



Radiative generation of non-zero θ_{13} in MSSM with broken A_4 flavor symmetry

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Abstract

We study the renormalization group effects on neutrino masses and mixing in Minimal Supersymmetric Standard Model (MSSM) by considering a μ - τ symmetric mass matrix at high energy scale giving rise to Tri-Bi-Maximal (TBM) type mixing. We outline a flavor symmetry model based on A_4 symmetry giving rise to the desired neutrino mass matrix at high energy scale. We take the three neutrino mass eigenvalues at high energy scale as input parameters and compute the neutrino parameters at low energy by taking into account of renormalization group effects. We observe that the correct output values of neutrino parameters at low energy are obtained only when the input mass eigenvalues are large $|m_{1,2,3}| = 0.08$ – 0.12 eV with a very mild hierarchy of either inverted or normal type. A large inverted or normal hierarchical pattern of neutrino masses is disfavored within our framework. We also find a preference towards higher values of $\tan\beta$, the ratio of vacuum expectation values (vev) of two Higgs doublets in MSSM in order to arrive at the correct low energy output. Such a model predicting large neutrino mass eigenvalues with very mild hierarchy and large $\tan\beta$ could have tantalizing signatures at oscillation, neutrino-less double beta decay as well as collider experiments.

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1. Introduction

Exploration of the origin of neutrino masses and mixing has been one of the major goals of particle physics community for the last few decades. The results of recent neutrino oscillation experiments have provided a clear evidence favoring the existence of tiny but non-zero neutrino masses [1]. Recent neutrino oscillation experiments like T2K [2], Double ChooZ [3], Daya-Bay [4] and RENO [5] have not only confirmed the earlier predictions for neutrino parameters, but also provided strong evidence for a non-zero value of the reactor mixing angle θ_{13} . The latest global fit values for 3σ range of neutrino oscillation parameters [6] are as follows:

$$\begin{aligned}
 \Delta m_{21}^2 &= (7.00\text{--}8.09) \times 10^{-5} \text{ eV}^2 \\
 \Delta m_{31}^2 (\text{NH}) &= (2.27\text{--}2.69) \times 10^{-3} \text{ eV}^2 \\
 \Delta m_{23}^2 (\text{IH}) &= (2.24\text{--}2.65) \times 10^{-3} \text{ eV}^2 \\
 \sin^2 \theta_{12} &= 0.27\text{--}0.34 \\
 \sin^2 \theta_{23} &= 0.34\text{--}0.67 \\
 \sin^2 \theta_{13} &= 0.016\text{--}0.030
 \end{aligned} \tag{1}$$

where NH and IH refer to normal and inverted hierarchy respectively. Another global fit study [7] reports the 3σ values as

$$\begin{aligned}
 \Delta m_{21}^2 &= (6.99\text{--}8.18) \times 10^{-5} \text{ eV}^2 \\
 \Delta m_{31}^2 (\text{NH}) &= (2.19\text{--}2.62) \times 10^{-3} \text{ eV}^2 \\
 \Delta m_{23}^2 (\text{IH}) &= (2.17\text{--}2.61) \times 10^{-3} \text{ eV}^2 \\
 \sin^2 \theta_{12} &= 0.259\text{--}0.359 \\
 \sin^2 \theta_{23} &= 0.331\text{--}0.637 \\
 \sin^2 \theta_{13} &= 0.017\text{--}0.031
 \end{aligned} \tag{2}$$

The observation of non-zero θ_{13} which is evident from the above global fit data can have non-trivial impact on neutrino mass hierarchy as studied in recent papers [8]. Non-zero θ_{13} can also shed light on the Dirac CP violating phase in the leptonic sector which would have remained unknown if θ_{13} were exactly zero. The detailed analysis of this non-zero θ_{13} have been demonstrated both from theoretical [9], as well as phenomenological [10] point of view, prior to and after the confirmation of this important result announced in 2012. It should be noted that prior to the discovery of non-zero θ_{13} , the neutrino oscillation data were compatible with the so called TBM form of the neutrino mixing matrix [11] given by

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{3}$$

which predicts $\sin^2 \theta_{12} = \frac{1}{3}$, $\sin^2 \theta_{23} = \frac{1}{2}$ and $\sin^2 \theta_{13} = 0$. However, since the latest data have ruled out $\sin^2 \theta_{13} = 0$, there arises the need to go beyond the TBM framework. In view of the importance of the non-zero reactor mixing and hence, CP violation in neutrino sector, the present work demonstrates how a specific $\mu\text{--}\tau$ symmetric mass matrix (giving rise to TBM type mixing)

Table 1
Character table of A_4 .

| Class | $\chi^{(1)}$ | $\chi^{(2)}$ | $\chi^{(3)}$ | $\chi^{(4)}$ |
|-------|--------------|--------------|--------------|--------------|
| C_1 | 1 | 1 | 1 | 3 |
| C_2 | 1 | ω | ω^2 | 0 |
| C_3 | 1 | ω^2 | ω | 0 |
| C_4 | 1 | 1 | 1 | -1 |

at high energy scale can produce non-zero θ_{13} along with the desired values of other neutrino parameters Δm_{21}^2 , Δm_{23}^2 , θ_{23} , θ_{12} at low energy scale through renormalization group evolution (RGE). We also outline how the μ - τ symmetric neutrino mass matrix with TBM type mixing can be realized at high energy scale within the framework of MSSM with an additional A_4 flavor symmetry at high energy scale. After taking the RGE effects into account, we observe that the output at TeV scale is very much sensitive to the choice of neutrino mass ordering at high scale as well as the value of $\tan\beta = \frac{v_u}{v_d}$, the ratio of vev's of two MSSM Higgs doublets $H_{u,d}$. We point out that this model allows only a very mild hierarchy of both inverted and normal type at high energy scale. We scan the neutrino mass eigenvalues at high energy and constrain them to be large $|m_{1,2,3}| = 0.08$ – 0.12 eV in order to produce correct neutrino parameters at low energy. We consider two such input values for mass eigenvalues, one with inverted hierarchy and the other with normal hierarchy and show the predictions for neutrino parameters at low energy scale. We also show the evolution of effective neutrino mass $m_{ee} = |\sum_i U_{ei}^2 m_i|$ (where U is the neutrino mixing matrix) that could be interesting from neutrino-less double beta decay point of view. Finally we consider the cosmological upper bound on the sum of absolute neutrino masses ($\sum_i |m_i| < 0.23$ eV) reported by the Planck collaboration [12] to check if the output at low energy satisfy this or not.

This article is organized as follows. In Section 2, we discuss briefly the A_4 model at high energy scale. In Section 3 we outline the RGE's of mass eigenvalues and mixing parameters. In Section 4 we discuss our numerical results, and finally conclude in Section 5.

2. A_4 model for neutrino mass

Type I seesaw framework is the simplest mechanism for generating tiny neutrino masses and mixing. In this seesaw mechanism neutrino mass matrix can be written as

$$m_{LL} = -m_{LR} M_R^{-1} m_{LR}^T. \quad (4)$$

Within this framework of seesaw mechanism neutrino mass has been extensively studied by discrete flavor groups by many authors [13] available in the literature. Among the different discrete groups the model by the finite group of even permutation, A_4 also can explain the μ - τ symmetric mass matrix obtained from this type I seesaw mechanism. This group has 12 elements having 4 irreducible representations, with dimensions n_i , such that $\sum_i n_i^2 = 12$. The characters of 4 representations are shown in Table 1. The complex number ω is the cube root of unity. In the present work we outline a neutrino mass model with A_4 symmetry given in Ref. [14]. This flavor symmetry is also accompanied by an additional Z_3 symmetry in order to achieve the desired leptonic mixing. In this model, the three families of left-handed lepton doublets $l = (l_e, l_\mu, l_\tau)$ transform as triplets, while the electroweak singlets e^c, μ^c, τ^c and the electroweak Higgs doublets $H_{u,d}$ transform as singlets under the A_4 symmetry. In order to break the flavor symmetry sponta-

neously, two A_4 triplet scalars $\phi_l = (\phi_{l1}, \phi_{l2}, \phi_{l3})$, $\phi_\nu = (\phi_{\nu 1}, \phi_{\nu 2}, \phi_{\nu 3})$ and three scalars $\zeta_1, \zeta_2, \zeta_3$ transforming as $\mathbf{1}, \mathbf{1}', \mathbf{1}''$ under A_4 are introduced. The Z_3 charges for $l, H_{u,d}, \phi_l, \phi_\nu, \zeta_{1,2,3}$ are $\omega, 1, 1, \omega, \omega$ respectively.

Under the electroweak gauge symmetry as well as the flavor symmetry mentioned above, the superpotential for the neutrino sector can be written as

$$W_\nu = (y_{\nu\phi}\phi_\nu + y_{\nu\zeta 1}\zeta_1 + y_{\nu\zeta 2}\zeta_2 + y_{\nu\zeta 3}\zeta_3) \frac{l l H_u H_u}{\Lambda^2} \tag{5}$$

where Λ is the cutoff scale and y 's are dimensionless couplings. Decomposing the first term (which is in a $\mathbf{3} \times \mathbf{3} \times \mathbf{3}$ form of A_4) into A_4 singlets, we get

$$l l \phi_\nu = (2l_e l_e - l_\mu l_\tau - l_\tau l_\mu)\phi_{\nu 1} + (2l_\mu l_\mu - l_e l_\tau - l_\tau l_e)\phi_{\nu 2} + (2l_\tau l_\tau - l_e l_\mu - l_\mu l_e)\phi_{\nu 3}$$

Similarly, the decomposition of the last three terms into A_4 singlet gives

$$l l \zeta_1 = (l_e l_e + l_\mu l_\tau + l_\tau l_\mu)\zeta_1$$

$$l l \zeta_2 = (l_\mu l_\mu + l_e l_\tau + l_\tau l_e)\zeta_2$$

$$l l \zeta_3 = (l_\tau l_\tau + l_e l_\mu + l_\mu l_e)\zeta_3$$

Assuming the vacuum alignments of the scalars as $\langle \phi_\nu \rangle = \alpha_\nu \Lambda(1, 1, 1)$, $\langle \zeta_1 \rangle = \alpha_\zeta \Lambda$, $\langle \zeta_{2,3} \rangle = 0$, the neutrino mass matrix can be written as

$$m_{LL} = \frac{v_u^2}{\Lambda} \begin{pmatrix} a + 2d/3 & -d/3 & -d/3 \\ -d/3 & 2d/3 & a - d/3 \\ -d/3 & a - d/3 & 2d/3 \end{pmatrix}, \tag{6}$$

where $d = y_{\nu\phi}\alpha_\nu$, $a = y_{\nu\zeta 1}\alpha_\zeta$ and v_u is the vev of H_u . The above mass matrix has eigenvalues $m_1 = \frac{v_u^2}{\Lambda}(a + d)$, $m_2 = \frac{v_u^2}{\Lambda}a$ and $m_3 = \frac{v_u^2}{\Lambda}(-a + d)$. Without adopting any un-natural fine tuning condition to relate the mass eigenvalues further, we wish to keep all the three neutrino mass eigenvalues as free parameters in the A_4 symmetric theory at high energy and determine the most general parameter space at high energy scale which can reproduce the correct neutrino oscillation data at low energy through renormalization group evolution (RGE).

Such a parameterization of the neutrino mass matrix however, does not disturb the generic features of the model for example, the μ - τ symmetric nature of m_{LL} , TBM type mixing as well the diagonal nature of the charged lepton mass matrix, which at leading order (LO) is given by [14,15]

$$m_l = v_d \alpha_l \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \tag{7}$$

Here v_d is the vev of H_d ; y_e, y_μ, y_τ and α_l are dimensionless couplings. These matrices in the leptonic sector given by (6) and (7) are used in the next section for numerical analysis.

3. RGE for neutrino masses and mixing

The left-handed Majorana neutrino mass matrix m_{LL} which is generally obtained from see-saw mechanism at high scale M_R , is usually expressed in terms of $K(t)$, the coefficient of the dimension five neutrino mass operator [16,17] in a scale-dependent manner [18],

$$m_{LL}(t) = v_u^2 K(t), \tag{8}$$

where $t = \ln(\mu/1 \text{ GeV})$ and the vev is $v_u = v_0 \sin \beta$ with $v_0 = 174 \text{ GeV}$ in MSSM. The neutrino mass eigenvalues m_i and the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrix U_{PMNS} [19] are then extracted through the diagonalization of $m_{LL}(t)$ at every point in the energy scale t using Eqs. (8),

$$m_{LL}^{\text{diag}} = \text{diag}(m_1, m_2, m_3) = V_{\nu L}^T m_{LL} V_{\nu L}, \quad (9)$$

and $U_{PMNS} = V_{\nu L}$ in the basis where the charged lepton mass matrix is diagonal. The PMNS mixing matrix,

$$U_{PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}, \quad (10)$$

is usually parameterized in terms of the product of three rotations $R(\theta_{23})$, $R(\theta_{13})$ and $R(\theta_{12})$, (neglecting CP violating phases) by

$$U_{PMNS} = U_l^\dagger U_\nu = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23} & c_{12}c_{23} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23} & -c_{12}s_{23} - c_{23}s_{13}s_{12} & c_{13}c_{23} \end{pmatrix}, \quad (11)$$

where U_l is unity in the basis where charge lepton mass matrix is diagonal, $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$ respectively.

The RGE's for v_u and the eigenvalues of coefficient $K(t)$ in Eq. (8), defined in the basis where the charged lepton mass matrix is diagonal, can be expressed as [20,22]

$$\frac{d}{dt} \ln v_u = \frac{1}{16\pi^2} \left[\frac{3}{20} g_1^2 + \frac{3}{4} g_2^2 - 3h_t^2 \right] \quad (12)$$

$$\frac{d}{dt} \ln K = -\frac{1}{16\pi^2} \left[\frac{6}{5} g_1^2 + 6g_2^2 - 6h_t^2 - \delta_{i3} h_\tau^2 - \delta_{3j} h_\tau^2 \right] \quad (13)$$

Neglecting h_μ^2 and h_e^2 compared to h_τ^2 , and taking scale-independent vev as in Eq. (8), we have the complete RGE's for three neutrino mass eigenvalues,

$$\frac{d}{dt} m_i = \frac{1}{16\pi^2} \left[\left(-\frac{6}{5} g_1^2 - 6g_2^2 + 6h_t^2 \right) + 2h_\tau^2 U_{\tau i}^2 \right] m_i. \quad (14)$$

The above equations together with the evolution equations for mixing angles (23)–(24), are used for the numerical analysis in our work.

The approximate analytical solution of Eq. (14) can be obtained by taking static mixing angle $U_{\tau i}^2$ in the integration range as [21]

$$m_i(t_0) = m_i(t_R) \exp\left(\frac{6}{5} I_{g1} + 6I_{g2} - 6I_t\right) \exp(-2U_{\tau i}^2 I_\tau) \quad (15)$$

The integrals in the above expression are usually defined as [18,21]

$$I_{gi}(t_0) = \frac{1}{16\pi^2} \int_{t_0}^{t_R} g_i^2(t) dt \quad (16)$$

and

$$I_f(t_0) = \frac{1}{16\pi^2} \int_{t_0}^{t_R} h_f^2(t) dt \tag{17}$$

where $i = 1, 2, 3$ and $f = t, b, \tau$ respectively. For a two-fold degenerate neutrino masses that is, $m_{LL}^{\text{diag}} = \text{diag}(m, m, m') = U_{PMNS}^T m_{LL} U_{PMNS}$, Eq. (15) is further simplified to the following expressions

$$m_1(t_0) \approx m(t_R) (1 + 2\delta_\tau (c_{12}s_{13}c_{23} - s_{12}s_{23})^2) + O(\delta_\tau^2) \tag{18}$$

$$m_2(t_0) \approx m(t_R) (1 + 2\delta_\tau (c_{23}s_{13}s_{12} + c_{12}s_{23})^2) + O(\delta_\tau^2) \tag{19}$$

$$m_3(t_0) \approx m'(t_R) (1 + 2\delta_\tau (c_{13}c_{23})^2) + O(\delta_\tau^2). \tag{20}$$

While deriving the above expressions, the following approximations are used

$$\exp(-2|U_{\tau i}|^2 I_\tau) \simeq 1 - 2|U_{\tau i}|^2 I_\tau = 1 + 2|U_{\tau i}|^2 \delta_\tau$$

$$-\delta_\tau = I_\tau \simeq \frac{1}{\cos^2 \beta} (m_\tau / 4\pi v)^2 \ln(M_R / m_i)$$

The sign of the quantity δ_τ in MSSM depends on the neutrino mixing matrix parameters and the approximation on δ_τ taken here is valid only if t_0 is associated with the top quark mass. From Eqs. (18) and (19), the low energy solar neutrino mass scale is then obtained as

$$\Delta m_{21}^2(t_0) = m_2^2 - m_1^2 \approx 4\delta_\tau m^2 (\cos 2\theta_{12} (s_{23}^2 - s_{13}^2 c_{23}^2) + s_{13} \sin 2\theta_{12} \sin 2\theta_{23}) + O(\delta_\tau^2) \tag{21}$$

3.1. Evolution equations for mixing angles

The corresponding evolution equations for the PMNS matrix elements U_{fi} are given by [20]

$$\frac{dU_{fi}}{dt} = -\frac{1}{16\pi^2} \sum_{k \neq i} \frac{m_k + m_i}{m_k - m_i} U_{fk} (U^T H_e^2 U)_{ki}, \tag{22}$$

where $f = e, \mu, \tau$ and $i, k = 1, 2, 3$ respectively. Here H_e is the Yukawa coupling matrices of the charged leptons in the diagonal basis and

$$(U^T H_e^2 U)_{ki} = h_\tau^2 (U_{k\tau}^T U_{\tau i}) + h_\mu^2 (U_{k\mu}^T U_{\mu i}) + h_e^2 (U_{ke}^T U_{ei})$$

Neglecting h_μ^2 and h_e^2 as before and denoting $A_{ki} = \frac{m_k + m_i}{m_k - m_i}$, Eq. (22) simplifies to [20]

$$\frac{ds_{12}}{dt} = \frac{1}{16\pi^2} h_\tau^2 c_{12} [c_{23}s_{13}s_{12} U_{\tau 1} A_{31} - c_{23}s_{13}c_{12} U_{\tau 2} A_{32} + U_{\tau 1} U_{\tau 2} A_{21}], \tag{23}$$

$$\frac{ds_{13}}{dt} = \frac{1}{16\pi^2} h_\tau^2 c_{23} c_{13}^2 [c_{12} U_{\tau 1} A_{31} + s_{12} U_{\tau 2} A_{32}], \tag{24}$$

$$\frac{ds_{23}}{dt} = \frac{1}{16\pi^2} h_\tau^2 c_{23}^2 [-s_{12} U_{\tau 1} A_{31} + c_{12} U_{\tau 2} A_{32}]. \tag{25}$$

These equations are valid for a generic MSSM with the minimal field content and are independent of the flavor symmetry structure at high energy scale.

4. Numerical analysis and results

For the analysis of the RGE's, Eqs. (14), (23)–(25) for neutrino masses and mixing angles, here we follow two consecutive steps (i) bottom-up running [21] in the first place, and then (ii) top-down running [18] in the next. In the first step (i), the running of the RGE's for the third family Yukawa couplings (h_t, h_b, h_τ) and three gauge couplings (g_1, g_2, g_3) in MSSM, are carried out from top-quark mass scale ($t_0 = \ln m_t$) at low energy end to high energy scale M_R [21,22]. In the present analysis we consider the high scale value as the unification scale $M_R = 1.6 \times 10^{16}$ GeV, with different $\tan\beta$ input values to check the stability of the model at low energy scale. For simplicity of the calculation, the SUSY breaking scale is taken at the top-quark mass scale $t_0 = \ln m_t$ [18,21]. We adopt the standard procedure to get the values of gauge couplings at top-quark mass scale from the experimental CERN-LEP measurements at M_Z , using one-loop RGE's, assuming the existence of a one-light Higgs doublet and five quark flavors below m_t scale [21,22]. Using CERN-LEP data, $M_Z = 91.187$ GeV, $\alpha_s(M_Z) = 0.118 \pm 0.004$, $\alpha_1^{-1}(M_Z) = 127.9 \pm 0.1$, $\sin^2\theta_W(M_Z) = 0.2316 \pm 0.0003$, and SM relations,

$$\frac{1}{\alpha_1(M_Z)} = \frac{3}{5} \frac{(1 - \sin^2\theta_W(M_Z))}{\alpha(M_Z)}, \quad \frac{1}{\alpha_2(M_Z)} = \frac{\sin^2\theta_W(M_Z)}{\alpha(M_Z)}, \quad g_i^2 = 4\pi\alpha_i, \quad (26)$$

we calculate the gauge couplings at M_Z scale, $\alpha_1(M_Z) = 0.0169586$, $\alpha_2(M_Z) = 0.0337591$, $\alpha_3(M_Z) = 0.118$. As already mentioned, we consider the existence of one light Higgs doublet ($n_H = 1$) and five quark flavors ($n_F = 5$) in the scale $M_Z - m_t$. Using one-loop RGE's of gauge couplings, we get $g_1(m_t) = 0.463751$, $g_2(m_t) = 0.6513289$ and $g_3(m_t) = 1.1891996$. Similarly, the Yukawa couplings are also evaluated at top-quark mass scale for input values of $m_t(m_t) = 174$ GeV, $m_b(m_b) = 4.25$ GeV, $m_\tau(m_\tau) = 1.785$ GeV and the QED-QCD rescaling factors $\eta_b = 1.55$, $\eta_\tau = 1.015$ in the standard fashion [22],

$$\begin{aligned} h_t(m_t) &= \frac{m_t(m_t)\sqrt{1 + \tan^2\beta}}{174 \tan\beta}, \\ h_b(m_t) &= \frac{m_b(m_t)\sqrt{1 + \tan^2\beta}}{174}, \\ h_\tau(m_t) &= \frac{m_\tau(m_t)\sqrt{1 + \tan^2\beta}}{174}, \end{aligned} \quad (27)$$

where $m_b(m_t) = \frac{m_b(m_b)}{\eta_b}$, $m_\tau(m_t) = \frac{m_\tau(m_\tau)}{\eta_\tau}$. The one-loop RGE's for top quark, bottom quark and τ -lepton Yukawa couplings in the MSSM in the range of mass scales $m_t \leq \mu \leq M_R$ are given by

$$\frac{d}{dt}h_t = \frac{h_t}{16\pi^2} \left(6h_t^2 + h_b^2 - \sum_{i=1}^3 c_i g_i^2 \right), \quad (28)$$

$$\frac{d}{dt}h_b = \frac{h_b}{16\pi^2} \left(6h_b^2 + h_\tau^2 - \sum_{i=1}^3 c'_i g_i^2 \right), \quad (29)$$

$$\frac{d}{dt}h_\tau = \frac{h_\tau}{16\pi^2} \left(4h_\tau^2 + 3h_b^2 - \sum_{i=1}^3 c''_i g_i^2 \right), \quad (30)$$

where

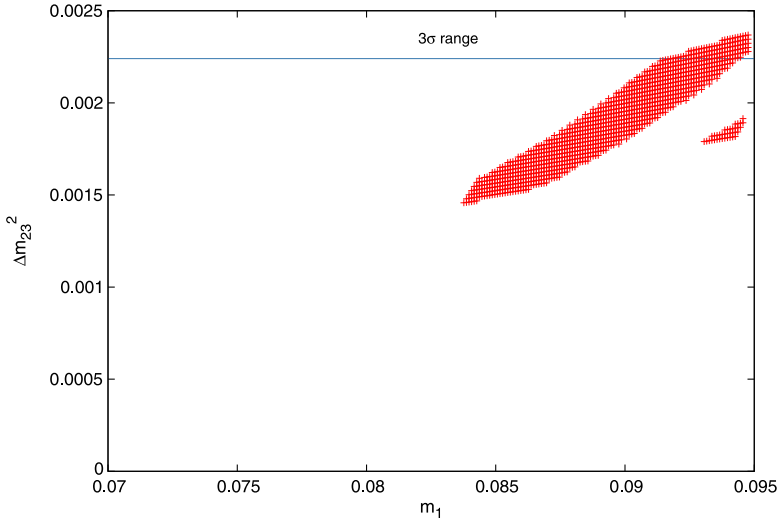


Fig. 1. Scatter plot showing Δm_{23}^2 at low energy versus initial value of m_1 at high energy for inverted hierarchy while keeping all other neutrino parameters at low energy within 3σ range.

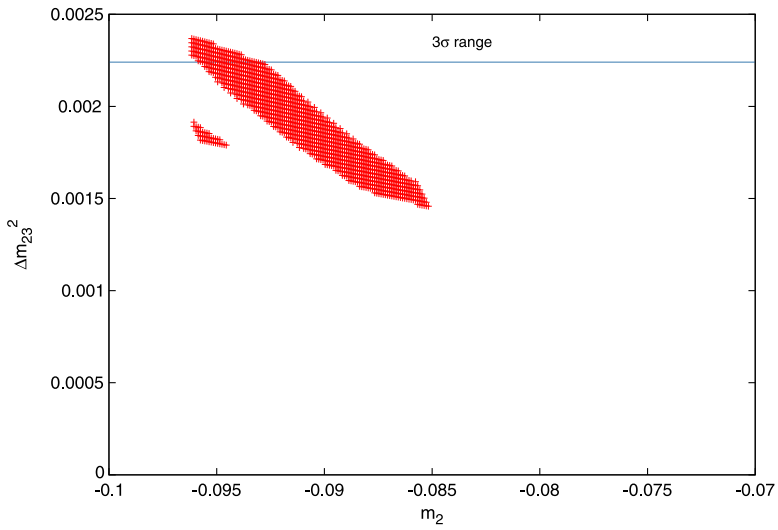


Fig. 2. Scatter plot showing Δm_{23}^2 at low energy versus initial value of m_2 at high energy for inverted hierarchy while keeping all other neutrino parameters at low energy within 3σ range.

$$c_i = \begin{pmatrix} \frac{13}{15} \\ 3 \\ \frac{16}{3} \end{pmatrix}, \quad c'_i = \begin{pmatrix} \frac{7}{15} \\ 3 \\ \frac{16}{3} \end{pmatrix}, \quad c''_i = \begin{pmatrix} \frac{9}{5} \\ 3 \\ 0 \end{pmatrix}. \quad (31)$$

The two-loop RGE's for the gauge couplings are similarly expressed in the range of mass scales $m_t \leq \mu \leq M_R$ as

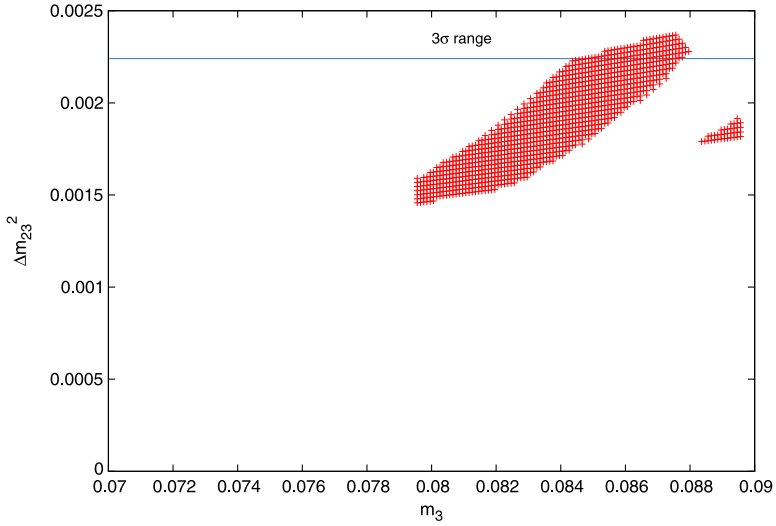


Fig. 3. Scatter plot showing Δm_{23}^2 at low energy versus initial value of m_3 at high energy for inverted hierarchy while keeping all other neutrino parameters at low energy within 3σ range.

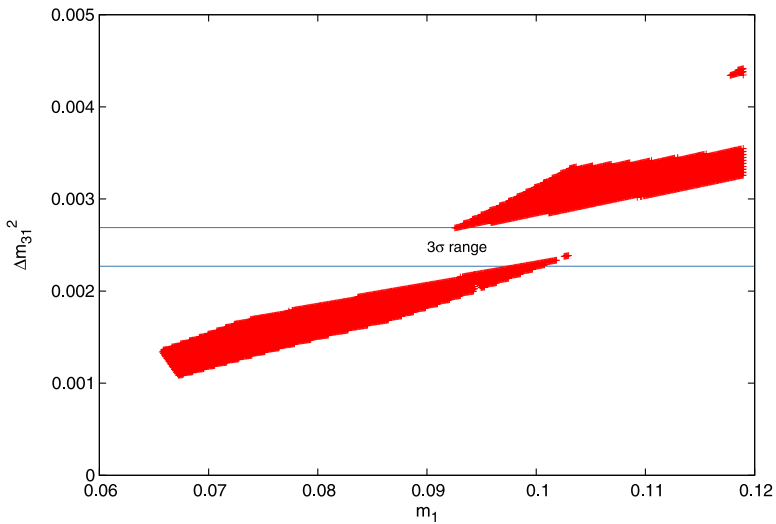


Fig. 4. Scatter plot showing Δm_{31}^2 at low energy versus initial value of m_1 at high energy for normal hierarchy while keeping all other neutrino parameters at low energy within 3σ range.

$$\frac{d}{dt}g_i = \frac{g_i}{16\pi^2} \left[b_i g_i^2 + \frac{1}{16\pi^2} \left(\sum_{j=1}^3 b_{ij} g_i^2 g_j^2 - \sum_{j=i,b,\tau}^3 a_{ij} g_i^2 h_j^2 \right) \right], \quad (32)$$

where

$$b_i = \begin{pmatrix} 6.6 \\ 1 \\ -3 \end{pmatrix}, \quad b_{ij} = \begin{pmatrix} 7.9 & 5.4 & 17 \\ 1.8 & 25 & 24 \\ 2.2 & 9 & 14 \end{pmatrix}, \quad a_{ij} = \begin{pmatrix} 5.2 & 2.8 & 3.6 \\ 6 & 6 & 2 \\ 4 & 4 & 0 \end{pmatrix}. \quad (33)$$

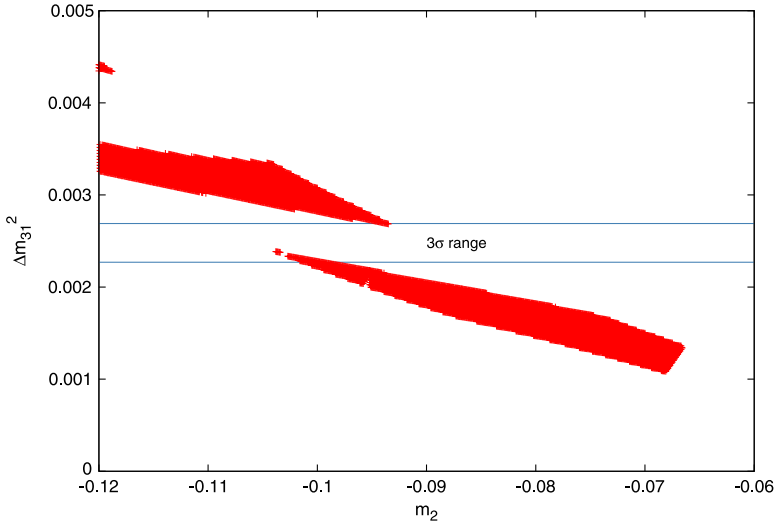


Fig. 5. Scatter plot showing Δm_{31}^2 at low energy versus initial value of m_2 at high energy for normal hierarchy while keeping all other neutrino parameters at low energy within 3σ range.

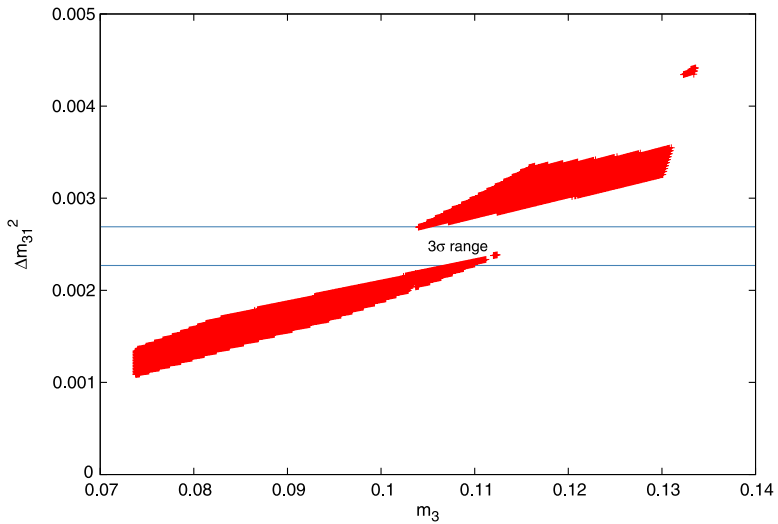


Fig. 6. Scatter plot showing Δm_{31}^2 at low energy versus initial value of m_3 at high energy for normal hierarchy while keeping all other neutrino parameters at low energy within 3σ range.

Values of $h_t, h_b, h_\tau, g_1, g_2, g_3$ evaluated at high scale $M_R = 1.6 \times 10^{16}$ from Eqs. (28)–(30) and (32) are

$$\begin{aligned}
 h_t(M_R) &= 0.142685458, & h_b(M_R) &= 0.378832042, \\
 h_\tau(M_R) &= 0.380135357, & g_1(M_R) &= 0.381783873, \\
 g_2(M_R) &= 0.377376229, & g_3(M_R) &= 0.374307543.
 \end{aligned}$$

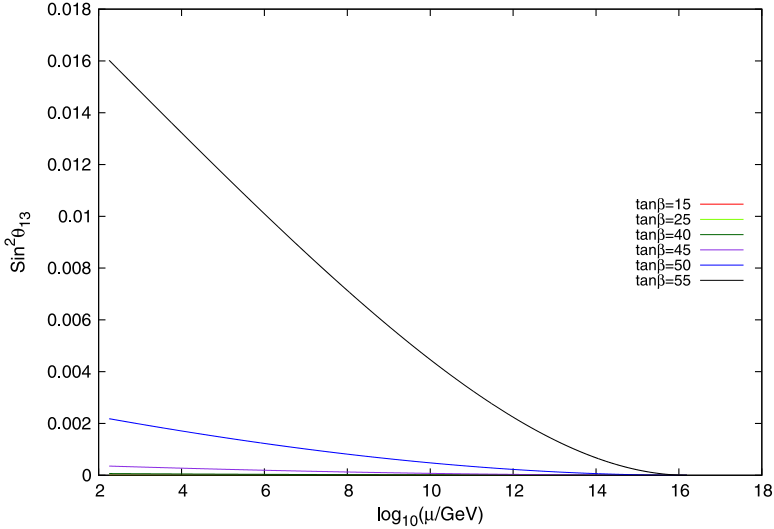


Fig. 7. Radiative generation of $\sin^2 \theta_{13}$ for $\tan \beta = 15, 25, 40, 45, 50, 55$ for inverted hierarchy using input values given in Table 2.

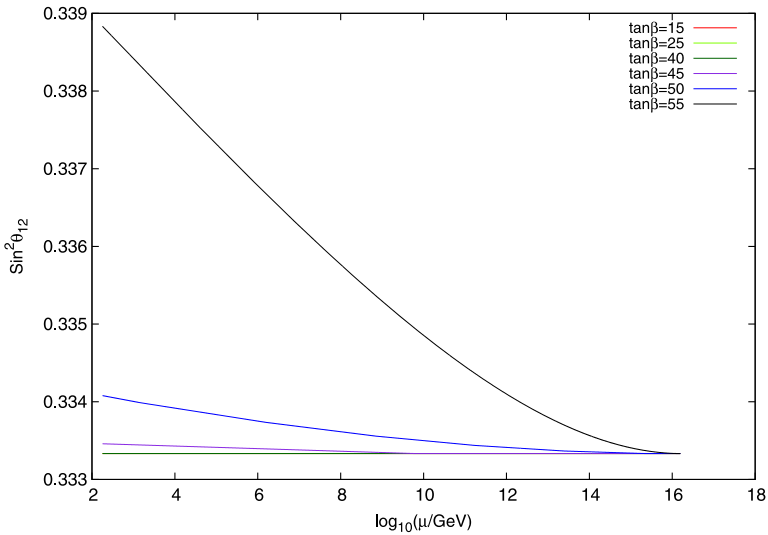


Fig. 8. Evolution of $\sin^2 \theta_{12}$ for $\tan \beta = 15, 25, 40, 45, 50, 55$ for inverted hierarchy using input values given in Table 2.

In the second step (ii), the running of three neutrino masses (m_1, m_2, m_3) and mixing angles (s_{12}, s_{23}, s_{13}) are carried out together with the running of Yukawa and gauge couplings, from high scale $t_R (= \ln M_R)$ to low scale t_o . In this case, we use the input values of Yukawa and gauge couplings evaluated earlier at scale t_R from the first stage running of RGE's in case (i). In principle, one can evaluate neutrino masses and mixing angles at every point of the energy scale. It can be noted that in the present problem, the running of other SUSY parameters such as $M_0, M_{1/2}, \mu$, are not required and hence, it is not necessary to supply their input values.

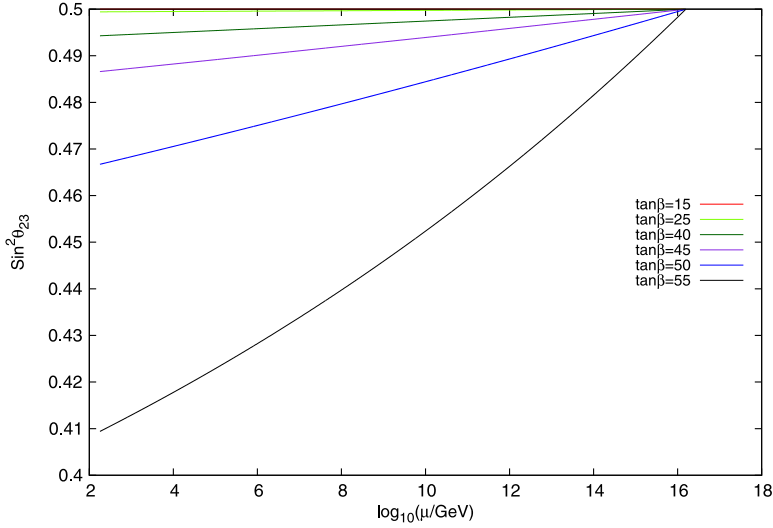


Fig. 9. Evolution of $\sin^2 \theta_{23}$ for $\tan \beta = 15, 25, 40, 45, 50, 55$ for inverted hierarchy using input values given in Table 2.

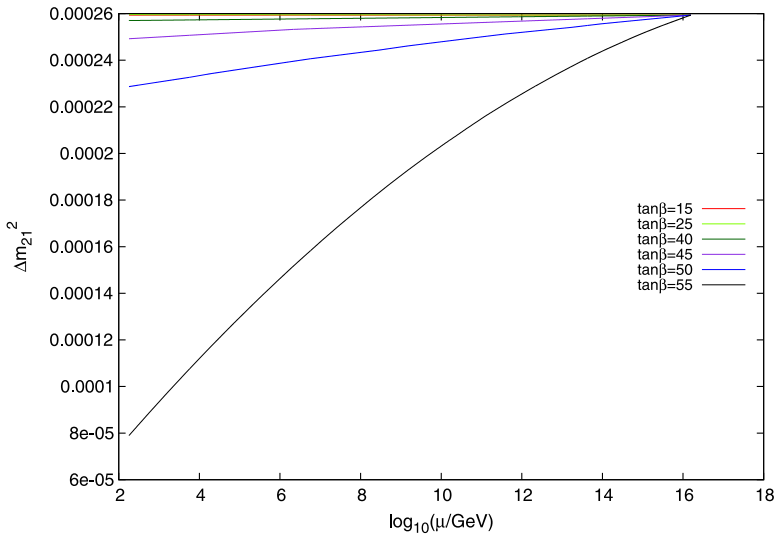


Fig. 10. Evolution of Δm_{21}^2 for $\tan \beta = 15, 25, 40, 45, 50, 55$ for inverted hierarchy using input values given in Table 2.

We are now interested in studying radiative generation θ_{13} for the case when $m_{1,2,3} \neq 0$ and $s_{13} = 0$ at high energy scale. Such studies can give the possible origin of the reactor angle in a broken A_4 model. During the running of mass eigenvalues and mixing angles from high to low scale, the non-zero input value of mass eigenvalues $m_{1,2,3}$ will induce radiatively a non-zero values of s_{13} . Similar approach was followed in [23] considering $m_3 = 0$. The authors in [23] used inverted hierarchy neutrino mass pattern $(m, -m, 0)$ at high scale. Such a specific structure of mass eigenvalues however, require fine tuning conditions in the flavor symmetry model at high energy. Instead of assuming a specific relation between mass eigenvalues at high energy scale,

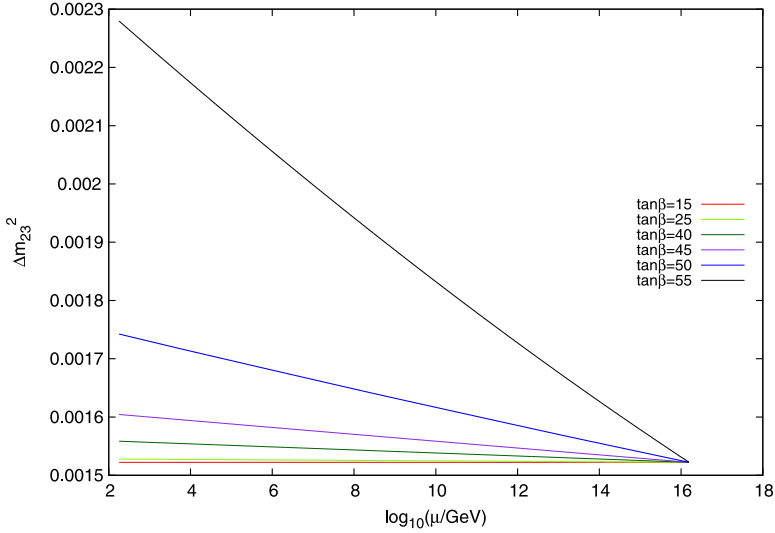


Fig. 11. Evolution of Δm_{23}^2 for $\tan \beta = 15, 25, 40, 45, 50, 55$ for inverted hierarchy using input values given in Table 2.

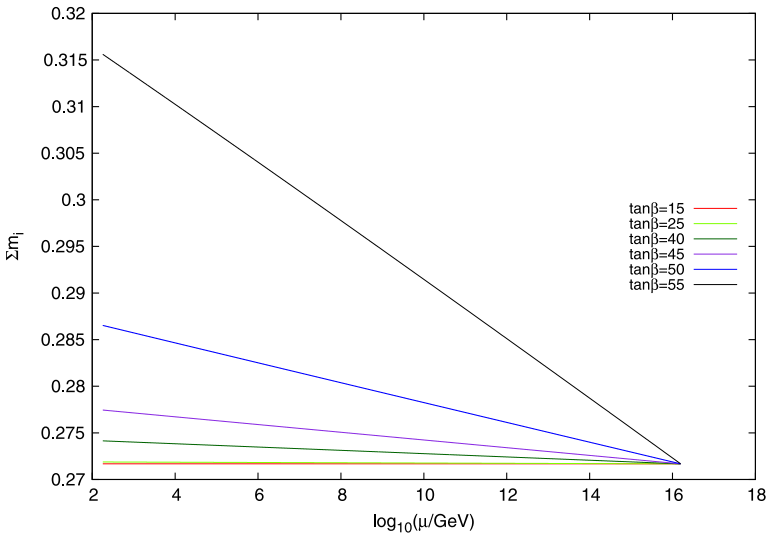


Fig. 12. Evolution of $\sum_i |m_i|$ for $\tan \beta = 15, 25, 40, 45, 50, 55$ for inverted hierarchy using input values given in Table 2.

here we attempt to find out the most general mass eigenvalues at high energy which can give rise to the correct neutrino data at low energy scale. The only assumption in our work is the opposite CP phases i.e. $(m_1, -m_2, m_3)$. In another work [24], authors have shown the radiative generation of Δm_{21}^2 considering the non-zero θ_{13} at high scale and $\tan \beta$ values lower than 50. They have also shown that Δm_{21}^2 can run from zero at high energy to the observed value at the low energy scale, only if θ_{13} is relatively large and the Dirac CP-violating phase is close to π . The running effects can be observed only when θ_{13} is non-zero at high-energy scale as per their analysis. In the present work, θ_{13} is assumed to be zero at high scale consistent with a TBM type mixing

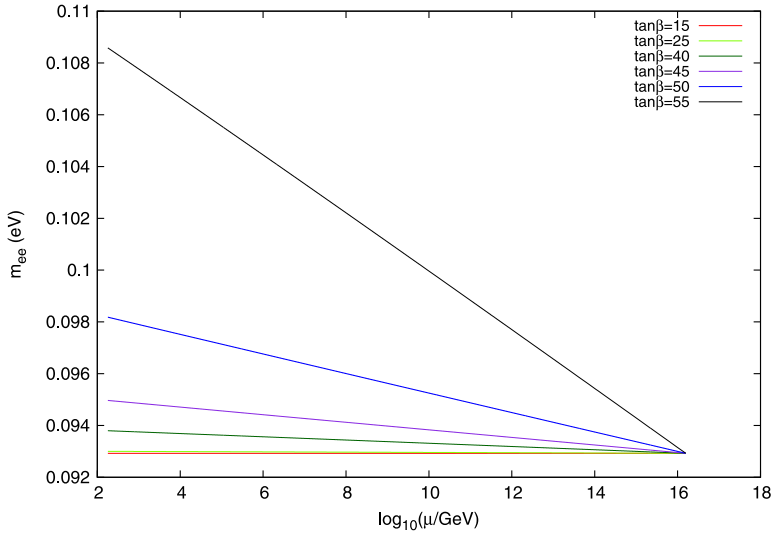


Fig. 13. Evolution of m_{ee} for $\tan \beta = 15, 25, 40, 45, 50, 55$ for inverted hierarchy using input values given in Table 2.

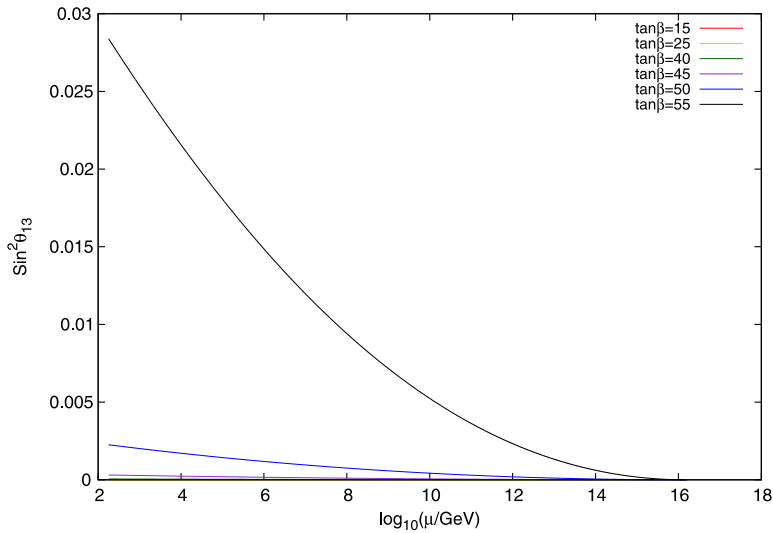


Fig. 14. Radiative generation of $\sin^2 \theta_{13}$ for $\tan \beta = 15, 25, 40, 45, 50, 55$ for normal hierarchy using input values given in Table 3.

within A_4 symmetric model. We also examine the running behavior of neutrino parameters in a neutrino mass model obeying special kind of μ - τ symmetry at high scale, which was not studied in the earlier work mentioned above.

For a complete numerical analysis, first we parameterize the neutrino mass matrix to have a TBM type structure with eigenvalues in the form $(m_1, -m_2, m_3)$. Since the mixing angles at high energy scale are fixed (TBM type), we only need to provide three input values namely, m_1, m_2, m_3 . Using these values at the high energy scale, neutrino parameters are computed at low energy scale by simultaneously solving the RGE's discussed above. We first allow moderate

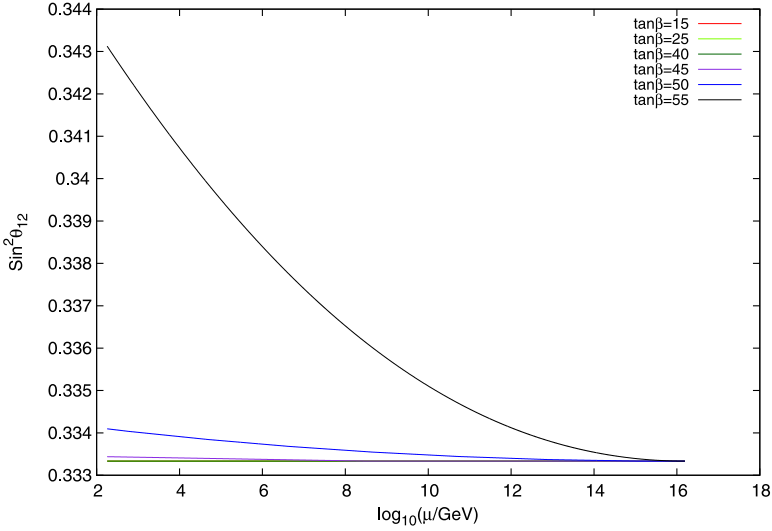


Fig. 15. Evolution of $\sin^2 \theta_{12}$ for $\tan \beta = 15, 25, 40, 45, 50, 55$ for normal hierarchy using input values given in Table 3.

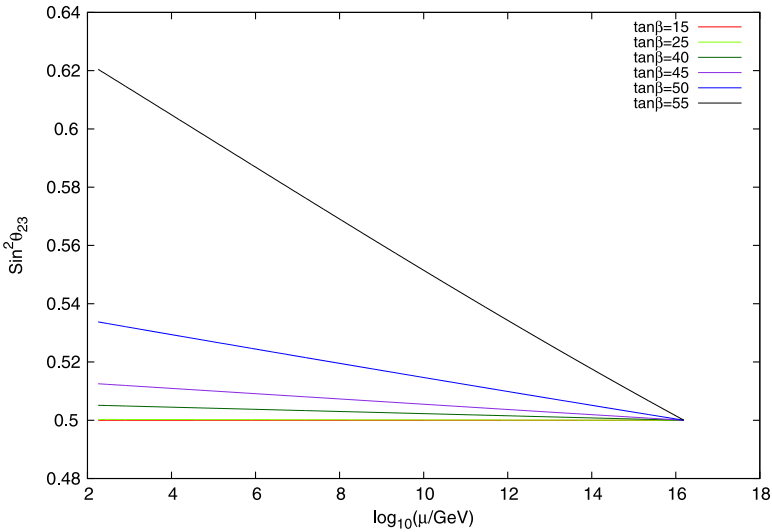


Fig. 16. Evolution of $\sin^2 \theta_{23}$ for $\tan \beta = 15, 25, 40, 45, 50, 55$ for normal hierarchy using input values given in Table 3.

as well as large hierarchies between the lightest and the heaviest mass eigenvalues (with the lighter being at least two orders of magnitudes smaller) of both normal and inverted type and find that the output values of θ_{13} do not lie in the experimentally allowed range for all values of $\tan \beta = 15, 25, 40, 45, 50, 55$ used in our analysis.

We then consider very mild hierarchical pattern of mass eigenvalues keeping them in the same order of magnitude range. We vary the neutrino mass eigenvalues at high energy scale in the range 0.01–0.12 eV and generate the neutrino parameters at low energy. We restrict the neutrino parameters $\theta_{13}, \theta_{12}, \theta_{23}$ and Δm_{21}^2 at low energy to be within the allowed 3σ range

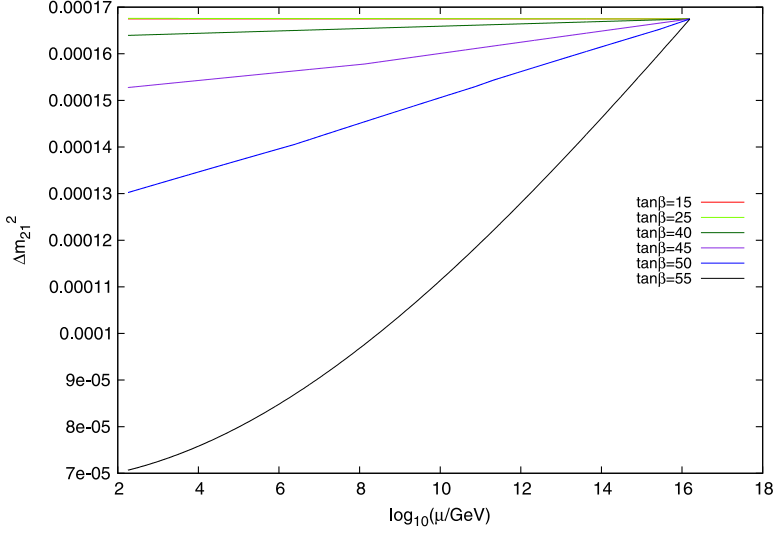


Fig. 17. Evolution of Δm_{21}^2 for $\tan \beta = 15, 25, 40, 45, 50, 55$ for normal hierarchy using input values given in Table 3.

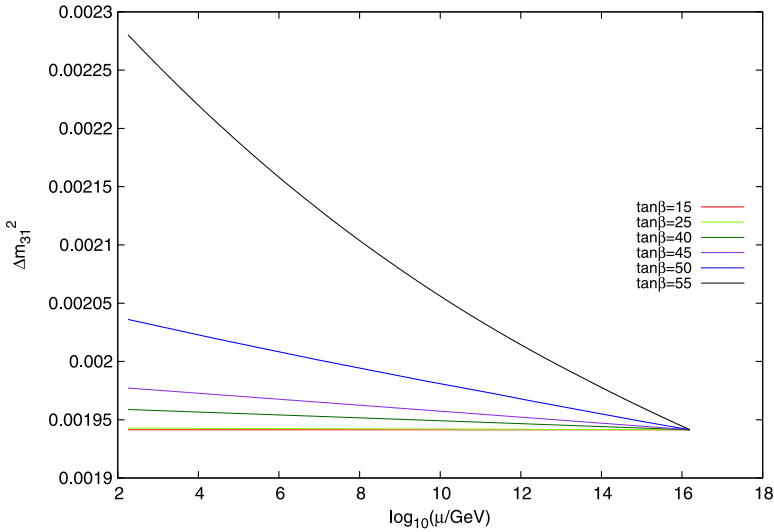


Fig. 18. Evolution of Δm_{31}^2 for $\tan \beta = 15, 25, 40, 45, 50, 55$ for normal hierarchy using input values given in Table 3.

and show the variation of Δm_{23}^2 (IH), Δm_{31}^2 (NH) at low energy with respect to the input mass eigenvalues at high energy. We show the results in Figs. 1–6 for a specific value of $\tan \beta = 55$. It can be seen from these figures that the correct value of neutrino parameters at low energy can be obtained only for large values of mass eigenvalues at high energy scale $|m_{1,2,3}| = 0.08\text{--}0.12$ eV. We then choose two specific sets of mass eigenvalues at high energy scale corresponding to inverted hierarchy and normal hierarchy respectively and show the evolution of several neutrino observables including oscillation parameters, effective neutrino mass $m_{ee} = |\sum_i U_{ei}^2 m_i|$, sum of absolute neutrino masses $\sum_i |m_i|$ in Figs. 7–20. It can be seen from Figs. 7 and 14 that the

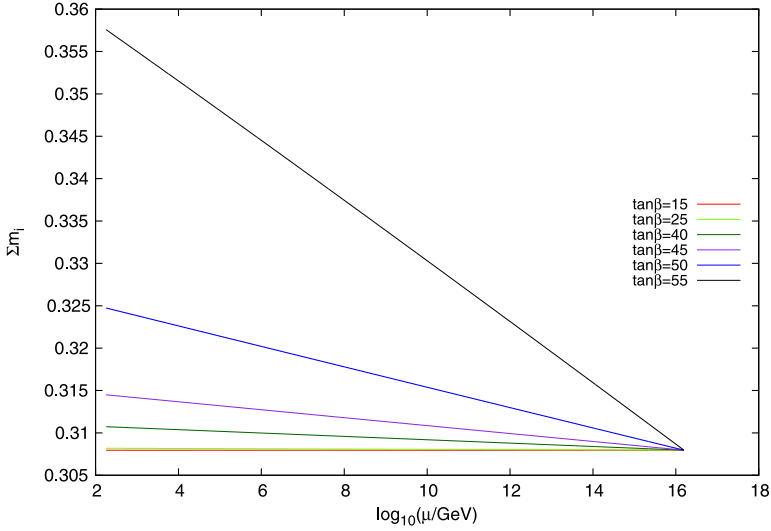


Fig. 19. Evolution of $\sum_i |m_i|$ for $\tan\beta = 15, 25, 40, 45, 50, 55$ for normal hierarchy using input values given in Table 3.

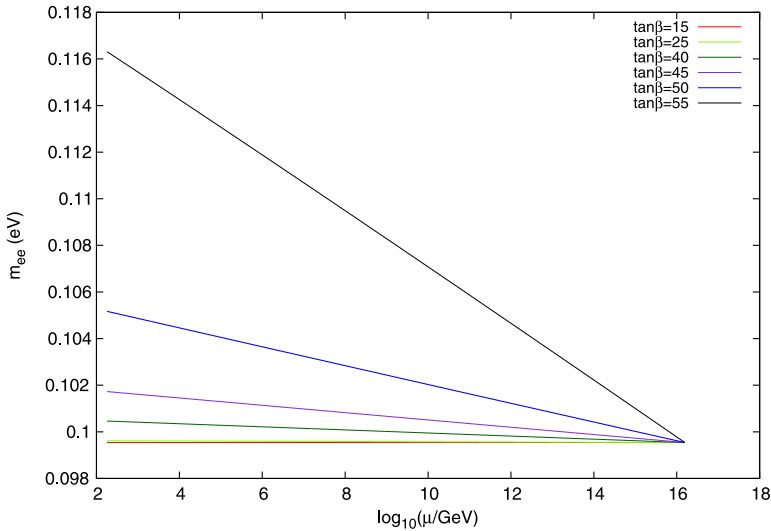


Fig. 20. Evolution of m_{ee} for $\tan\beta = 15, 25, 40, 45, 50, 55$ for normal hierarchy using input values given in Table 3.

correct value of θ_{13} can be obtained at low energy only for very high values of $\tan\beta = 55$. The other neutrino parameters also show a preference for higher $\tan\beta$ values. The output values of neutrino parameters at low energy are given in Tables 2 and 3 for both sets of input parameters. The large deviation of θ_{13} at low energy from its value at high energy ($\theta_{13} = 0$ for TBM at high energy) whereas smaller deviation of other two mixing angles can be understood from the RGE equations for mixing angles (23), (24), (25). Using the input values given in Tables 2 and 3, the slope of $\sin\theta_{13}$ can be calculated to be $\frac{h_\tau^2}{16\pi^2}(-5.88)$ and $\frac{h_\tau^2}{16\pi^2}(5.23)$ for inverted and normal hierarchies respectively. On the other hand, the slope of $\sin\theta_{23}$ at high energy scale is found

Table 2

Input and output values with different $\tan\beta$ values for Inverted Hierarchy.

| Input values | | Output values for different $\tan\beta$ | | | | | |
|-------------------|------------|---|------------------|------------------|------------------|------------------|------------------|
| | | $\tan\beta = 15$ | $\tan\beta = 25$ | $\tan\beta = 40$ | $\tan\beta = 45$ | $\tan\beta = 50$ | $\tan\beta = 55$ |
| m_1 (eV) | 0.0924619 | 0.0924619 | 0.0925375 | 0.0933433 | 0.0945343 | 0.0978126 | 0.1086331 |
| m_2 (eV) | -0.0938539 | -0.0938539 | -0.0939295 | -0.0947101 | -0.0958434 | -0.0989746 | -0.1089959 |
| m_3 (eV) | 0.0853599 | 0.0853599 | 0.0854102 | 0.0860902 | 0.0870723 | 0.0897417 | 0.0979824 |
| $\sin\theta_{23}$ | 0.707107 | 0.7070999 | 0.7066970 | 0.7030523 | 0.6975724 | 0.6831660 | 0.6398494 |
| $\sin\theta_{13}$ | 0.00 | 0.0000655 | 0.0006287 | 0.0081088 | 0.0188213 | 0.0467352 | 0.1265871 |
| $\sin\theta_{12}$ | 0.57735 | 0.57735 | 0.57735 | 0.57735 | 0.5774592 | 0.5779958 | 0.5820936 |

Table 3

Input and output values with different $\tan\beta$ values for Normal Hierarchy.

| Input values | | Output values for different $\tan\beta$ | | | | | |
|-------------------|------------|---|------------------|------------------|------------------|------------------|------------------|
| | | $\tan\beta = 15$ | $\tan\beta = 25$ | $\tan\beta = 40$ | $\tan\beta = 45$ | $\tan\beta = 50$ | $\tan\beta = 55$ |
| m_1 (eV) | 0.0992596 | 0.0992596 | 0.0993352 | 0.1001914 | 0.1014757 | 0.1049422 | 0.1159424 |
| m_2 (eV) | -0.1000997 | -0.1000996 | -0.1001752 | -0.1010062 | -0.1022256 | -0.1055608 | -0.1162467 |
| m_3 (eV) | 0.1085996 | 0.1085996 | 0.1086751 | 0.1095313 | 0.1107905 | 0.1142319 | 0.1253917 |
| $\sin\theta_{23}$ | 0.707107 | 0.7070999 | 0.7073014 | 0.7107263 | 0.7159094 | 0.7305902 | 0.7876961 |
| $\sin\theta_{13}$ | 0.00 | 0.0000582 | 0.0005604 | 0.0073647 | 0.0176199 | 0.0474922 | 0.1684841 |
| $\sin\theta_{12}$ | 0.57735 | 0.57735 | 0.57735 | 0.57735 | 0.5774410 | 0.5780104 | 0.5857702 |

to be $\frac{h_\tau^2}{16\pi^2}(2.95)$ and $\frac{h_\tau^2}{16\pi^2}(-2.63)$ for inverted and normal hierarchies respectively. Thus, the lower value of slope for $\sin\theta_{23}$ results in smaller deviation from TBM values compared to that of $\sin\theta_{13}$. We also note from Figs. 12, 19 that the sum of the absolute neutrino masses at low energy is 0.315 eV and 0.3555 eV for inverted and normal hierarchy respectively. This lies outside the limit set by the Planck experiment $\sum |m_i| < 0.23$ eV [12]. However, there still remains a little room for the sum of absolute mass to lie beyond this limit depending on the cosmological model, as suggested by several recent studies [25]. Ongoing as well as future cosmology experiments should be able to rule out or confirm such a scenario.

It is interesting to note that, our analysis shows a preference for very mild hierarchy of either inverted or normal type at high energy scale which also produces a very mild hierarchy at low energy. This can have interesting consequences in the ongoing neutrino oscillation as well as neutrino-less double beta decay experiments. Also, the large $\tan\beta$ region of MSSM (which gives better results in our model) will undergo serious scrutiny at the collider experiments making our model falsifiable both from neutrino as well as collider experiments. We note that the present analysis will be more accurate if the two loop contributions [26] RGE's are taken into account.

5. Conclusion

We have studied the effect of RGE's on neutrino masses and mixing in MSSM with μ - τ symmetric neutrino mass model giving TBM type mixing at high energy scale. We incorporate an additional flavor symmetry A_4 at high scale to achieve the desired structure of the neutrino mass matrix. The RGE equations for different neutrino parameters are numerically solved simultaneously for different values of $\tan\beta$ ranging from 15 to 55. We take the three neutrino mass eigenvalues at high energy scale as free parameters and determine the parameter space that

can give rise to correct values of neutrino oscillation parameters at low energy. We make the following observations

- Moderate or large hierarchy (both normal and inverted) of neutrino masses at high energy scale does not give rise to correct output at low energy scale.
- Very mild hierarchy (with all neutrino mass eigenvalues having same order of magnitude values and $|m_{1,2,3}| = 0.08\text{--}0.12$ eV) give correct results at low energy provided the $\tan\beta$ values are kept high, close to 55. Such a preference towards large mass eigenvalues with all eigenvalues having same order of magnitude values can have tantalizing signatures at oscillation as well as neutrino-less double beta decay experiments.
- No significant changes in running of $\sin^2\theta_{23}$, $\sin^2\theta_{12}$ with $\tan\beta$ are observed.
- Sum of absolute neutrino masses at low energy lie above the Planck upper bound $\sum |m_i| < 0.23$ eV [12] hinting towards non-standard cosmology to accommodate a larger $\sum |m_i|$ or more relativistic degrees of freedom [25].
- The preference for high $\tan\beta$ regions of MSSM could go through serious tests at collider experiments pushing the model towards verification or falsification.

Although we have arrived at some allowed parameter space in our model giving rise to correct phenomenology at low energy with the additional possibility that many or all of these parameter space might get ruled out in near future, we also note that it would have been more interesting if the running of the Dirac and Majorana CP violating phases [27] were taken into account. We also have not included the seesaw threshold effects and considered all the right handed neutrinos to decouple at the same high energy scale. Such threshold effects could be important for large values of $\tan\beta$ as discussed in [28]. We leave such a detailed study for future investigations.

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