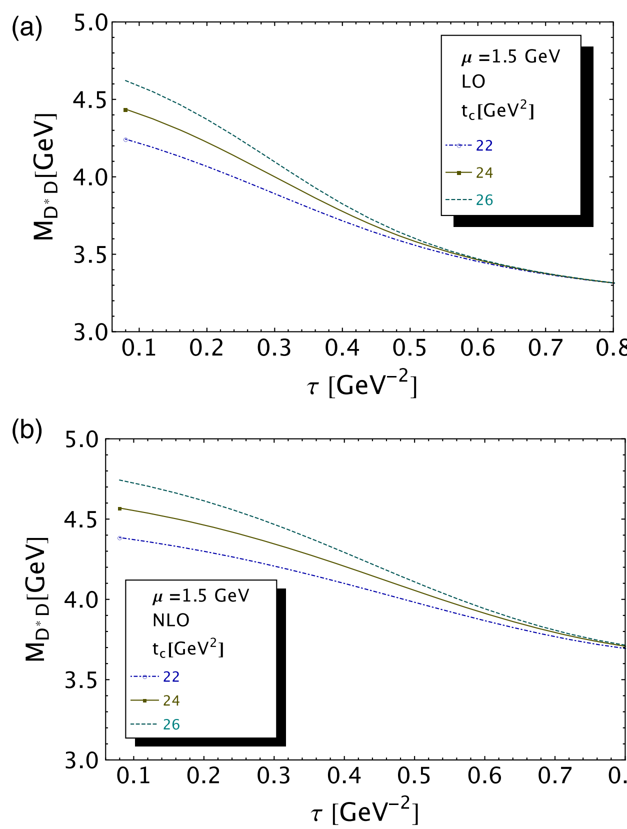

FIG. 4. $M_{(D^*D)}$ as a function of τ at (a) LO and (b) NLO for different values of t_c and for $\mu = 1.5$ GeV in the case of factorization of the four-quark condensate.

$\mu = 1.5$ GeV in the case of the factorization of the four-quark condensate used by the author in Ref. [13].

From Fig. 3 one can see that the NLO correction is huge for the favored choice $\mu = 1.5$ GeV of Ref. [13]:

$$f^{\rm LO}_{D^*D} = 110(5)_{t_c}\ {\rm keV}, \qquad f^{\rm NLO}_{D^*D} = 41(1)_{t_c}\ {\rm keV}, \qquad (21)$$

for $t_c = 24(2)$ GeV^2 and $\tau \simeq 0.4(0.6)$ GeV^{-2}, respectively, for LO (NLO) where only the error induced by t_c has been quoted. It indicates that the PT series is unreliable.

For the choice $\mu = 2.7$ GeV used to extract the $Z_c(4430)$ parameter, the correction to the coupling of about 10% is more reasonable. In the case of the optimal value $\mu = 4.65$ GeV obtained in Ref. [1] and in Eq. (19), the correction to the coupling is (almost) negligible. The τ behavior of the mass is shown in Fig. 4 for $\mu = 1.5$ GeV. One obtains in units of MeV:

$$M^{\rm LO}_{D^*D} = 3777(61)_{t_c}, \qquad M^{\rm NLO}_{D^*D} = 3913(46)_{t_c}. \qquad (22)$$

The NLO corrections are moderate due to the cancellation of these contributions in the ratio of moments. However, this result obtained from unreliable individual expressions of the moments is misleading and should not be (seriously) considered.

VIII. SUMMARY AND CONCLUSIONS

In this paper, we have used FESR to test the reliability of the LSR results [1] within the NLO corrections and where the OPE is truncated at the $d = 6$ condensates.

Compared to LSR, the OPE of FESR is more convergent while the t_c (continuum threshold) behavior of the result does not present stability. The common solution of the two approaches shown in Fig. 2 favors a value of t_c around 32 GeV^2, which restricts the conservative t_c range from LSR inside the τ to t_c stability region given in Ref. [1].

Attempting to identify this t_c value with the mass of the radial excitation $(D^*D)_1$, we obtain the one in Eq. (15), which confirms the direct LSR extraction in Ref. [1].

Assuming that the $Z_c(4430)$ is the first radial excitation of the $Z_c(3900)$ [named $(D^*D)_0$ in Ref. [1]] as expected from quark model [28] and from an extrapolation of $\psi' - J/\psi$ mass splitting [29], we find the value 46(56) keV of its coupling [1]. A such coupling is relatively weak compared to the one of the ground state 153 keV and of the second radial excitation $(D^*D)_1$ of 197(25) keV extracted directly from LSR [1]. This feature may explain why the $(D^*D)_0$ has been masked from a direct LSR analysis. It may also signal the different dynamics of the four-quark states compared to ordinary mesons. Using a Golberger-Treiman-like relation where the hadronic width behaves as $1/f^2_{\mathcal{H}}$, then, one may expect that the $Z_c(4430)$ is wider than the $Z_c(3900)$ as indicated by the data [20].

[1] R. Albuquerque, S. Narison, and D. Rabetiarivony, preceding paper, Phys. Rev. D **103**, 074015 (2021).
[2] J. S. Bell and R. A. Bertlmann, Nucl. Phys. **B177**, 218 (1981); **B187**, 285 (1981).
[3] R. A. Bertlmann, Acta Phys. Austriaca **53**, 305 (1981).
[4] C. Becchi, S. Narison, E. de Rafael, and F. J. Yndurain, Z. Phys. C **8**, 335 (1981).
[5] S. Narison and E. de Rafael, Phys. Lett. B **522**, 266 (2001).
[6] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979).
[7] V. I. Zakharov, Int. J. Mod. Phys. A **14**, 4865 (1999).
[8] S. Narison, *QCD Spectral Sum Rules*, World Scientific Lecture Notes in Physics Vol. 26 (World Scientific Company, Singapore, 1989), pp. 1–527, ISBN 9971-50-653-X.
[9] S. Narison, *QCD as a Theory of Hadrons*, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology Vol. 17 (Cambridge University Press, Cambridge, England, 2004), pp. 1–779, ISBN 9780521 811644.
[10] S. Narison, Phys. Rep. **84**, 263 (1982); Acta Phys. Pol. B **26**, 687 (1995); Riv. Nuovo Cimento **10N2**, 1 (1987); Nucl. Part. Phys. Proc. **258–259**, 189 (2015); **207–208**, 315 (2010).
[11] S. Narison, Int. J. Mod. Phys. A **33**, 1850045 (2018); **33**, 1850045(A) (2018), and references therein.
[12] S. Narison, Phys. Lett. B **707**, 259 (2012).
[13] Z.-G. Wang, arXiv: 2202.06058; Commun. Theor. Phys. **63**, 325 (2015).
[14] R. M. Albuquerque, S. Narison, F. Fanomezana, A. Rabemananjara, D. Rabetiarivony, and G. Randriamanatrika, Int. J. Mod. Phys. A **31**, 1650196 (2016).
[15] R. M. Albuquerque, S. Narison, D. Rabetiarivony, and G. Randriamanatrika, Int. J. Mod. Phys. A **33**, 1850082 (2018); Nucl. Part. Phys. Proc. **282–284**, 83 (2017).
[16] R. Albuquerque, S. Narison, D. Rabetiarivony, and G. Randriamanatrika, Nucl. Phys. **A1007**, 122113 (2021); Nucl. Part. Phys. Proc. **312-317**, 125 (2021).
[17] S. Narison and D. Rabetiarivony, arXiv:2110.05956.

[18] R. Albuquerque, S. Narison, and D. Rabetiarivony, Nucl. Phys. **A1023**, 122451 (2022).
[19] S. Narison, Phys. Lett. B **721**, 269 (2013); **718**, 1321 (2013); S. Narison, Nucl. Phys. B. Proc. Suppl. **234**, 187 (2013).
[20] P. A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020).
[21] R. A. Bertlmann, G. Launer, and E. de Rafael, Nucl. Phys. **B250**, 61 (1985); R. A. Bertlmann, C. A. Dominguez, M. Loewe, M. Perrottet, G. Launer, and E. de Rafael, Z. Phys. C **39**, 231 (1988).
[22] G. Launer, S. Narison, and R. Tarrach, Z. Phys. C **26**, 433 (1984).
[23] S. Narison, Phys. Lett. B **673**, 30 (2009).
[24] Y. Chung, H. G. Dosch, M. Kremer, and D. Schall, Z. Phys. C **25**, 151 (1984).
[25] H. G. Dosch, *Non-Perturbative Methods*, edited by S. Narison (World Scientific Company, Singapore, 1985), p. 40, ISBN 9789971500122.
[26] H. G. Dosch, M. Jamin, and S. Narison, Phys. Lett. B **220**, 251 (1989).
[27] S. Narison and R. Tarrach, Phys. Lett. **125B**, 217 (1983).
[28] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, Phys. Rev. D **89**, 114010 (2014).
[29] M. Nielsen and F. S. Navarra, Mod. Phys. Lett. A **29**, 1430005 (2014).