Runaway potentials in warm inflation satisfying the swampland conjectures

Suratna Das[1,*] and Rudnei O. Ramos[2,†]

[1]*Department of Physics, Indian Institute of Technology, Kanpur 208016, India*
[2]*Departamento de Fisica Teorica, Universidade do Estado do Rio de Janeiro,*
20550-013 Rio de Janeiro, RJ, Brazil

The runaway potentials, which do not possess any critical points, are viable potentials which befit the recently proposed de Sitter swampland conjecture very well. In this work, we embed such potentials in the warm inflation scenario motivated by quantum field theory models generating a dissipation coefficient with a dependence cubic in the temperature. It is demonstrated that such models are able to remain in tune with the present observations and they can also satisfy all three swampland conjectures, namely, the swampland distance conjecture, the de Sitter conjecture, and the trans-Planckian censorship conjecture, simultaneously. These features make such models viable from the point of view of effective field theory models in quantum gravity and string theory, away from the swampland.

I. INTRODUCTION

The fact that achieving a stable or metastable de Sitter vacuum in M or string theory has proven to be a tasking job for decades (see, e.g., Ref. [1] for a recent review)[1] has led to the recently proposed de Sitter swampland criterion [5–7], which is conjectured to constrain constructions of the de Sitter vacuum within string landscapes. This conjecture has created a lot of discussion in the recent literature and for good reason, since the early and the late time cosmologies are in need of phases where the Universe evolves like close to a de Sitter state [8,9]. Another decade-old swampland criterion, known as the swampland distance conjecture [10] and which was formulated to restrict tower of masses from appearing in a low-energy effective field theory, has cosmological implications as well and where the cosmological dynamics involves evolution of scalar fields, like inflation. The latest addition to this list of swampland criteria is the trans-Planckian censorship conjecture (TCC) [11,12], which has been devised to refrain

any sub-Planckian primordial modes from leaving the causal horizon during the inflationary phase and which would seed the structures in our Universe at later stages.

In short, all three above-mentioned swampland criteria restrict the dynamics of inflation in one way or another. The de Sitter conjecture, which puts bounds on the slope of the scalar potentials in an effective field theory [6,7],

$$|\nabla V| \geq \frac{c}{M_{\text{Pl}}} V \quad \text{or} \quad \min(\nabla_i \nabla_j V) \leq -\frac{c'}{M_{\text{Pl}}^2} V, \quad (1.1)$$

where c and c' are both constants of the order of unity, essentially restricts the slow-roll parameters to become smaller than unity [6]:

$$\epsilon_V \equiv \frac{M_{\text{Pl}}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2 \geq \frac{c^2}{2} \quad \text{or} \quad \eta_V \equiv M_{\text{Pl}}^2 \frac{V_{,\phi\phi}}{V} \leq -c', \tag{1.2}$$

with $M_{\text{Pl}} \equiv 1/\sqrt{8\pi G} \simeq 2.4 \times 10^{18}$ GeV the reduced Planck mass. However, it is essential to have the slow-roll parameters much smaller than unity for inflation to take place in a canonical cold inflationary paradigm.[2] The swampland distance conjecture, which restricts the excursion of a scalar field in an effective field theory [10],

$$\frac{\Delta\phi}{M_{\text{Pl}}} < d, \tag{1.3}$$

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*suratna@iitk.ac.in
†rudnei@uerj.br

[1]Note that, despite existing proposals for constructing de Sitter states in string theory, like, e.g., the Kachru-Kallosh-Linde-Trivedi scenario [2], there are still concerns about that (see, e.g., Ref. [3]). For a recent discussion about the difficulties in constructing de Sitter states in string theory, see, for instance, Ref. [4].

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[2]An analysis presented in Ref. [13] has shown how the above conjectures put strong constraints in cold inflation models as far the cosmic microwave background data are concerned.

where d is another constant of the order of unity, essentially favors the small-field models of inflation over the large-field ones. On the other hand, the TCC, which is a bound on the length scales leaving the causal horizon [12],

$$\left(\frac{a_f}{a_i}\right)\ell_{\mathrm{Pl}} < \frac{1}{H_f}, \qquad (1.4)$$

where a_i and a_f are, respectively, the scale factors at the beginning and at the end of the evolution, H_f is the Hubble parameter at the end of that evolution, and ℓ_{Pl} is any length scale of the order of Planck scale, yields a bound on the scale of inflation[3]:

$$V^{1/4} < 6 \times 10^8 \text{ GeV} \sim 3 \times 10^{-10} M_{\mathrm{Pl}}. \qquad (1.5)$$

If inflation takes place at such low-energy scales, then the observed scalar power amplitude can be obtained in cold inflation only if $\epsilon_V \sim 10^{-31}$, which yields a too small tensor-to-scalar ratio ($r = 16\epsilon \sim 10^{-30}$) to be detected by any of the near-future observation [12]. Moreover, in such a scenario, one requires $\eta_V \sim \mathcal{O}(10^{-2})$ so that the observed scalar spectral tilt ($n_s - 1 = 2\eta_V - 6\epsilon_V \sim 2\eta_V$) can be explained [18]. It is rather impossible to construct a potential which yields $\epsilon_V \sim 10^{-31}$ and $\eta_V \sim 10^{-2}$ so that cold inflation can be made in tune with the trans-Planckian censorship conjecture. Therefore, as it is essential to fulfill all the above three swampland criteria in order to realize an inflationary phase in any effective field theory consistent with the string landscape, it turns out to be a challenging task to realize canonical single-field slow-roll inflationary dynamics in a string vacuum [12,18–20].

It was pointed out in Refs. [20–23] that warm inflation (WI) [24], a variant inflationary paradigm to the standard cold inflation scenario, can accommodate the de Sitter conjecture quite easily due to its very construction. In particular, the de Sitter conjecture was explicitly analyzed in the WI context in Ref. [25], where it was demonstrated that this conjecture remains robust in WI. In WI, the inflaton field can continuously transfer its energy to a radiation bath during inflation, inducing an extra frictional term in the inflaton dynamics and resulting the field to roll slower than in the cold paradigm (for reviews on WI, see, e.g., Refs. [26,27]). In that case, WI takes place when the slow-roll parameters ϵ_V and η_V are smaller than $1 + Q$, with Q being the ratio of the frictional terms in the inflaton dynamics due to the dissipation, denoted by Υ, and the

expansion of the Universe, $Q \equiv \Upsilon/(3H)$, which can be much greater than unity. Therefore, WI can easily take place with steeper potentials ($\epsilon_V > 1$, $\eta_V > 1$) (which the cold paradigm fails to attain), and, thus, WI can satisfy the de Sitter conjecture with ease. However, it was shown in Refs. [18,28] that, in order to maintain the TCC, the scale of WI has to be as low as in the cold paradigm [as given in Eq. (1.5)].

Since then, at least two attempts have been made to construct viable WI models with steep potentials which can be realized in string landscapes by satisfying all three swampland conjectures mentioned above. The first one [29] was constructed in the Randall-Sundrum braneworld scenario, where the dissipative coefficient was taken to depend on both the inflaton field and the temperature of the radiation bath existing during inflation, and the steep potential was considered to be of the exponential form

$$V(\phi) = V_0 e^{-\alpha\phi/M_{\mathrm{Pl}}}. \qquad (1.6)$$

Such a steep potential in cold inflation leads to power-law-type inflation [30], where inflation does not exit gracefully in standard general relativity. However, as has been recently shown in Ref. [31], such a potential can gracefully exit in WI if the dissipative coefficient has a dependence on the temperature of the radiation bath like $\Upsilon \propto T^p$, with power $p > 2$. This was, for instance, the case studied in the model of Ref. [29], where the dissipation coefficient had a T^3 dependence on the temperature.[4] In another recent study [32], the potential (1.6) was also studied in the context of the WI model proposed by the authors of Ref. [33]. In Ref. [33], a WI model, namely, the minimal warm inflation (MWI) model, was constructed where the inflaton was an axionlike field coupled to gauge bosons in the usual way and whose derived dissipation coefficient turned out to be of the form $\Upsilon \propto T^3$. In the study done in Ref. [32], which is the second study where WI with steep potentials has been put to the test against the swampland conjectures, the runaway exponential potential (1.6) was embedded in the MWI model. However, the authors of that work have shown that such a combination yields too much red tilt in the scalar spectrum to be in accordance with the observations.[5]

The aim of the present paper is to study a generalized form of the runaway potential given by [34–37]

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[3]A modified version of the TCC has recently been suggested in Ref. [14] and which proposes that N is bounded only by the logarithm of the de Sitter entropy, i.e., $N < 2 \ln(M_{\mathrm{Pl}}/H_f)$. This allows for larger values of H during inflation, $H_f/M_{\mathrm{Pl}} < 10^{-12}$, which substantially alleviates the bound in Eq. (1.5) by some 4 orders of magnitude (see also Refs. [15–17] for other discussions on how to relax the TCC bound).

[4]Note that, as also shown in Ref. [31], an additional dependence on the inflaton field amplitude does not affect this conclusion.

[5]A steep runaway potential of the type $V(\phi) = V_0[1 + \exp(-\alpha\phi/M_{\mathrm{Pl}})]$ was also studied in Ref. [32] when embedding it in the MWI model. However, it was shown that, although such a combination can satisfy all three swampland conjectures while being in accordance with observations, inflation fails to the exit gracefully within the parameter range studied.

$$V(\phi) = V_0 e^{-\alpha(\phi/M_{\mathrm{Pl}})^n} \qquad (1.7)$$

with $n > 1$. We note that with $n > 1$ there is no graceful exit problem even for the case of cold inflation. Furthermore, as shown in Ref. [32] that the $n = 1$ case produces a way too red-tilted spectrum in WI with $p > 2$ (in the standard general relativity context), this will compel us to go beyond $n = 1$ and study the cases with $n > 1$. One study of the generalized potentials of the form of Eq. (1.7) has been performed recently in Ref. [37] in the WI context and as a quintessential inflation model. However, in that reference, only the weak dissipative regime of WI, $Q \ll 1$, has been analyzed. Motivated by the previous studies indicating that WI in the strong dissipative regime can be consistent with the swampland criteria, in the present work we reconsider this type of models in this regime of WI. It is worth recalling here that constructing WI models in the strong dissipative regime has been historically a challenge [38]. Here we will show that the model (1.7) can support strong dissipation with the well-motivated type of dissipation coefficients behaving like $\Upsilon \propto T^3$ but also lead to a dynamics that is consistent both from the observational as well as from the effective field theory (as defined by the swampland program) points of view.

This paper is organized as follows. In Sec. II, we briefly review the generics of the WI dynamics for completeness. In Sec. III, we present some useful analytical studies to determine the field ranges which are suitable for our analysis and used in the subsequent section. In Sec. IV, we then perform a full numerical study covering the appropriate parameter ranges leading to a consistent inflationary dynamics in the strong dissipative regime. Our conclusions are presented in Sec. V. Two Appendixes are also included to describe some of the technical details.

II. BRIEF REVIEW OF WARM INFLATION

First, let us briefly review the background dynamics of a generic WI paradigm. In WI, the inflaton dissipates its energy to a constant radiation bath throughout inflation. Thus, the background dynamics involve the evolutions equations for the inflaton field $\phi(t)$, for the radiation energy density $\rho_R(t)$ [or, equivalently, for the temperature $T(t)$ of the thermal bath as $\rho_R \propto T^4$] and the Friedmann equation, which accounts for the evolution of the scale factor:

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -\Upsilon(\phi, T)\dot{\phi}, \qquad (2.1)$$

$$\dot{\rho}_R + 4H\rho_R = \Upsilon(\phi, T)\dot{\phi}^2, \qquad (2.2)$$

$$H^2 = \frac{1}{3M_{\mathrm{Pl}}^2}\left[\frac{\dot{\phi}^2}{2} + V(\phi) + \rho_R\right]. \qquad (2.3)$$

Here, Υ is the rate of dissipation at which the inflaton decays to the radiation bath. In general, Υ can be a function

of both ϕ and T. Some details of the derivation of these dissipation coefficients in the context of WI have been given in Appendix A. The dimensionless ratio of the two frictional terms in the inflaton equation of motion, the one due to dissipation and the other due to the expansion of the Universe, is defined as

$$Q \equiv \frac{\Upsilon(\phi, T)}{3H}, \qquad (2.4)$$

which broadly classifies WI models into two classes: WI taking place in the weak dissipative regime, where $Q \ll 1$, and WI taking place in the strong dissipative regime, when $Q \gg 1$. The slow-roll parameters in WI are modified with respect to the ones in the cold inflation scenario to

$$\epsilon_{\mathrm{WI}} = \frac{\epsilon_V}{1 + Q}, \qquad (2.5)$$

$$\eta_{\mathrm{WI}} = \frac{\eta_V}{1 + Q}, \qquad (2.6)$$

and WI ends when $\epsilon_V \sim 1 + Q$. The fact that during inflation the energy density would be dominated by the potential energy density of the inflaton field and the radiation bath produced would be of (approximately) constant energy density helps us to reduce the above dynamical equations to the approximate ones:

$$3H(1 + Q)\dot{\phi} \approx V_{,\phi}, \qquad (2.7)$$

$$\rho_R \approx \frac{3Q}{4}\dot{\phi}^2, \qquad (2.8)$$

$$H^2 \approx \frac{V}{3M_{\mathrm{Pl}}^2}, \qquad (2.9)$$

where we must note that standard slow-roll approximations, like $\epsilon_V \ll 1$ or $\eta_V \ll 1$, have not been employed in getting the above approximated results. Hence, these approximations hold true even in cases of steep potentials for which $\epsilon_V > 1$ and/or $\eta_V > 1$ and, yet, an inflationary regime can still be supported provided that Q is large enough.

Let us now briefly discuss the perturbations generated during WI. Some details of the complete set of perturbation equations considered in WI have been given in Appendix B. In the cold inflation scenario, i.e., in the absence of dissipative effects and no radiation bath during inflation, the primordial scalar curvature power spectrum $\Delta_{\mathcal{R}}$ and the primordial tensor power spectrum Δ_T are given by the standard expressions [39]

$$\Delta_{\mathcal{R}} = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2, \qquad (2.10)$$

$$\Delta_T = \frac{2H^2}{\pi^2 M_{\mathrm{Pl}}^2}, \qquad (2.11)$$

respectively. Because of dissipation and the presence of a radiation bath, the primordial scalar power spectrum given by Eq. (2.10) changes, while the tensor spectrum Eq. (2.11) remains unchanged. The primordial power spectrum for WI at horizon crossing is given by [40,41]

$$\Delta_{\mathcal{R}}(k/k_*) = \left(\frac{H_*^2}{2\pi\dot{\phi}_*}\right)^2 \mathcal{F}(k/k_*), \qquad (2.12)$$

where the function $\mathcal{F}(k/k_*)$ in Eq. (2.12) is (see Appendix B for details)

$$\mathcal{F}(k/k_*) \equiv \left(1 + 2n_* + \frac{2\sqrt{3}\pi Q_*}{\sqrt{3 + 4\pi Q_*}}\frac{T_*}{H_*}\right)G(Q_*), \quad (2.13)$$

where n_* denotes the thermal distribution of the inflaton field due to the presence of the radiation bath and $G(Q_*)$ accounts for the effect of the coupling of the inflaton and radiation fluctuations [42–44]. The function $G(Q_*)$, in general, can be determined only by numerically solving the set of perturbation equations in WI. The specific form for the function $G(Q_*)$ depends mostly on the type of dissipation coefficient appearing in a WI model and weakly on the inflaton potential chosen, at least for $Q \gg 1$. The tensor-to-scalar ratio r and the spectral tilt n_s are defined in a standard way as

$$r = \frac{\Delta_T}{\Delta_{\mathcal{R}}} \qquad (2.14)$$

and

$$n_s - 1 = \lim_{k \to k_*} \frac{d\ln\Delta_{\mathcal{R}}(k/k_*)}{d\ln(k/k_*)}, \qquad (2.15)$$

respectively. A subindex $*$ means that the quantities are evaluated at the Hubble radius crossing of the pivot scale k_* ($k_* = a_*H_*$). The Planck Collaboration [45] puts an upper bound on the tensor-to-scalar ratio r as $r < 0.056$ (95% C.L., Planck TT, TE, EE + lowE + lensing + BK15, at the pivot scale $k_p = 0.002$ Mpc^{-1}), while the spectral tilt is measured to be $n_s = 0.9658 \pm 0.0040$ (95% C.L., Planck TT, TE, EE + lowE + lensing + BK15 + BAO + running) at pivot scale $k_* = 0.05$ Mpc^{-1}. Furthermore, the normalization of the primordial scalar curvature power spectrum, at the pivot scale k_*, is given by $\ln(10^{10}\Delta_{\mathcal{R}}) \simeq 3.047$ (TT, TE, EE − lowE + lensing + BAO 68% limits), according to the Planck Collaboration [46], and this is the value we will assume in all our numerical simulations, in particular, for finding the normalization V_0 of the potential in Eq. (1.7).

III. ANALYTICAL DETERMINATION OF THE FIELD RANGES

As discussed in the introduction, in this work, we are interested in studying WI for the class of runaway potentials given by the generalized exponential potentials of the form of Eq. (1.7) with $n > 1$. As already pointed out in Ref. [37] (see also Ref. [31]), for a simple functional form for the dissipation coefficient in terms of the temperature T and the inflaton field amplitude ϕ, given by[6]

$$\Upsilon(\phi, T) = C_\Upsilon T^p \phi^c M^{1-p-c}, \qquad (3.1)$$

where C_Υ is a constant and M some appropriate mass scale associated with the microscopic model leading to Eq. (3.1) and using the approximations in Eq. (2.9), which are valid during the WI dynamics, we find that the dissipation ratio Q evolves with the number of e-folds N like

$$\frac{d\ln Q}{dN} = -\frac{n\alpha(\frac{\phi}{M_{\mathrm{Pl}}})^{n-2}}{4 - p + (4 + p)Q} \\ \times \left[-2p(n-1) - 4c + (p-2)n\alpha\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^n\right]. \qquad (3.2)$$

Since the power p in the temperature satisfies $-4 < p < 4$ (see Refs. [49–51]), we find that only those cases of dissipation coefficient having a power in the temperature with $p > 2$ will lead to a dissipation ratio decreasing with the number of e-folds, which ensures that WI can gracefully exit in this class of models [31]. Having a decreasing dissipation ratio Q is also crucial in our derivation that follows below and will allow us to work with very steep potentials, yet keeping consistency with observations, in the large dissipation regime of WI. For instance, WI models with a dissipation coefficient $\Upsilon \propto T^3$ fit this condition. A dissipation coefficient with a cubic dependence in the temperature is also particularly well motivated by both early and recent models of WI (see Appendix A for some examples of particle physics quantum field theory interaction schemes leading to this type of dissipation coefficient), and, therefore, it is quite suitable for the study we have performed in the present work. Thus, henceforth, we will consider in all of our analysis a dissipation coefficient given simply by

$$\Upsilon = C_\Upsilon \frac{T^3}{M^2}. \qquad (3.3)$$

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[6]For some earlier studies also considering this functional form for the dissipation coefficient in WI, see, e.g., Refs. [47,48].

A. Background dynamics with the generalized exponential potential in WI

The dynamics of the model given by the potential in Eq. (1.7) can be divided in two regimes, depending on which region of the potential inflation starts and ends. The potential Eq. (1.7) has a plateau region at around $\phi = 0$ and an inflection point located at (for $n > 1$)

$$\frac{\phi_{\text{inflection}}}{M_{\text{Pl}}} = \left(\frac{n-1}{n\alpha}\right)^{1/n}. \tag{3.4}$$

In the region $0 < \phi < \phi_{\text{inflection}}$, the dynamics is similar to that of a hilltop inflation [52]. WI happening in this region favors the weak dissipative regime, $Q \ll 1$, as shown for the hilltop type of potentials, in general [41] (this was also the regime studied in Ref. [37] for the generalized exponential potential). On the other hand, when WI happens entirely in the runaway part of the potential, $\phi > \phi_{\text{inflection}}$, the strong dissipative regime of WI is favored. This is the part of the potential we are interested in to explore in this work, as motivated by the swampland program. The form of the potential in Eq. (1.7), for different cases for the exponent n, is shown in Fig. 1.

Let us now confirm that WI can indeed gracefully exit in the runaway part of the potential. WI ends when $\epsilon_V \approx 1 + Q$ or $\epsilon_{\text{WI}} \approx 1$. Therefore, to end inflation, ϵ_{WI} should be an increasing function of number of e-foldings yielding a condition [31]

$$\frac{d \ln \epsilon_V}{dN} > \frac{Q}{1+Q} \frac{d \ln Q}{dN}. \tag{3.5}$$

For the runaway potential in Eq. (1.7), the slow-roll parameters become

$$\epsilon_V = \frac{\alpha^2 n^2}{2} \left(\frac{\phi}{M_{\text{Pl}}}\right)^{2n-2}, \tag{3.6}$$
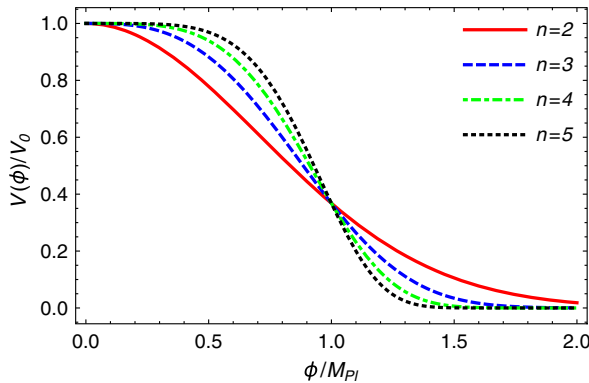

FIG. 1. The form of the generalized exponential potential (1.7) for different values of n and by choosing $\alpha = 1$.

$$\eta_V = \alpha^2 n^2 \left(\frac{\phi}{M_{\text{Pl}}}\right)^{2n-2} - \alpha n(n-1) \left(\frac{\phi}{M_{\text{Pl}}}\right)^{n-2}. \tag{3.7}$$

The slow-roll parameter ϵ_V evolves with the number of e-foldings as

$$\frac{d \ln \epsilon_V}{dN} = \frac{4\epsilon_V - 2\eta_V}{1+Q} = \frac{\alpha n(n-1)}{1+Q} \left(\frac{\phi}{M_{\text{Pl}}}\right)^{n-2}. \tag{3.8}$$

This shows that ϵ_V is a constant and does not evolve when $n = 1$, and it does evolve for $n > 1$. We are interested in the part of the potential for which $\phi > \phi_{\text{inflection}}$. We then get from Eq. (3.8) that

$$\left(\frac{\phi}{M_{\text{Pl}}}\right)^n = \frac{d \ln \epsilon_V}{dN} \frac{1+Q}{\alpha n(n-1)} \left(\frac{\phi}{M_{\text{Pl}}}\right)^2. \tag{3.9}$$

The left-hand side of the above equation has to be greater than $(n-1)/n\alpha$ such that inflation takes place in the steep slope of the potential. Thus, in this range, we get

$$\frac{d \ln \epsilon_V}{dN} > \frac{(n-1)^2}{1+Q} \left(\frac{\phi}{M_{\text{Pl}}}\right)^{-2}. \tag{3.10}$$

As the right-hand side of the above inequality is always positive, ϵ_V would always increase when inflation is taking place in the steep region of the slope. Thus, inflation is assured to end whenever Q is constant or decreases with e-foldings. However, inflation can also end when Q increases with N but with a slower rate than the evolution of ϵ_V as shown in Eq. (3.5). This is, however, a more difficult condition to achieve, in general.

With the form of the dissipative coefficient given in Eq. (3.3), the dissipation ratio Q evolves as

$$\frac{d \ln Q}{dN} = \frac{10\epsilon_V - 6\eta_V}{1 + 7Q}. \tag{3.11}$$

In the strong dissipative regime ($Q \gg 1$), the condition to end inflation given in Eq. (3.5) then becomes

$$9\epsilon_V > 4\eta_V, \tag{3.12}$$

which yields

$$\left(\frac{\phi}{M_{\text{Pl}}}\right)^n > -\frac{8(n-1)}{n\alpha}. \tag{3.13}$$

As we are interested only in the region beyond the inflection point, this condition will always be satisfied in the region of our interest. Thus, inflation will always end in the steep potential region beyond the inflection point.

B. The perturbations

Now, let us consider the perturbations in this theory. The form of the scalar power spectrum in WI has already been discussed in Eqs. (2.12) and (2.13). In this case, the form for the function $G(Q_*)$, valid for the generalized exponential potential Eq. (1.7), is found to be given by (see also discussion concerning this point in Appendix B)

$$G(Q_*) = \frac{1 + 6.12Q_*^{2.73}}{(1 + 6.96Q_*^{0.78})^{0.72}} + \frac{0.01Q_*^{4.61}(1 + 4.82 \times 10^{-6}Q_*^{3.12})}{(1 + 6.83 \times 10^{-13}Q_*^{4.12})^2}, \quad (3.14)$$

which is found to hold up to rather large values of Q_* ($Q_* \sim 3 \times 10^3$). This will be enough to cover the range of values of Q to be considered in the next section.

Note that the scalar power spectrum used in Ref. [33] was of the form [see, e.g., Eq. (12) in that reference]

$$\Delta_{\mathcal{R}} \approx \frac{\sqrt{3}}{4\pi^{3/2}} \frac{H^3 T}{\dot{\phi}^2} \left(\frac{Q}{Q_3}\right)^9 Q^{1/2}, \quad (3.15)$$

where $Q_3 \approx 7.3$. It can be easily shown that in the strong dissipative regime a more accurate form of the power spectrum given in Eq. (2.12), along with the relations given in Eqs. (2.13) and (3.14), does not differ much from the above equation. As the above equation is presented in a much simpler form for the strong dissipative regime, we will derive the scalar spectral index analytically using the form of the power spectrum given in Eq. (3.15) for simplicity [while numerically we will use the full form of the function $G(Q)$ given in Eq. (3.14) to calculate the spectral index]. We see that

$$n_s - 1 = 3\frac{d\ln H}{dN} - 2\frac{d\ln\dot{\phi}}{dN} + \frac{d\ln T}{dN} + \frac{19}{2}\frac{d\ln Q}{dN}. \quad (3.16)$$

To determine the quantities on the right-hand side of the above equations, we will use the approximated background equations given in Eqs. (2.9), along with the approximated forms of Q and T valid for the form of the dissipation coefficient given by Eq. (3.3) (see also Ref. [33]):

$$Q^7 \approx \tilde{C}_Q \frac{V'^6}{V^5}, \qquad T^7 \approx \tilde{C}_T \frac{V'^2}{V^{1/2}}, \quad (3.17)$$

which are valid in a strong dissipative regime. From these equations, we see that

$$\frac{d\ln H}{dN} \sim -\frac{\epsilon_V}{Q}, \qquad \frac{d\ln Q}{dN} \sim \frac{10\epsilon_V - 6\eta_V}{7Q},$$

$$\frac{d\ln T}{dN} \sim \frac{\epsilon_V - 2\eta_V}{7Q}, \qquad \frac{d\ln\dot{\phi}}{dN} \sim -\frac{3\epsilon_V + \eta_V}{7Q}. \quad (3.18)$$

Inserting all these in the expression of n_s, we get [33]

$$n_s - 1 = \frac{3}{7Q}(27\epsilon_V - 19\eta_V). \quad (3.19)$$

It is to note that, to obtain a red tilt of the scalar spectrum, one requires a potential yielding $\eta_V > \epsilon_V$, as has also been observed in Ref. [33]. To be more precise, we need $19\eta_V > 27\epsilon_V$, which yields a condition

$$\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^n > \frac{38(n-1)}{11n\alpha}. \quad (3.20)$$

This is a stronger bound on the field range than the one required for field ranges beyond the inflection point. Therefore, beyond the above range, inflation ends as well as we get the desired spectral index with appropriate choices of α for a given n. These findings are explicitly checked in our numerical examples considered in the next section.

IV. NUMERICAL ANALYSIS OF THE PARAMETER RANGES

We have numerically evolved the full background equations given in Eqs (2.3) for the cases $n = 2$–5, and the findings are furnished in Table I. In principle, we can tune appropriately both Q_* and the constant α of the potential, for a given value of n, such as to produce results consistent with the observable quantities for either smaller or larger values of Q_* than the ones shown in Table I. Our criteria for choosing the value of Q_* was that it would be large enough such that all the swampland criteria could be met and also to have n_s around the central value from the Planck analysis. The tensor-to-scalar ratio r is naturally very much suppressed in the large Q regime of WI, as already seen in other cases (see, e.g., Refs. [29,38]). The second and third columns containing the values of n_s and r, respectively, ensures that the models with different values of n (and the chosen values of α accordingly) are in accordance with the observations. The fifth column shows the amount of field traversed, $\Delta\phi$, which turns out to be sub-Planckian for all the cases, and it confirms that these examples obey the swampland distance conjecture. The tenth and the eleventh columns, which contain the values of the slow-roll parameters ϵ_V and η_V, respectively, which are much larger than unity, ensure that the swampland de Sitter conjecture is maintained. The last column, which quotes the values of V_*, i.e., the scale of inflation, confirms that the TCC is obeyed as well. Hence, Table I confirms that the runaway potentials with $n > 1$ not only remain in tune with observations but also obey all three swampland conjectures, making these models prime candidates as consistent effective field theory models in string landscapes.

For completeness, we have also shown in Fig. 2 how the potential, kinetic, and radiation energy densities evolve with the number of e-foldings for the $n = 3$ case.

TABLE I. Numerical estimation of respective parameters and the relevant cosmological quantities obtained from them.

Model	n_s	r	N_*	$\Delta\phi/M_{\rm Pl}$	C_Υ	$M/M_{\rm Pl}$	$T_{\rm end}$ (GeV)	V_0 (GeV)^4	ϵ_{V_*}	η_{V_*}	$V_*^{1/4}/M_{\rm Pl}$
$n=2$ $\alpha=9.6$ $Q_*=850.96$	0.9648	4.89×10^{-29}	48.0	0.98	6.04×10^{-11}	4.0×10^{-13}	1.52×10^7	3.07×10^{38}	31.7	44.2	1.52×10^{-11}
$n=3$ $\alpha=2.5$ $Q_*=740.15$	0.9689	1.44×10^{-28}	48.2	0.97	4.19×10^{-11}	4.0×10^{-13}	2.49×10^7	1.82×10^{39}	25.9	37.2	2.49×10^{-11}
$n=4$ $\alpha=0.45$ $Q_*=719.68$	0.9655	1.83×10^{-28}	48.1	0.97	3.86×10^{-11}	4.0×10^{-13}	2.86×10^7	3.59×10^{39}	25.0	36.5	2.85×10^{-11}
$n=5$ $\alpha=0.06$ $Q_*=699.53$	0.9645	2.37×10^{-28}	48.2	0.97	3.53×10^{-11}	4.0×10^{-13}	3.18×10^7	6.01×10^{39}	24.1	35.6	3.18×10^{-11}

The situation for the other models with $n = 2, 4$, and 5 are very much similar, so we do not show them explicitly here. It can be seen that inflation ends when the radiation energy density starts to dominate over the potential energy density, while the kinetic energy density remains small all along.

As an additional point to be commented, and also related to the result shown in Fig. 2, is the number of e-folds shown in the fourth column of Table I. As noticed from Fig. 2, WI exists gracefully at around 48 e-foldings, when the energy scale of inflation is as low as $10^{-11}M_{\rm Pl}$. In WI we can precisely determine the number of e-folds of inflation N_* between the moment the relevant scales with wave number k_* leaving the Hubble radius and reentering around today. This is due to the fact that WI is not followed by a reheating phase, and, thus, there is no uncertainty associated with the total number of e-folds which generally appears due to the uncertainty in the number of e-foldings of the reheating phase. By relating the comoving Hubble scale at N_* when the mode with comoving wave number k_* crossed the horizon, $a_*H_* = k_*$, with the one at present time, a_0H_0, we have that [53]

$$\frac{k_*}{a_0H_0} = \frac{a_*}{a_{\rm end}}\frac{a_{\rm end}}{a_{\rm reh}}\frac{a_{\rm reh}}{a_0}\frac{H_*}{H_0},\qquad(4.1)$$

where $a_*/a_{\rm end} = \exp(-N_*)$ and we can use the fact that at the end of WI the Universe smoothly transits to the radiation-dominated regime with no intermediary reheating phase; hence, $a_{\rm end}/a_{\rm reh} = 1$. This is particularly well satisfied for all the models exemplified in Table I. Furthermore, as also seen in Fig. 2, inflation ends when $V(\phi)$ drops below ρ_R, while the kinetic energy density $\dot\phi^2/2$ remains always subdominant, even after inflation. Therefore, there is no kination period as is observed, in general, for this type of runaway exponential potentials. Finally, we can relate $a_{\rm end} \equiv a_{\rm reh}$ with a_0 in Eq. (4.1) by assuming that after WI there are no additional sources of entropy and use then the entropy conservation result

$$g_s(T_{\rm end})T_{\rm end}^3 a_{\rm end}^3 = \left(2T_0^3 + \frac{21}{4}T_{\nu,0}^3\right)a_0^3,\qquad(4.2)$$

where T_0 and $T_{\nu,0} = (4/11)^{1/3}T_0$ are, respectively, today's [cosmic microwave background (CMB)] photon and neutrino temperatures and we have explicitly used the respective number of degrees of freedom, while $g_s(T_{\rm end})$ is the effective number of degrees of freedom at the end of WI.

Putting the above relations together, Eq. (4.1), in WI, becomes

$$\frac{k_*}{a_0H_0} = e^{-N_*}\left[\frac{43}{11g_s(T_{\rm end})}\right]^{1/3}\frac{T_0}{T_{\rm end}}\frac{H_*}{H_0}.\qquad(4.3)$$

Here, we take the convention that $a_0 = 1$. For the Hubble parameter today, we assume the Planck result $H_0 = 67.66$ km s^{-1} Mpc^{-1} [from the Planck Collaboration [46], TT, TE, EE $-$ lowE $+$ lensing $+$ BAO 68% limits, $H_0 = (67.66 \pm 0.42)$ km s^{-1} Mpc^{-1}]. Likewise, for the CMB temperature today, we assume the value $T_0 = 2.725{\rm K} = 2.349 \times 10^{-13}$ GeV, while for the pivot scale k_* we take
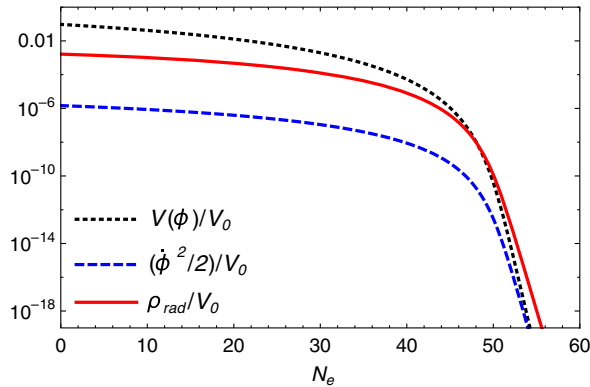

FIG. 2. The evolution of the potential, kinetic, and radiation energy densities for the model with $n = 3$ given in Table I.

the Planck value $k_* = 0.05/\text{Mpc}$. For $g_s(T_{\text{end}})$ we assumed the standard model value $g_s(T_{\text{end}}) = 106.75$ for definiteness (the results are very weakly dependent on g_s). Putting all these values in the above equation, N_* turns out to be of the order of 48, consistent with the results seen in Table I.

A final remark about the observational predictions of the models studied here concerns the level of non-Gaussianity that they produce. For WI in the large dissipative regime, as is the case for all models we have considered, a non-Gaussianity coefficient of the warm shape is predicted (see, e.g., Ref. [54] for details) as large as $|f_{\text{NL}}^{\text{warm}}| \sim 5$ in the case of a dissipation coefficient $\Upsilon \propto T^3$. This is still within the range of the results obtained by the Planck 2018 team using the WI shape [55], $f_{\text{NL}}^{\text{warmS}} = -48 \pm 27$ (from SMICA + T + E, 68% C.L.) and $f_{\text{NL}}^{\text{warmS}} = -39 \pm 44$ (from SMICA + T, 68% C.L.). But the expected result that we have for f_{NL}, for all the cases here studied, is still in a magnitude that can be large enough to possibly be probed from, e.g., the future fourth-generation CMB observatories [56] or from future large-scale structure surveys. Both of these are expected to bring down the present upper bounds on non-Gaussianities. Thus, non-Gaussianity can be one of the indicators differentiating WI in the strong dissipative regime from the weak dissipative one, through their distinct non-Gaussianity shapes and predictions for f_{NL} [54].

V. CONCLUSION

The proposed swampland conjectures, namely, the swampland distance conjecture, the de Sitter conjecture, and the trans-Planckian censorship conjecture, have severely constrained the construction of viable inflation models in any string landscape. Thus, the swampland program strongly restricts the class of possible effective field theory models of inflation that are consistent with quantum gravity. It is thus worthwhile to look for constructions where the inflationary dynamics can be accommodated away from swamplands. It has been pointed out previously that the WI scenario befits the swampland conjectures, especially the de Sitter one, much better than its counterpart, the cold inflation paradigm. The de Sitter conjecture is also better suited with the runaway type of potentials [like, e.g., Eq. (1.7)] which do not have any critical points. In particular, it has already been demonstrated that the de Sitter conjecture also remains robust in WI [25]. Besides of this, it has been a challenge, even for WI, to satisfy the TCC, which in its original formulation [12] requires inflation to happen at sufficiently small scales. The difficulty is associated with the construction of WI models able to support strong enough dissipation and at the same time to be consistent with the observations [38]. Even models motivated by WI, like the ones studied in Ref. [57], reflect well this difficulty. It is thus an important task to find appropriate inflation models that are able to satisfy all the

swampland criteria. In the present paper, we have explicitly considered the validity of the runaway type of potentials when embedding them in WI.

It was previously shown that the runaway potential with exponent $n = 1$ can be embedded in a Randall-Sundrum braneworld inflation successfully where it can observe all three swampland conjectures [29]. However, when such a potential is embedded in a standard general relativity context with WI, it yields too much red tilt in the scalar power spectrum to be in accordance with the observations [32]. In this work, we have examined the runaway potentials with exponents $n > 1$ when embedded in WI models characterized by a cubic in the temperature dependence of the dissipation coefficient to show that (a) such models gracefully exit inflation when inflation takes place in the runaway part of the potential; (b) they can remain in tune with the current observations by yielding the correct scalar spectral index; and (c) they can also simultaneously satisfy all three swampland conjectures as a consequence of supporting a strong enough dissipative regime of WI. The combination of all these features makes these types of models viable inflation models when constructed in the WI picture and which can be realized within string landscapes.

ACKNOWLEDGMENTS

APPENDIX A: THE DISSIPATION COEFFICIENT

Dissipative effects are expected to be experienced by systems, when displaced from their state of equilibrium and interacting with an environment. We can consider the case of a background scalar field ϕ initially displaced from its equilibrium state and interacting with other fields X. Given an interaction Lagrangian density like

$$\mathcal{L}_{\text{int}}(\phi, X) = -f(\phi)g(X), \tag{A1}$$

a proper study of the evolution of the background field can be performed in the context of the in-in, or the closed-time path functional formalism [58]. By integrating over the X field, a nonlocal effective equation of motion for ϕ can be derived, and the ensemble-averaged effective equation of motion for ϕ can be generically expressed like [58]

$$\partial_\mu \frac{\partial \mathcal{L}_{\text{eff},r}[\phi]}{\partial(\partial_\mu \phi)} - \frac{\partial \mathcal{L}_{\text{eff},r}[\phi]}{\partial \phi} - i \frac{\partial f(\phi)}{\partial \phi} \int d^4x' \theta(t - t')[f(\phi(x')) - f(\phi(x))] \langle [g(X(x)), g(X(x'))] \rangle = 0, \qquad (A2)$$

where $\mathcal{L}_{\text{eff},r}[\phi]$ is the renormalized effective Lagrangian density for ϕ and $\langle \cdots \rangle$ are ensemble averages with respect to an equilibrium (quantum or thermal) state. Equation (A2) forms the basis of many earlier works [59–63] (for a review, see also Ref. [26]) that evolved to warm inflation model realizations. The nonlocal term in Eq. (A2) represents a transfer of energy from the ϕ field into radiation. The nonlocal term in Eq. (A2) can be localized and expressed in the form of a proper dissipation term when there is a separation of timescales between the system and environment; e.g., given a response timescale τ related to the plasma interactions and ϕ slowly varying on the response timescale τ, $\dot{\phi}/\phi \ll \tau^{-1}$, which is typically referred to as the adiabatic approximation, then we can write [63]

$$-i \frac{\partial f(\phi)}{\partial \phi} \int d^4x' \theta(t - t')[f(\phi(x')) - f(\phi(x))] \langle [g(X(x)), g(X(x'))] \rangle \approx \Upsilon \dot{\phi}, \qquad (A3)$$

where Υ is the dissipation coefficient defined as

$$\Upsilon = \int d^4x' \Sigma_R(x, x')(t' - t) = -\lim_{\omega \to 0} \frac{\text{Im}\Sigma_R(\mathbf{k} = \mathbf{0}, \omega)}{\omega}, \qquad (A4)$$

where $\Sigma_R(\omega)$ is the Fourier transform of the retarded correlation function:

$$\Sigma_R(x, x') = -i \left[\frac{\partial f(\phi)}{\partial \phi} \right]^2 \theta(t - t') \langle [g(X(x)), g(X(x'))] \rangle. \qquad (A5)$$

Many examples of dissipation coefficients were, for example, derived in Ref. [64]. As discussed in the introduction, in this work, we are particularly interested in models leading to a dissipation coefficient that scales with the cubic power in the temperature, $\Upsilon \propto T^3$. Let us briefly review viable interaction schemes leading to such a dissipation coefficient.

1. Dissipation through a catalyst heavy field

One of the first working field theory models for WI has been constructed in the case of the inflaton field dissipating to light radiation fields intermediate by a heavy catalyst field. The implementation is based on a supersymmetric model with chiral superfields Φ, X, and Y_i, $i = 1, \ldots, N_Y$, described by the superpotential [26]

$$W = \frac{g}{2} \Phi X^2 + \frac{h_i}{2} X Y_i^2 + f(\Phi), \qquad (A6)$$

where a sum over the index i is implicit. The scalar component of the superfield Φ describes the inflaton field, with an expectation value $\phi = \phi/\sqrt{2}$, and $f(\Phi)$ describes the self-interactions in the inflaton sector. The X fields are assumed to be heavy fields with respect to the radiation bath temperature produced by the light fields Y, i.e., $m_X \gg T$ and $m_Y \ll T$. Under these circumstances, the dissipation

coefficient in the inflaton's equation of motion can be shown to be given by [64,65]

$$\Upsilon \simeq C_\Upsilon \frac{T^3}{\phi^2}, \qquad C_\Upsilon \simeq \frac{\alpha_h}{4} N_X N_Y, \qquad (A7)$$

where $\alpha_h = h^2 N_Y/(4\pi)$, N_X is the number of fields in the X heavy sector, and it was assumed $h = h_i$ for all the light fields for simplicity. Despite the dependence on the inflaton field in Eq. (A7), the results obtained for this model do not differ much from the ones we have obtained in Sec. IV. It can likewise support strong dissipation and satisfy all the swampland criteria and with an observationally consistent n_s. However, for $Q \sim \mathcal{O}(700)$ as in Table I, we have instead that $C_\Upsilon \sim 10^{14}$, which, from Eq. (A7) and assuming $h \sim 1$, implies the need of a huge number of heavy and/or light fields, $N_X N_Y \sim 5 \times 10^{15}$. Such a large number might be a technical challenge associated with this model, from both the perturbativity and the unitarity points of view, associated with this model for the present analysis (see, however, Ref. [66] for a possible scenario where these issues can be overcome and that uses brane constructions or also the proposal in Ref. [67] where large field multiplicities can be allowed due to a Kaluza-Klein tower in extra-dimensional scenarios).

2. Dissipation in the minimal warm inflation model

The MWI model was proposed by the authors in Ref. [33]. In the MWI model, the inflaton field has axionlike couplings to non-Abelian gauge fields, which yields a viable model of the thermal bath that can exist during WI. The inflaton field is coupled to a Yang-Mills field A_μ^a in an axionlike form:

$$\mathcal{L}_{\text{int}} = \frac{\alpha_g}{8\pi} \frac{\phi}{f} \tilde{F}^{a\mu\nu} F_{\mu\nu}^a, \qquad (A8)$$

where the dual gauge field strength $\tilde{F}^{a\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}^a$, $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g C^{abc} A_\mu^b A_\nu^c$, with g the Yang-Mills

coupling and C^{abc} the structure constant of the non-Abelian group. In Eq. (A8), one also has that $\alpha_g \equiv g^2/(4\pi)$ and f is a scale analogous to the axion decay constant. The corresponding dissipative coefficient that the interaction produces has been shown to be related to the Chern-Simons diffusion rate and is given by [68,69]

$$\Upsilon = C_\Upsilon \frac{T^3}{f^2}, \qquad C_\Upsilon = \kappa(\alpha_g, N_c, N_f)\alpha_g^5, \quad \text{(A9)}$$

where N_c is the dimension of the gauge group, N_f is the representation of the fermions if any, and κ is a dimensional quantity depending on N_c, N_f, and α_g.

One of the advantages of this model is that the shift symmetry satisfied by the inflaton naturally protects it from large thermal corrections that might undermine the slow-roll conditions during inflation.

It is useful to estimate the scale f appearing in Eq. (A9) from our numerical results given in Table I. Using that $\kappa \sim 100$ and $g \sim 0.1$, then from the values for C_Υ and M given in Table I, we find that for all models analyzed $f \sim 5 \times 10^4$ GeV. It is clear that in the present context we cannot associate ϕ with the QCD axion, since f is much below the astrophysical lower bound $f \gtrsim 10^9$ GeV set in for the QCD axion decay constant [70].

3. Dissipation through derivative couplings with the inflaton field

A third option to produce dissipation coefficients behaving like $\Upsilon \propto T^3$ is motivated from the previous example. We consider the case where the inflaton has a modulilike (or dilatonlike) derivative coupling with other radiation fields, which can be, for example, other scalar fields and with an interaction Lagrangian density given by

$$\mathcal{L}_{\text{int}} = g^2 \frac{\phi}{2M}(\partial_\mu\chi)^2. \quad \text{(A10)}$$

The scalar field χ is supposed to remain in thermal equilibrium through either its self-coupling or couplings to other radiation fields (e.g., additional gauge or other light fields that could be added to the model). It has been realized in Ref. [71] that the dissipation coefficient in this model can be precisely related to the bulk viscosity calculation for a scalar field [72], leading then to the result for the dissipation coefficient given by

$$\Upsilon \simeq 4.5g^4 \frac{\ln^2(\xi\lambda)}{\lambda}\frac{T^3}{M^2}, \quad \text{(A11)}$$

where ξ is a numerical constant, $\xi = \exp[15\zeta(3)/\pi^2] \simeq 0.064736$, and λ is the quartic self-coupling for the χ field, $-\lambda\chi^4/4!$. It is interesting to observe that this connection of the dissipation coefficient in WI at high temperature with a viscosity coefficient was already noticed in Ref. [60].

An additional interaction that now makes the connection with the bulk viscosity calculation in a pure gauge theory is by coupling the moduli field now to pure Yang-Mills gauge fields through a coupling like [note that this is different from the interaction term in Eq. (A8)]

$$\mathcal{L}_{\text{int}} = -g^2 \frac{\phi}{2M}F^{a\mu\nu}F^a_{\mu\nu}, \quad \text{(A12)}$$

which gives for the dissipation coefficient the result [69,73]

$$\Upsilon \simeq \frac{(12\pi\alpha_g)^2}{\ln(1/\alpha_g)}\frac{T^3}{M^2}, \quad \text{(A13)}$$

where, as before, $\alpha_g \equiv g^2/(4\pi)$.

It remains, as possible future work, to see how the above modulilike interactions can be implemented in an explicit quantum field theory model-building construction for WI.

APPENDIX B: PERTURBATIONS IN WARM INFLATION

We review here the first-order perturbation equations for WI, consisting of the inflaton perturbations $\delta\phi$, the radiation energy density perturbation $\delta\rho_r$, and the radiation momentum perturbation Ψ_r. The notation that we follow is the one given in Refs. [74,75].

The perturbed Friedmann-Lemaître-Robertson-Walker metric is given by

$$ds^2 = -(1 + 2\alpha)dt^2 - 2a\partial_i\beta dx^i dt$$
$$+ a^2[\delta_{ij}(1 + 2\varphi) + 2\partial_i\partial_j\gamma]dx^i dx^j, \quad \text{(B1)}$$

where α, β, γ, and φ are the spacetime-dependent perturbed-order variables. These metric perturbation functions are related to the complete set of equations (when Fourier transforming to space momentum) [74,75]

$$\chi = a(\beta + a\dot{\gamma}), \quad \text{(B2)}$$

$$\kappa = 3(H\alpha - \dot{\varphi}) + \frac{k^2}{a^2}\chi, \quad \text{(B3)}$$

$$-\frac{k^2}{a^2}\varphi + H\kappa = -\frac{1}{2M_{\text{Pl}}^2}\delta\rho, \quad \text{(B4)}$$

$$\kappa - \frac{k^2}{a^2}\chi = -\frac{3}{2M_{\text{Pl}}^2}\Psi, \quad \text{(B5)}$$

$$\dot{\chi} + H\chi - \alpha - \varphi = 0, \quad \text{(B6)}$$

$$\dot{\kappa} + 2H\kappa + \left(3\dot{H} - \frac{k^2}{a^2}\right)\alpha = \frac{1}{2M_{\text{Pl}}^2}(\delta\rho + 3\delta p), \quad \text{(B7)}$$

where $\delta\rho$, δp, and Ψ are, respectively, the total density, pressure, and momentum perturbations. In our two-fluid

system (inflaton plus radiation), they are given in terms of the inflaton field and radiation perturbations, e.g.,

$$\delta\rho = \delta\rho_\phi + \delta\rho_r, \tag{B8}$$

$$\delta p = \delta p_\phi + \delta p_r, \tag{B9}$$

$$\Psi = \Psi_\phi + \Psi_r, \tag{B10}$$

with $\delta\rho_\phi = \dot\phi\delta\dot\phi - \dot\phi^2\alpha + V_{,\phi}\delta\phi$, $\delta p_\phi = \dot\phi\delta\dot\phi - \dot\phi^2\alpha - V_{,\phi}\delta\phi$, $\delta p_r = \omega_r\delta\rho_r$, and $\Psi_\phi = -\dot\phi\delta\phi$ (with a "dot" always denoting derivative with respect to the cosmic time).

The evolution equations for the field and radiation perturbation quantities follow from the conservation of the energy-momentum tensor. The complete equations have been given in Ref. [43], which we indicate the interested reader for more details. Working in momentum space, defining the Fourier transform with respect to the comoving coordinates, the equations of motion for the radiation and momentum fluctuations with comoving wave number k are given by

$$\delta\dot\rho_r + 4H\delta\rho_r = (1+\omega_r)\rho_r(\kappa - 3H\alpha) + \frac{k^2}{a^2}\Psi_r$$
$$+ \delta Q_r + Q_r\alpha, \tag{B11}$$

$$\dot\Psi_r + 3H\Psi_r + \omega_r\delta\rho_r + \delta\Pi = -(1+\omega_r)\rho_r\alpha + J_r, \tag{B12}$$

where

$$Q_r = \Upsilon\dot\phi^2, \tag{B13}$$

$$\delta Q_r = \delta\Upsilon\dot\phi^2 + 2\Upsilon\dot\phi\delta\dot\phi - 2\alpha\Upsilon\dot\phi^2, \tag{B14}$$

$$J_r = -\Upsilon\dot\phi\delta\phi. \tag{B15}$$

In addition to Eqs. (B11) and (B12), there is also the evolution equation for the field fluctuations $\delta\phi$, which is described by a stochastic evolution determined by a Langevin-like equation [42]:

$$\delta\ddot\phi + 3H\delta\dot\phi + \left(\frac{k^2}{a^2} + V_{,\phi\phi}\right)\delta\phi = \xi_q + \xi_T - \delta\Upsilon\dot\phi$$
$$+ \dot\phi(\kappa + \dot\alpha) + (2\ddot\phi + 3H\dot\phi)\alpha - \Upsilon(\delta\dot\phi - \alpha\dot\phi), \tag{B16}$$

where $\xi_{q,T} \equiv \xi_{q,T}(\mathbf{k}, t)$ are stochastic Gaussian sources related to quantum and thermal fluctuations with appropriate amplitudes (for details and for their complete definitions, see Ref. [40]).

To complete the specification of the fluctuation equations, we need $\delta\Upsilon$, the fluctuation of the dissipation coefficient. For a general temperature T and field ϕ dependent dissipative coefficient, given by Eq. (3.1), we have that

$$\delta\Upsilon = \Upsilon\left[p\frac{\delta T}{T} + c\frac{\delta\phi}{\phi}\right]. \tag{B17}$$

Although dissipation implies departures from thermal equilibrium in the radiation fluid, the system has to be close to equilibrium for the calculation of the dissipative coefficient to hold; therefore, we assume $p_r \simeq \rho_r/3$ and, hence, $\omega_r = 1/3$. Then, with $\rho_r \propto T^4$, we have that $\delta T/T \simeq \delta\rho_r/(4\rho_r)$ and δQ_r in Eq. (B14) can be expressed as

$$\delta Q_r = 3HQ\dot\phi^2\left(\frac{p\delta\rho_r}{4\rho_r} + \frac{c\delta\phi}{\phi}\right) + 6HQ\dot\phi\delta\dot\phi - 6HQ\dot\phi^2\alpha. \tag{B18}$$

From the above relations, the complete system of first-order perturbation equations for WI becomes

$$\delta\ddot\phi = -3H(1+Q)\delta\dot\phi - \left(\frac{k^2}{a^2} + V_{,\phi\phi} + \frac{3cHQ\dot\phi}{\phi}\right)\delta\phi + \xi_q$$
$$+ \xi_T - \frac{pH}{\dot\phi}\delta\rho_r + \dot\phi(\kappa + \dot\alpha) + [2\ddot\phi + 3H(1+Q)\dot\phi]\alpha, \tag{B19}$$

$$\delta\dot\rho_r = -H\left(4 - \frac{3pQ\dot\phi^2}{4\rho_r}\right)\delta\rho_r + \frac{k^2}{a^2}\Psi_r + 6HQ\dot\phi\delta\dot\phi$$
$$+ \frac{3cHQ\dot\phi^2}{\phi}\delta\phi + \frac{4\rho_r}{3}\kappa - 3H\left(Q\dot\phi^2 + \frac{4\rho_r}{3}\right)\alpha, \tag{B20}$$

$$\dot\Psi_r = -3H\Psi_r - 3HQ\dot\phi\delta\phi - \frac{1}{3}\delta\rho_r - 4\rho_r\frac{\alpha}{3}. \tag{B21}$$

Equations (B19)–(B21), together with the metric perturbations Eqs. (B2)–(B7), form a complete set of equations in a "gauge-ready" form. From this point on, we can either choose to work in terms of gauge-invariant quantities [74,76] or, equivalently, just choose an appropriate gauge directly. Even though any appropriate gauge can be chosen, a convenient one showing good numerical stability when numerically integrating the full set of differential equations is the Newtonian slicing (or zero shear) gauge $\chi = 0$. In the $\chi = 0$ gauge, the relevant metric equations become

$$\kappa = \frac{3}{2M_{\rm Pl}^2}(\dot\phi\delta\phi - \Psi_r), \tag{B22}$$

$$\alpha = -\varphi, \tag{B23}$$

$$\dot\varphi = -H\varphi - \frac{1}{3}\kappa. \tag{B24}$$

Finally, the power spectrum is determined from the comoving curvature perturbation $\mathcal{R}$, defined as

$$\Delta_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \langle |\mathcal{R}|^2 \rangle, \tag{B25}$$

where "$\langle \cdots \rangle$" means average over different realizations of the noise terms in Eq. (B19) (see, for instance, Refs. [42–44] for details of the numerical procedure). Finally, the comoving curvature perturbation $\mathcal{R}$ is composed of contributions not only from the metric perturbations and the inflaton momentum perturbations, but also from the radiation momentum perturbations,

$$\mathcal{R} = \sum_{i=\phi,r} \frac{\rho_i + \bar{p}_i}{\rho + \bar{p}} \mathcal{R}_i, \tag{B26}$$

$$\mathcal{R}_i = -\varphi - \frac{H}{\rho_i + \bar{p}_i} \Psi_i, \tag{B27}$$

with $\bar{p} = p_\phi + p_r$, $\bar{p}_\phi \equiv p_\phi$, and $\bar{p}_r = p_r$.

Note that in the literature there are different forms for which the resulting curvature perturbations are presented. For instance, by neglecting the explicit coupling between the inflaton and radiation perturbations, e.g., by setting the temperature power of the dissipation coefficient to zero, $p = 0$, and dropping the metric perturbations (which are first order in the slow-roll coefficients), Eq. (B19) can be explicitly solved [40], leading to the result, computed at Hubble radius crossing $k = aH$,

$$\Delta_{\mathcal{R}} = \frac{H^3 T}{4\pi^2 \dot{\phi}^2} \left[\frac{3Q}{2\sqrt{\pi}} 2^{2\alpha} \frac{\Gamma(\alpha)^2 \Gamma(\nu-1) \Gamma(\alpha-\nu+3/2)}{\Gamma(\nu-\frac{1}{2}) \Gamma(\alpha+\nu-1/2)} \right. $$
$$\left. + \frac{H}{T} \coth\left(\frac{H}{2T}\right) \right], \tag{B28}$$

where $\nu = 3(1+Q)/2$, $\alpha = \sqrt{\nu^2 + 3\beta Q/(1+Q) - 3\eta_V}$, $\beta = M_{\text{Pl}}^2 \Upsilon_{,\phi} V_{,\phi}/(\Upsilon V)$, and $\Gamma(x)$ is the Γ function. By dropping slow-roll coefficients, $\alpha \approx \nu$ and Eq. (B28) can be very well approximated by the result

$$\Delta_{\mathcal{R}} \simeq \frac{H^4}{4\pi^2 \dot{\phi}^2} \left[1 + 2n_{\text{BE}}(H/T) + \frac{2\sqrt{3}\pi Q}{\sqrt{3+4\pi Q}} \frac{T}{H} \right], \tag{B29}$$

where n_{BE} is the Bose-Einstein distribution. In general, we can also replace n_{BE} by n_*, representing the statistical distribution state of the inflaton at the Hubble radius crossing, which might not be necessarily that of thermal equilibrium. The form given by Eq. (B29) is typically the result used in most of the recent literature in WI. When including the coupling between the inflaton and radiation perturbations shown in Eqs. (B19)–(B21), these equations can be solved only numerically. The result is a correction to, e.g., Eq. (B29), which can be expressed in the form of a function $G(Q)$ of the dissipation coefficient and determined by a proper fitting of the numerical result for the curvature perturbation. In particular, for the cubic in the temperature dissipation coefficient studied in this work, we obtain Eq. (3.14). Note that there are varied ways in how the perturbation equations are solved, which lead to differences on how this function $G(Q)$ is presented in the literature. For instance, in Ref. [42], where this effect of the coupling between inflaton and radiation perturbations in WI was first studied, an approximation to $G(Q)$ was given by neglecting both metric perturbations and other terms proportional to slow-roll coefficients in the perturbation equations, and only the leading -order dependence on $G(Q)$ through a simplified fitting was presented. Simpler fittings were also presented in Ref. [43].

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