



More lepton flavor violating observables for LHCb's run 2



Diego Guadagnoli^{a,*}, Dmitri Melikhov^{b,c}, M eril Reboud^{a,d}

^a Laboratoire d'Annecy-le-Vieux de Physique Th eorique UMR5108, Universit  de Savoie Mont-Blanc et CNRS, B.P. 110, F-74941, Annecy-le-Vieux Cedex, France

^b Institute of Nuclear Physics, Moscow State University, 119992, Moscow, Russia

^c Faculty of Physics, University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austria

^d  cole Normale Sup rieure de Lyon, 46 all e d'Italie, F-69364, Lyon Cedex 07, France

ARTICLE INFO

Article history:

Received 25 May 2016

Received in revised form 9 July 2016

Accepted 9 July 2016

Available online 14 July 2016

Editor: G.F. Giudice

ABSTRACT

The R_K measurement by LHCb suggests non-standard lepton non-universality (LNU) to occur in $b \rightarrow s\ell^+\ell^-$ transitions, with effects in muons rather than electrons. A number of other measurements of $b \rightarrow s\ell^+\ell^-$ transitions by LHCb and B -factories display disagreement with the SM predictions and, remarkably, these discrepancies are consistent in magnitude and sign with the R_K effect. Non-standard LNU suggests non-standard lepton flavor violation (LFV) as well, for example in $B \rightarrow K\ell\ell'$ and $B_s \rightarrow \ell\ell'$. There are good reasons to expect that the new effects may be larger for generations closer to the third one. In this case, the $B_s \rightarrow \mu e$ decay may be the most difficult to reach experimentally. We propose and study in detail the radiative counterpart of this decay, namely $B_s \rightarrow \mu e \gamma$, whereby the chiral-suppression factor is replaced by a factor of order α/π . A measurement of this mode would be sensitive to the same physics as the purely leptonic LFV decay and, depending on experimental efficiencies, it may be more accessible. A realistic expectation is a factor of two improvement in statistics for either of the $B_{d,s}$ modes.

  2016 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP³.

1. Introduction

During run 1, the LHCb experiment performed a number of measurements of $b \rightarrow s$ and $b \rightarrow c$ transitions, finding less than perfect agreement with the Standard Model (SM). On the same timescale, updates on some of the most interesting of these measurements were published by the B factories as well, with results consistent with LHCb's.

In more detail, the experimental situation can be summarized as follows. The most striking effect is that measured by LHCb in the ratio known as R_K [1]

$$R_K \equiv \frac{\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)}{\mathcal{B}(B^+ \rightarrow K^+ e^+ e^-)} = 0.745_{-0.074}^{+0.090} (\text{stat}) \pm 0.036 (\text{syst}), \quad (1)$$

in the di-lepton invariant-mass-squared range $[1, 6] \text{ GeV}^2$. The SM predicts unity with percent-level corrections [2–5], implying a 2.6σ discrepancy. Two convincing aspects of this discrepancy are the theoretical cleanness of R_K , and the fact that the measurement in the electron channel, the one more subject to large systematics, is, within errors, in agreement with the SM prediction [1]. On the

other hand, the muon-channel measurement, expected to be experimentally more solid, yields [6,7]

$$\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)_{[1,6]} = (1.19 \pm 0.03 \pm 0.06) \times 10^{-7}, \quad (2)$$

which is about 30% lower than the SM prediction, $\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)_{[1,6]}^{\text{SM}} = (1.75_{-0.29}^{+0.60}) \times 10^{-7}$ [8–10].

The very same pattern, with data lower than the SM prediction, is actually also observed in the $B_s \rightarrow \phi \mu^+ \mu^-$ channel and in the same range $m_{\mu\mu}^2 \in [1, 6] \text{ GeV}^2$, as initially found in 1/fb of LHCb data [11] and then confirmed by a full run-1 analysis [12]. This discrepancy is estimated to be more than 3σ [12].

Additional support comes from the $B \rightarrow K^* \mu\mu$ decay, for which LHCb can perform a full angular analysis. The quantity known as P'_5 , designed to have reduced sensitivity to form-factor uncertainties [13], exhibits a discrepancy in two bins, again in the low- $m_{\mu\mu}^2$ range. The effect was originally found in 1/fb of LHCb data [14], and confirmed by a full run-1 analysis [15] as well as, very recently, by a Belle analysis [16]. The P'_5 discrepancy as estimated by LHCb amounts to 3.4σ , and is in the 2σ -ballpark from Belle (2.1σ as compared to [17] and 1.7σ as compared to [18–20]). The theoretical error is, however, still debated, see in particular [20–23].

Further interesting results come from measurements of the ratios $R(D^{(*)}) \equiv \mathcal{B}(B \rightarrow D^{(*)} \tau \nu) / \mathcal{B}(B \rightarrow D^{(*)} \ell \nu)$. They were initially reported by BaBar [24] to be in excess of the SM prediction [25,26].

* Corresponding author.

E-mail address: diego.guadagnoli@laph.cnrs.fr (D. Guadagnoli).

The excess in the $R(D^*)$ channel was recently confirmed by LHCb in 3/fb of run-1 data [27]. Consistent results were also reported by Belle in two analyses, using respectively hadronically- [28] and semileptonically-decaying [29] taus.

On the theoretical side, it is likewise remarkable that a consistent picture of all the above-mentioned effects is possible already within an effective-theory approach. Global fits to the Wilson coefficients of the general $b \rightarrow s$ effective Hamiltonian point towards new-physics (NP) shifts either in C_9 only, or in the $SU(2)_L$ -invariant direction $C_9^{\text{NP}} = -C_{10}^{\text{NP}}$, with comparable χ^2 between the two cases [30,31]. (See also [32–36].) The corresponding terms in the effective Hamiltonian read

$$\begin{aligned} \mathcal{H}_{\text{SM+NP}}(\bar{b} \rightarrow \bar{s}\ell^+\ell^-) \\ = -\frac{4G_F}{\sqrt{2}} V_{tb}^* V_{ts} \frac{\alpha_{\text{em}}(m_b)}{4\pi} \bar{b}_L \gamma^\lambda s_L \bar{\ell} (C_9^\ell \gamma_\lambda + C_{10}^\ell \gamma_\lambda \gamma_5) \ell + \text{H.c.}, \end{aligned} \quad (3)$$

where $C_{9,10}^\ell = C_{9,10}^{\text{SM}} + C_{9,10}^{\text{NP},\ell}$. The ℓ label takes into account that the NP contributions to the Wilson coefficients may depend on the lepton flavor. This possibility, suggested by the R_K result and referred to as lepton-flavor non-universality (LNU), has inspired a number of SM extensions where new LNU interactions are introduced via the exchange of multi-TeV particles [30,37–71].

If data confirm the presence of beyond-SM LNU, in general – namely, in the absence of further assumptions – we also expect non-standard LFV [45].¹ One may object that this is not the case already in the SM plus any minimal mechanism for neutrino masses, as in this case LFV in decay is suppressed by m_ν . However, there is no compelling argument why the new physics explaining R_K should have neutrino masses as the only LFV spurions.

More specifically, ref. [45] proposed that the R_K deviation from unity be due to an effective interaction involving dominantly quarks and leptons of the third generation, namely

$$\mathcal{H}_{\text{NP}} = G \bar{b}'_L \gamma^\lambda b'_L \bar{\tau}'_L \gamma_\lambda \tau'_L. \quad (4)$$

If the scale of this interaction is above the electroweak-symmetry breaking one, quarks and leptons are, in general, in the gauge basis, indicated with a prime. After rotation to the mass eigenbasis, one can then expect effects that are largest for third-generation quarks and leptons, and suppressed accordingly for lighter generations.²

Assuming the interaction (4), the amount of LNU pointed to by R_K actually allows to quantify rather generally [45] the expected amount of LFV. In fact, R_K yields the ratio

$$\rho_{\text{NP}} = -0.159^{+0.060}_{-0.070} \quad (5)$$

between the NP and the SM+NP contribution to C_9^μ . Then

$$\frac{\mathcal{B}(B \rightarrow K \ell_i^\pm \ell_j^\mp)}{\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)} \simeq 2\rho_{\text{NP}}^2 \frac{|(U_L^\ell)_{3i}|^2 |(U_L^\ell)_{3j}|^2}{|(U_L^\ell)_{32}|^4}, \quad (6)$$

implying

$$\begin{aligned} \mathcal{B}(B \rightarrow K \ell_i^\pm \ell_j^\mp) &\simeq 5\% \cdot \mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-) \cdot \frac{|(U_L^\ell)_{3i}|^2 |(U_L^\ell)_{3j}|^2}{|(U_L^\ell)_{32}|^4} \\ &\simeq 2.2 \times 10^{-8} \cdot \frac{|(U_L^\ell)_{3i}|^2 |(U_L^\ell)_{3j}|^2}{|(U_L^\ell)_{32}|^4}, \end{aligned} \quad (7)$$

where we used $\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-) \simeq 4.3 \times 10^{-7}$ [6], and neglected phase-space differences across the different possible lepton final states.³ Eq. (7) tells us that LFV $B \rightarrow K$ decays are expected to be in the ballpark of 10^{-8} times an unknown factor involving U_L^ℓ matrix entries. In the $\ell_i \ell_j = e\mu$ case, this ratio reads $|(U_L^\ell)_{31}|/(U_L^\ell)_{32}| \lesssim 3.7$ [45], implying that the $B \rightarrow K\mu e$ rate may be around 10^{-8} , or much less if $|(U_L^\ell)_{31}|/(U_L^\ell)_{32}| \ll 1$. The latter possibility would suggest U_L^ℓ entries that decrease in magnitude with the distance from the diagonal. But then one may expect the ratio $|(U_L^\ell)_{33}|/(U_L^\ell)_{32}| > 1$, implying a $B \rightarrow K\mu\tau$ rate of $O(10^{-8})$ or above. In short, assuming the interaction (4), one can hope that at least one LFV $B \rightarrow K$ decay rate be in the ballpark of 10^{-8} [45], which happens to be within reach at LHCb's run 2. An entirely analogous reasoning applies for the purely leptonic modes $B_s \rightarrow \ell_i^\pm \ell_j^\mp$. Similarly as eq. (7) one has

$$\mathcal{B}(B_s \rightarrow \ell_i^\pm \ell_j^\mp) \simeq 5\% \cdot \mathcal{B}(B_s \rightarrow \mu^+ \mu^-) \cdot \frac{|(U_L^\ell)_{3i}|^2 |(U_L^\ell)_{3j}|^2}{|(U_L^\ell)_{32}|^4}. \quad (8)$$

Therefore, purely leptonic LFV decays of the B_s may well be within reach of LHCb during run 2, if the U -matrix factor on the r.h.s. is of order unity (or larger!) for at least one LFV mode.⁴

From the previous line of argument about the U_L^ℓ -matrix suppression, one may expect the $B_s \rightarrow e\mu$ to be the most difficult to access among the purely leptonic LFV modes. It is therefore useful to search for additional decays, that can give access to the same physics, while being comparably (or, hopefully, more) accessible experimentally.

Here we would like to point out that, in the $B_s \rightarrow \mu e$ channel, one such ‘proxy’ decay is provided by the inclusion of an additional hard photon in the final state. In fact, the additional photon replaces the chiral-suppression factor, of order $\max(m_{\ell_1}, m_{\ell_2})^2/m_{B_s}^2$, with a factor of order α_{em}/π . In the case of the μe channel these two factors are respectively 4×10^{-4} and 2 per mil. The actual enhancement of $\mathcal{B}(B_s \rightarrow \mu e \gamma)$ over the non-radiative counterpart needs to be worked out by explicit calculation.

We thus compute the decay of a generic pseudo-scalar meson to $\ell_1^+ \ell_2^- \gamma$, and study in detail the cases of B_s, B_d, K as initial state and of μe as final state. (To fix notation, formulae are given for B_q .) The photon energy E_γ is integrated above an experimental cut around 60 MeV, comparable to the experimental resolution on the total invariant mass of the final states, which has to yield the decaying-meson mass. We subsequently compare the resulting radiative rates with the non-radiative ones. We find an $O(1)$ factor for the B_q cases and at the percent level for the kaon case. Therefore, if experimental efficiencies for the radiative vs. the non-radiative B_q decays are comparable, measurements of the radiative counterparts of LFV decays will provide crucial quantitative tests of the new-physics scenario responsible for a possible LFV signal.

¹ Forbidding non-standard LFV within models able to explain R_K requires a dynamical or a symmetry mechanism, that for example extends the SM lepton-flavor symmetries to the new model. Attempts in this direction are in refs. [52,61].

² Actually, given the $O(\text{TeV})$ -scale of the new interactions, it is expected that the fermionic d.o.f. involved be complete multiplets under the unbroken EW symmetry [72]. This observation establishes correlations [73] between (among the others) effects in $b \rightarrow s$ and in $b \rightarrow c$ transitions, thus allowing a common origin for the $b \rightarrow s\ell\ell$ discrepancies and those in $R(D)$ and $R(D^*)$. For further quantitative studies see [62,65,68,74–76].

³ These differences are of the order of 30% [59,64], hence they are not important for the present argument.

⁴ For a (rough) comparison, we should keep in mind that at run 2 the LHCb is expected [77] to provide a first measurement of $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$, which in the SM is about 3% of $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$.

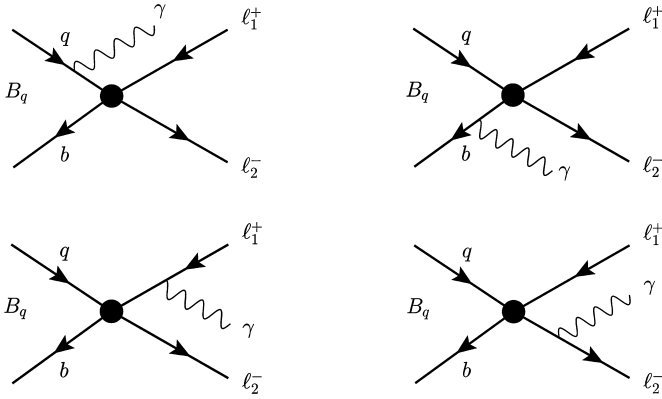


Fig. 1. Diagrams contributing to $B_q \rightarrow \bar{l}_1 l_2 \gamma$, within the interaction in eq. (4). The black dot denotes the insertion of \mathcal{O}_9 or \mathcal{O}_{10} .

2. Observables

Within the interaction in eq. (4) the contributions to the process $B_q \rightarrow \ell_1^+ \ell_2^- \gamma$ are given by the diagrams in Fig. 1, where the black dot denotes the insertion of the (LFV counterparts of the) operators \mathcal{O}_9 or \mathcal{O}_{10} , in the following normalization [78]

$$\begin{aligned} \mathcal{O}_9 &= \frac{\alpha}{4\pi} (\bar{l}_L \gamma^\lambda q_L) (\bar{l}_2 \gamma_\lambda \ell_1), \\ \mathcal{O}_{10} &= \frac{\alpha}{4\pi} (\bar{l}_L \gamma^\lambda q_L) (\bar{l}_2 \gamma_\lambda \gamma_5 \ell_1). \end{aligned} \quad (9)$$

The diagrams in Fig. 1 correspond to two amplitudes: the one where the photon is emitted from the B -meson quarks, denoted as $A^{(1)}$, and the one due to Bremsstrahlung from the final-state leptons, denoted as $A^{(2)}$. Accordingly, the decay width is the sum of three terms, those coming from the magnitudes squared of the above amplitudes, and that due to the interference between $A^{(1)}$ and $A^{(2)}$. In a notation close to ref. [79], these three contributions read:

$$\frac{d^2\Gamma^{(1)}}{d\hat{s}d\hat{t}} = \frac{G_F^2 \alpha_{\text{em}}^3 M_{B_q}^5}{2^{10} \pi^4} |V_{tb} V_{tq}^*|^2 \left[x^2 B_0^{(1)} + x \xi B_1^{(1)} + \xi^2 B_2^{(1)} \right], \quad (10)$$

$$\begin{aligned} \frac{d^2\Gamma^{(2)}}{d\hat{s}d\hat{t}} &= \frac{G_F^2 \alpha_{\text{em}}^3 M_{B_q}^5}{2^{10} \pi^4} |V_{tb} V_{tq}^*|^2 \left(\frac{f_{B_q}}{M_{B_q}} \right)^2 \frac{1}{(\hat{t} - \hat{m}_2^2)(\hat{u} - \hat{m}_1^2)^2} \times \\ &\left[x^2 B_0^{(2)} + x \xi B_1^{(2)} + \xi^2 B_2^{(2)} \right], \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{d^2\Gamma^{(12)}}{d\hat{s}d\hat{t}} &= \frac{G_F^2 \alpha_{\text{em}}^3 M_{B_q}^5}{2^{10} \pi^4} |V_{tb} V_{tq}^*|^2 \left(\frac{f_{B_q}}{M_{B_q}} \right) \frac{1}{(\hat{t} - \hat{m}_2^2)(\hat{u} - \hat{m}_1^2)} \times \\ &\left[x^2 B_0^{(12)} + x \xi B_1^{(12)} + \xi^2 B_2^{(12)} \right], \end{aligned} \quad (12)$$

where

$$\hat{s} = \frac{(p-k)^2}{M_{B_q}^2}, \quad \hat{t} = \frac{(p-p_1)^2}{M_{B_q}^2}, \quad \hat{u} = \frac{(p-p_2)^2}{M_{B_q}^2}, \quad (13)$$

and the $\hat{s}, \hat{t}, \hat{u}$ variables fulfill the constraint

$$\hat{s} + \hat{t} + \hat{u} = 1 + \hat{m}_1^2 + \hat{m}_2^2. \quad (14)$$

Here p, k, p_1 and p_2 denote the momenta of, respectively, the initial meson, the emitted photon, and the leptons $\ell_{1,2}$, whose masses are $m_{1,2}$. The hat denotes that the given variable has been made dimensionless by normalizing it to an appropriate power of M_{B_q} . Furthermore

$$x \equiv 1 - \hat{s}, \quad \xi \equiv \hat{u} - \hat{t} + \frac{\hat{m}_2^2 - \hat{m}_1^2}{\hat{s}}. \quad (15)$$

The $B_i^{(j)}$ functions are defined as follows:

$$\begin{aligned} B_0^{(1)} &= \left(F_V^2(\hat{s}) + F_A^2(\hat{s}) \right) \left[\left(\hat{s} - \frac{\hat{M}^2 \hat{m}^2}{\hat{s}} \right) (|C_9|^2 + |C_{10}|^2) + \right. \\ &\quad \left. 4\hat{m}_1 \hat{m}_2 (|C_9|^2 - |C_{10}|^2) \right], \\ B_1^{(1)} &= 8\hat{s} F_V(\hat{s}) F_A(\hat{s}) \text{Re}(C_9 C_{10}^*), \\ B_2^{(1)} &= \hat{s} \left(F_V^2(\hat{s}) + F_A^2(\hat{s}) \right) (|C_9|^2 + |C_{10}|^2), \end{aligned} \quad (16)$$

$$\begin{aligned} B_0^{(2)} &= 2\hat{M}^2 \left(2\hat{s} \rho (1 - \hat{m}^2) + x^2 \left(1 - \frac{\hat{M}^2 \hat{m}^2}{\hat{s}^2} \right) \right) |C_{10}|^2 + \\ &\quad 2\hat{m}^2 \left(2\hat{s} \rho (1 - \hat{M}^2) + x^2 \left(1 - \frac{\hat{M}^2 \hat{m}^2}{\hat{s}^2} \right) \right) |C_9|^2, \\ B_1^{(2)} &= \frac{4x^2 \hat{M} \hat{m}}{\hat{s}} \left[\hat{M}^2 |C_{10}|^2 + \hat{m}^2 |C_9|^2 \right], \\ B_2^{(2)} &= 2\hat{M}^2 \left(2\hat{s}(\hat{m}^2 - 1) - x^2 \right) |C_{10}|^2 + \\ &\quad 2\hat{m}^2 \left(2\hat{s}(\hat{M}^2 - 1) - x^2 \right) |C_9|^2, \end{aligned} \quad (17)$$

$$\begin{aligned} B_0^{(12)} &= -8x F_V(\hat{s}) \left(\hat{m}_1^2 + \hat{m}_2^2 - \frac{\hat{M}^2 \hat{m}^2}{\hat{s}} \right) \text{Re}(C_9 C_{10}^*) + \\ &\quad \frac{4\hat{M} \hat{m}}{\hat{s}} F_A(\hat{s}) \left[(\hat{M}^2 - \hat{s})(1 - \hat{m}^2) |C_{10}|^2 + \right. \\ &\quad \left. (\hat{m}^2 - \hat{s})(1 - \hat{M}^2) |C_9|^2 \right], \\ B_1^{(12)} &= -8x \hat{M} \hat{m} F_V(\hat{s}) \text{Re}(C_9 C_{10}^*) - \\ &\quad 4x F_A(\hat{s}) \left[\hat{M}^2 |C_{10}|^2 + \hat{m}^2 |C_9|^2 \right], \\ B_2^{(12)} &= 4\hat{s} \hat{M} \hat{m} F_A(\hat{s}) (|C_{10}|^2 + |C_9|^2), \end{aligned} \quad (18)$$

where

$$\hat{M} = \hat{m}_1 + \hat{m}_2, \quad \hat{m} = \hat{m}_2 - \hat{m}_1, \quad \rho = \frac{(\hat{s} - \hat{M}^2)(\hat{s} - \hat{m}^2)}{\hat{s}^2}. \quad (19)$$

In the lepton-flavor conserving limit, these equations reproduce the results of ref. [79], apart from an overall sign typo in eqs. (2.13) and (3.3) of that paper.

The f_{B_q} decay constant and the $B_q \rightarrow \gamma$ form factors involved are defined by [80]

$$\langle 0 | \bar{b} \gamma^\mu \gamma_5 q | B_q(p) \rangle = i p^\mu f_{B_q}, \quad (20)$$

$$\langle \gamma(k, \epsilon) | \bar{b} \gamma^\mu q | B_q(p) \rangle = \epsilon_\nu^* \epsilon^{\mu\nu\rho\sigma} p_\rho k_\sigma \frac{F_V^{(B_q)}(\hat{s})}{M_{B_q}}, \quad (21)$$

$$\langle \gamma(k, \epsilon) | \bar{b} \gamma^\mu \gamma_5 q | B_q(p) \rangle = i \epsilon_\nu^* (g^{\mu\nu} p k - p^\nu k^\mu) \frac{F_A^{(B_q)}(\hat{s})}{M_{B_q}}. \quad (22)$$

For the $F_{V,A}^{(B_q)}$ form factors we use the parameterization in [79]. (Within few percent, the $B_s \rightarrow \gamma$ form factors coincide with the $B_d \rightarrow \gamma$ ones. Such differences are clearly negligible in our context.)

For the $K \rightarrow \gamma$ form factors, defined in complete analogy to eqs. (21)–(22) and denoted as $F_{V,A}^{(K)}$, we instead adapt to light quarks the recent analysis [81] of heavy-meson transition form factors. The latter is based on the relativistic constituent quark model [82,83]. This model makes no fundamental difference between heavy and light mesons, and as such it is applicable to either case. For heavy quarks, the analytic expressions for the form factors from the constituent quark model reproduce the known results from QCD for heavy-to-heavy and heavy-to-light form factors. For light quarks, properties known for QCD in the chiral limit constrain the structure of the axial current of the constituent quarks [84]. Form-factor predictions within this model are thereby expected to be within about 10%⁵ with respect to ones based on first-principle approaches.⁶ We calculated the $K \rightarrow \gamma$ form factors making use of the model parameters from ref. [86]. The calculation may be parameterized by the simple formula

$$F_{V,A}^{(K)}(\hat{s}) = \frac{Q_d F_{V,A}^{(d)}(0) \pm Q_s F_{V,A}^{(s)}(0)}{1 - \hat{s} \left(\frac{M_K}{M_{V,A}} \right)^2}, \quad (23)$$

with $Q_{d,s} = -1/3$, $F_V^{(d,s)}(0) = \{-0.216, -0.18\}$, $F_A^{(d,s)}(0) = \{0.201, 0.19\}$ and $M_{V,A} = 0.89$ GeV. For the meson decay constants we use $f_{B_{d,s}} = \{0.186, 0.224\}$ GeV [87] and $f_K = 0.155$ [88]. The rest of the relevant input parameters are taken from [89].

The branching ratio for the corresponding non-radiative decay $B_q \rightarrow \ell_1^+ \ell_2^-$ reads

$$\begin{aligned} \mathcal{B}(B_q \rightarrow \ell_1^+ \ell_2^-) &= \tau_{B_q} \frac{G_F^2 \alpha_{\text{em}}^2 M_{B_q}^3 f_{B_q}^2}{2^6 \pi^3} |V_{tb} V_{tq}^*|^2 \sqrt{\rho} \times \\ &\left[(1 - \hat{m}^2) |F_P + \hat{M} C_{10}|^2 + (1 - \hat{M}^2) |F_S - \hat{m} C_9|^2 \right], \end{aligned} \quad (24)$$

with

$$F_{S,P} = M_{B_q} \frac{m_b C_{S,P} - m_q C'_{S,P}}{m_b + m_q}. \quad (25)$$

In the lepton-flavor conserving limit this formula reproduces exactly the corresponding one in ref. [78].

We note that the above branching-ratio formulae refer to ‘instantaneous’ B_q decays. This observation is relevant for a precision calculation of B_s decay branching ratios. In this case, one should replace τ_{B_q} with τ_{B_H} (where B_H is the heavier of the $B_s - \bar{B}_s$ mass eigenstates), to account for the large width difference in the B_s system [90].

Furthermore, as already remarked, in our numerics we integrate the photon energy spectrum above $E_{\text{cut}} = 60$ MeV, comparable to the experimental resolution on the total invariant mass of the final states, which has to yield the decaying-meson mass.⁷ The above cut doesn’t completely exclude ‘soft’ photons, namely ones such that $E_\gamma \ll m_{B_q}/2$. Their effect can be summed to all orders in the soft-photon approximation, and leads to a multiplicative correction factor to the non-radiative rate, of the order of 10%, as discussed

⁵ Such accuracy is more than sufficient in our case, given that the $K \rightarrow \mu e \gamma$ mode will turn out to be less interesting than the B -decay counterparts, see eq. (27) below.

⁶ One such approach is to relate the $K \rightarrow \ell \ell' \gamma$ form factors to those of $K \rightarrow \ell \nu \gamma$ [85] using isospin symmetry. We thank the referee for a useful remark in this context.

⁷ We take $E_{\text{cut}} = 60$ MeV from the $B_s \rightarrow \mu \mu$ case [91]. When including an additional photon, the correct value to be taken for E_{cut} may be slightly higher [92]. However, the choice of its precise value has a minor impact on our predictions, as E_{cut} only modifies the \hat{s} integration endpoint $\hat{s}_{\text{max}} = 1 - 2E_{\text{cut}}/M_{B_q}$.

in [91]. However, within LHCb, the effect of soft final-state radiation is corrected for by an appropriate Monte Carlo.

3. Numerical analysis

One can now compute the predictions for the radiative and non-radiative cases within the interaction in eq. (4) [45], whereby the shifts to C_9 and C_{10} read

$$\delta C_9 = -\delta C_{10} = \frac{G}{2} \frac{(U_L^{d*})_{33} (U_L^d)_{3q} (U_L^{\ell*})_{3\ell_2} (U_L^\ell)_{3\ell_1}}{-\frac{4G_F}{\sqrt{2}} V_{tb}^* V_{tq} \frac{\alpha_{\text{em}}(m_b)}{4\pi}}. \quad (26)$$

These predictions will depend on two basic parameters, the overall strength G of the interaction in eq. (4), and the product of four chiral rotations turning the fermion fields $(\bar{b}'b')(\bar{\tau}'\tau')$ into the fields relevant for the process, $(\bar{b}q)(\bar{\ell}_2\ell_1)$. This product of four U -matrix entries will be denoted as U_4 for brevity.

The parameters G and U_4 are completely unknown and we have at best some guiding criteria to fix them to reasonable ranges:

- Since, for a given process, G and U_4 always appear as a product, it is always possible to shuffle an arbitrary numerical factor between G and U_4 . As a consequence, to fix a reasonable range for G with any confidence, one may consider predictive models for U_4 , as in ref. [64] (see also [58]). One obtains new-physics scales $\Lambda_{NP} = 1/G^2$ between 750 GeV and 5 TeV.⁸ We then assume $4 \times 10^{-8} \leq G \leq 2 \times 10^{-6}$.
- The PMNS-matrix anarchy suggests that the leptonic part of U_4 may be of $\mathcal{O}(1)$. As concerns the U_L^d matrix entries 33 and 3q, one can assume them to be close in magnitude to the CKM entries V_{tb} and V_{tq} respectively.⁹ We therefore consider the range $10^{-4} \leq U_4 \leq 0.05$, keeping in mind that B_s would correspond to $U_4 \sim |V_{ts}| \simeq 0.04$ and B_d to $U_4 \sim |V_{td}| \simeq 0.008$.

To get an idea of the resulting predictions, we note that the upper limit $\mathcal{B}(B_s \rightarrow \mu^\pm e^\mp) < 1.1 \times 10^{-8}$ from LHCb [93] corresponds to $G \times U_4 = 1.6 \times 10^{-8}$, which is 5 times smaller than the product of our highest allowed values for G and U_4 . A general picture of the predictions for the $B_s \rightarrow \mu e \gamma$ branching ratio and its non-radiative counterpart as a function of G vs. U_4 in the above-mentioned ranges is presented in Fig. 2. The gray area denotes the parameter space excluded by the LHCb $B_s \rightarrow \mu e$ search of ref. [93].

The figure shows that the radiative mode is slightly enhanced with respect to the non-radiative counterpart. Actually, within our considered model, where the shifts to C_9 and C_{10} differ only by a sign, the $|G \times U_4|^2$ dependence cancels altogether in the radiative over non-radiative ratio, and we find

$$\begin{aligned} \frac{\mathcal{B}(B_s \rightarrow \mu e \gamma)}{\mathcal{B}(B_s \rightarrow \mu e)} &= 1.3, & \frac{\mathcal{B}(B_d \rightarrow \mu e \gamma)}{\mathcal{B}(B_d \rightarrow \mu e)} &= 1.2, \\ \frac{\mathcal{B}(K \rightarrow \mu e \gamma)}{\mathcal{B}(K \rightarrow \mu e)} &= 2.7 \times 10^{-2}. \end{aligned} \quad (27)$$

As a consequence, assuming experimental efficiencies for radiative and non-radiative cases to be comparable, the measurement of the B_q radiative decay along with the non-radiative one offers a precise cross-check of the new-physics mechanism responsible for a possible LFV signal. In the K case instead, the radiative mode is too

⁸ As emphasized in [64], these mass scales may appear low for, say, a Z' as the underlying mediator of the interaction (4). However, it must be remembered that this interaction couples primarily to the third generation.

⁹ This assumption should actually hold to a good extent, provided that new interactions other than eq. (4) are indeed negligible, as assumed here.

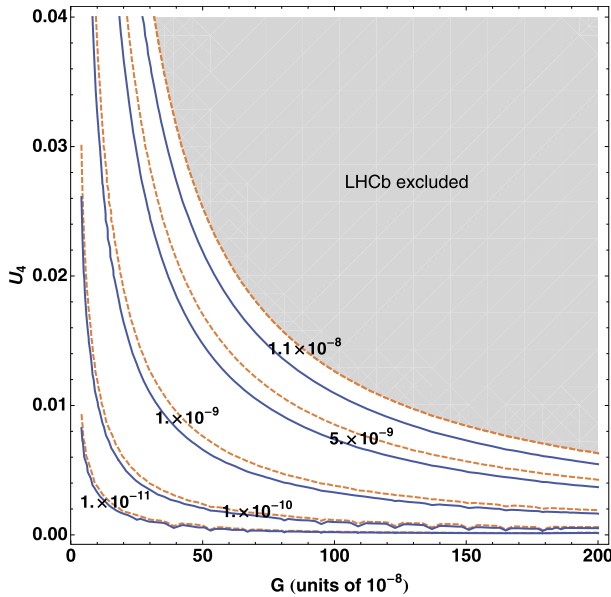


Fig. 2. $\mathcal{B}(B_s \rightarrow e^\pm \mu^\mp \gamma)$ (blue, solid) and $\mathcal{B}(B_s \rightarrow e^\pm \mu^\mp)$ (orange, dashed) as a function of G vs. the product of U -matrix entries appearing in eq. (26), and denoted as U_4 . (See text for more details.) The gray area is excluded by the LHCb upper limit on the non-radiative decay [93].

suppressed to be potentially interesting, unless the $K \rightarrow \mu e$ mode is found at an unexpectedly large rate.

The numbers in eq. (27) can be intuitively understood as follows. First note that the Bremsstrahlung contribution $d\Gamma^{(2)}$ to the radiative decay comes with a factor of $(f/M)^2$, with M the mass of the decaying meson and f its decay constant, as well as with a chiral suppression factor. On the other hand, both of these suppression factors are absent in the direct-emission contribution $d\Gamma^{(1)}$. Therefore, the radiative decay will be competitive with the non-radiative one – the latter also $(f/M)^2$ as well as chirally suppressed – to the extent that the direct-emission contribution can dominate, which occurs whenever f/M is small enough, that is the case for both B_d and B_s , but not for kaons. In other words, the larger the ratio M/f , with M the mass of the decaying meson and f its decay constant, the larger the parametric enhancement of the radiative decay over the non-radiative counterpart.

4. Conclusions

We studied in detail the prediction for the LFV decays of a K , B_d or $B_s \rightarrow \mu^\pm e^\mp \gamma$. These decays are ‘proxies’ to the corresponding non-radiative decays, in that LFV physics of the kind discussed in the text will produce both sets of decays, the chiral-suppression factor in the non-radiative modes being replaced by a factor of the order of α/π in the radiative ones. We found that predictions for the total branching ratios for $B_{d,s} \rightarrow \mu^\pm e^\mp \gamma$ exceed by about 30% those for the corresponding non-radiative modes. Taking into account that experimental efficiencies may be slightly lower for $\mu e \gamma$ than they are for just μe pairs, inclusion of the proposed radiative modes realistically corresponds to a doubling of the statistics as compared to the purely non-radiative modes, for either of B_d and B_s .

Acknowledgements

The work of D.G. is partially supported by the CNRS grant PICS07229. The authors acknowledge K. Lane for discussions on related subjects, and J.-F. Marchand for remarks on the manuscript.

References

- [1] LHCb Collaboration, R. Aaij, et al., Test of lepton universality using $B^+ \rightarrow K^+ \ell^+ \ell^-$ decays, Phys. Rev. Lett. 113 (2014) 151601, arXiv:1406.6482.
- [2] M. Bordone, G. Isidori, A. Pattori, On the Standard Model predictions for R_K and R_{K^*} , arXiv:1605.07633.
- [3] C. Bobeth, G. Hiller, G. Piranishvili, Angular distributions of $\bar{B} \rightarrow \bar{K} \ell^+ \ell^-$ decays, J. High Energy Phys. 12 (2007) 040, arXiv:0709.4174.
- [4] HPQCD Collaboration, C. Bouchard, G.P. Lepage, C. Monahan, H. Na, J. Shigemitsu, Standard Model predictions for $B \rightarrow K \ell^+ \ell^-$ with form factors from lattice QCD, Phys. Rev. Lett. 111 (2013) 162002, arXiv:1306.0434.
- [5] G. Hiller, F. Krüger, More model independent analysis of $b \rightarrow s$ processes, Phys. Rev. D 69 (2004) 074020, arXiv:hep-ph/0310219.
- [6] LHCb Collaboration, R. Aaij, et al., Differential branching fractions and isospin asymmetries of $B \rightarrow K^{(*)} \mu^+ \mu^-$ decays, J. High Energy Phys. 06 (2014) 133, arXiv:1403.8044.
- [7] LHCb Collaboration, R. Aaij, et al., Differential branching fraction and angular analysis of the $B^+ \rightarrow K^+ \mu^+ \mu^-$ decay, J. High Energy Phys. 02 (2013) 105, arXiv:1209.4284.
- [8] C. Bobeth, G. Hiller, D. van Dyk, More benefits of semileptonic rare B decays at low recoil: CP violation, J. High Energy Phys. 07 (2011) 067, arXiv:1105.0376.
- [9] C. Bobeth, G. Hiller, D. van Dyk, C. Wacker, The decay $B \rightarrow K \ell^+ \ell^-$ at low hadronic recoil and model-independent $\Delta B = 1$ constraints, J. High Energy Phys. 01 (2012) 107, arXiv:1111.2558.
- [10] C. Bobeth, G. Hiller, D. van Dyk, General analysis of $\bar{B} \rightarrow \bar{K}^{(*)} \ell^+ \ell^-$ decays at low recoil, Phys. Rev. D 87 (2013) 034016, arXiv:1212.2321.
- [11] LHCb Collaboration, R. Aaij, et al., Differential branching fraction and angular analysis of the decay $B_s^0 \rightarrow \phi \mu^+ \mu^-$, J. High Energy Phys. 07 (2013) 084, arXiv:1305.2168.
- [12] LHCb Collaboration, R. Aaij, et al., Angular analysis and differential branching fraction of the decay $B_s^0 \rightarrow \phi \mu^+ \mu^-$, J. High Energy Phys. 09 (2015) 179, arXiv:1506.08777.
- [13] S. Descotes-Genon, T. Hurth, J. Matias, J. Virto, Optimizing the basis of $B \rightarrow K^* \ell \ell$ observables in the full kinematic range, J. High Energy Phys. 05 (2013) 137, arXiv:1303.5794.
- [14] LHCb Collaboration, R. Aaij, et al., Measurement of form-factor-independent observables in the decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$, Phys. Rev. Lett. 111 (2013) 191801, arXiv:1308.1707.
- [15] LHCb Collaboration, R. Aaij, et al., Angular analysis of the $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decay using 3 fb $^{-1}$ of integrated luminosity, J. High Energy Phys. 02 (2016) 104, arXiv:1512.04442.
- [16] Belle Collaboration, A. Abdesselam, et al., Angular analysis of $B^0 \rightarrow K^{*}(892)^0 \ell^+ \ell^-$, arXiv:1604.04042, 2016.
- [17] S. Descotes-Genon, L. Hofer, J. Matias, J. Virto, On the impact of power corrections in the prediction of $B \rightarrow K^* \mu^+ \mu^-$ observables, J. High Energy Phys. 12 (2014) 125, arXiv:1407.8526.
- [18] A. Bharucha, D.M. Straub, R. Zwicky, $B \rightarrow V \ell \ell$ in the Standard Model from light-cone sum rules, arXiv:1503.05534.
- [19] S. Jäger, J. Martin Camalich, On $B \rightarrow V \ell \ell$ at small dilepton invariant mass, power corrections, and new physics, J. High Energy Phys. 05 (2013) 043, arXiv:1212.2263.
- [20] S. Jäger, J. Martin Camalich, Reassessing the discovery potential of the $B \rightarrow K^* \ell^+ \ell^-$ decays in the large-recoil region: SM challenges and BSM opportunities, Phys. Rev. D 93 (2016) 014028, arXiv:1412.3183.
- [21] S. Descotes-Genon, J. Matias, J. Virto, Understanding the $B \rightarrow K^* \mu^+ \mu^-$ anomaly, Phys. Rev. D 88 (2013) 074002, arXiv:1307.5683.
- [22] J. Lyon, R. Zwicky, Resonances gone topsy turvy – the charm of QCD or new physics in $b \rightarrow s \ell^+ \ell^-$?, arXiv:1406.0566.
- [23] M. Ciuchini, M. Fedele, E. Franco, S. Mishima, A. Paul, L. Silvestrini, et al., $B \rightarrow K^* \ell^+ \ell^-$ decays at large recoil in the Standard Model: a theoretical reappraisal, arXiv:1512.07157.
- [24] BaBar Collaboration, J.P. Lees, et al., Evidence for an excess of $\bar{B} \rightarrow D^{(*)} \tau^- \bar{\nu}_\tau$ decays, Phys. Rev. Lett. 109 (2012) 101802, arXiv:1205.5442.
- [25] S. Fajfer, J.F. Kamenik, I. Nisandzic, On the $B \rightarrow D^* \tau \bar{\nu}_\tau$ sensitivity to new physics, Phys. Rev. D 85 (2012) 094025, arXiv:1203.2654.
- [26] J.F. Kamenik, F. Mescia, $B \rightarrow D$ tau nu branching ratios: opportunity for lattice QCD and hadron colliders, Phys. Rev. D 78 (2008) 014003, arXiv:0802.3790.
- [27] LHCb Collaboration, R. Aaij, et al., Measurement of the ratio of branching fractions $\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \tau^- \bar{\nu}_\tau) / \mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \mu^- \bar{\nu}_\mu)$, Phys. Rev. Lett. 115 (2015) 111803, arXiv:1506.08614.
- [28] Belle Collaboration, M. Huschle, et al., Measurement of the branching ratio of $\bar{B} \rightarrow D^{(*)} \tau^- \bar{\nu}_\tau$ relative to $\bar{B} \rightarrow D^{(*)} \ell^- \bar{\nu}_\ell$ decays with hadronic tagging at Belle, Phys. Rev. D 92 (2015) 072014, arXiv:1507.03233.
- [29] P. Goldenzweig, Talk at the ‘‘Rencontres de Moriond EW interactions and unified theories’’, 12–19 March, 2016.
- [30] D. Ghosh, M. Nardecchia, S.A. Renner, Hint of lepton flavour non-universality in B meson decays, J. High Energy Phys. 12 (2014) 131, arXiv:1408.4097.
- [31] W. Altmannshofer, D.M. Straub, New physics in $b \rightarrow s$ transitions after LHC run 1, Eur. Phys. J. C 75 (2015) 382, arXiv:1411.3161.

- [32] F. Beaujean, C. Bobeth, D. van Dyk, Comprehensive Bayesian analysis of rare (semi)leptonic and radiative B decays, *Eur. Phys. J. C* 74 (2014) 2897, arXiv:1310.2478.
- [33] T. Hurth, F. Mahmoudi, S. Neshatpour, Global fits to $b \rightarrow s\ell\ell$ data and signs for lepton non-universality, *J. High Energy Phys.* 12 (2014) 053, arXiv:1410.4545.
- [34] F. Beaujean, C. Bobeth, S. Jahn, Constraints on tensor and scalar couplings from $B \rightarrow K\bar{\mu}\mu$ and $B_s \rightarrow \bar{\mu}\mu$, *Eur. Phys. J. C* 75 (2015) 456, arXiv:1508.01526.
- [35] S. Descotes-Genon, L. Hofer, J. Matias, J. Virto, Global analysis of $b \rightarrow s\ell\ell$ anomalies, arXiv:1510.04239.
- [36] T. Hurth, F. Mahmoudi, S. Neshatpour, On the anomalies in the latest LHCb data, arXiv:1603.00865.
- [37] W. Altmannshofer, D.M. Straub, New physics in $B \rightarrow K^*\mu\mu$?, *Eur. Phys. J. C* 73 (2013) 2646, arXiv:1308.1501.
- [38] R. Gauld, F. Goertz, U. Haisch, On minimal Z' explanations of the $B \rightarrow K^*\mu^+\mu^-$ anomaly, *Phys. Rev. D* 89 (2014) 015005, arXiv:1308.1959.
- [39] A.J. Buras, J. Girrbach, Left-handed Z' and Z FCNC quark couplings facing new $b \rightarrow s\mu^+\mu^-$ data, *J. High Energy Phys.* 1312 (2013) 009, arXiv:1309.2466.
- [40] R. Gauld, F. Goertz, U. Haisch, An explicit Z' -boson explanation of the $B \rightarrow K^*\mu^+\mu^-$ anomaly, *J. High Energy Phys.* 1401 (2014) 069, arXiv:1310.1082.
- [41] A.J. Buras, F. De Fazio, J. Girrbach, 331 models facing new $b \rightarrow s\mu^+\mu^-$ data, *J. High Energy Phys.* 1402 (2014) 112, arXiv:1311.6729.
- [42] W. Altmannshofer, S. Gori, M. Pospelov, I. Yavin, Quark flavor transitions in $L_\mu - L_\tau$ models, *Phys. Rev. D* 89 (2014) 095033, arXiv:1403.1269.
- [43] S. Biswas, D. Chowdhury, S. Han, S.J. Lee, Explaining the lepton non-universality at the LHCb and CMS within a unified framework, *J. High Energy Phys.* 02 (2015) 142, arXiv:1409.0882.
- [44] G. Hiller, M. Schmaltz, R_K and future $b \rightarrow s\ell\ell$ physics beyond the standard model opportunities, *Phys. Rev. D* 90 (2014) 054014, arXiv:1408.1627.
- [45] S.L. Glashow, D. Guadagnoli, K. Lane, Lepton flavor violation in B decays?, *Phys. Rev. Lett.* 114 (2015) 091801, arXiv:1411.0565.
- [46] G. Hiller, M. Schmaltz, Diagnosing lepton-nonuniversality in $b \rightarrow s\ell\ell$, *J. High Energy Phys.* 1502 (2015) 055, arXiv:1411.4773.
- [47] B. Gripaios, M. Nardecchia, S. Renner, Composite leptoquarks and anomalies in B -meson decays, *J. High Energy Phys.* 1505 (2015) 006, arXiv:1412.1791.
- [48] A. Crivellin, G. D'Ambrosio, J. Heeck, Explaining $h \rightarrow \mu^\pm\tau^\mp$, $B \rightarrow K^*\mu^+\mu^-$ and $B \rightarrow K\mu^+\mu^-/B \rightarrow Ke^+e^-$ in a two-Higgs-doublet model with gauged $L_\mu - L_\tau$, *Phys. Rev. Lett.* 114 (2015) 151801, arXiv:1501.00993.
- [49] A. Crivellin, G. D'Ambrosio, J. Heeck, Addressing the LHC flavor anomalies with horizontal gauge symmetries, *Phys. Rev. D* 91 (2015) 075006, arXiv:1503.03477.
- [50] C. Niehoff, P. Stangl, D.M. Straub, Violation of lepton flavour universality in composite Higgs models, *Phys. Lett. B* 747 (2015) 182–186, arXiv:1503.03865.
- [51] S.D. Aristizabal, F. Staub, A. Vicente, Shedding light on the $b \rightarrow s$ anomalies with a dark sector, arXiv:1503.06077.
- [52] A. Celis, J. Fuentes-Martin, M. Jung, H. Serodio, Family non-universal Z' models with protected flavor-changing interactions, arXiv:1505.03079.
- [53] D. Bečirević, S. Fajfer, N. Košnik, Lepton flavor non-universality in $b \rightarrow s\ell^+\ell^-$ processes, arXiv:1503.09024.
- [54] A. Carmona, F. Goertz, Lepton flavor and non-universality from minimal composite Higgs setups, arXiv:1510.07658.
- [55] A. Celis, W.-Z. Feng, D. Lüst, Stringy explanation of $b \rightarrow s\ell^+\ell^2$ anomalies, *J. High Energy Phys.* 02 (2016) 007, arXiv:1512.02218.
- [56] I. de Medeiros Varzielas, G. Hiller, Clues for flavor from rare lepton and quark decays, *J. High Energy Phys.* 1506 (2015) 072, arXiv:1503.01084.
- [57] C.-J. Lee, J. Tandean, Minimal lepton flavor violation implications of the $b \rightarrow s$ anomalies, arXiv:1505.04692.
- [58] S.M. Boucenna, J.W.F. Valle, A. Vicente, Are the B decay anomalies related to neutrino oscillations?, arXiv:1503.07099.
- [59] A. Crivellin, L. Hofer, J. Matias, U. Nierste, S. Pokorski, et al., Lepton-flavour violating B decays in generic Z' models, arXiv:1504.07928.
- [60] G. Bélanger, C. Delaunay, S. Westhoff, A dark matter relic from muon anomalies, *Phys. Rev. D* 92 (2015) 055021, arXiv:1507.06660.
- [61] R. Alonso, B. Grinstein, J.M. Camalich, Lepton universality violation and lepton flavor conservation in B -meson decays, arXiv:1505.05164.
- [62] A. Greljo, G. Isidori, D. Marzocca, On the breaking of lepton flavor universality in B decays, arXiv:1506.01705.
- [63] W. Altmannshofer, I. Yavin, Predictions for lepton flavor universality violation in rare B decays in models with gauged $L_\mu - L_\tau$, *Phys. Rev. D* 92 (2015) 075022, arXiv:1508.07009.
- [64] D. Guadagnoli, K. Lane, Charged-lepton mixing and lepton flavor violation, *Phys. Lett. B* 751 (2015) 54–58, arXiv:1507.01412.
- [65] L. Calibbi, A. Crivellin, T. Ota, Effective field theory approach to $b \rightarrow s\ell\ell^{(\prime)}$, $B \rightarrow K^{(*)}\nu\bar{\nu}$ and $B \rightarrow D^{(*)}\tau\nu$ with third generation couplings, arXiv:1506.02661.
- [66] S. Sahoo, R. Mohanta, Lepton flavour violating B meson decays via scalar leptoquark, arXiv:1512.04657.
- [67] A. Falkowski, M. Nardecchia, R. Ziegler, Lepton flavor non-universality in B -meson decays from a $U(2)$ flavor model, *J. High Energy Phys.* 11 (2015) 173, arXiv:1509.01249.
- [68] D. Buttazzo, A. Greljo, G. Isidori, D. Marzocca, Toward a coherent solution of diphoton and flavor anomalies, arXiv:1604.03940.
- [69] C.-W. Chiang, X.-G. He, G. Valencia, Z' model for $b \rightarrow s\ell\bar{\ell}$ flavor anomalies, *Phys. Rev. D* 93 (2016) 074003, arXiv:1601.07328.
- [70] G. Bélanger, C. Delaunay, A dark sector for $g_\mu - 2$, R_K and a diphoton resonance, arXiv:1603.03333.
- [71] D. Bečirević, O. Sumensari, R. Zukanovich Funchal, Lepton flavor violation in exclusive $b \rightarrow s$ decays, *Eur. Phys. J. C* 76 (2016) 134, arXiv:1602.00881.
- [72] R. Alonso, B. Grinstein, J. Martin Camalich, $SU(2) \times U(1)$ gauge invariance and the shape of new physics in rare B decays, *Phys. Rev. Lett.* 113 (2014) 241802, arXiv:1407.7044.
- [73] B. Bhattacharya, A. Datta, D. London, S. Shivashankara, Simultaneous explanation of the R_K and $R(D^{(*)})$ puzzles, *Phys. Lett. B* 742 (2015) 370–374, arXiv:1412.7164.
- [74] M. Bauer, M. Neubert, One leptoquark to rule them all: a minimal explanation for $R_{D^{(*)}}$, R_K and $(g-2)_\mu$, *Phys. Rev. Lett.* 116 (2016) 141802, arXiv:1511.01900.
- [75] S. Fajfer, N. Košnik, Vector leptoquark resolution of R_K and $R_{D^{(*)}}$ puzzles, *Phys. Lett. B* 755 (2016) 270–274, arXiv:1511.06024.
- [76] S.M. Boucenna, A. Celis, J. Fuentes-Martin, A. Vicente, J. Virto, Non-abelian gauge extensions for B -decay anomalies, arXiv:1604.03088.
- [77] LHCb Collaboration, R. Aaij, et al., Implications of LHCb measurements and future prospects, *Eur. Phys. J. C* 73 (2013) 2373, arXiv:1208.3355.
- [78] C. Bobeth, T. Ewerth, F. Krüger, J. Urban, Analysis of neutral Higgs boson contributions to the decays $\bar{B}_s \rightarrow \ell^+\ell^-$ and $\bar{B} \rightarrow K\ell^+\ell^-$, *Phys. Rev. D* 64 (2001) 074014, arXiv:hep-ph/0104284.
- [79] D. Melikhov, N. Nikitin, Rare radiative leptonic decays $B(d, s) \rightarrow \ell^+\ell^-\gamma$, *Phys. Rev. D* 70 (2004) 114028, arXiv:hep-ph/0410146.
- [80] F. Krüger, D. Melikhov, Gauge invariance and form-factors for the decay $B \rightarrow \gamma\ell^+\ell^-$, *Phys. Rev. D* 67 (2003) 034002, arXiv:hep-ph/0208256.
- [81] A. Kozachuk, D. Melikhov, N. Nikitin, Annihilation type rare radiative $B_{(s)} \rightarrow V\gamma$ decays, *Phys. Rev. D* 93 (2016) 014015, arXiv:1511.03540.
- [82] V.V. Anisovich, D.I. Melikhov, V.A. Nikonov, Photon-meson transition form-factors $\gamma\pi^0$, $\gamma\eta$ and $\gamma\eta'$ at low and moderately high Q^2 , *Phys. Rev. D* 55 (1997) 2918–2930, arXiv:hep-ph/9607215.
- [83] D. Melikhov, Dispersion approach to quark binding effects in weak decays of heavy mesons, *Eur. Phys. J. C* 4 (2002) 2, arXiv:hep-ph/0110087.
- [84] W. Lucha, D. Melikhov, S. Simula, Constituent quarks, chiral symmetry, and chiral point of the constituent quark model, *Phys. Rev. D* 74 (2006) 054004, arXiv:hep-ph/0606281.
- [85] V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, J. Portoles, Kaon decays in the Standard Model, *Rev. Mod. Phys.* 84 (2012) 399, arXiv:1107.6001.
- [86] D. Melikhov, B. Stech, Weak form-factors for heavy meson decays: an update, *Phys. Rev. D* 62 (2000) 014006, arXiv:hep-ph/0001113.
- [87] HPQCD Collaboration, R.J. Dowdall, C.T.H. Davies, R.R. Horgan, C.J. Monahan, J. Shigemitsu, B -meson decay constants from improved lattice nonrelativistic QCD with physical u , d , s , and c quarks, *Phys. Rev. Lett.* 110 (2013) 222003, arXiv:1302.2644.
- [88] N. Carrasco, et al., Leptonic decay constants f_K , f_D , and f_{D_s} with $N_f = 2 + 1 + 1$ twisted-mass lattice QCD, *Phys. Rev. D* 91 (2015) 054507, arXiv:1411.7908.
- [89] Particle Data Group, K.A. Olive, et al., Review of particle physics, *Chin. Phys. C* 38 (2014) 090001.
- [90] K. De Bruyn, R. Fleischer, R. Knegjens, P. Koppenburg, M. Merk, A. Pellegrino, et al., Probing new physics via the $B_s^0 \rightarrow \mu^+\mu^-$ effective lifetime, *Phys. Rev. Lett.* 109 (2012) 041801, arXiv:1204.1737.
- [91] A.J. Buras, J. Girrbach, D. Guadagnoli, G. Isidori, On the Standard Model prediction for $\mathcal{B}(B_{s,d} \rightarrow \mu^+\mu^-)$, *Eur. Phys. J. C* 72 (2012) 2172, arXiv:1208.0934.
- [92] J.-F. Marchand, private communication.
- [93] LHCb Collaboration, R. Aaij, et al., Search for the lepton-flavor violating decays $B_s^0 \rightarrow e^\pm\mu^\mp$ and $B^0 \rightarrow e^\pm\mu^\mp$, *Phys. Rev. Lett.* 111 (2013) 141801, arXiv:1307.4889.