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Nonlinearly charged AdS black hole solutions in three-dimensional massive gravity's rainbow

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ABSTRACT

Two new families of nonlinearly charged and asymptotically anti-de Sitter (AdS) rainbow black holes have been introduced, as the exact solutions to the coupled electromagnetic and gravitational field equations, in massive gravity theory and in the presence of power-law nonlinear electrodynamics. The conserved and thermodynamic quantities such as black hole mass, charge, temperature, entropy and electric potential have been calculated from geometric and thermodynamic approaches. Despite the fact that some of them receive corrections from rainbow functions and nonlinear electrodynamics, it has been proved that they fulfill the first law of black hole thermodynamics. Thermal stability of the black holes has been studied by use of the canonical ensemble and geometrical thermodynamics methods, separately. Regarding the black hole heat capacity and thermodynamic Ricci scalars, the points of type-one and type-two phase transitions and the conditions under which the black holes remain locally stable have been determined. Also, global stability of the novel rainbow black holes has been investigated from the viewpoint of the grand canonical ensemble. Through calculation of Gibbs free energy of the black holes, the points of Hawking-Page phase transition and the size of the black holes which are globally stable or in the radiative phase have been determined.

1. Introduction

Einstein's general relativity, as the relativistic theory of gravity, predicts the existence of massless spin-2 particles which are named as gravitons. Alternative models of the so-called modified gravity theory are proposed with the aim of solving the failures of this theory. It is well-known that the accelerated expansion of the universe in the large scale structure cannot be explained by Einstein's theory of gravity. Nowadays, it is believed that this phenomena can be explained if an unknown source of energy, named as dark energy, exist [1–4].

Recent studies on the gravitational waves show that propagation speed of gravitational waves is slightly different from the speed of light. Observation of the cosmic rays, originated in our Galaxy, on the earth constraints the gravitational wave speed to $c - v_g < 10^{-15}c$. Also, for the cosmic rays with extragalactic origin the speed of gravitational wave is bounded to $c - v_g < 10^{-19}c$ [5–7]. This implies that gravitons carry a nonzero amount of mass. According to the reports of LIGO scientific Collaboration and Virgo Collaboration, by assumption that gravitons are dispersed in vacuum like massive particles, the graviton mass is bounded to $m_g \leq 7.7 \times 10^{-23}\ ev/c^2$ [8,9]. Massive gravity theory is one of the alternatives that modifies Einstein gravity by giving the graviton a mass and provides a possible explanation for the accelerated expansion of the universe without requirement of dark energy component [10–12]. It is evident that massive gravity reduces to the usual gravity theory when the mass of graviton is taken equal to zero. Massive gravity theory is an interesting research topic and its gravitational and cosmological aspects have been studied by many authors [3,13,14] (See also [15] and references therein).

In addition, gravity's rainbow, as the generalization of doubly special relativity to the case of curved spacetimes, is an attempt to constructing the quantum theory of gravity. In this theory, the spacetime geometry depends on the energy of the test particle which is used to probe the gravity. Thus, the particles with different amount of energy experience different geometry. In this regard, there is a rainbow of energy-dependent metrics, and this theory of doubly general relativity is named as gravity's rainbow [16,17]. It must be noted that all the energy dependency of the spacetime is given by the rainbow functions. This theory is considerable in the high energy regime when the amount of energies are in the order of Planck energy. In the infrared limit, when the energies are negligible in comparison to

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the Planck energy rainbow functions reduce to unity and the usual theory of gravity is recovered [18–22]. Regarding the interesting results of this theory, such as black hole remnant and nonsingular universe, it is an attractive research area, and it is expected to explain some more failures of Einstein's original theory of gravity [23–27].

Although the successfulness of Maxwell's theory of classical electrodynamics has been confirmed by numerous experimental tests, this theory is confronted by some challenges. The appearance of infinite electric field and self-energy for the pointlike charged particles and violating the symmetry of conformal invariance in the spacetimes with dimensions other than four are the most famous failures of this theory [28–30]. In order to overcome these problems, alternative models of nonlinear electrodynamics have been proposed. Among the most important models of nonlinear electrodynamics are the Born-Infeld, logarithmic, exponential, quadratically-extended and power-law models have produced interesting results [31–37]. The Lagrangian of these models are written as the functions of Maxwell invariant $\mathcal{F}=F^{ab}F_{ab}$ and they can be expanded in powers of $\mathcal{F}$. In the case of weak electromagnetic fields the higher powers of $\mathcal{F}$ can be ignored and the Lagrangian of Maxwell theory is recovered. It means that nonlinear theories of electrodynamics are significant if the electromagnetic fields are strong, enough [38]. The higher powers of $\mathcal{F}$ are interpreted as the photon-photon interactions. In the case of strong electromagnetic fields, these powers are important and their effects cannot be ignored. Thus, Maxwell's electromagnetic theory must be promoted to the nonlinear electrodynamics for the case of strong fields [39–41]. Now, making use of the various models of nonlinear electrodynamics, study of charged black hole solutions in the usual and modified theories of gravity, has produced many interesting results.

In the present work, we are interested on study of the exact black hole solutions in the framework of a generalized theory composed of massive and rainbow gravity theories with the nonlinear electrodynamics. We obtain the exact black hole solutions of the coupled tensor and vector field equations. Then, we investigate the related thermodynamic quantities and validity of the first law of black hole thermodynamics. Thermodynamic phase transition, local and global stabilities will be studied by use of the canonical and grand canonical ensembles. This study can be considered as the extension of the subject of ref. [15] to the case of nonlinear electrodynamics.

The outline of this paper is as follows: In Sec. 2, the equations of motion are obtained by varying the action of three-dimensional massive gravity theory which is coupled to the power-law nonlinear electrodynamics. By introducing an energy-dependent geometry, in which the quantum gravitational effects are considered via rainbow functions, two new classes of rainbow black holes are obtained. Mathematical analysis of the solutions show that their asymptotic behavior is like AdS black holes. In sec. 3, thermodynamic and conserved quantities of the novel charged rainbow black holes are calculated and it is shown that, despite the fact that they get modified by rainbow functions and nonlinearity parameter p, they satisfy the first law of black hole thermodynamics. Sec. 4 is devoted to study of the thermodynamic stability or phase transition of the new AdS black holes by use of the canonical ensemble method. The points of type-one and type-two phase transitions and the ranges at which black holes are locally stable have been determined noting the black hole heat capacity. In sec. 5, geometrical thermodynamics of the black holes is investigated by use of a recently proposed thermodynamic line element. In this method, the location of the phase transition points are characterized noting the divergent points of the thermodynamic Ricci scalars. Through comparison of the results of canonical ensemble and geometrical thermodynamics approaches, we found that the results of these two alternative methods are identical. By calculating the Gibbs free energies, global stability and the Hawking-Page phase transition points of the black holes are studied in section 6. Sec. 7 is dedicated to summarizing and discussing the results.

2. The field equations in massive gravity's rainbow

The action of three-dimensional massive gravity theory including cosmological constant $\Lambda=-\ell^{-2}$, and in the presence of nonlinear electromagnetic theory may be written in the following general form [42]

$$I=-\frac{1}{16\pi}\int\sqrt{-g}\left[\mathcal{R}-2\Lambda+m_G^2\sum_{i=1}^{4}c_i\mathcal{U}_i+L(\mathcal{F})\right]d^3x, \tag{2.1}$$

where $\mathcal{R}$ is the Ricci scalar and m_G is related to the mass of graviton. The coefficients c_i's are some constants and $\mathcal{U}_i$'s are symmetric polynomials of eigenvalues of the 3×3 matrix $\mathcal{K}^a_a=\sqrt{g^{ac}f_{cb}}$, where f is a fixed symmetric tensor. The polynomials $\mathcal{U}_i$'s are given via the following relations [43,44]

$$\begin{aligned}&\mathcal{U}_1=[\mathcal{K}],\qquad \mathcal{U}_2=[\mathcal{K}]^2-[\mathcal{K}^2],\qquad \mathcal{U}_3=[\mathcal{K}]^3-3[\mathcal{K}][\mathcal{K}^2]+2[\mathcal{K}^3],\\&\mathcal{U}_4=[\mathcal{K}]^4-6[\mathcal{K}]^2[\mathcal{K}^2]+8[\mathcal{K}^3][\mathcal{K}]+3[\mathcal{K}^2]^2-6[\mathcal{K}^4],\quad \text{with}\quad [\mathcal{K}]\equiv\mathcal{K}^a_a.\end{aligned}$$

The last term in Eq. (2.1), denoted by $L(\mathcal{F})$, is the lagrangian of nonlinear electrodynamics which is assumed as a function of Maxwell's invariant $\mathcal{F}=F^{ab}F_{ab}$. In terms of electromagnetic four-vector A_a, $F_{ab}=\partial_aA_b-\partial_bA_a$ is the Faraday tensor. Here, we use the power-law model of nonlinear electrodynamics which is given by the following relation [45,46]

$$L(\mathcal{F})=(-\mathcal{F})^p. \tag{2.2}$$

Note that the power p sometimes is conventionally named as nonlinearity parameter. Also, it is worth mentioning that in the case $p=1$ Eq. (2.2) reduces to the Lagrangian of Maxwell's classical electrodynamics. By varying the action (2.1), with respect to g_{ab}, we obtain the following tensor equation [43,44]

$$\mathcal{R}_{ab}-\frac{1}{2}\mathcal{R}g_{ab}+\Lambda g_{ab}+m_G^2\chi_{ab}=\frac{1}{2}L(\mathcal{F})g_{ab}-2L'(\mathcal{F})F_{ac}F_b^{\ c}, \tag{2.3}$$

and the vector equation for the electromagnetic field is written as

$$\partial_a\left[\sqrt{-g}L'(\mathcal{F})F^{ab}\right]=0. \tag{2.4}$$

We use prime to show derivative with respect to the argument, in Eq. (2.3), χ_{ab} is the massive term with the following explicit form [43]

$$\chi_{ab} = -\frac{c_1}{2}\left(\mathcal{U}_1 g_{ab} - \mathcal{K}_{ab}\right) - \frac{c_2}{2}\left(\mathcal{U}_2 g_{ab} - 2\mathcal{U}_1\mathcal{K}_{ab} + 2\mathcal{K}_{ab}^2\right) - \frac{c_3}{2}\left(\mathcal{U}_3 g_{ab} - 3\mathcal{U}_2\mathcal{K}_{ab} + 6\mathcal{U}_1\mathcal{K}_{ab}^2 - 6\mathcal{K}_{ab}^3\right)$$
$$-\frac{c_4}{2}\left(\mathcal{U}_4 g_{ab} - 4\mathcal{U}_3\mathcal{K}_{ab} + 12\mathcal{U}_2\mathcal{K}_{ab}^2 - 24\mathcal{U}_1\mathcal{K}_{ab}^3 + 24\mathcal{K}_{ab}^4\right). \tag{2.5}$$

We are interested on solving the coupled tensor and vector field equations (2.3) and (2.4) in a spherically symmetric and energy-dependent spacetime identified by the following ansatz [19,20]

$$ds^2 = -\frac{G(r)}{f^2(\varepsilon)}dt^2 + \frac{1}{g^2(\varepsilon)}\left[\frac{dr^2}{G(r)} + r^2 d\theta^2\right], \tag{2.6}$$

where, $G(r)$ is an unknown function of radial component r, to be determined. It is conventionally is named as the metric function. The functions $f(\varepsilon)$ and $g(\varepsilon)$ are known as temporal and spacial rainbow functions, respectively. The energy scale ε is defined as the ratio of the test particle's energy to the Planck energy. The functions $f(\varepsilon)$ and $g(\varepsilon)$ go to unity in the infrared limit $\varepsilon \to 0$, and the Einstein-Λ geometry is achieved. The line element (2.6) indicates an energy-dependent metric with the energy-independent coordinates. All the energy dependencies are illustrated via the rainbow functions [17] (for more details see [21] and references therein).

By taking the symmetric tensor f in the following form [15]

$$f_{ab} = \mathrm{diag}\left(0,\ 0,\ \frac{c^2}{g^2(\varepsilon)}\right), \tag{2.7}$$

in terms of the positive constant c, one can show that

$$\mathcal{U}_1 = \frac{c}{r}, \quad \text{and} \quad \mathcal{U}_2 = \mathcal{U}_3 = \mathcal{U}_4 = 0. \tag{2.8}$$

Also, it is easily shown that the components of χ_{ab} take the following explicit forms

$$\chi_{tt} = -\frac{cc_1}{2r}g_{tt}, \qquad \chi_{rr} = \frac{cc_1}{2r}g_{rr}, \qquad \chi_{\theta\theta} = 0. \tag{2.9}$$

The only nonzero component of the Faraday tensor is $F_{tr} = -F_{rt}$. Taking it as a function of r, we have $F_{tr}(r) = -A_t'(r)$ and the Maxwell invariant $\mathcal{F}$ can be written as

$$\mathcal{F} = -2f^2(\varepsilon)g^2(\varepsilon)F_{tr}^2. \tag{2.10}$$

Regarding Eqs. (2.2) and (2.6), the electromagnetic field equation (2.4) can be written as

$$(A_t'(r))^{2p-2}\left[A_t'(r) + (2p-1)rA_t''(r)\right] = 0, \qquad p \neq \frac{1}{2}, \tag{2.11}$$

which can be solved as

$$A_t(r) = \begin{cases} -q\ln\left(\frac{r}{\ell}\right) & \text{for} \quad p = 1, \\ -q\left(\frac{2p-1}{2(p-1)}\right)r^{\frac{2(p-1)}{2p-1}}, & \text{for} \quad p \neq \frac{1}{2},\ 1, \end{cases} \tag{2.12}$$

where q is an integration constant of in terms of which the black hole electric charge can be calculated. Based on the fact that the electric potential $A_t(r)$ is required to be finite (or zero) at infinity, one can show that the following condition must be satisfied

$$\frac{p-1}{2p-1} < 0 \qquad \text{which implies that} \qquad \frac{1}{2} < p < 1. \tag{2.13}$$

Also, making use of the relation $F_{tr} = -\partial_r A_t(r)$, the nonzero component of the electromagnetic field is given by

$$F_{tr} = q\, r^{\frac{-1}{2p-1}} \qquad \text{for} \quad \frac{1}{2} < p \leq 1. \tag{2.14}$$

In the geometry, introduced by Eq. (2.6), the gravitational field equations (2.3) lead to the following differential equations

$$e_{tt} = e_{rr} = \frac{G'(r)}{r} + \frac{2\Lambda}{g^2(\varepsilon)} + \frac{(2p-1)}{g^2(\varepsilon)}\frac{(2q_\varepsilon)^p}{r^{\frac{2p}{2p-1}}} - \frac{cc_1 m_G^2}{rg^2(\varepsilon)} = 0, \qquad \text{for} \quad \frac{1}{2} < p \leq 1, \tag{2.15}$$

$$e_{\theta\theta} = G''(r) + \frac{2\Lambda}{g^2(\varepsilon)} - \frac{(2q_\varepsilon)^p}{g^2(\varepsilon)r^{\frac{2p}{2p-1}}} = 0, \qquad \text{for} \quad \frac{1}{2} < p \leq 1, \tag{2.16}$$

for the tt (rr) and $\theta\theta$ components, respectively. Here, q_ε is defined as $q_\varepsilon = f^2(\varepsilon)g^2(\varepsilon)q^2$. As a matter of calculation, one is able to show that these components fulfill the following relation

$$e_{\theta\theta} = \left(1 + r\frac{d}{dr}\right)e_{tt}. \tag{2.17}$$

Regarding Eq. (2.17), one can conclude that two differential equations (2.15) and (2.16) are not independent. Thus one can solve the first order differential equation (2.15) and ensure that the solution satisfies the second order differential equation (2.16).

Now, the metric function $G(r)$, as the solution to the differential equation (2.15), can be obtained as

$$G(r)=\begin{cases} -m-\frac{1}{g^2(\varepsilon)}\left[\Lambda r^2+\frac{(2p-1)^2}{2(p-1)}(2q_\varepsilon)^p r^{\frac{2(p-1)}{2p-1}}-m_G^2cc_1r\right], & \text{for } \frac{1}{2}<p<1, \\ -m-\frac{1}{g^2(\varepsilon)}\left[\Lambda r^2+2q_\varepsilon \ln\left(\frac{r}{\ell}\right)-m_G^2cc_1r\right], & \text{for } p=1. \end{cases} \tag{2.18}$$

The integration constant m, which is named as mass parameter, is related to the black hole mass. An important point is that in the case $p=\frac{3}{4}$, which is corresponding to the case of conformally invariant (CI) electromagnetic Lagrangian [29], the metric function (2.14) is written in the following form

$$G^{(CI)}(r)=-m-\frac{1}{g^2(\varepsilon)}\left[\Lambda r^2-\frac{(2q_\varepsilon)^{\frac{3}{4}}}{2r}-m_G^2cc_1r\right], \quad \text{for} \quad p=\frac{3}{4}. \tag{2.19}$$

Now, by introducing a new variable $\xi=r-\frac{m_G^2cc_1}{2\Lambda}$, the metric function $G(r)$ can be written as

$$G(\xi)=-\mathcal{M}-\Lambda_{eff}\xi^2-\frac{(2p-1)^2(2q_\varepsilon)^p}{2g^2(\varepsilon)(p-1)}\left(\xi+\frac{m_G^2cc_1}{2\Lambda}\right)^{\frac{2(p-1)}{2p-1}}, \quad \text{for} \quad \frac{1}{2}<p<1, \tag{2.20}$$

where, $\mathcal{M}=m+\left(\frac{m_G^2cc_1\ell}{2g(\varepsilon)}\right)^2$ is considered as a new constant, and the effective cosmological constant Λ_{eff} is defined as $\Lambda_{eff}=\frac{\Lambda}{g^2(\varepsilon)}$. By taking the limit $\xi\to\infty$, and noting the allowed range of p, we have

$$\lim_{\xi\to\infty} G(\xi)=-\mathcal{M}-\Lambda_{eff}\xi^2, \tag{2.21}$$

which confirms that the asymptotic behavior of the solutions is AdS. It is asymptotically BTZ for $p=0$. Also, letting $p=0$, the metric function takes the following form

$$G(\zeta)=-m_0-\Lambda_0\zeta^2, \quad \text{with} \quad m_0=m+\frac{(m_G^2cc_1)^2}{1-2\Lambda} \quad \text{and} \quad \zeta=r+\frac{m_G^2cc_1}{1-2\Lambda}, \tag{2.22}$$

which shows a pure AdS spacetime with the redefined cosmological constant $\Lambda_0=\frac{1}{g^2(\varepsilon)}\left(\Lambda-\frac{1}{2}\right)$.

In order to explore the singularities of spacetime under consideration, we need to calculate the curvature scalars. This information can be obtained from the behavior of Ricci and Kretschmann scalars. Thus, we proceed to calculate these curvature scalars. It is matter of calculation to show that

$$\mathcal{R}=6\Lambda+(4p-3)\frac{(2q_\varepsilon)^p}{r^{\frac{2p}{2p-1}}}-\frac{2m_G^2cc_1}{r}, \quad \text{for} \quad \frac{1}{2}<p\leq 1, \tag{2.23}$$

$$\begin{aligned}\mathcal{R}^{\mu\nu\rho\lambda}\mathcal{R}_{\mu\nu\rho\lambda}=&12\Lambda^2+4\Lambda(4p-3)\frac{(2q_\varepsilon)^p}{r^{\frac{2p}{2p-1}}}+\left(8p^2-8p+3\right)\frac{(2q_\varepsilon)^{2p}}{r^{\frac{4p}{2p-1}}}\\ &+\frac{2m_G^4c^2c_1^2}{r^2}-\frac{8\Lambda m_G^2cc_1}{r^2}-4(2p-1)m_G^2cc_1\frac{(2q_\varepsilon)^p}{r^{\frac{4p-1}{2p-1}}}, \quad \text{for} \quad \frac{1}{2}<p\leq 1.\end{aligned} \tag{2.24}$$

Noting the allowed domain of the p-values, an immediate consequence of Eqs. (2.23) and (2.24) is that

$$\lim_{r\longrightarrow\infty}\mathcal{R}=6\Lambda, \qquad \text{and} \qquad \lim_{r\longrightarrow 0}\mathcal{R}=\infty, \tag{2.25}$$

$$\lim_{r\longrightarrow\infty}\mathcal{R}^{\mu\nu\rho\lambda}\mathcal{R}_{\mu\nu\rho\lambda}=12\Lambda^2, \qquad \text{and} \qquad \lim_{r\longrightarrow 0}\mathcal{R}^{\mu\nu\rho\lambda}\mathcal{R}_{\mu\nu\rho\lambda}=\infty. \tag{2.26}$$

The results of Eqs. (2.25) and (2.26) show that: a) The new exact solutions presented here are asymptotically AdS, and b) There is an essential singularity located at the origin, which can be covered by existence of the event horizon.

The plots of $G(r)$ versus r are shown in Fig. 1 for the cases of $p\neq 1$, $p=0.75$ which is corresponding to the CI electrodynamics, and $p=1$ as the metric function of the Einstein-Maxwell-massive gravity theory, separately. They show that the black holes with two horizons, extreme black holes and naked singularity black holes can be obtained. It is worth mentioning that existence of the event horizons and curvature singularities together enable one to interpret the solutions as the new AdS massive black holes under the influence of rainbow functions, as quantum effects.

3. Thermodynamical first law

Here, we seek for satisfaction of the first law of thermodynamics for the novel AdS black holes introduced in previous section. To do this, we need to calculate the conserved and thermodynamic quantities of the black holes.

The black hole temperature $T <$ associated with the black hole horizon, can be obtained by use of the concept of surface gravity κ. Making use of the relation $T=\frac{\kappa}{2\pi}$, with $\kappa=\sqrt{-\frac{1}{2}(\nabla_\mu\chi_\nu)(\nabla^\mu\chi^\nu)}$, and taking $\chi^\mu=(-1,\ 0,\ 0)$, as a matter of calculation, one can show

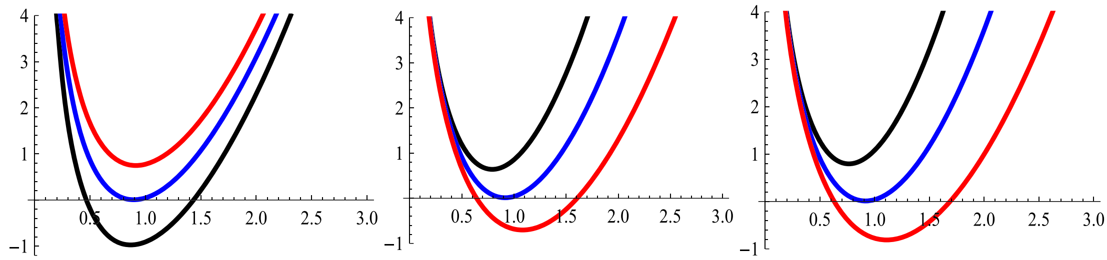

Fig. 1. $G(r)$ versus r for $\Lambda=-1$, $q=2$, $f(\varepsilon)=0.8$, $c=1$, $c_1=1$, $m_G=0.8$, Eq. (2.18). (a) $M=1$, $g(\varepsilon)=0.9$ and $p=0.8$ (black), 0.8185 (blue), 0.83 (red). (b) $M=0.5$, $p=0.75$ and $g(\varepsilon)=0.67$ (black), 0.76 (blue), 0.88 (red). (c) $M=0.5$, $p=1$ and $g(\varepsilon)=0.65$ (black), 0.76 (blue), 0.9 (red).

that $T=\frac{1}{4\pi}G'(r_+)$. r_+ is the black hole outer horizon radius which can be determined as the real root(s) of equation $G(r_+)=0$. Thus, starting from Eq. (2.18), we have

$$T=\frac{1}{4\pi f(\varepsilon)g(\varepsilon)}\left[m_G^2cc_1-2\Lambda r_+-(2p-1)\frac{(2q_\varepsilon)^p}{r_+^{\frac{1}{2p-1}}}\right],\quad \text{for}\quad \frac{1}{2}<p\leq 1. \tag{3.1}$$

The extreme black holes are defined as the black holes with zero temperature. They can occur provided that the black hole charge and size are fixed such that $T(r_{ext},q_{ext})=0$. In other words, extreme black holes can exist if the real $r_+=r_{ext}$ satisfy the following equation

$$2\Lambda r_{ext}+(2p-1)\left(2q_{ext}^2f^2(\varepsilon)g^2(\varepsilon)\right)^p r_{ext}^{\frac{-1}{2p-1}}=m_G^2cc_1. \tag{3.2}$$

Since it is not easy to determine the real roots Eq. (3.2) analytically, we study this by use of figures. The plots of T versus r_+ are shown in Fig. 2. The plots show that the physical black holes (i.e. the black holes with positive temperature) are those with $r_+>r_{ext}$. The black holes with negative temperature are those have horizon radius smaller than r_{ext}. They are not physically reasonable and we call unphysical black holes throughout the paper. These facts have been illustrated in Fig. 2.

According to the entropy-area law, the black hole entropy, as a pure geometrical quantity, is equal to one-fourth of the black hole horizon area. With the help of this law one can write

$$S=\frac{\pi r_+}{2g(\varepsilon)}. \tag{3.3}$$

The other thermodynamic quantity to be determined on the black hole horizon is the black hole electric potential Φ. The electric potential, relative to a reference point at a large distance from the outer event horizon, can be calculated by use of the following standard relation [28,47,48]

$$\Phi=A_\mu\chi^\mu|_{\text{reference}}-A_\mu\chi^\mu|_{r=r_+}, \tag{3.4}$$

where, A_t is given by Eq. (2.12) and χ^μ is the null generator of the horizon. Therefore, we obtain

$$\Phi(r_+)=\begin{cases}-q\ln\left(\frac{r_+}{\ell}\right), & \text{for}\quad p=1,\\ -q\left(\frac{2p-1}{2(p-1)}\right)r_+^{\frac{2(p-1)}{2p-1}}, & \text{for}\quad \frac{1}{2}<p<1.\end{cases} \tag{3.5}$$

The total electric charge of the black holes, as a conserved quantity, can be calculated by considering the electric flux at infinity (i.e. $r\to\infty$). Thus, by use of the Gauss's electric law one can show that [48]

$$Q=\frac{p}{g(\varepsilon)}(2)^{p-2}(q_\varepsilon)^{\frac{2p-1}{2}},\quad \text{for}\quad \frac{1}{2}<p\leq 1. \tag{3.6}$$

It is clear that, the total charge of the black holes is affected by the rainbow functions. It must be noted that, when $p=1$ is chosen, Eq. (3.6) recovers to the result of ref. [20].

The black hole mass is the other conserved quantity to be calculated. As a matter of calculation one is able to obtain the total black hole mass [20]. That is

$$M=\frac{m}{8f(\varepsilon)}. \tag{3.7}$$

Note that the integration constant m is obtained by use of the condition $G(r_+)=0$. Although the black hole mass M given in Eq. (3.7), has been corrected in the presence of rainbow functions, it reduces to the mass of BTZ black holes when the infrared limit is taken.

Now, we can check the first law of thermodynamics for the quantities obtained in this section. This is possible if we obtain the black hole mass as a function of the extensive thermodynamic quantities S and Q

$$M(Q,S)=\begin{cases}\frac{m_G^2cc_1S}{4\pi f(\varepsilon)g(\varepsilon)}-\frac{\Lambda S^2}{2\pi^2 f(\varepsilon)}-\frac{(2p-1)^2 2^p}{16(p-1)f(\varepsilon)}\left(\frac{Q}{p2^{p-2}}\right)^{\frac{2p}{2p-1}}\left(\frac{2S}{\pi}\right)^{\frac{2(p-1)}{2p-1}}, & \text{for } \frac{1}{2}<p<1,\\ \frac{m_G^2cc_1S}{4\pi f(\varepsilon)g(\varepsilon)}-\frac{\Lambda S^2}{2\pi^2 f(\varepsilon)}-\frac{Q^2}{f(\varepsilon)}\ln\left(\frac{2g(\varepsilon)S}{\pi\ell}\right), & \text{for } p=1,\end{cases} \tag{3.8}$$

where, Eqs. (2.18), (3.3), (3.6) and (3.7) have been used. By treating Q and S as the thermodynamical extensive variables, after some straightforward calculations, one can show that

$$\Phi=\left(\frac{\partial M}{\partial Q}\right)_S, \qquad T=\left(\frac{\partial M}{\partial S}\right)_Q, \qquad \text{for} \quad \frac{1}{2}<p\leq 1. \tag{3.9}$$

It confirms the validity of the thermodynamical first law in the form of

$$dM=TdS+\Phi dQ, \qquad \text{for} \quad \frac{1}{2}<p\leq 1. \tag{3.10}$$

The calculations show that, although almost all of the conserved and thermodynamic quantities get modified under the influence of rainbow functions and massive gravitons, they satisfy the first law of black hole thermodynamics in its standard form.

4. Thermal stability in the canonical ensemble

Thermal stability of the black holes is an important issue to be considered when the thermodynamic properties are studied. In this section, we investigate thermal stability of our new black hole solutions by considering the impacts of gravity's rainbow. For this purpose, we use the canonical ensemble as a fundamental thermodynamic method. By use of this approach and noting the signature of the black hole heat capacity, one can obtain informations about points of type-one and type-two phase transition as well as the local stability of the black holes. Thus, we need to calculate the heat capacity of the new black holes identified in this article. The black hole heat capacity, with the black hole charge as a constant, is given through the following relation [49–51]

$$\mathcal{C}_Q=T\left(\frac{\partial S}{\partial T}\right)_Q. \tag{4.1}$$

Regarding this definition and making use of Eqs. (3.1) and (3.3), after some algebraic calculations, one is able to show that

$$\mathcal{C}_Q=\frac{\pi r_+\left[m_G^2cc_1r_+-2\Lambda r_+^2-(2p-1)\eta_+\right]}{2g(\varepsilon)\left(\eta_+-2\Lambda r_+^2\right)}, \qquad \text{for} \quad \frac{1}{2}<p\leq 1, \tag{4.2}$$

where, $\eta_+=(2q_\varepsilon)^p\, r_+^{\frac{2(p-1)}{2p-1}}$. It is well-known that a physically reasonable black hole (i.e. black holes having positive temperature) is locally stable if its heat capacity is positive. The black holes with negative heat capacity are known as unstable. The unstable black holes undergo thermodynamic phase transition to be stabilized. Type-one phase transition points are characterized by vanishing points of heat capacity (or temperature). The divergent points of the black hole heat capacity are the location of the type-two phase transition points. Noting Eq. (4.2) one can conclude that the denominator of the heat capacity does not vanish and there is no point of type-one phase transition. Also, its numerator have only one positive vanishing point at $r_+=r_{ext}$ where type-one phase transition occurs. For more clarifying, we have plotted $\mathcal{C}_Q$ and T versus r_+ under the influence of rainbow functions in Fig. 2. The plots show that $\mathcal{C}_Q$ does not diverge and there is no point of type-two phase transition. The black holes undergo type-one phase transition at the horizon radius of the extreme black holes $r_+=r_{ext}$. Also, the black holes with the horizon radii grater than r_{ext}, have positive heat capacity and temperature, are locally stable.

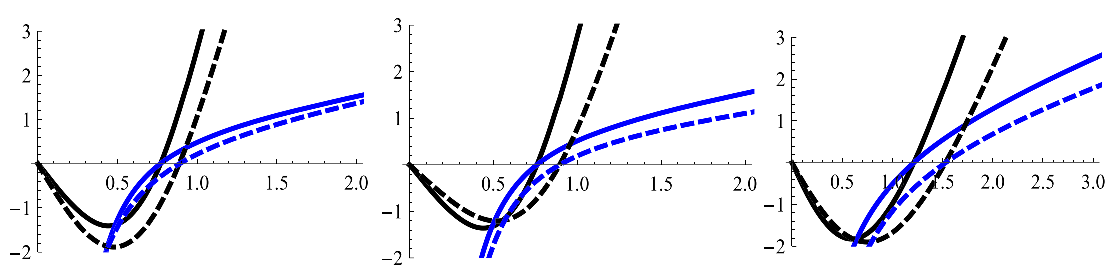

Fig. 2. $\mathcal{C}_Q$ (black) and T (blue) versus r_+ for $\Lambda=-1$, $q=2$, $f(\varepsilon)=0.8$, $c=1$, $c_1=1$, $m_G=1$, Eqs. (3.1) and (4.2). (a) $5\mathcal{C}_Q$ and $3T$: $g(\varepsilon)=0.9$, $p=0.76$ (continues), 0.85 (dashed). (b) $5\mathcal{C}_Q$ and $3T$: $p=0.75$, $g(\varepsilon)=0.9$ (continues), 1.2 (dashed). (c) $2\mathcal{C}_Q$ and $4T$: $p=1$, $g(\varepsilon)=0.9$ (continues), 1.1 (dashed).

5. Geometrical thermodynamics

The other method of studying the thermodynamic stability or finding the positions of the thermodynamic phase transition points is the geometrical thermodynamics (GTs). In this method, the points of thermodynamic phase transitions are identified by the divergent points of the thermodynamic Ricci scalar. Indeed, the information about thermodynamic phase transitions is extracted from Ricci scalar of a thermodynamic metric. There are a broad variety of proposed thermodynamic metrics. Among them the Weinhold, Ruppeiner and Quevedo thermodynamic metrics have attracted more attention [52–54]. Recently, it has been demonstrated that these metrics produce extra transition points, and a new thermodynamic metric has been proposed by Hendi, Panahiyan, Eslam Panah and Momennia (HPEM) [55] which can produce the exact locations of the phase transition points [55,56]. The proposed metric of HPEM is written as

$$[ds^{(HPEM)}]^2 = \frac{S M_S}{M_{QQ}^3}\left(-M_{SS}dS^2 + M_{QQ}dQ^2\right). \tag{5.1}$$

Here, M is considered as a function of black hole charge and entropy, and the indices Q and S indicate the quantities with respect to which derivatives of mass are taken. Making use of this thermodynamic metric, one is able to calculate the related Ricci scalar. It can be written in the following form

$$\begin{aligned}\mathcal{R}^{(HPEM)} = \frac{1}{2S^3M_S^3M_{SS}^2}\Big[&2S^2M_{SQ}^2M_{SS}^2M_{QQ}^2 - S^2M_SM_{QQ}^2M_{SQQ}M_{SS}^2 - S^2M_SM_{SS}M_{QQ}^2M_{SQ}M_{SSQ}\\ &+ S^2M_{QQQ}M_SM_{SQ}M_{SS}^2M_{QQ} - 6S^2M_{SS}M_{SSQQ}M_S^2M_{QQ}^2 - 9S^2M_S^2M_{QQQ}^2M_{SS}^2\\ &+ 6S^2M_{QQ}M_{QQQQ}M_{SS}^2M_S^2 + 4S^2M_{SS}M_{QQQ}M_{SSQ}M_{QQ}M_S^2 - 2S^2M_{SS}^3M_{QQ}^3\\ &+ 2S^2M_S^2M_{SS}M_{QQS}^2M_{QQ} + S^2M_{QQ}^3M_{SS}M_SM_{SSS} - 2M_S^2M_{QQ}^3M_{SS} - SM_S^2M_{QQ}^3M_{SSS}\\ &+ SM_S^2M_{QQ}^2M_{SS}M_{QQS} + 2S^2M_S^2M_{QQ}^2M_{QQS}M_{SSS} + S^2M_S^2M_{QQ}^2M_{SSQ}^2\Big].\end{aligned} \tag{5.2}$$

In order to find the divergent points of the above-given thermodynamical Ricci scalar, as the points of thermodynamic phase transitions, we have plotted $\mathcal{R}^{(HPEM)}$ and T versus r_+ together, in Fig. 3, to determine the phase transition points and to show that the results of GTs, by applying the HPEM metric, are completely consistent with those of canonical ensemble method. In addition, the plots show that there is only one point of type-one phase transition which coincides with the vanishing point of the black hole temperature (or heat capacity). There is no extra divergence point. Thus, there is no point of type-two and no extra type-one phase transition point.

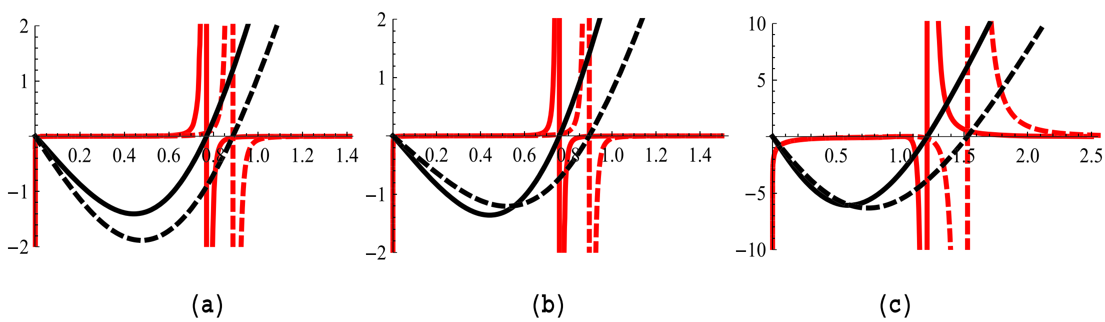

Fig. 3. $\mathcal{R}^{(HPEM)}$ (red) and $\mathcal{C}_Q$ (black) versus r_+ for $\Lambda=-1$, $q=2$, $f(\varepsilon)=0.8$, $c=1$, $c_1=1$, $m_G=1$, Eqs. (4.2) and (5.2). (a) $10^{-7}\mathcal{R}^{(HPEM)}$ and $5\mathcal{C}_Q$: $g(\varepsilon)=0.9$, $p=0.76$ (continues), 0.85 (dashed). (b) $5\times10^{-8}\mathcal{R}^{(HPEM)}$ and $5\mathcal{C}_Q$: $p=0.75$, $g(\varepsilon)=0.9$ (continues), 1.2 (dashed). (c) $5\times10^{-3}\mathcal{R}^{(HPEM)}$ and $10\mathcal{C}_Q$: $p=1$, $g(\varepsilon)=0.9$ (continues), 1 (dashed).

6. Global stability and Hawking-Page phase transition

The black hole global stability and the Hawking-Page phase transition points can be determined by analyzing the signature of Gibbs free energy. Hawking and Page argued that a black hole is globally stable provided that its Gibbs free energy is positive. The unstable black holes experience Hawking-Page phase transition. The critical temperature at which black holes undergo Hawking-Page phase transition is labeled by T_c. Any black hole with the negative free energy is assumed to be in the thermal (radiative) phase [57]. With these issues in mind, we proceed to calculate the Gibbs free energy ($\mathcal{G}$) for the novel energy-dependent massive black holes introduced in the present work. This can be down by use of the following definition [58,59]

$$\mathcal{G}(r_+) = M(r_+) - T(r_+)S(r_+) - Q\,\Phi(r_+). \tag{6.1}$$

Making use of Eqs. (3.3), (3.5), (3.6) and (3.8) into Eq. (6.1), after some algebraic manipulations, we obtain

$$\mathcal{G}(r_+) = \begin{cases} \frac{\Lambda r_+^2}{8f(\varepsilon)g^2(\varepsilon)} + \frac{f(\varepsilon)q^2}{4}\left[1+\ln\left(\frac{r_+}{\ell}\right)\right], & \text{for} \quad p=1, \\ \frac{1}{8f(\varepsilon)g^2(\varepsilon)}\left[\Lambda r_+^2 - \frac{(2p-1)(4p+1)}{2(p-1)}(2q_\varepsilon)^p\, r_+^{\frac{2(p-1)}{2p-1}}\right], & \text{for} \quad \frac{1}{2}<p<1. \end{cases} \tag{6.2}$$

An immediate consequence of Eq. (6.2) is that it does not depend on the mass term (m_G), and it is identical in both of the massive and massless gravity theories. For a more close analyzing the global stability of our solutions we need to determine the real roots of $\mathcal{G}(r_+)=0$. It is easily shown that the vanishing points of Gibbs free energy are identified by the following relation

$$r_+ \equiv \begin{cases} r_1 = \sqrt{\frac{q_\varepsilon}{\Lambda}L_W(\xi_1)}, & r_+<\ell, \\ r_2 = \sqrt{\frac{-q_\varepsilon}{\Lambda}L_W(\xi_2)}, & r_+>\ell, \end{cases} \quad \text{for} \quad p=1, \tag{6.3}$$

where $\xi_1 = -\left(e^2 q_\varepsilon\right)^{-1}$, $\xi_2 = e^2 q_\varepsilon^{-1}$, and $L_W(\xi)$ is the well-known Lambert function which satisfies the following equation [34]

$$L_W(\xi) exp[L_W(\xi)] = \xi. \tag{6.4}$$

The Lambert function $L_W(\xi)$ can be expanded as

$$L_W(\xi) = \xi - \xi^2 + \frac{3}{2}\xi^3 - \frac{8}{3}\xi^4 + \cdots. \tag{6.5}$$

Also, by solving the equation $\mathcal{G}(r_+) = 0$, one can show that

$$r_+ \equiv R_1 = \left[\frac{2\Lambda(p-1)}{(2p-1)(4p+1)}\right]^{\frac{2p-1}{2p}}, \quad \text{for} \quad \frac{1}{2} < p < 1. \tag{6.6}$$

Noting the fact that Λ is negative, both of these roots exist in the range of permitted p-values. The plots of $G(r_+)$ versus r_+ are shown in Fig. 4, for more clarifying. The plots show that, for the case $\frac{1}{2} < p < 1$, the physical black holes with the horizon radius in the interval $r_{ext} < r_+ < R_1$ are globally stable, and those having the horizon radius equal to R_1 experience Hawking-Page phase transition. Also, the black holes with the horizon radii greater than R_1 prefer to be in the radiative phase [panels (a) and (b)].

The black hole Gibbs free energy vanishes at the points $r_+ = r_1$ and $r_+ = r_2$ with $r_1 < r_2$. The plots of Fig. 4-(c) show that the physical black holes with horizon radii in the range $r_{ext} < r_+ < r_1$ are globally stable. Those with horizon radius equal to r_1 undergo Hawking-Page phase transition, and the physical black holes with the horizon radii grater than r_2 are in the phase of radiation.

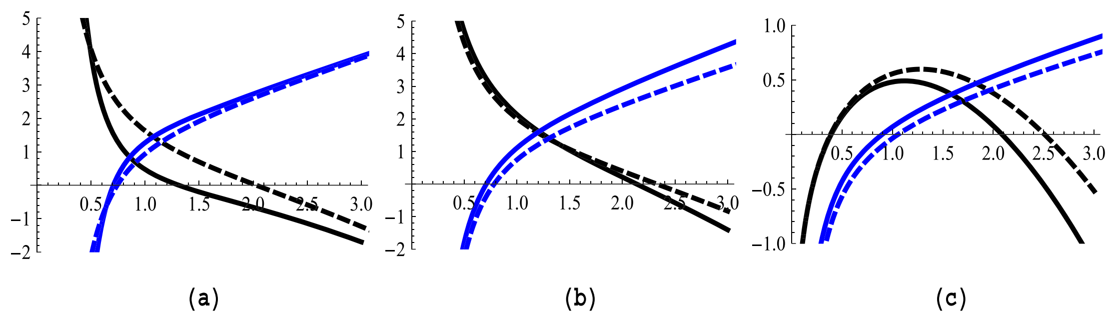

Fig. 4. $\mathcal{G}$(black) and T (blue) versus r_+ for $\Lambda = -1$, $q = 2$, $f(\varepsilon) = 0.8$, $c = 1$, $c_1 = 1$, $m_G = 1$, Eq. (6.2). (a) G and $5T$: $g(\varepsilon) = 0.9$, $p = 0.64$ (continues), 0.73 (dashed). (b) G and $5T$: $p = 0.75$, $g(\varepsilon) = 0.8$ (continues), 0.95 (dashed). (c) G and T : $p = 1$, $g(\varepsilon) = 0.7$ (continues), 0.8 (dashed).

7. Conclusion

Starting from a suitable three-dimensional action which, in addition to the Einstein-Λ terms, contains a mass term and the Lagrangian of power-law nonlinear electrodynamics which are coupled to gravity. By varying this action, with respect to the various fields, we obtained the related field equations in the massive gravity theory. The exact solutions of the field equations have been obtained in an energy-dependent and circularly symmetric spacetime. As the result two classes of charged rainbow black holes have been introduced in massive gravity theory and under the influence of nonlinear electrodynamics. Through mathematical properties of the solutions, and calculation of the curvature scalars, it has been found that the solutions have the same asymptotic behavior as AdS black holes.

Next, the black hole mass, electric charge, and the black hole temperature, entropy and electric potential, in order, as the conserved and thermodynamic quantities have been calculated by use of the geometric and thermodynamic approaches. The calculations show that almost all of these quantities are affected by the rainbow functions and nonlinearity parameter p. Validity of the first law of black hole thermodynamics has been proved, with the help of the obtained conserved and thermodynamic quantities, for both classes of AdS Massive black holes introduced here.

Then, local stability or phase transition of the nonlinearly charged rainbow black holes have been studied from the canonical ensemble and the geometrical thermodynamics approaches, separately. To do these, the heat capacity and the thermodynamic Ricci scalar of the black holes have been calculated. The results show that, for our new massive black holes, there is no type-two phase transition point. The black holes with the horizon radius equal to r_{ext} undergo type-one phase transition, and those with the horizon radii in the range $r_+ > r_{ext}$ are locally stable. It has been shown that the results of these two alternative approaches are identical (Figs. 2 and 3).

Finally, the global stability of the black holes has been investigated by use of the grand canonical ensemble and regarding the Gibbs free energies. By analyzing the Gibbs free of the black holes, the horizon radius of the black holes which undergo Hawking-page phase transition as well as the ranges at which the black holes are globally stable or are in the radiative phase have been identified (see Fig. 4 and its explanations).

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