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Letter

# On the cosmological constant, the KK mass scale, and the cut-off dependence in the dark dimension scenario

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**Abstract** In this short note we comment on the relation between the cosmological and the Kaluza–Klein mass scale in the dark dimension scenario [1], also in view of some recent claims [2] that would raise some doubts about the validity of this scenario. Here we argue that these claims have serious flaws and cannot be trusted.

The Swampland program aims to distinguish effective field theories (EFT) that can be completed to quantum gravity in the ultraviolet from those which cannot [3]. In this way it hints towards a new and deep interplay between physics in the UV and in the IR. One of the swampland conjectures is the anti-de Sitter distance conjecture [4], which relates the cosmological constant $\Lambda_{cc}$ to the mass scale $m$ of a tower of states:

$$m \sim \left|\frac{\Lambda_{cc}}{M_p^4}\right|^{\alpha} M_p , \tag{1}$$

where $\alpha$ is a positive order-one number. This distance conjecture was concretely applied [1] to the case of a positive cosmological constant leading to the dark dimension (DD) scenario with one extra dimension of micron size and a related Kaluza–Klein (KK) mass scale $m \equiv m_{KK} \sim \mathcal{O}(\text{meV})$, up to a numerical factor. As discussed for example in [5–8], the DD scenario has many interesting features for particle physics and cosmology. However, in a recent work [2] it is claimed that the basic prediction of the DD scenario, namely the relation between the cosmological constant and the tower mass scale is invalid. We will discuss that in our opinion the conclusion of [2] has serious flaws and cannot be trusted.

Let us first recall that the allowed range of the parameter $\alpha$ can be largely restricted. Since the KK tower contains massive spin-two bosons, the Higuchi bound [9] provides an absolute upper limit to $\alpha$, namely $\alpha \leq 1/2$. This bound must be respected in any unitary EFT.

A lower bound for $\alpha$ follows from explicit calculations of the vacuum energy in string theory (on which we will comment further below), and for a $d$-dimensional EFT it is given as $\alpha \geq 1/d$. So in total we have for $d = 4$

$$1/4 \leq \alpha \leq 1/2 . \tag{2}$$

However, note that this bound on $\alpha$, in particular $\alpha \leq 1/2$, might be refined in case the relation (1) contains a large constant on its right hand side.

As explained in [1], additional experimental arguments, like constraints from 5th-force experiments, then lead to the conclusion that there is one extra dimension of radius $R$ in the micron range, and that the lower bound for $\alpha = 1/4$ is basically saturated, i.e. there is the following parametric relation between the KK tower mass scale and the cosmological constant (up to another proportionality parameter $\lambda$):

$$\Lambda_{cc} \simeq m_{KK}^4 . \tag{3}$$

Note that for any value of $\alpha$ larger than 1/4, the measured value of $\Lambda_{cc}$ would imply a too small value for $m_{KK}$, incompatible with already existing bounds on the size of the fifth dimension; conversely starting with $m_{KK} \sim \mathcal{O}(\text{meV})$ and increased values of $\alpha > 1/4$ leads to too large contributions in the vacuum energy.

Now, it is claimed in [2] based on a specific EFT loop calculation that the dominating contribution to the vacuum

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energy corresponds in the nutshell to $\alpha = 1/2$ or to $\alpha = 3/2$, leading to a too large contribution to $\Lambda_{cc}$. Note that whereas $\alpha = 1/2$ just saturates the Higuchi bound, $\alpha = 3/2$ parametrically violates it and therefore is potentially inconsistent with unitarity. However, we will show that with a certain choice of the UV cut-off the Higuchi bound can be rescued. In other words, it will depend on the choice of the UV cut-off whether the Higuchi bound is violated or not.

Second, as already indicated, their results contain some particular UV cut-off dependence, where it all depends on how one regularizes the infinite integral. This makes the EFT result a priori ill defined and therefore the EFT calculation is potentially highly misleading. In particular, it also does not take into account quantum gravity effects, like UV-IR mixing, which we will mention later. In other words, without fixing the value of the cut-off in their formulae, it is meaningless to extract any relation between $\Lambda_{cc}$ and $m_{KK}$. On the other hand, in string theory or in quantum gravity, the cut-off is removed and the string computations are well defined and have been confirmed (see e.g. [10–13]) several times leading to a finite result behaving as in Eq. (3).

In field theory, a particular example is QFT at finite temperature leading to the same behaviour $T^4$ where $T = 1/R$. No string regularisation is used for that. Actually, the finite temperature example is very similar to the Casimir energy computation which can be computed with completely convergent diagrams yielding the result (3) (see for instance [14] for any $d$). Note that in both cases, the result should vanish at the decompactification limit or at $T = 0$, which is automatic if the higher-dimensional theory becomes supersymmetric; it is also a condition imposed by the swampland distance conjecture [15]. It is worth pointing out that in the string calculations modular invariance of the one-loop amplitudes plays a decisive role, which dictates a particular regularisation for the associated EFT computations. It amounts to perform first a Poisson resummation and then subtract the infinite zero mode contribution which is radius/temperature independent.

Let us finally have a more close look into the result of [2]. Their EFT calculation leads to the following two relations (see eqs. (26) and (27) in [2]), and, depending how supersymmetry is broken, the EFT relations are (up to a log correction in the second case):

$$\Lambda_{cc} \sim m_{KK}^2 R \Lambda^3 \quad \text{or} \quad \Lambda_{cc} \sim m_{KK}^{2/3} R^{5/3} \Lambda^5 . \tag{4}$$

The first case would correspond to $\alpha = 1/2$ and the second case to the ”forbidden” value $\alpha = 3/2$. However, these formulas still contain the radius $R$ of the 5th dimension and the UV cut-off $\Lambda$. The Higuchi bound will now put a bound on $R$ and $\Lambda$. Specifically, the Higuchi bound is satisfied in case the following relations hold for the two cases:

$$R\Lambda^3 \le M_p^2 \quad \text{or} \quad R\Lambda^3 \le (\Lambda_{cc} M_p)^{2/5} \simeq 10^{-48} M_p^2 . \tag{5}$$

In the first case $R\Lambda^3$ is not strongly restricted, but for the second case due to the smallness of $\Lambda_{cc} \simeq 10^{-120} M_p^4$ a very low cut-off is required by the Higuchi bound.

Finally, we will show how the relations (4) can be made consistent with the DD scenario by a certain, natural choice for $R$ and $\Lambda$. First these formulas still contain the radius $R$ of the 5th dimension, which should be replaced by $R = 1/m_{KK}$. Hence additional powers of $m_{KK}$ will show up in the result.

Secondly, the UV cut-off $\Lambda$ is not a constant but crucially depends also on the IR mass scale, given in terms of $\Lambda_{cc}$. This is precisely a manifestation of the before mentioned UV-IR mixing in quantum gravity or in string theory, which will eliminate the arbitrariness from the EFT calculation. Therefore, one has to determine what is the correct cut-off. Actually, consistency of black hole physics in the presence of $N$ particle species imposes the largest possible cut-off in any EFT coupled to quantum gravity, the so-called species scale $\Lambda_{sp} = M_p/\sqrt{N_{sp}}$ [16]. This is the scale where gravity becomes strongly coupled and the EFT necessarily breaks down. Since $\Lambda_{sp}^{-1}$ constitutes the smallest size black hole described by the EFT involving only the Einstein term, $\Lambda_{sp}$ codifies the “number of light degrees of freedom” (i.e., the number of KK excitations lighter than the cut-off). Alternatively, $\Lambda_{sp} = M_p/\sqrt{F_1}$ can be identified with the scale at which $R^2$ corrections to the Einstein action become important, with $F_1$ being the one-loop topological string free energy [17,18]. We stress once more that the UV-IR mixing arises because the UV species scale depends on the IR scale (namely the cosmological constant) in the following way: $\Lambda_{sp} = \Lambda_{cc}^{1/12} M_p^{2/3}$.

For the DD scenario, the species scale has the following dependence on $m_{KK}$: $\Lambda = \Lambda_{sp} \simeq m_{KK}^{1/3} M_p^{2/3}$. However, inserting $\Lambda_{sp}$ into Eq. (4) does not reproduce the string calculation. Specifically, one gets that

$$\Lambda_{cc} \sim m_{KK}^2 M_p^2 \quad \text{or} \quad \Lambda_{cc} \sim m_{KK}^{2/3} M_p^{10/3} . \tag{6}$$

The second relation now explicitly violates the Higuchi bound.

An alternative choice for the cut-off is to identify it with the KK scale, i.e. $\Lambda = \Lambda_{KK} \simeq m_{KK}$. This is the scale where the 4-dimensional EFT description breaks down and turns into a 5-dimensional EFT, at least for what the gravity interactions in the 5D bulk concern. This choice of the cut-off is also in-line with the corresponding string calculation, where only the lowest KK mode effectively contributes. Plugging this cut-off into both relations of Eq. (4), one immediately sees that the swampland relation (3) is indeed satisfied for both cases.

In summary, in this note we argue that there is no reason to conclude that the DD scenario is invalid. On the contrary, we pointed out the caveats of the EFT calculation in [2] with respect to the cut-off dependence and showed how, by the choice of a particular cut-off, it is consistent with the corresponding string calculation and swampland prediction. Indeed, the swampland guiding principles point to $\Lambda_{cc} \sim m_{KK}^{1/\alpha}$ while theoretical and experimental bounds single out the power $m_{KK}^{4}$, implying that the fundamental nature of the DD is not just a matter of summing EFT contributions, because only the string calculation can properly account for the massive states above the species scale and the UV-IR mixing as the EFT inevitably breaks down at $\Lambda_{sp}$.[1]

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## References

1. M. Montero, C. Vafa, I. Valenzuela, The dark dimension and the swampland. arXiv:2205.12293 [hep-th]
2. C. Branchina, V. Branchina, F. Contino, A. Pernace, Does the cosmological constant really indicate the existence of a dark dimension? arXiv:2308.16548 [hep-th]
3. C. Vafa, The string landscape and the swampland. arXiv:hep-th/0509212
4. D. Lüst, E. Palti, C. Vafa, AdS and the swampland. Phys. Lett. B **797**, 134867 (2019). https://doi.org/10.1016/j.physletb.2019.134867. arXiv:1906.05225 [hep-th]
5. L. Anchordoqui, I. Antoniadis, D. Lüst, The dark dimension, the swampland, and the dark matter fraction composed of primordial black holes. Phys. Rev. D **106**, 086001 (2022). https://doi.org/10.1103/PhysRevD.106.086001. arXiv:2206.07071 [hep-th]
6. E. Gonzalo, M. Montero, G. Obied, C. Vafa, Dark dimension gravitons as dark matter. arXiv:2209.09249 [hep-ph]
7. L.A. Anchordoqui, I. Antoniadis, D. Lüst, Aspects of the dark dimension in cosmology. Phys. Rev. D **107**(8), 083530 (2023). https://doi.org/10.1103/PhysRevD.107.083530. arXiv:2212.08527 [hep-ph]
8. L.A. Anchordoqui, I. Antoniadis, N. Cribiori, D. Lüst, M. Scalisi, The scale of supersymmetry breaking and the dark dimension. JHEP **05**, 060 (2023). https://doi.org/10.1007/JHEP05(2023)060. arXiv:2301.07719 [hep-th]
9. A. Higuchi, Forbidden mass range for spin-2 field theory in De Sitter space-time. Nucl. Phys. B **282**, 397–436 (1987). https://doi.org/10.1016/0550-3213(87)90691-2
10. H. Itoyama, T.R. Taylor, Supersymmetry restoration in the compactified O(16) x O(16)-prime heterotic string theory. Phys. Lett. B **186**, 129–133 (1987). https://doi.org/10.1016/0370-2693(87)90267-X
11. H. Itoyama, T.R. Taylor, Small Cosmological Constant in String Models. FERMILAB-CONF-87-129-T
12. I. Antoniadis, C. Kounnas, Superstring phase transition at high temperature. Phys. Lett. B **261**, 369–378 (1991). https://doi.org/10.1016/0370-2693(91)90442-S
13. Q. Bonnefoy, E. Dudas, S. Lüst, On the weak gravity conjecture in string theory with broken supersymmetry. Nucl. Phys. B **947**, 114738 (2019). https://doi.org/10.1016/j.nuclphysb.2019.114738. arXiv:1811.11199 [hep-th]
14. N. Arkani-Hamed, S. Dubovsky, A. Nicolis, G. Villadoro, Quantum horizons of the standard model landscape. JHEP **06**, 078 (2007). https://doi.org/10.1088/1126-6708/2007/06/078. arXiv:hep-th/0703067 [hep-th]
15. H. Ooguri, C. Vafa, On the geometry of the string landscape and the swampland. Nucl. Phys. B **766**, 21–33 (2007). https://doi.org/10.1016/j.nuclphysb.2006.10.033. arXiv:hep-th/0605264 [hep-th]
16. G. Dvali, Black holes and large *N* species solution to the hierarchy problem. Fortsch. Phys. **58**, 528–536 (2010). https://doi.org/10.1002/prop.201000009. arXiv:0706.2050 [hep-th]
17. D. van de Heisteeg, C. Vafa, M. Wiesner, D.H. Wu, Species scale in diverse dimensions. arXiv:2310.07213 [hep-th]
18. N. Cribiori, D. Lüst, G. Staudt, Black hole entropy and moduli-dependent species scale. Phys. Lett. B **844**, 138113 (2023). https://doi.org/10.1016/j.physletb.2023.138113. arXiv:2212.10286 [hep-th]
19. C.P. Burgess, F. Quevedo, Perils of towers in the swamp: dark dimensions and the robustness of effective field theories. arXiv:2304.03902 [hep-th]

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[1] This practical reasoning has also been missed in [19].