LANGEVIN DIFFUSION COEFFICIENTS RATIO IN STU MODEL WITH HIGHER DERIVATIVE CORRECTIONS*

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In this article, we study Langevin diffusion coefficients for the five-dimensional $\mathcal{N}=2$ STU model in the presence of higher derivative corrections. We obtained the effect of black hole charge, corresponding to the chemical potential, on the Langevin diffusion coefficients ratio. We confirm universal behavior of transverse-to-longitudinal ratio of coefficients.

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The study of quark–gluon plasma (QGP) using the AdS/CFT correspondence [1, 2] was in the last decade an important subject [3]. According to AdS/CFT correspondence, there is a relation between a conformal field theory (CFT) in d-dimensional space and a supergravity theory in $(d+1)$-dimensional anti-de Sitter (AdS) space. It is indeed a relation between the type IIB string theory in $\mathrm{AdS}_5 \times \mathrm{S}^5$ space and $\mathcal{N}=4$ super Yang–Mills theory on the 4-dimensional boundary of AdS_5 space. The study of the QGP is a testing ground for the finite temperature field theory, which is important to understand the early evolution of the universe. The most important quantities of QGP are drag force [4, 5] and jet-quenching parameter [6].

The jet-quenching parameter is obtained using the expectation value of a closed light-like Wilson loop in the dipole approximation [7]. In order to calculate the jet-quenching parameter in QCD, one needs to use perturbation

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theory. However, by using the AdS/CFT correspondence, it can be calculated in non-perturbative quantum field theory [8]. The motion of a heavy quark in context of QCD has a dual picture in the string theory, one can imagine an open string attached to the D-brane and stretched to the black hole horizon. The stochastic motion related to the fluctuation correlations of the trailing string [9, 10] can be obtained in terms of the Langevin coefficients [11, 12]. Therefore, the Langevin coefficients are important to study QGP. In Ref. [13], it has been found that the longitudinal Langevin diffusion coefficient along the string motion is larger than the transverse coefficient. Also, in Ref. [14], the Langevin diffusion of a relativistic heavy quark in a general anisotropic strongly coupled background has been studied.

In this article, we would like to study the Langevin diffusion coefficients in the STU model. The STU model includes a chemical potential to the model. For example, the presence of a baryon number chemical potential for heavy quark in the context of AdS/CFT correspondence yields to introducing a macroscopic density of heavy-quark baryons. The STU model is a kind of $D = 5$, $\mathcal{N} = 2$ gauged supergravity theory which is dual to the $\mathcal{N} = 4$ SYM theory with the finite chemical potential. The solutions of $\mathcal{N} = 2$ supergravity may be, however, also the solutions of supergravity theory with high supersymmetry. It has been found that the $\mathcal{N} = 2$ supergravity is an ideal laboratory [15]. Therefore, the STU model may be considered as a gravity dual of a strongly coupled plasma. Moreover, the $D = 5$, $\mathcal{N} = 2$ gauged supergravity theory is a natural way to explore gauge/gravity duality, and three-charge non-extremal black holes are important thermal background for this correspondence. The STU model describes a five-dimensional space-time whose four-dimensional boundary includes QCD. Drag force and jet-quenching parameter has been already obtained using the AdS/CFT in STU background, and such studies are called STU-QCD correspondence [16–20]. Shear viscosity-to-entropy ratio of dual QGP was also investigated in the STU model [21]. We will also study universal longitudinal and transverse Langevin coefficients ratio in the presence of higher derivative corrections. The STU model was already used to study holographic superfluids and superconductors [22, 23].

The STU model has generally an 8-charge non-extremal black hole. However, there are many situations of the black holes with four and three charges. In this case, there is a great difference between the three-charge and four-charge black holes. For example, if there are only 3 charges, then the entropy vanishes (except in the non-BPS case). So, one really needs four charges to get a regular black hole. In 5 dimensions, the situation is different, there is no distinction between BPS and non-BPS branch. So, in 5 dimensions, the three-charged configurations are the most interesting ones [24]. Hence, our interest is in the three-charged non-extremal black hole solution in $\mathcal{N} = 2$

gauged supergravity which is called the STU model described by the following solution [25]:

$$\mathrm{d}s^2 = -\frac{f_k}{\mathcal{H}^{\frac{2}{3}}}\mathrm{d}t^2 + \mathcal{H}^{\frac{1}{3}}\left(\frac{\mathrm{d}r^2}{f_k} + \frac{r^2}{R^2}\mathrm{d}\Omega^2_{3,k}\right), \tag{1}$$

where

$$\begin{aligned} f_k &= k - \frac{\mu}{r^2} + \frac{r^2}{R^2}\mathcal{H}, \\ \mathcal{H} &= \prod_{i=1}^{3} H_i, \\ H_i &= 1 + \frac{q_i}{r^2}, \qquad i = 1,2,3. \end{aligned} \tag{2}$$

Here, R is the constant AdS radius relating to the coupling constant via $R = 1/g$, and r is the radial coordinate along the black hole, so the boundary of AdS space is located at $r \to \infty$ (or $r = r_m$ on the D-brane). The black hole horizon is specified by $r = r_{\mathrm{h}}$ which is obtained from $f_k = 0$. In the STU model, there are three real scalar fields given by

$$X^i = \frac{\mathcal{H}^{\frac{1}{3}}}{H_i} \tag{3}$$

which is also a solution of metric (1) and satisfies the following condition $\prod_{i=1}^{3} X^i = 1$. In other words, if one sets $X^1 = S$, $X^2 = T$, and $X^3 = U$, then there is the special condition written as STU $= 1$. Finally, the factor of k indicates the space curvature. The special case of $k = 0$ corresponds to the black brane limit relevant to the thermal CFT in an infinite volume.

The Brownian motion of moving quark at fixed velocity can be understood using generalized Langevin coefficients. There are longitudinal and transverse Langevin coefficients which can be written in terms of world-sheet metric temperature given by [13, 14]

$$T = \frac{1}{4\pi}\sqrt{\frac{1}{G_{00}G_{rr}}\left(G_{00}G_{pp}\right)'\left(\frac{G_{00}}{G_{pp}}\right)'}\Bigg|_{r=r_0}, \tag{4}$$

where prime denote derivative with respect to r and

$$G_{00} = -\frac{f_k}{H^{\frac{2}{3}}} = -\frac{k+\frac{\left(1+\frac{q_1}{r^2}\right)\left(1+\frac{q_2}{r^2}\right)\left(1+\frac{q_3}{r^2}\right)r^2}{R^2}-\frac{\mu}{r^2}}{\left(\left(1+\frac{q_1}{r^2}\right)\left(1+\frac{q_2}{r^2}\right)\left(1+\frac{q_3}{r^2}\right)\right)^{\frac{2}{3}}}, \tag{5}$$

$$G_{rr} = \frac{H^{\frac{1}{3}}}{f_k} = \frac{\left(\left(1+\frac{q_1}{r^2}\right)\left(1+\frac{q_2}{r^2}\right)\left(1+\frac{q_3}{r^2}\right)\right)^{\frac{1}{3}}}{k+\frac{\left(1+\frac{q_1}{r^2}\right)\left(1+\frac{q_2}{r^2}\right)\left(1+\frac{q_3}{r^2}\right)r^2}{R^2}-\frac{\mu}{r^2}}, \tag{6}$$

$$G_{kk} = \frac{H^{\frac{1}{3}}r^2}{R^2} = \frac{\left(\left(1+\frac{q_1}{r^2}\right)\left(1+\frac{q_2}{r^2}\right)\left(1+\frac{q_3}{r^2}\right)\right)^{\frac{1}{3}}r^2}{R^2}, \tag{7}$$

$$G_{pp} = \frac{H^{\frac{1}{3}}r^2}{R^2} = \frac{\left(\left(1+\frac{q_1}{r^2}\right)\left(1+\frac{q_2}{r^2}\right)\left(1+\frac{q_3}{r^2}\right)\right)^{\frac{1}{3}}r^2}{R^2}. \tag{8}$$

r_0 is root of the following equation (obtained from the reality condition):

$$G_{00} + G_{pp}v^2 = 0 \tag{9}$$

which for the case of $k = 0$ reduces to the following equation:

$$\begin{aligned} 0 = & \left(1-v^2\right)r^6 + \left(R^2 + (q_1+q_2+q_3)\left(1-v^2\right)\right)r^4 \\ & + \left((q_1q_2+q_1q_3+q_2q_3)\left(1-v^2\right) - \mu R^2\right)r^2 + q_1q_2q_3\left(1-v^2\right). \end{aligned} \tag{10}$$

It is clear that at $v = 0$ limit $r_0 = r_{\mathrm{h}}$, where r_{h} is the black hole horizon radius given by $f_k = 0$. It has been argued that the longitudinal and transverse Langevin coefficients ratio can be written as follows:

$$\begin{aligned} \frac{\kappa_{\parallel}}{\kappa_{\perp}} &= \pm 16\pi^2 \left(\frac{G_{00}G_{rr}}{G_{kk}G_{pp}\left(\frac{G_{00}}{G_{pp}}\right)'\left|\left(\frac{G_{00}}{G_{pp}}\right)'\right|}\right)\Bigg|_{r=r_0} T^2 \\ &= \frac{(G_{00}G_{pp})'}{G_{kk}G_{pp}\left(\frac{G_{00}}{G_{pp}}\right)'}\Bigg|_{r=r_0}. \end{aligned} \tag{11}$$

After some calculation, one can obtain

$$\frac{\kappa_{\parallel}}{\kappa_{\perp}} = \frac{r_0^{10} + Ar_0^8 + Br_0^6 + Cr_0^4 + Dr_0^2 + E}{ar_0^8 + br_0^6 + cr_0^4 + dr_0^2 + e}, \tag{12}$$

where we defined the following coefficients:

$$
\begin{aligned}
A &= \tfrac{5}{3}\left(q_1+q_2+q_3\right)+\tfrac{1}{2}kR^2\,,\\
B &= \tfrac{2}{3}\left(q_1^2+q_2^2+q_3^2\right)+\tfrac{8}{3}\left(q_1q_2+q_1q_3+q_2q_3\right)+\tfrac{2}{3}kR^2\left(q_1+q_2+q_3\right)\,,\\
C &= \left(q_1+q_2\right)q_3^2+\left(q_1+q_3\right)q_2^2+\left(q_2+q_3\right)q_1^2+\tfrac{5}{6}kR^2\left(q_1q_2+q_1q_3+q_2q_3\right)\\
&\quad +4q_1q_2q_3-\tfrac{1}{6}\mu R^2\left(q_1+q_2+q_3\right)\,,\\
D &= \tfrac{4}{3}q_1q_2q_3\left(q_1+q_2+q_3\right)+\tfrac{1}{3}\left(q_1^2q_2^2+q_1^2q_3^2+q_2^2q_3^2\right)\\
&\quad -\tfrac{1}{3}\mu R^2\left(q_1q_2+q_1q_3+q_2q_3\right)+kR^2q_1q_2q_3\,,\\
E &= -\tfrac{1}{2}q_1q_2q_3\left(\mu R^2-\tfrac{2}{3}\left(q_1q_2+q_1q_3+q_2q_3\right)\right)\,,\\
a &= -\tfrac{1}{2}kR^2\,,\\
b &= \mu R^2\,,\\
c &= \tfrac{1}{2}\left(\mu\left(q_1+q_2+q_3\right)+k\left(q_1q_2+q_1q_3+q_2q_3\right)\right)R^2\,,\\
d &= kR^2q_1q_2q_3\,,\\
e &= -\tfrac{1}{2}\mu q_1q_2q_3\,.
\end{aligned}
\tag{13}
$$

It is already concluded that $\kappa_{\|}\geq\kappa_{\perp}$ is universal for isotropic backgrounds, where equality holds for $v=0$. It is also verified for the STU model with positive charge and illustrated for some situations summarized in Tables I, II and III. Without loss of generality, we set $R=\mu=1$ and vary black hole charges for three different cases of $k=0$, $k=1$ and $k=-1$. From Table I the case of $k=1$, Table II for the case of $k=0$ and Table III for the case of $k=-1$, we see universal behavior of $\frac{\kappa_{\|}}{\kappa_{\perp}}$ as expected. Here, we consider some unphysical cases of negative charge and take into account all possible mathematical situations, however universality holds for all the cases.

TABLE I

Value of $L=\frac{\kappa_{\|}}{\kappa_{\perp}}$ for $k=1$.

L	r_0	q_1	q_2	q_3	v	L	r_0	q_1	q_2	q_3	v
1.0000006	0.786	0	0	0	0.001	1.67	0.058	1	1	0	0.1
3	1	0	0	0	1	3.54	0.74	1	1	0	0.9
1.0000008	0.64	1	0	0	0.001	∞	1	-1	-1	0	0.1
2.6	0.866	1	0	0	0.9	∞	1	-1	0	0	0.99
4.3	1	1	0	0	1	∞	1	1	1	-1	0.99
4.7	0.58	10	0	0	0.9	1.0086	0.78	1	-1	-1	0.1
1.01	0.30	10	0	0	0.1	1.69	0.14	1	-1	-1	0.99
16.333	1	-1	-1	0	0.9	1.000086	0.786	1	-1	-1	0.01

TABLE II

Value of $L = \frac{\kappa_{\|}}{\kappa_{\perp}}$ for $k = 0$.

L	r_0	q_1	q_2	q_3	v	L	r_0	q_1	q_2	q_3	v
1.000001	1.00000025	0	0	0	0.001	1.68	0.07	1	1	0	0.1
∞	1	0	0	0	1	6.37	1.14	1	1	0	0.9
1.000001	0.786	1	0	0	0.001	1.0067	3.18	−10	−1	0	0.1
5.57	1.359	1	0	0	0.9	49.097	2.76	−1	0	0	0.99
∞	1	1	0	0	1	50.98	2.59	1	1	−1	0.99
6.56	0.7	10	0	0	0.9	1.008	1.3	1	−1	−1	0.1
1.01	0.3	10	0	0	0.1	48.87	2.77	1	−1	−1	0.99
4.8	1.8	−1	−1	0	0.9	1.00008	1.3	1	−1	−1	0.01

TABLE III

Value of $L = \frac{\kappa_{\|}}{\kappa_{\perp}}$ for $k = -1$.

L	r_0	q_1	q_2	q_3	v	L	r_0	q_1	q_2	q_3	v
1.000001	1.27	0	0	0	0.001	1.69	0.1	1	1	0	0.1
∞	0	0	0	0	1	10.2	2.06	1	1	0	0.9
1.0000017	1.0000005	1	0	0	0.001	1.01	1.7	−1	−1	0	0.1
9.07	2.29	1	0	0	0.9	96.5	7.2	−1	0	0	0.99
∞	0	1	0	0	1	98.8	7.09	1	1	−1	0.99
10.8	0.96	10	0	0	0.9	1.01	1.6	1	−1	−1	0.1
1.01	0.33	10	0	0	0.1	96.5	7.2	1	−1	−1	0.99
7.65	2.79	−1	−1	0	0.9	1.01	1.6	1	−1	−1	0.01

Hence, for the positive charges in relativistic regime (in the case of $k = 1$) the increasing value of charges and number of charges, which are corresponding to chemical potential of QGP, increase the value of ratio $\frac{\kappa_{\|}}{\kappa_{\perp}}$.

The higher derivative corrections of the STU model can be found in Ref. [20]

$$
\begin{aligned}
f_k &= k - \frac{\mu}{r^2} + \frac{r^2}{R^2}\prod_i \left(1 + \frac{q_i}{r^2}\right) + c_1 \left(\frac{\mu^2}{96 r^6 \prod_i \left(1 + \frac{q_i}{r^2}\right)} - \frac{\prod_i q_i (q_i + \mu)}{9 R^2 r^4}\right), \\
\mathcal{H} &= \prod_{i=1}^{3} H_i , \\
H_i &= 1 + \frac{q_i}{r^2} - \frac{c_1 q_i \left(q_i + \mu\right)}{72 r^2 \left(r^2 + q_i\right)^2}, \qquad i = 1, 2, 3 ,
\end{aligned}
\tag{14}
$$

where c_1 is the small constant parameter corresponding to the higher derivative terms and a_1 is q_i-dependent quantity which parameterizes the corrections to the background geometry [26]. In that case the modified horizon radius is given by the following expression:

$$\begin{aligned} r_{\mathrm{h}} = r_{0\mathrm{h}} \; &+ \; \frac{c_1 \prod_i \left(1+\frac{q_i}{r_{0\mathrm{h}}^2}\right)\left(\sum q_i^2 - \frac{26 r_{0\mathrm{h}}^2}{3}\sum q_i + 3 r_{0\mathrm{h}}^4\right)}{576 R^2 \left[\left(\prod_i \left(1+\frac{q_i}{r_{0\mathrm{h}}^2}\right)\right)^{\frac{2}{3}}\left(\frac{1}{3}\sum q_i - 2 r_{0\mathrm{h}}^2\right) - R^2\right]} \\ &+ \; c_1 \frac{2\left(\prod_i \left(1+\frac{q_i}{r_{0\mathrm{h}}^2}\right)\right)^{\frac{1}{3}}\left(\frac{13}{3}\sum q_i - 3 r_{0\mathrm{h}}^2\right) + 3R^2}{576\left[\left(\prod_i \left(1+\frac{q_i}{r_{0\mathrm{h}}^2}\right)\right)^{\frac{2}{3}}\left(\frac{1}{3}\sum q_i - 2 r_{0\mathrm{h}}^2\right) - R^2\right]} \,, \end{aligned} \tag{15}$$

where $r_{0\mathrm{h}}$ is the horizon radius without higher derivative corrections. It should be noted that in order to obtain expression (15), we removed μ by using $f_k = 0$.

As before, we can calculate components G_{00}, G_{rr}, G_{kk} and G_{pp} to obtain $\frac{\kappa_\parallel}{\kappa_\perp}$ and investigate universal behavior $\kappa_\parallel \geq \kappa_\perp$. Numerically, we find that the relation $\frac{\kappa_\parallel}{\kappa_\perp} \geq 1$ is valid in the presence of higher order corrections. The effect of c_1 is a reduction of this ratio, for example, in the case of three-charge black hole with $q_1 = 10^6$, $q_2 = q_3 = 1$, $c_1 = 0.01$, and $v = 0.9$, we have $\frac{\kappa_\parallel}{\kappa_\perp} = 3.8$. Hence, we confirmed universal properties of the Langevin diffusion coefficients in the STU model with higher derivative terms. Already thermodynamical and statistical analysis of STU black holes have been given in Ref. [27]. Now, it may be interesting to consider the logarithmically corrected STU model [28] and investigate its Langevin diffusion coefficients.

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REFERENCES

[1] J.M. Maldacena, *Adv. Theor. Math. Phys.* **2**, 231 (1998); *Int. J. Theor. Phys.* **38**, 1113 (1999).

[2] E. Witten, *Adv. Theor. Math. Phys.* **2**, 253 (1998).

[3] S.S. Gubser, *Nucl. Phys. A* **830**, 657 (2009).

[4] C.P. Herzog, *J. High Energy Phys.* **0609**, 032 (2006).

[5] S.S. Gubser, *Phys. Rev. D* **74**, 126005 (2006).

[6] H. Liu, K. Rajagopal, U.A. Wiedemann, *Phys. Rev. Lett.* **97**, 182301 (2006).

[7] B.G. Zakharov, *J. Exp. Theor. Phys. Lett.* **65**, 615 (1997).

[8] F.L. Lin, T. Matsuo, *Phys. Lett. B* **641**, 45 (2006).

[9] S.S. Gubser, *Nucl. Phys. B* **790**, 175 (2008).

[10] J. Casalderrey-Solana, D. Teaney, *J. High Energy Phys.* **0704**, 039 (2007).

[11] J. de Boer, V.E. Hubeny, M. Rangamani, M. Shigemori, *J. High Energy Phys.* **0907**, 094 (2009).

[12] D.T. Son, D. Teaney, *J. High Energy Phys.* **0907**, 021 (2009).

[13] D. Giataganas, H. Soltanpanahi, *Phys. Rev. D* **89**, 026011 (2014).

[14] D. Giataganas, H. Soltanpanahi, *J. High Energy Phys.* **1406**, 047 (2014).

[15] C. Hoyos-Badajoz, *J. High Energy Phys.* **0909**, 068 (2009).

[16] J. Sadeghi, B. Pourhassan, *J. High Energy Phys.* **0812**, 026 (2008).

[17] J. Sadeghi, M.R. Setare, B. Pourhassan, S. Hashmatian, *Eur. Phys. J. C* **61**, 527 (2009).

[18] J. Sadeghi, M.R. Setare, B. Pourhassan, *J. Phys. G: Nucl. Part. Phys.* **36**, 115005 (2009).

[19] K. Bitaghsir Fadafan, B. Pourhassan, J. Sadeghi, *Eur. Phys. J. C* **71**, 1785 (2011).

[20] B. Pourhassan, J. Sadeghi, *Can. J. Phys.* **91**, 995 (2013).

[21] J. Sadeghi, B. Pourhassan, A.R. Amani, *Int. J. Theor. Phys.* **52**, 42 (2013).

[22] H. Saadat, B. Pourhassan, *Int. J. Theor. Phys.* **52**, 997 (2013).

[23] B. Pourhassan, M.M. Bagheri-Mohagheghi, `arXiv:1609.08402 [hep-th]`.

[24] V. Balasubramanian, F. Larsen, *Nucl. Phys. B* **478**, 199 (1996).

[25] D.T. Son, A.O. Starinets, *J. High Energy Phys.* **0603**, 052 (2006).

[26] S. Cremonini, K. Hanaki, J.T. Liu, P. Szepietowski, *Phys. Rev. D* **80**, 025002 (2009).

[27] A. Pourdarvish, B. Pourhassan, M. Tabassomi, *Int. J. Theor. Phys.* **53**, 1814 (2014).

[28] B. Pourhassan, M. Faizal, *Eur. Phys. J. C* **77**, 96 (2017).