Decays of fully beauty scalar tetraquarks to $B_q\bar{B}_q$ and $B_q^*\bar{B}_q^*$ mesons

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Decays of the fully beauty four-quark structures X_{4b} and T_{4b} to B meson pairs are investigated in the framework of the QCD three-point sum rule method. We model the scalar exotic mesons X_{4b} and T_{4b} as diquark-antidiquark systems composed of the axial-vector and pseudoscalar diquarks, respectively. The masses $m = (18540 \pm 50)$ MeV and $\tilde{m} = (18858 \pm 50)$ MeV of these compounds calculated in our previous articles fix possible decay channels of these particles. In the present work, we consider their decays to $B_q\bar{B}_q$ and $B_q^*\bar{B}_q^*$ ($q = u, d, s, c$) mesons. In the case of X_{4b}, the mass of which is below the $2\eta_b$ threshold, these channels determine essential part of its full width Γ_{4b}. The tetraquark T_{4b} can decay to the pair $\eta_b\eta_b$; therefore, partial widths of processes with $B(B^*)$ mesons in the final state permit us to refine our estimate for the full width of this particle. The predictions $\Gamma_{\mathrm{4b}} = (9.6 \pm 1.1)$ MeV and $\tilde{\Gamma}_{\mathrm{4b}}^{\mathrm{Full}} = (144 \pm 29)$ MeV obtained in this article can be used in future experimental investigations of four b-quark mesons.

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I. INTRODUCTION

Interest in four-quark exotic mesons containing heavy c and b quarks appeared in the first years of the parton model and QCD [1–6]. Close attention to these hypothetical particles was inspired by many reasons. First of all, fundamental laws of QCD allow the existence of multiquark hadrons; therefore, such states became objects for intensive theoretical studies. The second reason was a possibility to find multiquark particles stable against strong decays, hence with a long mean lifetime. Investigations showed that tetraquarks, i.e., four-quark mesons built of a heavy bb diquark and light antidiquark, may have desired features. Such candidates to strong-interaction stable particles were analyzed in various publications by means of different models and methods (see Refs. [7–10], and references therein).

Fully heavy tetraquarks were also considered in numerous papers aimed to reveal their properties. Recent data of the LHCb-ATLAS-CMS Collaborations provided new experimental information [11–13], which is important for physics of heavy exotic mesons. These experiments discovered four X resonances in the invariant mass distributions of the di-J/ψ and $J/\psi\psi'$ mesons. The X particles have masses in the range 6.2–7.3 GeV and presumably are fully charmed states, though alternative explanations were suggested as well.

In our articles [14–17], we studied the X structures as fully charmed scalar particles using both the diquark-antidiquark and hadronic molecule models. We calculated their masses and full width by employing the QCD two- and three-point sum rule (SR) methods and compared obtained results with the LHCb-ATLAS-CMS data. In accordance with our predictions, the resonance $X(6600)$ is the tetraquark composed of axial-vector diquarks [14], whereas $X(6200)$ may be considered as a hadronic molecule $\eta_c\eta_c$ [15]. The structure $X(6900)$ can be interpreted as a superposition of a diquark-antidiquark state built of pseudoscalar components and a molecule $\chi_{c0}\chi_{c0}$ [15,16]. In Ref. [17], we explained $X(7300)$ by employing the superposition of a molecule $\chi_{c1}\chi_{c1}$ and a radially excited diquark-antidiquark.

It is interesting that even in the framework of the four-quark picture there are competing explanations for X states. Thus, the resonance $X(6200)$ was considered as the ground-state tetraquark with $J^{\mathrm{PC}} = 0^{++}$ or 1^{+-}. The first radially excited state of this tetraquark was assigned to be $X(6600)$ [18]. The X resonances were interpreted as different radially and orbitally excited diquark-antidiquark states also in Refs. [19,20].

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In most articles devoted to analysis of fully charmed tetraquarks, the authors investigated also their beauty partners $bb\bar{b}\,\bar{b}$ by computing the masses and other parameters of these particles. Such structures, produced in pp and $p\bar{p}$ collisions, may be discovered in the mass distributions of the $\eta_b\eta_b$, $\eta_b\Upsilon$, and $\Upsilon\Upsilon$ mesons: In fact, $\Upsilon\Upsilon$ pairs were detected and studied by the CMS Collaboration [21].

Predictions for parameters of $bb\bar{b}\,\bar{b}$ with different quantum numbers were made in Refs. [22–25] using various methods and schemes. Results of these articles sometimes contradict each another. For instance, in Ref. [22], it was shown that the mass 18754 MeV of the scalar exotic meson $X_{4\mathrm{b}}$ is below the $\eta_b\eta_b$ and $\Upsilon\Upsilon$ thresholds; therefore, it cannot be fixed in these mass distributions. A similar problem was addressed in Ref. [23], where the mass of $X_{4\mathrm{b}}$ was found equal to (18826 ± 25) MeV, which is less than $\Upsilon\Upsilon$ but higher than $\eta_b\eta_b$ thresholds. The masses of exotic $bb\bar{b}\,\bar{b}$ mesons with different spin-parities were extracted from the sum rule analyses in Ref. [25]. In accordance with this paper, the scalar tetraquarks have masses (18.45–18.59) GeV and cannot be observed in two-bottomonia final states. Only the scalar particle made of pseudoscalar diquarks can decay to $\eta_b\eta_b$ and $\Upsilon\Upsilon$ mesons, because its mass (19640 ± 140) MeV considerably exceeds relevant limits.

The scalar diquark-antidiquark states $X_{4\mathrm{b}}$ and $T_{4\mathrm{b}}$ with axial-vector and pseudoscalar diquarks were explored also in our articles [14,16]. To this end, we used the QCD two-point SR method [26,27], which is one of the effective tools to investigate spectroscopic parameters and strong couplings of conventional hadrons. But, it is also suitable to study multiquark structures [28,29]. The masses $m = (18540 \pm 50)$ MeV and $\tilde{m} = (18858 \pm 50)$ MeV of the tetraquarks $X_{4\mathrm{b}}$ and $T_{4\mathrm{b}}$ found by this way allowed us to fix their possible strong decay modes. Because m resides below both $\eta_b\eta_b$ and $\Upsilon\Upsilon$ thresholds, this particle cannot decay to two bottomonia final states. The exotic meson $T_{4\mathrm{b}}$ falls apart to a pair $\eta_b\eta_b$ and has width equal to $\tilde{\Gamma}_{4\mathrm{b}} = (94 \pm 28)$ MeV [16].

But, fully beauty tetraquarks can also decay through alternative mechanisms [23,24,30]. Thus, $X_{4\mathrm{b}}$ and $T_{4\mathrm{b}}$ can transform to 2γ, Υl^+l^-, or to four leptons $l_1^+l_1^-l_2^+l_2^-$ due to annihilation of valence b and $\bar{b}$ quarks and related processes. The $b\bar{b}$ annihilations to gluons followed by appearance of quark-antiquark pairs can generate processes with $\eta_b + H$, $B_q\bar{B}_q$, and $B_q^*\bar{B}_q^*$ ($q = u, d, s, c$) final states. It is clear that thresholds for these decays are considerably smaller than masses of the tetraquarks $X_{4\mathrm{b}}$ and $T_{4\mathrm{b}}$. They are crucial for tetraquarks which are below the $\eta_b\eta_b$ threshold and cannot dissociate to these bottomonia.

In the present work, we explore strong decays of the tetraquarks $X_{4\mathrm{b}}$ and $T_{4\mathrm{b}}$ to $B_q\bar{B}_q$ and $B_q^*\bar{B}_q^*$ mesons. In the case of $T_{4\mathrm{b}}$, they are necessary to refine $\tilde{\Gamma}_{4\mathrm{b}}$. But the aforementioned processes form a considerable part of the $X_{4\mathrm{b}}$ tetraquark's full width, because a decay $X_{4\mathrm{b}} \to \eta_b\eta_b$ is forbidden kinematically. Widths of decays under consideration are determined by the strong couplings of particles at the vertices $X(T)_{4\mathrm{b}}B_q\bar{B}_q$ and $X(T)_{4\mathrm{b}}B_q^*\bar{B}_q^*$. In the current article, we evaluate strong couplings of interest in the context of the QCD three-point SR method.

This article is structured in the following manner: In Sec. II, we explore the decay channels of the tetraquark $X_{4\mathrm{b}}$ and compute partial widths of the processes $X_{4\mathrm{b}} \to B_q\bar{B}_q$. The decays of $X_{4\mathrm{b}}$ to final states $B^{*+}B^{*-}$, $\bar{B}^{*0}B^{*0}$, and $\bar{B}_s^{*0}B_s^{*0}$ are studied in Sec. III. Here, we also evaluate the full width $X_{4\mathrm{b}}$. The similar investigation for the diquark-antidiquark state $T_{4\mathrm{b}}$ is performed in Sec. IV, in which we estimate contributions of the processes $T_{4\mathrm{b}} \to B_q\bar{B}_q$ and $T_{4\mathrm{b}} \to B_q^*\bar{B}_q^*$ to the full width of $T_{4\mathrm{b}}$. In the last Sec. V, we compare obtained predictions with available ones and make our brief conclusions.

II. DECAYS $X_{4\mathrm{b}} \to B_q\bar{B}_q$

As we have noted above, $X_{4\mathrm{b}}$ cannot decay to meson pairs $\eta_b\eta_b$ and $\Upsilon\Upsilon$; its full width is primarily determined by the processes $X_{4\mathrm{b}} \to B_q\bar{B}_q$ and $X_{4\mathrm{b}} \to B_q^*\bar{B}_q^*$. Here, we evaluate the partial widths of the decays $X_{4\mathrm{b}} \to B^+B^-$, $\bar{B}^0B^0$, $\bar{B}_s^0B_s^0$, and $B_c^+B_c^-$, where $B_{(s,c)}$ are pseudoscalar mesons.

The partial widths of these processes depend on the strong couplings g_l, $l = 1$–4, of the tetraquark $X_{4\mathrm{b}}$ and final state mesons at the corresponding three-particle vertices. Therefore, the main problem to be considered in this section is computation of g_l. In the case of the channel $X_{4\mathrm{b}} \to B^+B^-$, this is a coupling g_1 of particles at the vertex $X_{4\mathrm{b}}B^+B^-$. We are going to analyze the decay $X_{4\mathrm{b}} \to B^+B^-$ in a detailed manner and write down only essential expressions and numerical results for other processes.

The strong coupling g_1 can be extracted from the three-point correlation function

$$\Pi(p, p') = i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \langle 0|\mathcal{T}\{J^{B^+}(y) \times J^{B^-}(0) J^\dagger(x)\}|0\rangle, \quad (1)$$

where $J(x)$ is the interpolating current for the scalar tetraquark $X_{4\mathrm{b}}$:

$$J(x) = b_a^T(x) C\gamma_\mu b_b(x) \bar{b}_a(x) \gamma^\mu C \bar{b}_b^T(x), \quad (2)$$

with C being the charge conjugation matrix.

The currents $J^{B^+}(x)$ and $J^{B^-}(x)$ for the B mesons are given by the formulas

$$J^{B^+}(x) = \bar{b}_j(x) i\gamma_5 u_j(x), \quad J^{B^-}(x) = \bar{u}_i(x) i\gamma_5 b_i(x), \quad (3)$$

where i, $j = 1$, 2, 3 are color indices.

In accordance with the sum rule approach, we have to express the function $\Pi(p, p')$ in terms of involved particles' parameters. By this way, we determine the physical side of the sum rule. For these purposes, we write down the $\Pi(p,p')$ in the following form:

$$\Pi^{\rm Phys}(p,p') = \frac{\langle 0|J^{B^+}|B^+(p')\rangle}{p'^2-m_B^2}\frac{\langle 0|J^{B^-}|B^-(q)\rangle}{q^2-m_B^2} \times \langle B^+(p')B^-(q)|X_{4\rm b}(p)\rangle \frac{\langle X_{4\rm b}(p)|J^\dagger|0\rangle}{p^2-m^2} + \cdots, \tag{4}$$

where only the contribution of ground-level particles is presented explicitly: Effects of higher resonances and continuum states are shown as ellipses. It is evident that four momenta of $X_{4\rm b}$ and B^+ are p and p', respectively. Therefore, the momentum of B^- is equal to $q = p - p'$.

To simplify the correlation function $\Pi^{\rm Phys}(p,p')$, we express the matrix elements which enter to Eq. (4), using the masses and current couplings (decay constants) of involved particles. For the scalar tetraquark $X_{4\rm b}$, the matrix element $\langle 0|J|X_{4\rm b}\rangle$ can be replaced by a product of its mass m and current coupling f:

$$\langle 0|J|X_{4\rm b}\rangle = fm. \tag{5}$$

The matrix element of the pseudoscalar B mesons is determined by the formula

$$\langle 0|J^B|B\rangle = \frac{f_B m_B^2}{m_b}, \tag{6}$$

with m_B and f_B being their mass and decay constant, respectively. Here, m_b is the mass of the b quark.

The vertex $\langle B^+(p')B^-(q)|X_{4\rm b}(p)\rangle$ is modeled in the following form:

$$\langle B^+(p')B^-(q)|X_{4\rm b}(p)\rangle = g_1(q^2) p\cdot p'. \tag{7}$$

Here, $g_1(q^2)$ is the form factor which at the mass shell of the B^- meson, i.e., at $q^2 = m_B^2$, fixes the strong coupling g_1.

By taking into account these expressions, it is not difficult to recast $\Pi^{\rm Phys}(p,p')$ into the form

$$\Pi^{\rm Phys}(p,p') = g_1(q^2)\frac{fmf_B^2m_B^4}{2m_b^2(p^2-m^2)(p'^2-m_B^2)} \times \frac{(m^2+m_B^2-q^2)}{(q^2-m_B^2)} + \cdots, \tag{8}$$

where the dots denote contributions of higher resonances and continuum states. The correlator $\Pi^{\rm Phys}(p,p')$ is simply proportional to I. Therefore, the whole expression in the right-hand side of Eq. (8) is the invariant amplitude $\Pi^{\rm Phys}(p^2,p'^2,q^2)$ which can be applied to derive the form factor $g_1(q^2)$.

The second component which is required to get the sum rule for $g_1(q^2)$ is the correlator Eq. (1) computed using the quark propagators, which reads

$$\Pi^{\rm OPE}(p,p') = \frac{16}{3}\int d^4xd^4y e^{ip'y}e^{-ipx}\langle \bar{b}b\rangle \times {\rm Tr}[S_u^{ij}(y)\gamma_5 S_b^{ja}(-x)S_b^{ai}(x-y)\gamma_5]. \tag{9}$$

In Eq. (9), $S_{u(b)}(x)$ are u and b quark propagators:

$$S_u^{ab}(x) = i\frac{\not{x}}{2\pi^2x^4}\delta_{ab} - \frac{m_u}{4\pi^2x^2}\delta_{ab} - \frac{\langle \bar{u}u\rangle}{12}\delta_{ab} + i\langle \bar{u}u\rangle\frac{m_u\not{x}}{48}\delta_{ab} + \cdots \tag{10}$$

and

$$S_b^{ab}(x) = \frac{i}{(2\pi)^4}\int d^4k e^{-ikx}\left\{\frac{\delta_{ab}(\not{k}+m_b)}{k^2-m_b^2} - \frac{g_sG_{ab}^{\alpha\beta}}{4}\frac{\sigma_{\alpha\beta}(\not{k}+m_b)+(\not{k}+m_b)\sigma_{\alpha\beta}}{(k^2-m_b^2)^2} + \mathcal{O}\langle \alpha_sG^2/\pi\rangle\delta_{ab} + \cdots\right\}. \tag{11}$$

Here, we have used the notation

$$G_{ab}^{\alpha\beta} \equiv G_A^{\alpha\beta}\lambda_{ab}^A/2, \tag{12}$$

where $G_A^{\alpha\beta}$ is the gluon field-strength tensor and λ^A are the Gell-Mann matrices. The index A runs in the range $1,2,\ldots,8$. The propagators $S_{u(b)}(x)$ are known with considerably higher accuracy, but in Eqs. (10) and (11) we keep only a few terms: Arguments in favor of such choices will be provided below.

The correlator $\Pi^{\rm OPE}(p,p')$ contains three quark propagators and vacuum condensate $\langle \bar{b}b\rangle$ of b quarks. The function $\Pi^{\rm OPE}(p,p')$ differs from a standard one which in the case, for instance, of the decay $T_{4\rm b}\to\eta_b\eta_b$ depends on four propagators $S_b(x)$. The reason is that to calculate $\Pi^{\rm OPE}(p,p')$ one contracts heavy and light quark fields, and, because a pair of B^+B^- mesons contains only b and $\bar{b}$ quarks, the remaining $\bar{b}b$ fields in $X_{4\rm b}$ constitute a heavy quark condensate.

Using the relation between the heavy quark and gluon condensates

$$m_b\langle \bar{b}b\rangle = -\frac{1}{12\pi}\left\langle\frac{\alpha_sG^2}{\pi}\right\rangle, \tag{13}$$

we get

$$\Pi^{\rm OPE}(p,p') = -\frac{4}{9m_b\pi}\left\langle\frac{\alpha_sG^2}{\pi}\right\rangle\int d^4xd^4y e^{ip'y}e^{-ipx} \times {\rm Tr}[S_u^{ij}(y)\gamma_5 S_b^{ja}(-x)S_b^{ai}(x-y)\gamma_5]. \tag{14}$$

In other words, the correlator $\Pi^{\rm OPE}(p,p')$ is suppressed by the dimension-four factor $\langle\alpha_s G^2/\pi\rangle$. In what follows, we denote by $\Pi^{\rm OPE}(p^2,p'^2,q^2)$ the corresponding invariant amplitude.

In calculation of $\Pi^{\rm OPE}(p,p')$, we set $m_u=0$. The perturbative terms in all propagators lead to a contribution which is proportional to $\langle\alpha_s G^2/\pi\rangle$. A dimension-7 term in $\Pi^{\rm OPE}(p,p')$ arising from the component $\sim\langle\bar{u}u\rangle$ in $S_u(x)$ and perturbative ones in $S_b(x)$ vanishes. A contribution $\sim\langle\alpha_s G^2/\pi\rangle^2$ generated by components $g_s G_{ab}^{\alpha\beta}$ in b propagators and the perturbative term in $S_u(x)$ can be safely neglected. Higher-dimensional pieces in the quark propagators omitted in Eqs. (10) and (11) give effects suppressed by additional factors. As a result, $\Pi^{\rm OPE}(p,p')$ calculated with dimension-7 accuracy actually contains a dimension-4 term proportional to $\langle\alpha_s G^2/\pi\rangle$.

To derive the sum rule for the form factor $g_1(q^2)$, we equate the invariant amplitudes $\Pi^{\rm Phys}(p^2,p'^2,q^2)$ and $\Pi^{\rm OPE}(p^2,p'^2,q^2)$ and get the sum rule equality. Contributions of the higher resonances and continuum terms can be suppressed by applying Borel transformations over variables $-p^2$ and $-p'^2$ to both sides of this expression and removed using an assumption on the quark-hadron duality. After these operations, we find

$$g_1(q^2)=\frac{2m_b^2}{fmf_B^2m_B^4}\frac{q^2-m_B^2}{m^2+m_B^2-q^2}e^{m^2/M_1^2}e^{m_B^2/M_2^2}\times\Pi(\mathbf{M}^2,\mathbf{s}_0,q^2), \tag{15}$$

where $\Pi(\mathbf{M}^2,\mathbf{s}_0,q^2)$ is the amplitude $\Pi^{\rm OPE}(p^2,p'^2,q^2)$ after Borel transformations and continuum subtractions. It can be expressed by means of the spectral density $\rho(s,s',q^2)$:

$$\Pi(\mathbf{M}^2,\mathbf{s}_0,q^2)=\int_{16m_b^2}^{s_0}ds\int_{m_b^2}^{s_0'}ds'\rho(s,s',q^2)\times e^{-s/M_1^2}e^{-s'/M_2^2}. \tag{16}$$

Here, (M_1^2,s_0) and (M_2^2,s_0') are the Borel and continuum subtraction parameters for the $X_{4\rm b}$ and B^+ channels, respectively. It is worth noting that $\rho(s,s',q^2)$ is computed as an imaginary part of the correlation function $\Pi^{\rm OPE}(p,p')$.

As is seen, $g_1(q^2)$ contains the mass m and current coupling f of the tetraquark $X_{4\rm b}$. These quantities were found in Ref. [14]:

$$m=(18540\pm50)\ \text{MeV},\qquad f=(6.1\pm0.4)\times10^{-1}\ \text{GeV}^4. \tag{17}$$

To this end, we used the two-point SR method and applied for the Borel and continuum subtraction parameters the following regions:

$$M^2\in[17.5,18.5]\ \text{GeV}^2,\qquad s_0\in[375,380]\ \text{GeV}^2. \tag{18}$$

The sum rule Eq. (15) depends also on the mass $m_B=(5279.25\pm0.26)$ MeV and decay constant $f_B=(206\pm7)$ MeV of the $B^{\pm}$ mesons borrowed from Refs. [31,32], respectively. The values of the gluon condensate and b and c quarks' masses are well known:

$$\left\langle\frac{\alpha_s G^2}{\pi}\right\rangle=(0.012\pm0.004)\ \text{GeV}^4,\qquad m_b=4.18^{+0.03}_{-0.02}\ \text{GeV},\qquad m_c=(1.27\pm0.02)\ \text{GeV}. \tag{19}$$

To perform numerical analysis, one has to fix the working windows for the parameters (M_1^2,s_0) and (M_2^2,s_0'). For M_1^2 and s_0 connected with the tetraquark $X_{4\rm b}$, we employ the regions Eq. (18). It is worth noting that the working windows in Eq. (18) meet all constraints imposed on them by SR method. Thus, the pole contribution PC in the relevant mass calculations changes within limits $0.72\geq\text{PC}\geq0.66$. In other words, for all s_0 the pole contribution exceeds 0.5. At $M_1^2=17.5\ \text{GeV}^2$, a dimension-4 term constitutes $\simeq-1.5\%$ of the result, ensuring convergence of the operator product expansion. Such strong constraints naturally lead to rather narrow regions for M_1^2 and s_0. Because $g_1(q^2)$ depends also on fm, another choice for (M_1^2,s_0) may generate additional, uncontrollable ambiguities.

The parameters (M_2^2,s_0') for the B^+ channel are chosen within limits:

$$M_2^2\in[5.5,6.5]\ \text{GeV}^2,\qquad s_0'\in[33.5,34.5]\ \text{GeV}^2. \tag{20}$$

The s_0' is limited by the mass $m_{B^+(2S)}=5976$ MeV of the excited $B^+(2S)$ meson [33] and satisfies $s_0'<m_{B^+(2S)}^2$. The Borel parameter M_2^2 also complies with constraints of SR analysis. But these two sets (M_1^2,s_0) and (M_2^2,s_0') should lead to relatively stable regions where $g_1(q^2)$ can be evaluated.

It is known that the sum rule method gives credible results only in the Euclidean region $q^2<0$. Therefore, we introduce a new variable $Q^2=-q^2$ and denote the obtained function by $g_1(Q^2)$. We compute $g_1(Q^2)$ by varying Q^2 within the boundaries $Q^2=1$–$10\ \text{GeV}^2$ and depict the obtained results in Fig. 1. Let us emphasize that at each Q^2 calculations performed here meet constraints imposed on parameters $\mathbf{M}^2$ and $\mathbf{s}_0$ by the SR method. For example, in Fig. 2, the coupling $g_1(2\ \text{GeV}^2)$ is plotted as a function of the parameters M_1^2 and M_2^2 at the middle of the regions s_0 and s_0', where it demonstrates a relative stability: Indeed, upon changing M_1^2 and M_2^2 inside of explored regions, variations of $g_1(2\ \text{GeV}^2)$ do not exceed $\pm12\%$ of the central value. At the point $Q^2=2\ \text{GeV}^2$, we get

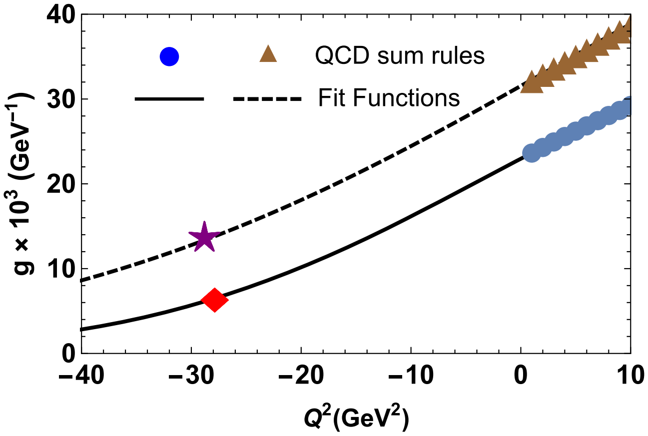

FIG. 1. QCD predictions and $\mathcal{F}(Q^2)$ functions for the form factors $g_1(Q^2)$ (solid line) and $g_5(Q^2)$ (dashed line). The couplings g_1 and g_5 are extracted at the points $Q^2 = -m_B^2$ and $Q^2 = -m_{B^*}^2$ labeled on the plot by the red diamond and star, respectively.

$$g_1(2\ \mathrm{GeV}^2) = (2.30 \pm 0.26) \times 10^{-2}\ \mathrm{GeV}^{-1}. \quad (21)$$

To calculate the partial width of the process $X_{4b} \to B^+B^-$, one needs the value of the form factor $g_1(q^2)$ at the mass shell of the B^- meson $q^2 = m_B^2$. To this end, it is necessary to introduce a fit function $\mathcal{F}_1(Q^2)$ that at momenta $Q^2 > 0$ leads to the same data as the SR computations but can be extrapolated to a region of $Q^2 < 0$ and employed to fix $\mathcal{F}_1(-m_B^2)$.

In present article, we use the functions $\mathcal{F}_l(Q^2)$:

$$\mathcal{F}_l(Q^2) = \mathcal{F}_l^0 \exp\left[c_l^1 \frac{Q^2}{m^2} + c_l^2 \left(\frac{Q^2}{m^2}\right)^2\right], \quad (22)$$

with parameters $\mathcal{F}_l^0$, c_l^1, and c_l^2. They should be fixed by comparing SR predictions and $\mathcal{F}_1(Q^2)$. Analysis carried out in the case of the form factor $g_1(q^2)$ gives $\mathcal{F}_1^0 = 0.02\ \mathrm{GeV}^{-1}$, $c_1^1 = 10.07$, and $c_1^2 = -68.03$. This function is depicted in Fig. 1, where one can be convinced in nice agreement of $\mathcal{F}_1(Q^2)$ and QCD SR data.

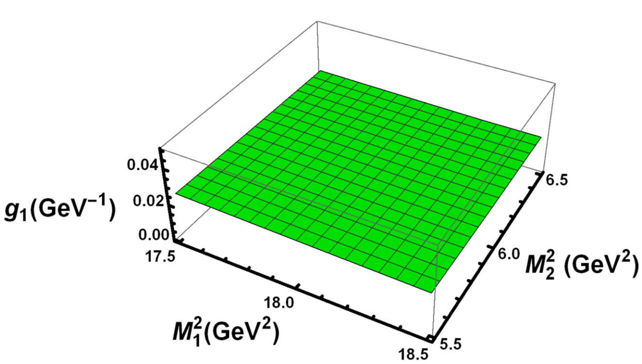

FIG. 2. The strong coupling $g = g_1(2\ \mathrm{GeV}^2)$ as a function of the parameters M_1^2 and M_2^2 at $s_0 = 377.5\ \mathrm{GeV}^2$ and $s_0' = 34\ \mathrm{GeV}^2$.

For the coupling g_1, we find

$$g_1 \equiv \mathcal{F}_1(-m_B^2) = (6.49 \pm 1.41) \times 10^{-3}\ \mathrm{GeV}^{-1}, \quad (23)$$

where ambiguities in Eq. (28) are generated mainly by the choice of the parameters $\mathbf{M}^2$ and $\mathbf{s}_0$. Let us note that we calculate the correlation function $\Pi^{\mathrm{OPE}}(p, p')$ and g_1 (and other strong couplings) at leading order of QCD. In general, the next-to-leading-order (NLO) perturbative contributions improve accuracy of theoretical analysis and are necessary to fix a scale μ in heavy quark masses and vacuum condensates. Depending on the problem under consideration, NLO terms may affect the final results. Indeed, NLO corrections to parameters of light four-quark mesons are significant [34]. At the same time, similar contributions to masses of doubly heavy tetraquarks calculated in Ref. [35] by means of the inverse Laplace SR approach were found to be numerically small. The smallness of NLO corrections in mass computations may be explained by an analytic form of the relevant SR given as a ratio of two-point correlation functions. This is not the case for vertex functions, where NLO effects may be large. But this problem requires detailed studies, which are beyond the scope of our paper.

The width of the channel $X_{4b} \to B^+B^-$ is given by the formula

$$\Gamma[X_{4b} \to B^+B^-] = g_1^2 \frac{m_B^2 \lambda}{8\pi}\left(1 + \frac{\lambda^2}{m_B^2}\right), \quad (24)$$

where $\lambda = \lambda(m, m_B, m_B)$ and

$$\lambda(x,y,z) = \frac{\sqrt{x^4 + y^4 + z^4 - 2(x^2y^2 + x^2z^2 + y^2z^2)}}{2x}. \quad (25)$$

We find

$$\Gamma_1[X_{4b} \to B^+B^-] = (1.10 \pm 0.34)\ \mathrm{MeV}. \quad (26)$$

The parameters of the decay $X_{4b} \to \bar{B}^0B^0$ almost coincide with ones for the process $X_{4b} \to B^+B^-$, though there is a small gap between the masses $m_{B_0} = (5279.63 \pm 0.20)$ MeV and m_B of the mesons B^0 and $B^\pm$. As a result, one obtains $g_2(q^2) \approx g_1(q^2)$ and $\Gamma_2[X_{4b} \to \bar{B}^0B^0] \approx \Gamma_1[X_{4b} \to B^+B^-]$.

The analysis of the process $X_{4b} \to \bar{B}_s^0B_s^0$ requires some modifications. First of all, the interpolating currents of the mesons $\bar{B}_s^0$ and B_s^0 are, respectively,

$$J^{\bar{B}_s}(x) = \bar{s}_j(x)i\gamma_5 b_j(x), \quad J^{B_s}(x) = \bar{b}_i(x)i\gamma_5 s_i(x). \quad (27)$$

The matrix element of $\bar{B}_s^0$ and B_s^0 mesons is

$$\langle 0|J^{B_s}|B_s^0\rangle = \frac{f_{B_s}m_{B_s}^2}{m_b + m_s}, \quad (28)$$

TABLE I. Decay modes of the tetraquarks X_{4b} and T_{4b}, strong couplings g_l and G_l, and partial widths Γ_l and $\tilde{\Gamma}_l$. For all decays, the Borel and continuum subtraction parameters in the X_{4b} channel are $M_1^2 \in [17.5, 18.5]$ GeV^2 and $s_0 \in [375, 380]$ GeV^2, whereas in the T_{4b} channel $\tilde{M}_1^2 \in [17.5, 18.5]$ GeV^2 and $\tilde{s}_0 \in [380, 385]$ GeV^2 have been used.

l	Modes	M_2^2 (GeV^2)	s_0' (GeV^2)	g_l (GeV^{-1})	Γ_l (MeV)	G_l (GeV^{-1})	$\tilde{\Gamma}_l$ (MeV)
1	B^+B^-	5.5–6.5	33.5–34.5	$(6.49 \pm 1.41) \times 10^{-3}$	1.10 ± 0.34	$(1.02 \pm 0.21) \times 10^{-2}$	2.92 ± 0.89
2	$\bar{B}^0B^0$	5.5–6.5	33.5–34.5	$(6.49 \pm 1.41) \times 10^{-3}$	1.10 ± 0.34	$(1.02 \pm 0.21) \times 10^{-2}$	2.92 ± 0.89
3	$\bar{B}_s^0B_s^0$	5.5–6.5	34–35	$(5.61 \pm 1.21) \times 10^{-3}$	0.81 ± 0.25	$(0.85 \pm 0.17) \times 10^{-2}$	1.99 ± 0.59
4	$B_c^+B_c^-$	6.5–7.5	45–47	$(4.67 \pm 0.89) \times 10^{-3}$	0.51 ± 0.14	$(4.87 \pm 0.95) \times 10^{-3}$	0.59 ± 0.16
5	$B^{*+}B^{*-}$	5.5–6.5	34–35	$(1.36 \pm 0.26) \times 10^{-2}$	2.26 ± 0.62	$(1.75 \pm 0.35) \times 10^{-2}$	4.05 ± 1.15
6	$\bar{B}^{*0}B^{*0}$	5.5–6.5	34–35	$(1.36 \pm 0.26) \times 10^{-2}$	2.26 ± 0.62	$(1.75 \pm 0.35) \times 10^{-2}$	4.05 ± 1.15
7	$\bar{B}_s^{*0}B_s^{*0}$	6–7	35–36	$(1.14 \pm 0.22) \times 10^{-2}$	1.58 ± 0.43	$(1.59 \pm 0.32) \times 10^{-2}$	3.32 ± 0.95

with $m_s = 93.4^{+8.6}_{-3.4}$ MeV being the s quark's mass. In Eq. (28), $m_{B_s} = (5366.91 \pm 0.11)$ MeV and $f_{B_s} = (234 \pm 5)$ MeV are the mass and decay constant, respectively, of these mesons. Therefore, Eqs. (15) and (16) change accordingly, where one should replace $m_b^2 \to (m_b + m_s)^2$.

In computations of $g_3(q^2)$, we use the following Borel and continuum subtraction parameters: for the X_{4b} channel the parameters (M_1^2, s_0) from Eq. (18) and

$$M_2^2 \in [5.5, 6.5]\ \text{GeV}^2, \qquad s_0' \in [34, 35]\ \text{GeV}^2, \tag{29}$$

for the $\bar{B}_s^0$ channel. The fit function $\mathcal{F}_3(Q^2)$ necessary for further computations is determined by the parameters $\mathcal{F}_3^0 = 0.02$ GeV^{-1}, $c_3^1 = 9.67$, and $c_3^2 = -63.84$. Then, for the coupling g_3 and width of the decay $X_{4b} \to \bar{B}_s^0 B_s^0$, we find

$$g_3 \equiv \mathcal{F}_3(-m_{B_s}^2) = (5.61 \pm 1.21) \times 10^{-3}\ \text{GeV}^{-1},$$
$$\Gamma_3[X_{4b} \to \bar{B}_s^0 B_s^0] = (0.81 \pm 0.25)\ \text{MeV}. \tag{30}$$

The investigations of the fourth decay $X_{4b} \to B_c^+ B_c^-$ can be carried out in the standard way. In this case, we consider the correlation function, in which the interpolating currents for the mesons B_c^+ and B_c^- have the forms, respectively,

$$J^{B_c^+}(x) = \bar{b}_j(x) i\gamma_5 c_j(x), \quad J^{B_c^-}(x) = \bar{c}_i(x) i\gamma_5 b_i(x). \tag{31}$$

The matrix element of these mesons is

$$\langle 0|J^{B_c}|B_c\rangle = \frac{f_{B_c} m_{B_c}^2}{m_b + m_c}, \tag{32}$$

with $m_{B_c} = (6274.47 \pm 0.27)$ MeV and $f_{B_c} = (476 \pm 27)$ MeV being the mass and decay constant, respectively, of $B_c^{\pm}$ [31,36]. The form factor $g_4(q^2)$ is extracted from the SRs using the following parameters:

$$M_2^2 \in [6.5, 7.5]\ \text{GeV}^2, \qquad s_0' \in [45, 47]\ \text{GeV}^2. \tag{33}$$

The fit function $\mathcal{F}_4(Q^2)$ is given by the parameters $\mathcal{F}_4^0 = 0.009$ GeV^{-1}, $c_4^1 = 4.58$, and $c_4^2 = -8.89$. Then, for the coupling g_4 and width of the decay $X_{4b} \to B_c^+ B_c^-$, we get

$$g_4 \equiv \mathcal{F}_4(-m_{B_c}^2) = (4.67 \pm 0.89) \times 10^{-3}\ \text{GeV}^{-1},$$
$$\Gamma_4[X_{4b} \to B_c^+ B_c^-] = (0.51 \pm 0.14)\ \text{MeV}. \tag{34}$$

Predictions obtained for parameters of these decays are collected in Table I.

III. CHANNELS $X_{4b} \to B^{*+}B^{*-}$, $\bar{B}^{*0}B^{*0}$, AND $\bar{B}_s^{*0}B_s^{*0}$

The decays of the tetraquark X_{4b} to vector mesons $B_q^* \bar{B}_q^*$, with some modifications, can be analyzed as the ones studied in the previous section. Let us consider the process $X_{4b} \to B^{*+}B^{*-}$ and evaluate the form factor $g_5(q^2)$ corresponding to the vertex $X_{4b}B^{*+}B^{*-}$.

The correlation function required to derive SR for $g_5(q^2)$ is

$$\Pi_{\mu\nu}(p, p') = i^2 \int d^4x d^4y e^{ip'y} e^{-ipx} \langle 0|\mathcal{T}\{J_\mu^{B^{*+}}(y) \times J_\nu^{B^{*-}}(0) J^\dagger(x)\}|0\rangle, \tag{35}$$

where

$$J_\mu^{B^{*+}}(x) = \bar{b}_j(x)\gamma_\mu u_j(x), \quad J_\nu^{B^{*-}}(x) = \bar{u}_i(x)\gamma_\nu b_i(x) \tag{36}$$

are interpolating currents for the vector mesons B^{*+} and B^{*-}, respectively.

The $\Pi_{\mu\nu}(p, p')$ in terms of physical parameters of the particles X_{4b}, B^{*+}, and B^{*-} has the decomposition

$$\Pi_{\mu\nu}^{\text{Phys}}(p, q) = \frac{g_5(q^2) f m f_{B^*}^2 m_{B^*}^2}{(p^2 - m^2)(p'^2 - m_{B^*}^2)(q^2 - m_{B^*}^2)} \times \left(\frac{m^2 - m_{B^*}^2 - q^2}{2} g_{\mu\nu} - p_\mu p'_\nu\right) + \cdots. \tag{37}$$

Equation (37) is obtained using the matrix elements

$$\langle 0|J_\mu^{B^{*+}}|B^{*+}(p')\rangle = f_{B^*} m_{B^*} \varepsilon_\mu(p'),$$
$$\langle 0|J_\nu^{B^{*-}}|B^{*-}(q)\rangle = f_{B^*} m_{B^*} \varepsilon'_\nu(q), \tag{38}$$

where $m_{B^*} = (5324.71 \pm 0.21)$ MeV and $f_{B^*} = (210 \pm 6)$ MeV are the mass and decay constants, respectively, of the $B^{*\pm}$ mesons [31,37]. Here, $\varepsilon_\mu(p')$ and $\varepsilon'_\nu(q)$ are the polarization vectors of B^{*+} and B^{*-}, respectively. The vertex $X_{4b}B^{*+}B^{*-}$ is modeled by the expression

$$\langle B^{*+}(p')B^{*-}(q)|X_{4b}(p)\rangle = g_5(q^2)[(q\cdot p')\varepsilon^*(p')\cdot\varepsilon'^*(q) - p'\cdot\varepsilon'^*(q)q\cdot\varepsilon^*(p')]. \tag{39}$$

The QCD side of SRs is determined by the formula

$$\Pi^{\mathrm{OPE}}_{\mu\nu}(p,p') = -\frac{4}{9m_b\pi}\left\langle\frac{\alpha_s G^2}{\pi}\right\rangle\int d^4x d^4y e^{ip'y}e^{-ipx} \times \mathrm{Tr}[S_u^{ij}(y)\gamma_\nu S_b^{ja}(-x)S_b^{ai}(x-y)\gamma_\mu]. \tag{40}$$

In the SR computations, we make use of invariant amplitudes $\hat{\Pi}^{\mathrm{Phys}}(p^2,p'^2,q^2)$ and $\hat{\Pi}^{\mathrm{OPE}}(p^2,p'^2,q^2)$ which correspond to terms $g_{\mu\nu}$ in the physical and QCD sides, respectively. After Borel transformations and continuum subtractions, the SR for the form factor $g_5(q^2)$ reads

$$g_5(q^2) = \frac{2}{fmf_{B^*}^2 m_{B^*}^2}\frac{q^2 - m_{B^*}^2}{m^2 - m_{B^*}^2 - q^2}e^{m^2/M_1^2} \times e^{m_{B^*}^2/M_2^2}\hat{\Pi}(\mathbf{M}^2,\mathbf{s}_0,q^2), \tag{41}$$

with $\hat{\Pi}(\mathbf{M}^2,\mathbf{s}_0,q^2)$ being the amplitude $\hat{\Pi}^{\mathrm{OPE}}(p^2,p'^2,q^2)$ after relevant manipulations.

The partial width of the decay $X_{4b}\to B^{*+}B^{*-}$ can be evaluated by means of the expression

$$\Gamma_5[X_{4b}\to B^{*+}B^{*-}] = g_5^2\frac{m_{B^*}^2\hat{\lambda}}{8\pi}\left(\frac{m_{B^*}^2}{m^2} + \frac{2\hat{\lambda}^2}{3m_{B^*}^2}\right), \tag{42}$$

where $\hat{\lambda} = \lambda(m, m_{B^*}, m_{B^*})$.

The coupling g_5 is determined in accordance with a scheme explained above. In Fig. 1, we provide the SR data and extrapolating function $\mathcal{F}_5(Q^2)$ employed to find g_5. To extract the SR data, we have used the working regions Eq. (29). The coupling g_5 has been evaluated at the mass shell of B^{*-} meson. This coupling and partial width of the decay $X_{4b}\to B^{*+}B^{*-}$ are equal, respectively, to

$$g_5 \equiv \mathcal{F}_5(-m_{B^*}^2) = (1.36\pm 0.26)\times 10^{-3}\ \mathrm{GeV}^{-1},$$
$$\Gamma_5[X_{4b}\to B^{*+}B^{*-}] = (2.26\pm 0.62)\ \mathrm{MeV}. \tag{43}$$

Predictions obtained for parameters of other modes are presented in Table I. Here, one can find couplings g_6 and g_7, as well as partial widths of the decays $X_{4b}\to\bar{B}^{*0}B^{*0}$ and $\bar{B}_s^{*0}B_s^{*0}$. The strong coupling g_6 of particles at the vertex $X_{4b}\bar{B}^{*0}B^{*0}$ and width of the channel $X_{4b}\to\bar{B}^{*0}B^{*0}$ do not differ numerically from those for the process $X_{4b}\to B^{*+}B^{*-}$. To calculate parameters of the mode $X_{4b}\to\bar{B}_s^{*0}B_s^{*0}$, we have used the following input information: the mass of the B_s^{*0} meson $m_{B_s^*} = (5415.8\pm 1.5)$ MeV and its decay constant $f_{B_s^*} = (221\pm 7)$ MeV. In this case the regions for M_2^2 and s_0' are chosen as

$$M_2^2\in[6,7]\ \mathrm{GeV}^2,\qquad s_0'\in[35,36]\ \mathrm{GeV}^2. \tag{44}$$

For the width of the decay $X_{4b}\to\bar{B}_s^{*0}B_s^{*0}$, we obtain

$$\Gamma_7[X_{4b}\to\bar{B}_s^{*0}B_s^{*0}] = (1.58\pm 0.43)\ \mathrm{MeV}. \tag{45}$$

Information about partial widths of the decays obtained in last two sections allows us to estimate the full width of the tetraquark X_{4b}:

$$\Gamma_{4b} = (9.62\pm 1.13)\ \mathrm{MeV}. \tag{46}$$

IV. PROCESSES $T_{4b}\to B_q\bar{B}_q$ AND $T_{4b}\to B_q^*\bar{B}_q^*$

The scalar tetraquark T_{4b} was studied in Ref. [16], in which we calculated the mass $\tilde{m}$ and full width $\tilde{\Gamma}_{4b}$ of this particle. It was modeled as a diquark-antidiquark compound built of pseudoscalar constituents. The interpolating current for such a state has the form

$$\tilde{J}(x) = b_a^T(x)Cb_b(x)\bar{b}_a(x)C\bar{b}_b^T(x). \tag{47}$$

The spectroscopic parameters of T_{4b}

$$\tilde{m} = (18858\pm 50)\ \mathrm{MeV},$$
$$\tilde{f} = (9.54\pm 0.71)\times 10^{-2}\ \mathrm{GeV}^4 \tag{48}$$

were extracted from the two-point SRs by employing the following parameters $\tilde{M}^2$ and $\tilde{s}_0$:

$$\tilde{M}^2\in[17.5,18.5]\ \mathrm{GeV}^2,$$
$$\tilde{s}_0\in[380,385]\ \mathrm{GeV}^2. \tag{49}$$

In accordance with these results, the tetraquark T_{4b} can decay through the channel $T_{4b}\to\eta_b\eta_b$. The full width of T_{4b} was estimated using this decay mode and found equal to

$$\tilde{\Gamma}_{4b} = (94\pm 28)\ \mathrm{MeV}. \tag{50}$$

The processes $T_{4b}\to B_q\bar{B}_q$ are additional decay channels for the tetraquark T_{4b}: By taking into account these modes, we are going to refine our previous prediction for $\tilde{\Gamma}_{4b}$.

The treatment of these processes does not differ from our analysis presented above. There are only some differences generated by the interpolating current of the tetraquark T_{4b} and its parameters. For instance, in the case of the channel $T_{4b}\to B^+B^-$, the phenomenological side of the required

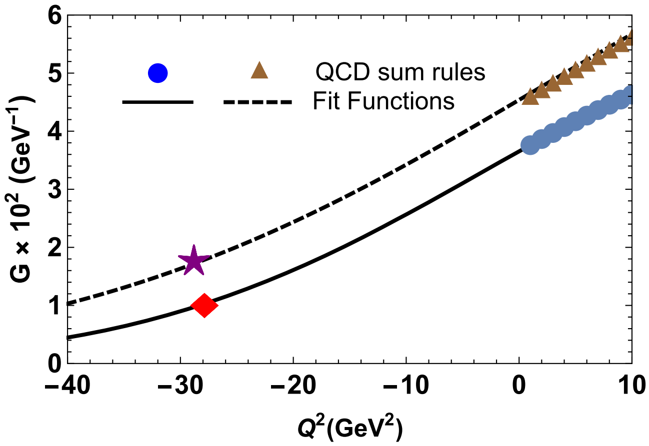

FIG. 3. The sum rule results and extrapolating functions for the form factors $G_1(Q^2)$ (solid line) and $G_5(Q^2)$ (dashed line). The strong couplings G_1 and G_5 are found at the points $Q^2 = -m_B^2$ and $Q^2 = -m_{B^*}^2$ and denoted by the red diamond and star, respectively.

sum rule after substitutions $m, f \to \tilde{m}, \tilde{f}$ and $g_1(q^2) \to G_1(q^2)$ is given by Eq. (8). The mass and current coupling of T_{4b} emerge in this expression through the matrix element

$$\langle 0|\tilde{J}|T_{4b}\rangle = \tilde{f}\,\tilde{m}, \tag{51}$$

whereas $G_1(q^2)$ arises from the vertex

$$\langle B^+(p')B^-(q)|T_{4b}(p)\rangle = G_1(q^2)p\cdot p'. \tag{52}$$

The form factor $G_1(q^2)$ describes strong interaction of particles at the vertex $T_{4b}B^+B^-$ and is a quantity which should be estimated at $q^2 = m_B^2$.

The correlation function $\tilde{\Pi}^{\rm OPE}(p,p')$ in terms of quark propagators reads

$$\tilde{\Pi}^{\rm OPE}(p,p') = -\frac{1}{9m_b\pi}\left\langle\frac{\alpha_s G^2}{\pi}\right\rangle \int d^4x d^4y e^{ip'y}e^{-ipx} \times {\rm Tr}[S_u^{ij}(y)S_b^{ja}(-x)S_b^{ai}(x-y)]. \tag{53}$$

Then, the sum rule for $G_1(q^2)$ has the form

$$G_1(q^2) = \frac{2m_b^2}{\tilde{f}\,\tilde{m}\,f_B^2 m_B^4}\frac{q^2-m_B^2}{\tilde{m}^2+m_B^2-q^2}e^{\tilde{m}^2/M_1^2}e^{m_B^2/M_2^2} \times \tilde{\Pi}(\mathbf{M}^2,\mathbf{s}_0,q^2), \tag{54}$$

with $\tilde{\Pi}(\mathbf{M}^2,\mathbf{s}_0,q^2)$ being the Borel transformed and subtracted invariant amplitude $\tilde{\Pi}^{\rm OPE}(p^2,p'^2,q^2)$.

The remaining manipulations are similar to ones explained above. In numerical computations of $G_1(q^2)$ as the parameters (M_1^2, s_0) for the T_{4b} tetraquark's channel, we employ regions Eq. (49), whereas (M_2^2, s_0') in the B^+ channel are the same as in Eq. (20). Results of computations are plotted in Fig. 3.

The extrapolating functions $\tilde{\mathcal{F}}_l(Q^2)$ employed for analysis of the T_{4b} tetraquark's decays have the same functional dependence on the momentum $Q^2 = -q^2$ with replacement $m^2 \to \tilde{m}^2$ in $\mathcal{F}_l(Q^2)$. Its parameters, in the case of the vertex $T_{4b}B^+B^-$, are

$$\tilde{\mathcal{F}}_1^0 = 0.04~{\rm GeV}^{-1},\quad \tilde{c}_1^1 = 10.47,\quad \tilde{c}_1^2 = -72.81. \tag{55}$$

Then, one can easily evaluate the coupling G_1 and partial width of the decay $T_{4b} \to B^+B^-$:

$$G_1 \equiv \tilde{\mathcal{F}}_1(-m_B^2) = (1.02\pm 0.21)\times 10^{-2}~{\rm GeV}^{-1} \tag{56}$$

and

$$\tilde{\Gamma}_1[T_{4b}\to B^+B^-] = (2.92\pm 0.89)~{\rm MeV}. \tag{57}$$

The remaining processes $T_{4b} \to \bar{B}^0B^0$, $\bar{B}_s^0B_s^0$, and $B_c^+B_c^-$ are explored by a similar manner. The parameters of the second decay $T_{4b} \to \bar{B}^0B^0$ are approximately the same as the ones for the first channel. In the case of $T_{4b} \to \bar{B}_s^0B_s^0$ and $T_{4b} \to B_c^+B_c^-$, we obtain

$$G_3 \equiv \tilde{\mathcal{F}}_3(-m_{B_s}^2) = (0.85\pm 0.17)\times 10^{-2}~{\rm GeV}^{-1}, \quad \tilde{\Gamma}_3[T_{4b}\to \bar{B}_s^0B_s^0] = (1.99\pm 0.59)~{\rm MeV} \tag{58}$$

and

$$G_4 \equiv \tilde{\mathcal{G}}_4(-m_{B_c^+}^2) = (4.87\pm 0.95)\times 10^{-3}~{\rm GeV}^{-1}, \quad \tilde{\Gamma}_4[T_{4b}\to B_c^+B_c^-] = (0.59\pm 0.16)~{\rm MeV}, \tag{59}$$

respectively.

Parameters of the decays $T_{4b} \to B_q^*\bar{B}_q^*$ are collected in Table I. In Fig. 3, we plot also the function $G_5 \equiv \tilde{\mathcal{G}}_5(Q^2)$ and corresponding SR predictions for the form factor $G_5(Q)$.

The sum of partial widths of the decays considered in the present section,

$$\tilde{\Gamma}_{4b}^{\rm BB} = (19.84\pm 2.35)~{\rm MeV}, \tag{60}$$

as well as parameters of the process $T_{4b} \to \eta_b\eta_b$, allows us to evaluate new prediction for the full width of T_{4b}:

$$\tilde{\Gamma}_{4b}^{\rm Full} = (114\pm 29)~{\rm MeV}. \tag{61}$$

The main contribution to $\tilde{\Gamma}_{4b}^{\rm Full}$ comes from the channel $T_{4b} \to \eta_b\eta_b$ with the branching ratio

$$\mathcal{B}(T_{4b}\to\eta_b\eta_b) = \tilde{\Gamma}_{4b}/\tilde{\Gamma}_{4b}^{\rm Full} \approx 0.82. \tag{62}$$

The remaining modes constitute ≈0.18 part of the full width and can be considered as sizable corrections to $\tilde{\Gamma}_{4b}^{\rm Full}$.

V. SUMMARY

In the present article, we have explored decays of fully beauty scalar tetraquarks $X_{4\mathrm{b}}$ and $T_{4\mathrm{b}}$ to $B_q\bar{B}_q$ and $B_q^*\bar{B}_q^*$ mesons in the context of the QCD three-point sum rule method. These decays are very important for exotic mesons with masses below $2\eta_b$ threshold, because they form an essential part of their full widths.

We have modeled $X_{4\mathrm{b}}$ and $T_{4\mathrm{b}}$ as diquark-antidiquark states built of diquarks with different spins. Thus, ingredients of the tetraquark $X_{4\mathrm{b}}$ are axial-vector diquarks, whereas $T_{4\mathrm{b}}$ is composed of pseudoscalar ones. The masses and current couplings of $X_{4\mathrm{b}}$ and $T_{4\mathrm{b}}$ were computed in the context of SR method in our articles [14,16].

The mass m of $X_{4\mathrm{b}}$ is less than the $2\eta_b$ limit; therefore, the full width of this tetraquark $\Gamma_{4\mathrm{b}} = (9.62 \pm 1.13)$ MeV has been estimated in the current article using namely these processes. The tetraquark $T_{4\mathrm{b}}$ with the mass $\tilde{m}$ above $2\eta_b$ threshold dissociates to $\eta_b\eta_b$ mesons which form an important part of its full width. Nevertheless, contributions of the channels $T_{4\mathrm{b}} \to B_q\bar{B}_q$ and $B_q^*\bar{B}_q^*$ to the full width of $T_{4\mathrm{b}}$ are sizable.

Decays of the tetraquark $bb\bar{b}\bar{b}$ with the spin-parities $J^{\mathrm{PC}} = 0^{++}$ and mass below $2\eta_b$ were explored using different models in Refs. [23,30] as well. In these articles, the full width of such a state was estimated as 1.2 and 8.5 MeV, respectively. The channel $\Upsilon\mu^+\mu^-$ was analyzed in Ref. [24], where the width was found in the range 10^{-3}–10 MeV.

As is seen, our prediction for $\Gamma_{4\mathrm{b}}$ is consistent with result in Ref. [30]. But, there are other decays of $X_{4\mathrm{b}}$ which may contribute to $\Gamma_{4\mathrm{b}}$ and modify it considerably. In the context of the method used in Ref. [30], the process $\eta_b + H$ seems important to estimate $\Gamma_{4\mathrm{b}}$. This decay is definitely beyond reach of the sum rule method and has not been considered here. In other words, additional efforts are necessary to make model-independent predictions for widths of fully beauty tetraquarks lying below the $2\eta_b$ threshold.

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